PHYSICAL REVIEW C **69**, 021301(R) (2004)

Spin-orbit splitting in low-*j* neutron orbits and proton densities in the nuclear interior

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(Received 12 June 2003; published 23 February 2004)

On the basis of relativistic mean field calculations, we demonstrate that the spin-orbit splitting of $p_{3/2}$ and $p_{1/2}$ neutron orbits depends sensitively on the magnitude of the proton density near the center of the nucleus, and in particular on the occupation of $s_{1/2}$ proton orbits. We focus on two exotic nuclei, ^{46}Ar and ^{206}Hg, in which the presence of a pair of $s_{1/2}$ proton holes would cause the spin-orbit splitting between the $p_{3/2}$ and $p_{1/2}$ neutron orbits near the Fermi surface to be much smaller than in the nearby doubly magic nuclei ^{48}Ca and ^{208}Pb. We also explore how partial occupancy of the $s_{1/2}$ proton orbits affects this quenching. We note that these two exotic nuclei depart from the long-standing paradigm of a central potential proportional to the ground state baryon density and a spin-orbit potential proportional to the derivative of the central potential.

DOI: 10.1103/PhysRevC.69.021301 PACS number(s): 21.10.Ft, 21.10.Pc

One of the primary motivations for the study of exotic nuclei is to search for novel shell structure effects. A large amount of attention has been paid to the possibility that the spin-orbit force on high-*j* neutron orbits weakens in nuclei near the neutron drip line [1–14]. The neutron magic numbers for stable nuclei rely on the effect of the strong spin-orbit force on high-*j* orbits, so the weakening of this force has the potential to change the neutron magic numbers in neutron-rich nuclei. The possibility of the narrowing or collapse of the $N=28$ major shell closure in neutron-rich nuclei near ^{42}Si has attracted considerable attention because these isotopes are becoming accessible to experiments [15–18]. The two most important reasons generally given for the decline of the spin-orbit force on high-*j* neutron orbits near the neutron drip line are the large neutron surface diffuseness and the influence of the continuum in these nuclei [19,20].

In the present communication, we discuss a novel shell structure effect having to do with spin-orbit splitting in low-*j* neutron orbits—namely, *p* orbits. The dramatic decrease in the spin-orbit splitting described here is *not* caused by the neutron density near the nuclear surface, but rather by the *proton density in the nuclear interior*. The two specific nuclei we discuss here, ^{46}Ar and ^{206}Hg, are exotic but within two protons of the valley of stability. Our study uses the relativistic mean field theory, which has also been used to study the spin-orbit splitting of high-*j* orbits in exotic nuclei [4,8,9,11,13].

The relativistic mean field calculation reported here is identical to the calculation used in Ref. [21] to predict the properties of neutron-rich nuclei over a wide mass range. The model used in Ref. [21] is based on a Lagrangian developed in Refs. [22,23] that includes novel nonlinear couplings between the isoscalar and isovector mesons. These new terms, which supplement the phenomenologically successful Lagrangians of Refs. [24–26], modify the density dependence of the symmetry energy without changing ground state properties that are well established experimentally. Modifications to the poorly known density dependence of the symmetry energy induces interesting correlations between the neutron skin of heavy nuclei and a variety of neutron-star properties [22,23,27,28].

In both doubly magic nuclei ^{48}Ca and ^{208}Pb, the highest lying proton orbits below the Fermi surface (or the lowest energy proton hole states in ^{47}K and ^{207}Tl) are $s_{1/2}$ orbits. The effect of removing a pair of $s_{1/2}$ protons from ^{48}Ca and ^{208}Pb is illustrated in Fig. 1, which compares the proton densities of ^{46}Ar and ^{48}Ca (upper panel), and the proton densities of ^{206}Hg and ^{208}Pb (lower panel). The root-mean-square charge radii predicted for ^{48}Ca and ^{208}Pb with the present

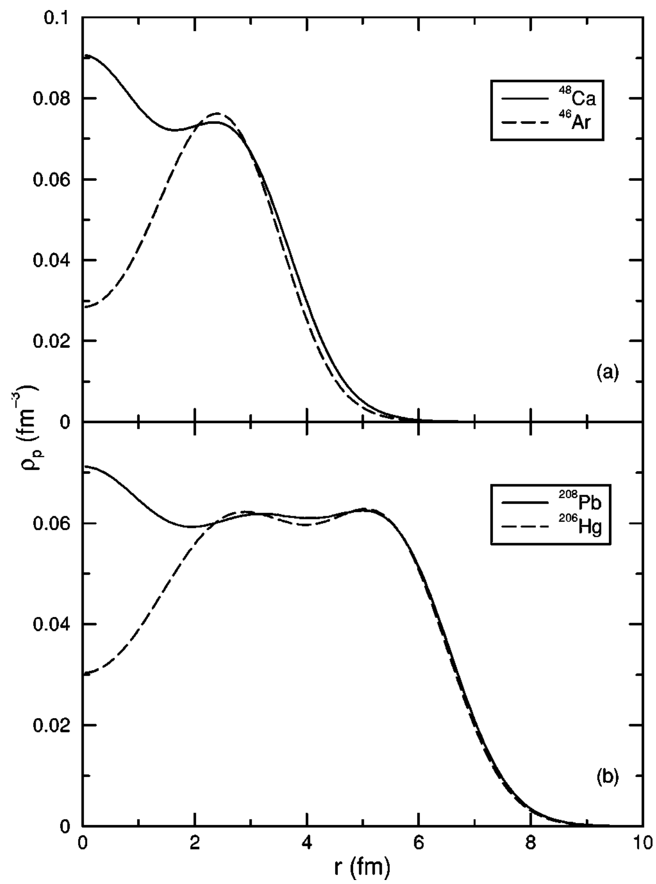

FIG. 1. Proton (point) densities for (a) ^{46}Ar and ^{48}Ca and for (b) ^{206}Hg and ^{208}Pb computed using the relativistic parametrization of Ref. [24]. The development of a "proton hole" in the interior of the nucleus is readily observed.

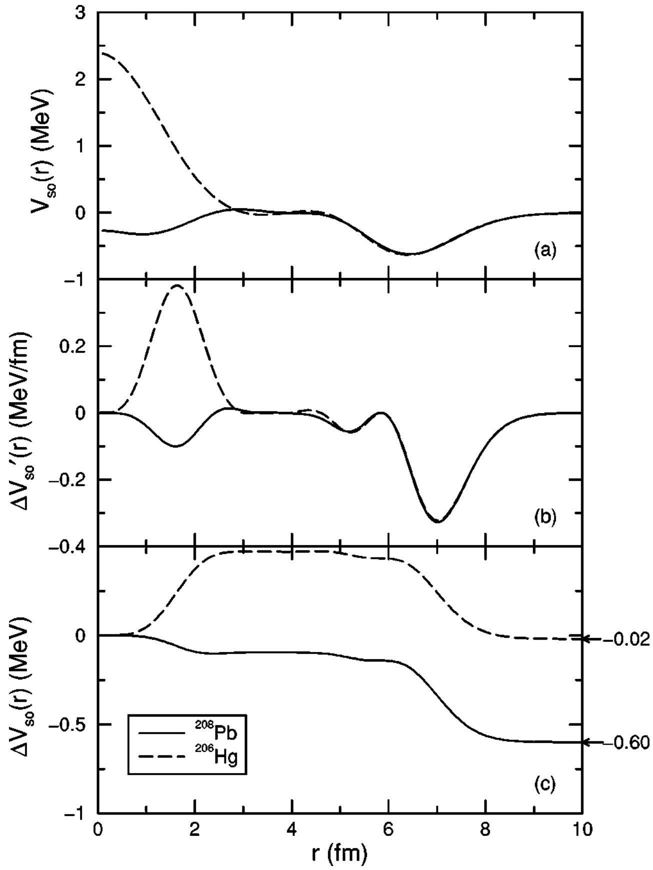

FIG. 2. (a) The Schrödinger-equivalent spin-orbit potential for ^{208}Pb (solid line) and ^{206}Hg (dashed line). Panels (b) and (c) display the effect of folding the spin-orbit potential with the Schrödinger-equivalent p orbitals, as defined in Eq. (1). The arrows point to a first-order estimate of the spin-orbit splitting.

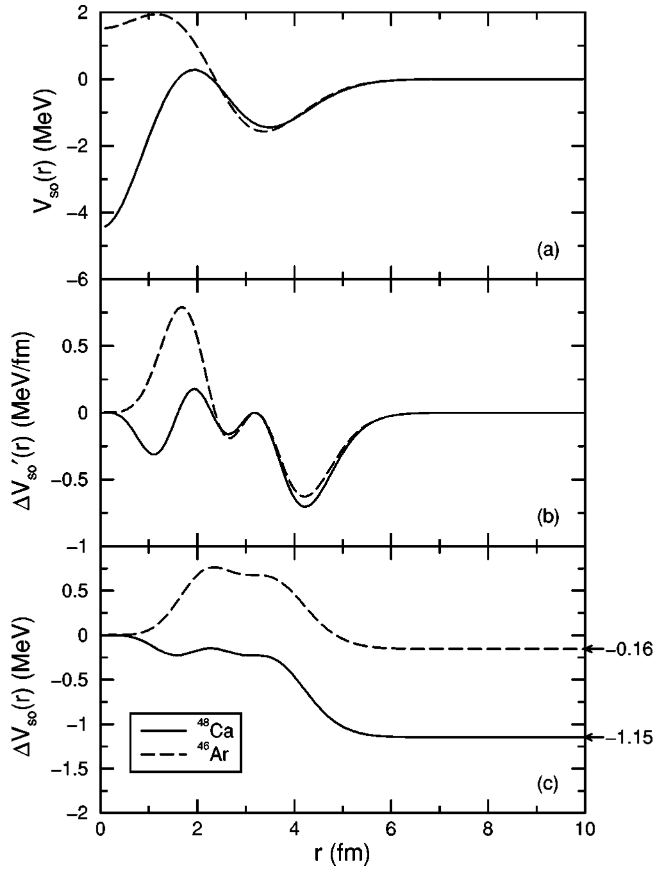

FIG. 3. (a) The Schrödinger-equivalent spin-orbit potential for ^{48}Ca (solid line) and ^{46}Ar (dashed line). Panels (b) and (c) display the effect of folding the spin-orbit potential with the Schrödinger-equivalent p orbitals, as defined in Eq. (1). The arrows point to a first-order estimate of the spin-orbit splitting.

calculation are in excellent agreement with experiment [29]. As the $s_{1/2}$ wave functions are strongly peaked in the center of the nucleus, the removal of these protons from ^{48}Ca and ^{208}Pb results in sharply reduced proton densities in the centers of ^{46}Ar and ^{206}Hg. This, in turn, causes a sharp increase in the magnitude of the spin-orbit interaction in the nuclear interior. Figure 2(a) illustrates this effect in ^{208}Pb and ^{206}Hg; inside of 2 fm, V_{so} is much stronger—and of the opposite sign—in the $s_{1/2}^{-2}$ nucleus ^{206}Hg than in the doubly magic ^{208}Pb core. This unconventional behavior of the spin-orbit potential is intimately related to the Lorentz structure of the Dirac mean fields. While the depletion of $s_{1/2}$ proton strength manifests itself in both the vector and scalar densities, the "proton-hole" disappears from the central potential as a result of the sensitive cancellation between the attractive scalar and the repulsive vector potentials [30]. In contrast, (the derivatives of) the scalar and vector potentials add constructively in the spin-orbit potential and the development of a nontrivial spin-orbit structure in the interior of the nucleus ensues.

Figures 2(b) and 2(c) illustrate the effect of folding the spin-orbit potential in ^{208}Pb and ^{206}Hg with the $3p_{1/2}$ and $3p_{3/2}$ neutron wave functions. That is, we display

$$\Delta V_{so}(r) \equiv \int_0^r dr' V_{so}(r')[2u_{p_{3/2}}^2(r') + u_{p_{1/2}}^2(r')]. \qquad (1)$$

Expressions for the Schrödinger-equivalent spin-orbit potential and wave functions $[u(r)]$ may be found in Ref. [31]. For the purpose of this study, normalized wave functions have been used. Note that the above quantity, while not exact, provides an accurate (first-order) estimate of the $p_{3/2}$-$p_{1/2}$ spin-orbit splitting $\Delta V_{so} \equiv \Delta V_{so}(r \to \infty)$.

The combined effect of a strong increase in V_{so} in the interior of ^{206}Hg together with neutron wave functions that are much larger at small radii than larger j orbits, yields a large effect on the integrated spin-orbit energy and, therefore, on the spin-orbit splitting for the p neutrons, νp. Indeed, this effect leads to the collapse of the νp spin-orbit splitting: from −0.60 MeV in ^{208}Pb to −0.02 MeV in ^{206}Hg. A similar effect occurs in ^{46}Ar relative to ^{48}Ca: the spin-orbit splitting is reduced by almost an order of magnitude, as shown in Fig. 3. Figure 4 displays the exact values. We conclude that in the two exotic nuclei ^{46}Ar and ^{206}Hg, the spin-orbit interaction ceases to be a surface-dominated phenomenon.

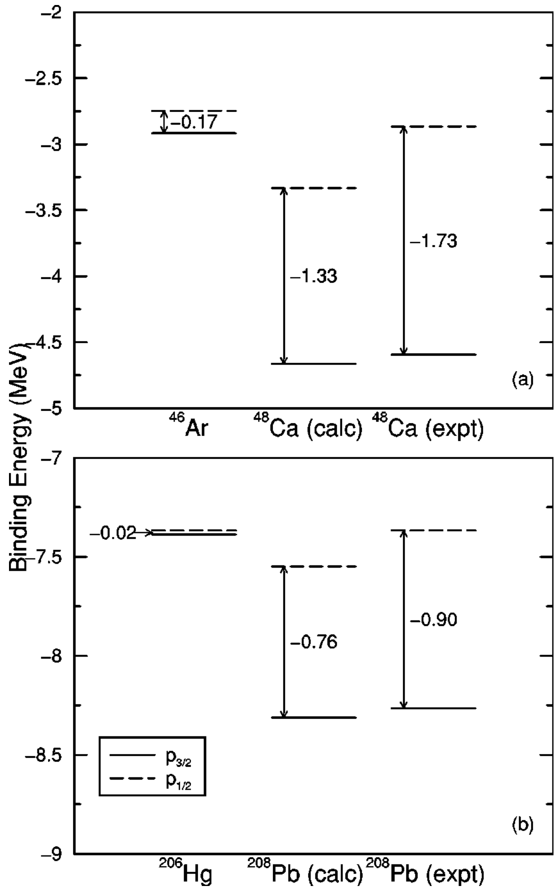

FIG. 4. Comparison between the experimental and calculated $p_{3/2}$-$p_{1/2}$ spin-orbit splitting for the doubly magic nuclei ^{48}Ca and ^{208}Pb. Also shown is the predicted collapse of the spin-orbit splitting in the two exotic nuclei ^{46}Ar and ^{206}Hg.

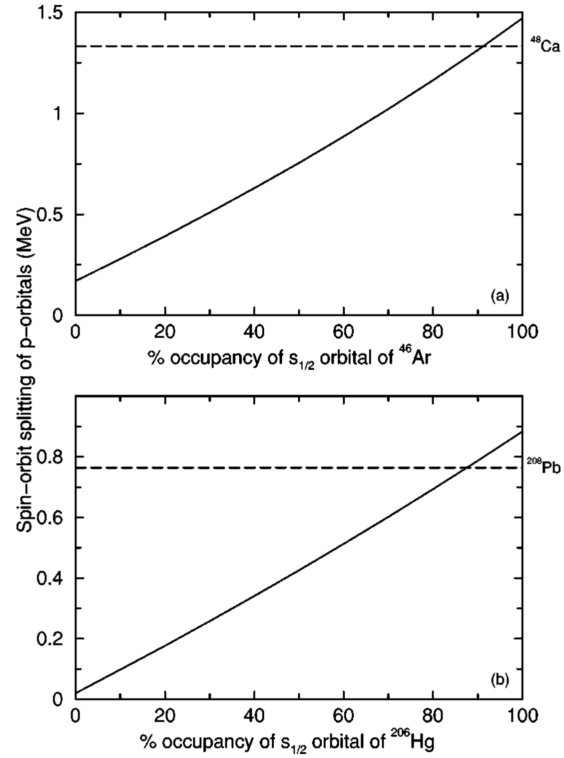

FIG. 5. Spin-orbit splittings of the p orbitals (in MeV) for ^{46}Ar and ^{206}Hg as a function of the occupancy of the $s_{1/2}$ proton orbital. The dashed lines show spin-orbit splittings for the two doubly magic nuclei ^{48}Ca and ^{208}Pb, respectively.

Figure 4 summarizes the calculations and compares the experimental and calculated binding energies for νp orbits in ^{48}Ca and ^{208}Pb. It should be emphasized that while models with nonlinear couplings between the isoscalar and isovector mesons change the energy of the individual $p_{3/2}$ and $p_{1/2}$ orbitals slightly, the prediction for their spin-orbit splitting is largely model independent. Thus, we limit ourselves to the original NL3 set of Ref. [24]. The experimental binding energies for ^{48}Ca are taken from the ^{48}Ca(d,p) measurement of Uozumi *et al.* [32] and the mass compilation of Audi and collaborators [33]. The ^{208}Pb(p,d) data used to extract the binding energies for ^{208}Pb are taken from the compilation of Martin [34]; the mass data are taken from Ref. [33].

Pairing correlations affect the occupancies of the $s_{1/2}$ proton orbits discussed here. Therefore, they also affect the proton densities and the strength of the νp spin-orbit splitting. In both ^{207}Tl and ^{47}K (one proton less than ^{208}Pb and ^{48}Ca, respectively), the $\pi d_{3/2}$ orbit is approximately 350 keV more tightly bound than the $\pi s_{1/2}$ orbit as measured by the ^{208}Pb$(d,{}^3$He$)$ and ^{48}Ca$(d,{}^3$He$)$ reactions [34,35]. A pairing interaction of reasonable strength can result in significant occupancy in the $\pi s_{1/2}$ orbits in ^{206}Hg and ^{46}Ar.

The effect of the occupancy of the $\pi s_{1/2}$ orbits on the νp spin-orbit splitting is illustrated in Fig. 5. The dashed lines in the two panels show the spin-orbit splitting calculated for ^{48}Ca (top panel) and ^{208}Pb (bottom panel). The solid lines calculated for ^{46}Ar (top) and ^{206}Hg (bottom) demonstrate that the magnitude of the νp spin-orbit splitting is approximately proportional to the occupancy of the $\pi s_{1/2}$ orbit.

For stable nuclei, the standard experimental technique for mapping single neutron strength in a nucleus is to use a stripping reaction such as (d,p). To differentiate between spin-orbit partners (such as $p_{3/2}$ and $p_{1/2}$) a polarized deuteron beam would be used (as in Ref. [32]). For the exotic nucleus ^{46}Ar, such a measurement would be performed in inverse kinematics with a ^{46}Ar beam and polarized deuteron target. The measurement would further be complicated by the likelihood that the $p_{3/2}$ and $p_{1/2}$ strengths would be somewhat fragmented, as they are in ^{51}Ti, ^{53}Cr, and ^{55}Fe [36]. In ^{206}Hg, the $p_{3/2,1/2}$ orbits would be observed as holes, requiring the use of the pickup reaction (p,d) in inverse kinematics, once again with a polarized target to differentiate between spin-orbit partners. For example, the normal kinematics experiment ^{208}Pb(p,d) with a polarized beam is reported in [37]. Even with the challenges presented by these experiments, the goal of measuring the νp spin-orbit splitting seems reasonable with a $\pi s_{1/2}$ occupancy as large as 50 %.

In summary, we have used relativistic mean field calculations to demonstrate that the spin-orbit splitting of $p_{3/2}$ and $p_{1/2}$ neutron orbits depends sensitively on the magnitude of the proton density near the center of the nucleus, and in particular on the occupation of $s_{1/2}$ proton orbits. The quenching (or collapse) of the spin-orbit splitting in high-j neutron orbits has been advertised as the hallmark for novel nuclear-structure effects in neutron-rich nuclei. This collapse is associated with the development of a diffuse neutron-rich surface. In this communication we have proposed a new mechanism for the collapse of the spin-orbit splitting—but among low-j neutron orbits. This mechanism is based, not on a rearrangement of the neutron density at the surface of the nucleus, but rather, on a depletion of the proton density in the nuclear interior. Two exotic nuclei, ^{46}Ar and ^{206}Hg, may be accessible for the study of this effect. In these nuclei we show that the presence of a pair of $s_{1/2}$ proton holes causes the splitting between the $p_{3/2}$ and $p_{1/2}$ neutron orbits near the Fermi surface to be much smaller than in the nearby doubly magic nuclei ^{48}Ca and ^{208}Pb. Furthermore, partial occupancy of the $\pi s_{1/2}$ orbits can still result in significant quenching of spin-orbit splitting between the νp orbits. Thus these two exotic nuclei, only two protons away from being doubly magic, deviate from a long-standing paradigm that has been applied with enormous success in both structure and reaction calculations, namely, that of a central potential proportional to the ground state baryon density and a spin-orbit potential proportional to the derivative of the central potential.

One of the authors (J.P.) thanks Dr. M. Centelles, Dr. Tapas Sil, and Dr. X. Viñas for many stimulating discussions. This work was supported in part by the U.S. Department of Energy through Grant No. DE-FG05-92ER40750, the National Science Foundation through Grant No. PHY-0139950, and the State of Florida.

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