Density and temperature dependence of nucleon-nucleon elastic cross section

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The in-medium neutron-proton, proton-proton (neutron-neutron) elastic scattering cross sections (σ^*_{np}, $\sigma^*_{pp(nn)}$) are studied based on the effective Lagrangian of density dependent relativistic hadron theory in which the $\delta[a_0(980)]$ meson is included. Our study shows that at low densities the σ^*_{np} is about three to four times larger than $\sigma^*_{pp(nn)}$ and at densities higher than the normal density the isospin effect is almost washed out. Because of coupling to δ meson the σ^*_{nn} and σ^*_{pp} are different in isospin asymmetric medium following the splitting of the proton and neutron masses. The isospin effect on the density dependence of the in-medium nucleon elastic cross section is dominantly contributed by the isovector δ and ρ mesons. The temperature effect on the σ^*_{np} and $\sigma^*_{pp(nn)}$ is studied. It is shown that the temperature effect is weaker compared with the density effect but it becomes obvious as density increases.

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The rapid advance in nuclear reactions using rare isotopes has opened up several new frontiers in nuclear science [1–6]. In particular, the intermediate energy heavy rare isotopes currently available at National Superconducting Cyclotron Laboratory (NSCL/MSU) and the future new facilities RIA in USA and SIS-200 at GSI in Germany provide a unique opportunity to explore novel properties of dense neutron-rich matter that was not in reach in terrestrial laboratories before.

In order to study theoretically the neutron-rich nuclear collisions at intermediate energy within a microscopic transport model approach, both the isospin-dependent mean field and the two-body scattering cross sections should be introduced. The isospin-dependent mean field has been studied extensively with both non-relativistic and relativistic theories. Concerning the nucleon-nucleon elastic cross sections (ECS), it is already known that up to 100 MeV the free proton-neutron cross section is about two to three times larger than that of proton-proton (neutron-neutron) [7]. However, it is not clear how the nuclear medium corrects the neutron-proton and proton-proton (or neutron-neutron) ECS. The isospin-dependent in-medium ECS were calculated based on QHD-II model in which the isospin-vector channel was introduced through a coupling to the vector-isovector ρ meson field [8]. In this work we intend to introduce the scalar-isovector $\delta[a_0(980)]$ meson into the effective Lagrangian to calculate the in-medium two-body scattering elastic cross sections. In free nucleon-nucleon potentials the δ meson is a standard and an essential ingredient. The importance of the δ meson in nuclear matter with extreme neutron-to-proton ratios was conjectured in Refs. [9–11]. It was pointed out that conventional mean-field models, neglecting the δ meson, are likely to miss an important contribution to the isospin degree of freedom, which ought to be necessary for a proper description of strongly asymmetric matter. Recently, the role of the δ meson for asymmetric nuclear matter was studied in Ref. [12]. Some calculation results for the structure of exotic nuclei showed the importance of the inclusion of the δ meson for the stability conditions of drip-line nuclei [13,14]. Therefore it would be worthwhile to study how the inclusion of the δ meson changes the medium effect on nucleon-nucleon cross sections. We take the same approach as in Refs. [8,15,16] for the formalism and the effective Lagrangian employed in the density dependent relativistic Hadron theory for asymmetric nuclear matter and exotic nuclei given in Ref. [13].

The effective Lagrangian density is taken as

$$\begin{aligned} L = {} & \overline{\Psi}[i\gamma_\mu\partial^\mu - M_N]\Psi + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\boldsymbol{\delta}\partial^\mu\boldsymbol{\delta} \\ & - \tfrac{1}{4}L_{\mu\nu}\cdot L^{\mu\nu} - \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{2}m_\delta^2\boldsymbol{\delta}^2 + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu \\ & + g_\sigma\overline{\Psi}\Psi\sigma - g_\omega\overline{\Psi}\gamma_\mu\Psi\omega^\mu + g_\delta\overline{\Psi}\boldsymbol{\tau}\cdot\Psi\boldsymbol{\delta} \\ & - \tfrac{1}{2}g_\rho\overline{\Psi}\gamma_\mu\boldsymbol{\tau}\cdot\Psi\boldsymbol{\rho}^\mu, \end{aligned}$$

where g_σ, g_ω, g_ρ, and g_δ are density dependent and $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $L_{\mu\nu} \equiv \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$.

The distribution functions for fermions $[f(\mathbf{p})]$ and antifermions $[\bar{f}(\mathbf{p})]$, respectively, are $f_i(\mathbf{p}) = 1/(1+e^{[E_i^*(\mathbf{p})-\mu_i^*]/T})$ and $\bar{f}_i(\mathbf{p}) = 1/(1+e^{[E_i^*(\mathbf{p})+\mu_i^*]/T})$, where i represents proton $(i=p)$ or neutron $(i=n)$ and $E^* = \sqrt{\mathbf{p}^2 + M_i^{*2}}$. The effective nucleon mass M^* is given by

$$M^* = M_0 + \Sigma_{H(\sigma)}(x,\tau) + \Sigma_{H(\delta^0)}(x,\tau). \qquad (1)$$

$\Sigma_{H(\sigma)}$ and $\Sigma_{H(\delta)}$ are the self-energy parts of nucleon contributed from σ and δ mesons, respectively. Because the self-energy $\Sigma_{H(\delta^0)}(x,\tau)$ has opposite sign for neutron and proton for isospin asymmetric medium the correction of the nuclear medium to proton mass and neutron mass from δ meson is of opposite sign. Thus the proton and neutron effective masses differ for isospin asymmetric systems. The effective chemical potential μ^* is

$$\mu_i^* = \mu_i + \Sigma^0_{H(\omega)}(x,\tau) + \Sigma^0_{H(\rho^0)}(x,\tau). \qquad (2)$$

TABLE I. Parametrization of the density dependent couplings taken from Ref. [13].

Meson α	σ	ω	δ	ρ
m_α (MeV)	550	783	983	770
A_α	13.1334	15.1640	19.1023	19.6270
B_α	0.4258	0.3474	1.3653	1.7566
C_α	0.6578	0.5152	2.3054	8.5541
D_α	0.7914	0.5989	0.0693	0.7783
E_α	0.7914	0.5989	0.5388	0.5746
ρ_0, 0.18 fm^{-3}; E/A, 16 MeV; K, 282 MeV; M^*/M_0, 0.554; a_{Sym}, 26.1 MeV.				

$\Sigma_{H(\omega)}$ and $\Sigma_{H(\rho)}$ are the self-energy parts of nucleon contributed from ω and ρ mesons respectively. Within the mean-field approximation the equation of state can be written as

$$\epsilon = \sum_{i=n,p} 2\int \frac{d^3\mathbf{p}}{(2\pi)^3} E_i^*(\mathbf{p})[f_i(\mathbf{p}) + \bar{f}_i(\mathbf{p})] + U(\sigma) + \tfrac{1}{2}\Sigma^0_{H(\omega)}(x,\tau)\rho_B + \tfrac{1}{2}\Sigma^0_{H(\rho^0)}(x,\tau)\rho_{B3} - \tfrac{1}{2}\Sigma_{H(\delta^0)}(x,\tau)\rho_{S3}, \tag{3}$$

where the baryon and scalar densities ρ_B and ρ_S are

$$\rho_B = \sum_{i=n,p} 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}[f_i(\mathbf{p}) - \bar{f}_i(\mathbf{p})], \tag{4}$$

$$\rho_S = \sum_{i=n,p} 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{M_i^*}{E_i^*}[f_i(\mathbf{p}) + \bar{f}_i(\mathbf{p})], \tag{5}$$

while $\rho_{B3}=\rho_{Bp}-\rho_{Bn}$ and $\rho_{S3}=\rho_{Sp}-\rho_{Sn}$.

The functional form of the density dependent coupling for σ, ω, δ, and ρ mesons taken from Ref. [13] is $g_\alpha(\rho)=A_\alpha[1+B_\alpha(\rho/\rho_0+D_\alpha)^2]/[1+C_\alpha(\rho/\rho_0+E_\alpha)^2]$, where α represents σ, ω, δ, or ρ, and the parameters A_α, B_α, C_α, D_α, and E_α are listed in Table I.

By solving Eq. (1) self-consistently we obtain the effective proton and neutron masses. We illustrate the density dependence of the neutron and proton effective masses at $T=0$ MeV for $Y_p=0.3$ and $Y_p=0.5[Y_p=\rho_p/(\rho_p+\rho_n)]$ in Fig. 1(a). The proton and neutron effective masses differ at $Y_p=0.3$ case due to the inclusion of the δ meson. The proton effective mass is larger than the neutron effective mass in neutron-rich nuclear medium. With the increase of density, the splitting of the proton and neutron masses increases, which will affect the in-medium ECS.

Figure 1(b) shows the temperature dependence of the neutron and proton effective masses for $Y_p=0.3$. The ratios $M^*(T)/M^*(T=0)$ for different cases are also shown in the upper-right plot. One can see from this figure that the effective nuclear mass first increases slightly until $T=150$ MeV, and then decreases quickly as temperature increases further, which has also been observed in Refs. [17,18]. The upper-right plot shows that increasing slope of $M^*(T)/M^*(T=0)$ depends on density as well as on the species of nucleons. The behavior of the density and temperature dependence of the effective mass should influence the density and temperature dependence of in-medium ECS which will be discussed later.

With the same approach as in Refs. [8,15,16] we obtain the expressions of $\sigma_{pp(nn)}$ and $\sigma_{pp(nn)}$ with the inclusion of the individual δ term and the crossing terms of δ-σ, δ-ω as well as δ-ρ. Because of the coupling to δ meson the collisions of $np\to np$ and $np\to pn$ should be treated differently, which makes the calculations of in-medium ECS much more complicated. For the same reason, σ^*_{nn} is not equal to σ^*_{pp} for isospin asymmetric systems.

Before calculating the ECS, we have to introduce the commonly used form factor of nucleon-meson-nucleon vertex $F_i(t)=\Lambda_i^2/(\Lambda_i^2-t)$, where the subscripts i denote the vertices for different meson-nucleon couplings and Λ_i is the cutoff mass for i meson species. In this work, the values of $\Lambda_\sigma=1200$ MeV, $\Lambda_\omega=808$ MeV (which are same as those chosen in Ref. [19]) $\Lambda_\delta=1000$ MeV, and $\Lambda_\rho=800$ MeV are used. We find there is very weak influence on the results when taking $\Lambda_\delta=800-1000$ MeV and $\Lambda_\rho=600-800$ MeV. Again, the $F_i(t)$ is different for $np\to np$ and $np\to pn$ channels when $M_n^*\neq M_p^*$.

The main contributions to the in-medium ECS come from the σ-σ, ω-ω and the crossing term σ-ω. Figure 2(a) shows the contributions of σ-σ, ω-ω, and σ-ω terms to $\sigma_{pp(nn)}$ and σ_{np} at $T=0$ MeV, $E_K=10$ MeV for $Y_p=0.5$. The ratio be-

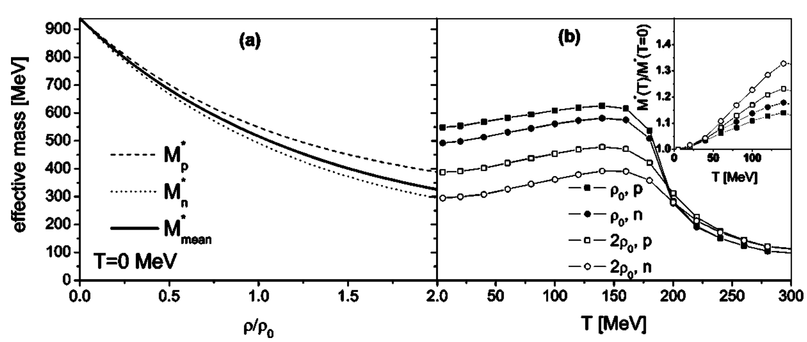

FIG. 1. (a) The nucleon effective mass as a function of reduced nuclear density at $T=0$ MeV. The effective masses for $Y_p=0.5$ are plotted as thick line, while the proton and neutron effective masses for $Y_p=0.3$ are demonstrated as light ones. (b) The neutron effective mass (circles) and proton effective mass (rectangles) as a function of temperature for $Y_p=0.3$. The lines with solid symbols are for the case when $\rho/\rho_0=1$, while those with open symbols are for the case when $\rho/\rho_0=2$. The ratios $M^*(T)/M^*(T=0)$ for different cases are shown in the upper-right plot.

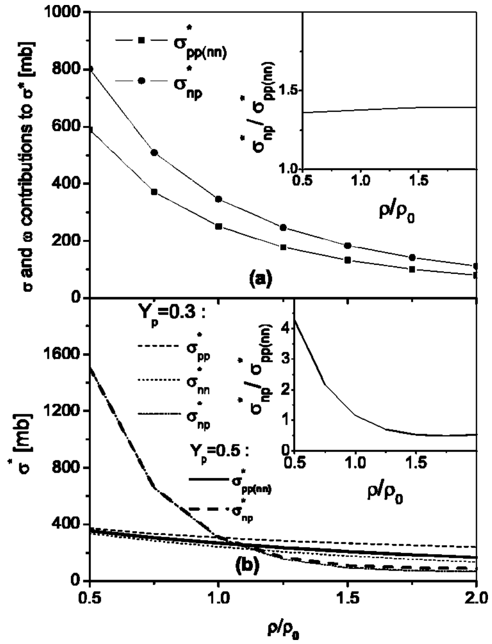

FIG. 2. (a) The σ and ω contributions to $\sigma_{pp(nn)}$ and σ_{np} at $T=0$ MeV, $E_K=10$ MeV for $Y_p=0.5$. The ratio between σ_{np} and $\sigma_{pp(nn)}$ is illustrated in the upper-right plot. (b) The density dependence of σ^*_{pp}, σ^*_{nn}, and σ^*_{np} in isospin-symmetric ($Y_p=0.5$, thick lines) and asymmetric ($Y_p=0.3$, light lines) nuclear medium. $T=0$ MeV and $E_K=10$ MeV are chosen. The ratio between σ^*_{np} and $\sigma^*_{pp(nn)}$ for $Y_p=0.5$ is shown in the upper-right plot.

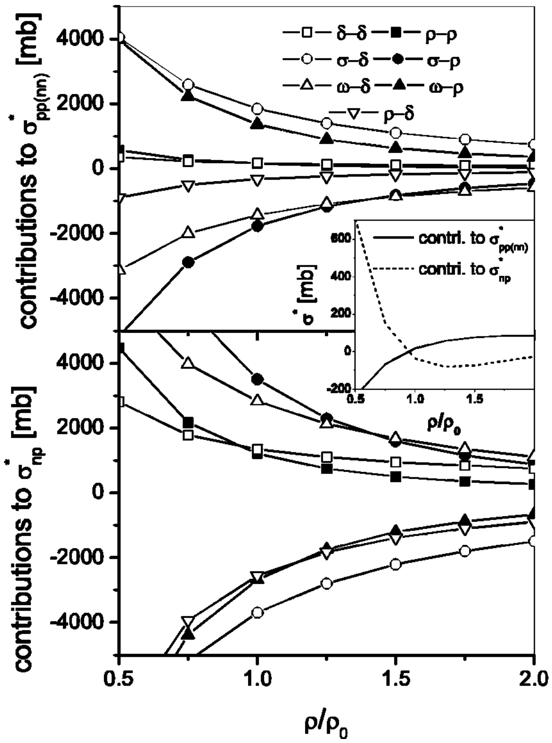

FIG. 3. The contributions from the δ, ρ meson related terms to $\sigma_{pp(nn)}$ (in upper plot) and σ_{np} (in lower plot) at $T=0$ MeV and $E_K=10$ MeV for $Y_p=0.5$. The central-right small plot shows the sum of the contributions from the δ and ρ meson related terms.

tween σ_{np} and $\sigma_{pp(nn)}$ is also shown in the upper-right plot. From this figure one can find that both $\sigma_{pp(nn)}$ and σ_{np} have similar shape of density dependence, and the $\sigma_{np}/\sigma_{pp(nn)}$ ratio is almost density independent.

The contributions from the δ and ρ mesons and related crossing terms are shown in Fig. 3. In order to explore how δ and ρ meson fields affect the in-medium ECS, we show the individual contributions from each term. The central-right plot shows the sum of the seven terms contributing to $\sigma_{pp(nn)}$ and σ_{np}. This figure shows rich information about the contributions of each term to the in-medium ECS as well as the cancellation effect among the seven terms. First, the magnitude of all individual contributions to the in-medium ECS decreases with density. This density dependence results from both the density dependence of the meson-nucleon couplings and the effective masses of protons and neutrons. Among the 7 terms, the crossing terms of the σ meson to δ and ρ mesons provide the most important contributions and the crossing terms of ω to δ and ρ mesons are the next important terms. Further, the cancellation among the contributions from different terms are very strong so the final results are the delicate balance of all contributed terms.

In Fig. 2(b) we show the density dependence of σ^*_{pp}, σ^*_{nn}, and σ^*_{np} at $T=0$ MeV for $Y_p=0.3$ and $Y_p=0.5$. The $\sigma^*_{pp(nn)}$ and σ^*_{np} for $Y_p=0.5$ are shown with thick lines while those for $Y_p=0.3$ are illustrated with light lines. One can see from the figure that σ^*_{pp}, σ^*_{nn} and σ^*_{np} decrease with the increase of densities. This behavior is consistent with other theoretical calculations, for instance, Refs. [18,20,21] and the phenomenological expression of $\sigma^*=[1-\alpha(\rho/\rho_0)]\sigma^{free}$. The most impressive thing shown in the figure is that the density dependence of σ^*_{np} and $\sigma^*_{pp(nn)}$ is very different: At low densities, the σ^*_{np} is about three to four times larger than $\sigma^*_{pp(nn)}$ and then decreases quickly with density and finally the $\sigma^*_{np}/\sigma^*_{pp(nn)}$ ratio approaches ~ 1 as density reaching about the normal density. The $\sigma^*_{np}/\sigma^*_{pp(nn)}$ ratio is shown in the small top-right plot. It means that at dense nuclear matter the isospin effect on in-medium ECS almost washes out. For $Y_p=0.3$ case, σ^*_{pp} and σ^*_{nn} differ. The value of σ^*_{pp} is larger than that of σ^*_{nn} because $M^*_p > M^* > M^*_n$ [see Fig. 1(a)]. The splitting of σ^*_{pp} and σ^*_{nn} increases with density and the degree of isospin asymmetry of the medium following the splitting of the proton and neutron effective masses.

In Ref. [8] the in-medium σ^*_{np} and $\sigma^*_{pp(nn)}$ are calculated based on QHD-II model with the medium correction of the effective mass of ρ meson taken into account. We found in Ref. [8] that $\sigma^*_{pp(nn)}$ depends on density obviously but σ^*_{np} depends on density weakly. The different results can be attributed to the inclusion of the δ mesons. In the mean field level, the contributions of the δ and ρ mesons compensate each other in central potential, producing an effective isovector potential that is comparable in strength to the one obtained in calculations that include only the ρ meson. While for the calculations of the in-medium cross sections, which are beyond the mean field, the direct compensation of the contributions from δ and ρ mesons is lost. So we may not expect that the comparable results for the in-medium cross

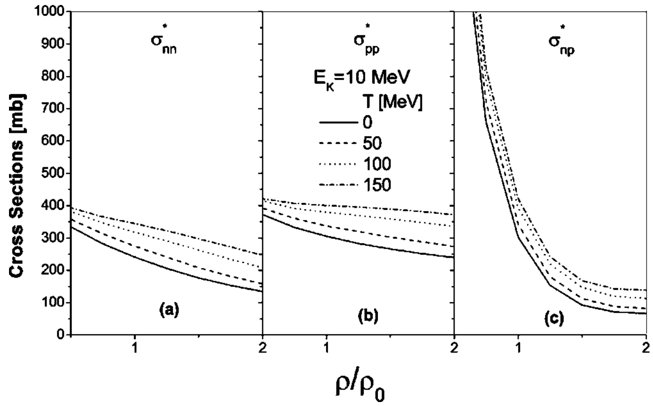

FIG. 4. $\sigma^*_{pp(nn)}$ and σ^*_{np} at $T=0, 50, 100$, and 150 MeV as function of density for $Y_p=0.3$ and $E_K=10$ MeV.

sections should be obtained for the cases of inclusion and noninclusion of the δ meson. The different results for the density dependence of $\sigma^*_{pp(nn)}$ and σ^*_{np} obtained in two different models may provide us with a more strict check for the model.

Now let us investigate the temperature effect on $\sigma^*_{pp(nn)}$ and σ^*_{np}. Figure 4 shows σ^*_{nn} [in (a)], σ^*_{pp} [(b)], and σ^*_{np} [(c)] at $T=0$, 50, 100, and 150 MeV. In Fig. 4 one sees that the in-medium ECS increases with temperature and the influence of temperature on in-medium ECS increases with density. We should point out that this temperature effect is model dependent. With the model similar to that used in Ref. [8] we find the temperature effect on the medium correction of nuclear mass and then on the ECS is weaker than the present results. One of the most important information contained in Fig. 4 is that the influence of temperature on in-medium cross sections is much weaker than that of density. The cause of this difference can be attributed to the different behavior of the density and temperature dependence of the nucleon effective mass. This tells us that one should consider the effect of medium density on the in-medium cross section more seriously than that of medium temperature.

In summary, in this work we have studied the density and temperature effects on in-medium elastic cross sections based on the density dependent relativistic Hadron field theory in which the δ meson is included. Our results show that at low densities the σ^*_{np} is about three to four times larger than $\sigma^*_{pp(nn)}$ and then decreases quickly with density and finally the $\sigma^*_{np}/\sigma^*_{pp(nn)}$ ratio approaches ~ 1 as densities reaching and exceeding the normal density. It means that the isospin effect on in-medium elastic cross section almost washes out at high densities. The analysis about the individual contributions from each term contributing to $\sigma^*_{pp(nn)}$ and σ^*_{pn} indicates that the isospin effect on the density dependence of the in-medium elastic cross sections is dominantly caused by the delicate balance of the isovector δ and ρ mesons. For isospin asymmetric medium, the σ^*_{nn} and σ^*_{pp} differ following the splitting of the proton and neutron masses due to the inclusion of the δ meson. Our calculation results show that the in-medium ECS increases with the temperature, and the temperature effect of nuclear medium on σ^*_{np} and $\sigma^*_{pp(nn)}$ is weaker compared with the density effect. The temperature effect enhances as density increases so the influence of temperature on in-medium ECS may have to be taken into account at higher nuclear density.

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