Pion parameters in nuclear medium from chiral perturbation theory and virial expansion

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We consider two methods to find the effective parameters of the pion traversing a nuclear medium. One is the first order chiral perturbation theoretic evaluation of the pion pole contribution to the two-point function of the axial-vector current. The other is the exact, first order virial expansion of the pion self-energy. We find that, although the results of chiral perturbation theory are not valid at normal nuclear density, those from the virial expansion may be reliable at such density. The latter predicts both the mass shift and the in-medium decay width of the pion to be small, of about a few MeV.

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I. INTRODUCTION

A considerable amount of work at finite temperature and chemical potential has been devoted to determining the effective parameters of strongly interacting particles in different media [1–6]. The results obtained are useful not only in analyzing the heavy ion collision experiments and properties of the early universe at different epochs, but also in extracting indications of an eventual phase transition.

The case of pion appears to be the simplest to study. Being the Goldstone boson of the spontaneously broken chiral symmetry of QCD, its interactions with itself and other particles are highly restricted by this symmetry, leading to the effective theory of QCD, called chiral perturbation theory (χPT) [7,8]. At finite temperature, one has the further advantage of having again only pions dominating the heat bath. Thus χPT provides a reliable method to calculate the pion parameters at finite temperature [1–3].

It is natural to apply χPT to calculate the pion parameters in nuclear medium [9,10]. Although the method parallels that followed for the case in a heat bath, the results calculated here to leading order may have restricted validity, due to the presence of baryonic resonances close to the πN threshold. Similar difficulties also appear in determining the nucleon parameters at finite temperature [4].

In this work we compare the χPT result with that of the (first order) virial expansion of the pion self-energy [4,11,12]. The latter gives the shifted pole position in terms of an integral over the product of the density distribution function times the πN scattering amplitude obtainable from experiment. It is thus free from the difficulty encountered in calculating the amplitude and is valid as long as the nucleon "gas" is dilute enough.

The virial formula has also been employed earlier to the same problem, but only in an approximated version [13–15]. But if the amplitude varies appreciably in the range of integration, in particular, if it changes sign — as is the case here — this version is not justified.

We first rederive the results of χPT by evaluating the axial-vector current correlation function to one loop, using the in-medium Feynman rules for the original chiral Lagrangian in presence of external fields. Besides completeness, its purpose is to show that this conventional framework is quite simple, without requiring functional integration over the nucleon field to produce a "new" effective Lagrangian [10]. We then derive the exact, first order virial expansion for the pion self-energy and evaluate it with experimental data.

Section II reviews briefly χPT, constructing the Lagrangian for the πN system [16,17]. In Sec. III we work out the shift in the mass and the decay constant of the pion using this Lagrangian. Next we derive the virial formula and evaluate the pole shift in Sec. IV. In Sec. V we discuss the limitations of these methods.

II. CHIRAL PERTURBATION THEORY

The Lagrangian of QCD with two massless quark flavors is

$$\mathcal{L}^{(0)}_{QCD} = i\bar{q}\gamma^\mu\partial_\mu q + \cdots, \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}, \tag{2.1}$$

where the dots denote terms involving other fields. If we split the quark field into its right and left handed parts, $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$, it is clear that $\mathcal{L}^{(0)}_{QCD}$ is invariant under the symmetry group $G = \mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ of independent, global SU(2) transformations on q_R and q_L,

$$q_R \to g_R q_R, \quad q_L \to g_L q_L, \quad g_{R,L} \in \mathrm{SU}(2)_{R,L}. \tag{2.2}$$

Phenomenology suggests strongly that the symmetry of the Lagrangian is broken spontaneously by the vacuum state to the diagonal subgroup $H = \mathrm{SU}(2)_V$, giving rise to the pionic degrees of freedom.

In χPT one derives the transformation rules for the observed Goldstone and nonGoldstone fields from the above symmetry of the underlying QCD theory. It turns out that the Goldstone fields $\pi^i(x)$, $i = 1, 2, 3$ are collected in the form of a unitary matrix

$$u(x)=e^{i\pi(x)/2F_\pi},\quad \pi(x)=\sum_{i=1}^{3}\pi^i(x)\tau^i, \tag{2.3}$$

where the constant F_π can be identified with the pion decay constant F_π=92.4 MeV and τ^i are the Pauli matrices. Then the matrix u transforms under G according to

$$u\to g_R u h^\dagger = h u g_L^\dagger, \tag{2.4}$$

where the group element $h(\pi)\in \mathrm{SU}(2)_V$. Notice that h is x dependent due to its dependence on $\pi^i(x)$. However the square of this matrix $u^2=U$ has the global transformation rule

$$U\to g_R U g_L^\dagger. \tag{2.5}$$

On the other hand, the non-Goldstone, nucleon doublet field $\psi(x)$ transforms as

$$\psi\to h\psi,\quad \psi=\begin{pmatrix}p\\ n\end{pmatrix}. \tag{2.6}$$

There are two Noether currents following from the symmetry of $\mathcal{L}_{QCD}^{(0)}$, namely, the vector and axial vector currents,

$$V_\mu^i(x)=\bar{q}(x)\gamma_\mu\frac{\tau^i}{2}q(x),\quad A_\mu^i(x)=\bar{q}(x)\gamma_\mu\gamma_5\frac{\tau^i}{2}q(x). \tag{2.7}$$

The evaluation of the correlation functions of the currents is most conveniently carried out in the external field method [8]. Although we are interested here in such a function of the axial-vector current only, we couple both the currents to the external fields $v_\mu^i(x)$ and $a_\mu^i(x)$ to reveal the full symmetry of the underlying theory. Thus the original Lagrangian extends to

$$\begin{aligned}\mathcal{L}_{QCD}^{(0)}&+v_\mu^i(x)V_i^\mu(x)+a_\mu^i(x)A_i^\mu(x)\\ &=i\bar{q}_R\gamma^\mu\{\partial_\mu-i(v_\mu+a_\mu)\}q_R\\ &\quad+i\bar{q}_L\gamma^\mu\{\partial_\mu-i(v_\mu-a_\mu)\}q_L+\cdots,\end{aligned} \tag{2.8}$$

where $v_\mu(x)$ and $a_\mu(x)$ are the matrix valued external vector and axial vector fields,

$$v_\mu(x)=\sum_{i=1}^{3}v_\mu^i(x)\frac{\tau^i}{2},\quad a_\mu(x)=\sum_{i=1}^{3}a_\mu^i(x)\frac{\tau^i}{2}. \tag{2.9}$$

The extended Lagrangian is now invariant locally, i.e., under x-dependent symmetry transformations on q_R and q_L, if the external vector and axial-vector fields are also subjected to the appropriate gauge transformations.

The presence of the external fields in the underlying theory and the associated gauge invariance can be readily incorporated in the effective theory. All we have to do is to replace the ordinary derivatives by the covariant derivatives [18]. Thus we have for U,

$$D_\mu U=\partial_\mu U-i(v_\mu+a_\mu)U+iU(v_\mu-a_\mu), \tag{2.10}$$

and for ψ,

$$\mathcal{D}_\mu\psi=\partial_\mu\psi+\Gamma_\mu\psi,$$

$$\Gamma_\mu=\tfrac{1}{2}\{u^\dagger[\partial_\mu-i(v_\mu+a_\mu)]u+u[\partial_\mu-i(v_\mu-a_\mu)]u^\dagger\}. \tag{2.11}$$

The building elements for the effective Lagrangian at this stage are thus U, $D_\mu U$, ψ, and $\mathcal{D}_\mu\psi$.

A simplification in the construction of the effective Lagrangian emerges by noting that the variables $(U,D_\mu U)$ transform under the full group G, while $(\psi,\mathcal{D}_\mu\psi)$ transform only under the unbroken subgroup H. We may take advantage of the mixed transformation property of u to redefine the former type of variables so as to transform under H only. Thus one introduces the variable [16],

$$u_\mu=iu^\dagger D_\mu U u^\dagger=u_\mu^\dagger, \tag{2.12}$$

replacing U and $D_\mu U$ [18]. Any term in the Lagrangian that is constructed out of these variables so as to be invariant under H will also be automatically invariant under G.

We now write the effective Lagrangian of χPT for the πN system as

$$\mathcal{L}_{eff}=\mathcal{L}_\pi+\mathcal{L}_N,$$

where $\mathcal{L}_\pi$ is the well-known pion Lagrangian [8], which to leading order is given by

$$\mathcal{L}_\pi=\frac{F_\pi^2}{4}\{\langle D_\mu U D^\mu U^\dagger\rangle+m_\pi^2\langle U+U^\dagger\rangle\}, \tag{2.13}$$

$\langle\cdots\rangle$ denoting trace over the 2×2 isospin matrices. The pieces in $\mathcal{L}_N$ to first and second order,

$$\mathcal{L}_N=\mathcal{L}_N^{(1)}+\mathcal{L}_N^{(2)}, \tag{2.14}$$

are

$$\mathcal{L}_N^{(1)}=\bar{\psi}(i\not{\mathcal{D}}-m_N)\psi+\frac{g_A}{2}\bar{\psi}\not{u}\gamma_5\psi, \tag{2.15}$$

and

$$\begin{aligned}\mathcal{L}_N^{(2)}&=c_1 m_\pi^2\langle U+U^\dagger\rangle\bar{\psi}\psi-\frac{c_2}{4m_N^2}\langle u_\mu u_\nu\rangle(\bar{\psi}\mathcal{D}^\mu\mathcal{D}^\nu\psi+\mathrm{H.c.})\\ &\quad+\frac{c_3}{2}\langle u_\mu u^\mu\rangle\bar{\psi}\psi-\frac{c_4}{4}\bar{\psi}\gamma^\mu\gamma^\nu[u_\mu,u_\nu]\psi.\end{aligned} \tag{2.16}$$

We shall use vertices in $\mathcal{L}_N^{(1)}$ to second order and those in $\mathcal{L}_N^{(2)}$ to first order in our perturbative calculations. Here g_A turns out to be the axial-vector coupling constant appearing in the neutron beta decay, g_A=1.27. The coupling constants c_1, c_2, and c_3 are determined from the experimental data for πN scattering in the low energy region and its extrapolation inside the Mandelstam triangle, where it is compared with the χPT evaluation, getting [19,17],

$$c_1=-0.81\pm 0.12\ \mathrm{GeV}^{-1},\quad c_2=3.2\pm 0.25\ \mathrm{GeV}^{-1},$$
$$c_3=-4.66\pm 0.36\ \mathrm{GeV}^{-1}. \tag{2.17}$$

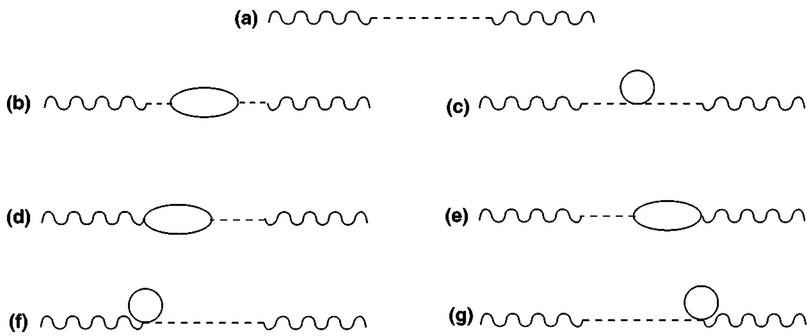

FIG. 1. Feynman diagrams for the two-point function to one loop. Only loops with nucleons are considered.

Expanding out in the pion field and setting $v_\mu=0$, we bring out explicitly the vertices, contributing to the pion pole diagrams to one loop [20],

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(0)} - F_\pi \partial_\mu \boldsymbol{\pi}\cdot \boldsymbol{a}^\mu,$$

$$\mathcal{L}_N^{(1)} = \mathcal{L}_N^{(0)} - \frac{g_A}{2F_\pi}\bar{\psi}\gamma^\mu\gamma_5\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau}\psi + \frac{g_A}{2}\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{a}_\mu\cdot\boldsymbol{\tau}\psi,$$

$$\mathcal{L}_N^{(2)} = -\frac{2m_\pi^2 c_1}{F_\pi^2}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\bar{\psi}\psi - \frac{c_2}{m_N^2}\left(\frac{1}{F_\pi^2}\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi} - \frac{2}{F_\pi}\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{a}_\nu\right)\bar{\psi}\partial^\mu\partial^\nu\psi + c_3\left(\frac{1}{F_\pi^2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{2}{F_\pi}\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{a}_\mu\right)\bar{\psi}\psi. \tag{2.18}$$

where the superscript "0" indicates free Lagrangian densities.

III. MASS AND COUPLING SHIFTS

The two point correlation function of the axial vector current in a medium is given by the ensemble average,

$$i\int d^4x e^{iq\cdot x}\mathrm{Tr}[e^{-\beta(H-\mu N)}A_\lambda^i(x)A_\sigma^{i'}(0)]/\mathrm{Tr}[e^{-\beta(H-\mu N)}]. \tag{3.1}$$

Here H is the QCD Hamiltonian, β is the inverse temperature, and N is the number operator for nucleons with chemical potential μ. Our aim is to find the corrections to the pion pole in this correlation function due to interaction of pion with nucleons in medium at zero temperature.

We shall work in the real time formulation of the field theory in medium [21]. Here the perturbation expansion proceeds as in the conventional (vacuum) field theory, except that the propagators assume the form of 2×2 matrices. For the pole term to one loop, it however suffices to work as in the vacuum field theory, only replacing the vacuum propagators by the 11-component of the corresponding propagators in medium.

So we first calculate the vacuum correlation function,

$$i\int d^4x e^{iq\cdot x}\langle 0|TA_\lambda^i(x)A_\sigma^{i'}(0)|0\rangle, \tag{3.2}$$

in χPT. We recall that the generating functional of QCD,

$$\langle 0|Te^{i\int d^4x a_\mu^i(x)A_i^\mu(x)}|0\rangle, \tag{3.3}$$

is represented in χPT by

$$\langle 0|Te^{i\int d^4x \mathcal{L}_{int}(\pi,\psi,a)}|0\rangle, \tag{3.4}$$

where $\mathcal{L}_{int}$ is obtained from Eqs. (2.17). Since the two-point function (3.2) is the coefficient of the term quadratic in $a_\mu(x)$ in the expansion of the generating functional (3.3), we find these quadratic terms from the functional (3.4) of the effective theory.

Figure 1 shows the free pion pole diagram (a) giving the amplitude,

$$\delta^{ii'}q_\lambda q_\sigma iF_\pi^2\Delta(q), \quad \Delta(q) = i/(q^2 - m_\pi^2 + i\epsilon), \tag{3.5}$$

together with all one loop corrections to it, relevant in the nuclear medium. Let us calculate the self-energy diagram (b) to illustrate the method. Its contribution may be written in the form

$$\delta^{ii'}q_\lambda q_\sigma iF_\pi^2\Delta(q)\{-i\Pi(q)\}\Delta(q), \tag{3.6}$$

which modifies the free pion pole term (3.5) to

$$\delta^{ii'}q_\lambda q_\sigma\frac{-F_\pi^2}{q^2 - m_\pi^2 - \Pi(q)}. \tag{3.7}$$

To calculate the self-energy function $\Pi(q)$ of the pion in the nuclear medium, we first write it in vacuum,

$$\Pi^{(0)}(q) = \frac{ig_A^2}{2F_\pi^2}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}[\not{q}\gamma_5(\not{p}+m_N)\not{q}\gamma_5(\not{p}-\not{q}+m_N)] \times\Delta^N(p)\Delta^N(p-q), \tag{3.8}$$

where $\Delta^N(p)$ is the vacuum nucleon propagator after extracting the factor $(\not{p}+m_N)$, $\Delta^N(p)=i/(p^2-m_N^2+i\epsilon)$. Following our discussion above, we now replace the vacuum nucleon propagator in Eq. (3.8) by its 11-component in nuclear medium ($E_p=\sqrt{\vec{p}^2+m_N^2}$),

$$\Delta_{11}^N(p) = \frac{i}{p^2 - m_N^2 + i\epsilon} - 2\pi\{n^+(E_p)\theta(p_0) + n^-(E_p)\theta(-p_0)\}\delta(p^2 - m_N^2), \tag{3.9}$$

where $n^\pm(E_p)$ are the distribution functions for the nucleon and the antinucleon, respectively,

$$n^{\pm}(E_p) = \frac{1}{e^{\beta(E_p \mp \mu)} + 1}. \tag{3.10}$$

As the temperature goes to zero, we get for $\mu > 0$,

$$n^{+}(E_p) \to \theta(\mu - E_p), \quad n^{-}(E_p) \to 0. \tag{3.11}$$

The density dependent part of the self-energy in the medium is then obtained as

$$\Pi^{(n)}(q) = -8g_A^2 \frac{m_N^2}{F_\pi^2} q^4 \int \frac{d^4p}{(2\pi)^3} \frac{\delta(p^2 - m_N^2)\theta(\mu - p_0)\theta(p_0)}{-4(p \cdot q)^2 + q^4}. \tag{3.12}$$

Consider the pion to be at rest ($\vec{q}=0$) in the medium, when it simplifies to

$$\Pi^{(n)}(q_0, \vec{q}=0) = -8g_A^2 \frac{m_N^2}{F_\pi^2} q_0^2 \int \frac{d^3p}{(2\pi)^3 2E_p} \frac{\theta(p_F - |\vec{p}|)}{-4E_p^2 + q_0^2} = \frac{g_A^2 \bar{n}}{4F_\pi^2 m_N} q_0^2, \tag{3.13}$$

where $\bar{n}$ is the nucleon number density in symmetric nuclear matter,

$$\bar{n} = 4\int \frac{d^3p}{(2\pi)^3}\theta(p_F - |\vec{p}|) = \frac{2p_F^3}{3\pi^2}, \tag{3.14}$$

p_F being the Fermi momentum, $p_F = \sqrt{\mu^2 - m_N^2}$. Assuming that the vacuum part has already been taken care of to define the physical parameters, it is $\Pi^{(n)}(q_0)$ which is relevant in Eq. (3.7).

It is now simple to calculate the remaining self-energy and vertex diagrams. A special feature is presented, however, by the constant vertex diagrams (f,g) arising from the vertex proportional to c_2: While all other contributions are proportional to $q_\lambda q_\sigma$, this one is not, being given by

$$\delta^{ii'} i\Delta(q)(q_\lambda q_0 \delta_{\sigma 0} + q_\sigma q_0 \delta_{\lambda 0}) 2c_2 \bar{n}. \tag{3.15}$$

It reflects the fact that our treatment breaks Lorentz invariance to $O(3)$. Thus while the matrix element

$$\langle 0|A_\mu^a|\pi^b(q)\rangle = i\delta^{ab} f_\mu(q), \tag{3.16}$$

is defined in vacuum as $f_\mu = q_\mu F_\pi$, it must be expressed in medium as [22,9]

$$f_\mu = \delta_{\mu 0} q_0 F_\pi^t + \delta_{\mu i} q_i F_\pi^s, \quad i = 1,2,3. \tag{3.17}$$

The results of calculating all the diagrams of Fig. 1 with vertices given by (2.18) may now be expressed in terms of the effective parameters,

$$m_\pi^{(n)} = m_\pi \left\{ 1 + \left(2c_1 - c_2 - c_3 + \frac{g_A^2}{8m_N} \right) \frac{\bar{n}}{F_\pi^2} \right\}, \tag{3.18}$$

$$F_\pi^t = F_\pi \left\{ 1 + \left(c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \frac{\bar{n}}{F_\pi^2} \right\}, \tag{3.19}$$

$$F_\pi^s = F_\pi \left\{ 1 + \left(-c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \frac{\bar{n}}{F_\pi^2} \right\}. \tag{3.20}$$

These results were obtained earlier in this form in Ref. [10] by integrating out the nucleon field in the generating functional in presence of the external field a_μ. We postpone discussing the validity of these results until Sec. V.

IV. VIRIAL EXPANSION

We next turn to a different approach to the problem, namely, the virial expansion for the self-energy of the particle in question [4,11,12]. The resulting (first order) formula is valid, if the medium is sufficiently dilute. As we shall discuss below, its range of validity is, in general, different from that calculated above using χPT.

Let us derive the formula for the case at hand. A simple derivation follows, if we recognize that the self-energy function is an S-matrix element [12]. Consider first the process in vacuum. Just as the amplitude T for the two body πN scattering,

$$\pi(k,i) + N(p,s) \to \pi(k',i') + N(p',s'),$$

is given by the S-matrix element,

$$\begin{aligned} i(2\pi)^4\delta^4(p + k - p' - k')T_{i'i;s's} &= \langle k',i';p',s'|S-1|k,i;p,s\rangle \\ &= \langle 0|a(k',i')b(p',s')(S-1)b^\dagger(p,s)a^\dagger(k,i)|0\rangle, \end{aligned} \tag{4.1}$$

we may express the self-energy $\Pi^{(0)}$ of the pion (in vacuum) by the one-body matrix element,

$$\begin{aligned} -i(2\pi)^4\delta^4(k-k')\delta_{i'i}\Pi^{(0)}(k) &= \langle k',i'|S-1|k,i\rangle \\ &= \langle 0|a(k',i')(S-1)a^\dagger(k,i)|0\rangle, \end{aligned} \tag{4.2}$$

where S is the familiar scattering matrix operator,

$$S = e^{i\int d^4x \mathcal{L}_{int}(x)}.$$

Here $i(i')$ and $s(s')$ are indices denoting the pion isospin and the nucleon spin projection in the initial (final) state respectively. Following the usual practice, the amplitude T is regarded as a 2×2 matrix in the nucleon isospin space. The operators $a(k,i)$ and $b(p,s)$ annihilate respectively a pion of momentum k and isospin i and a nucleon of momentum p and spin s.

The corresponding self-energy $\Pi(k)$ in nuclear medium is obtained simply by replacing the vacuum expectation value in Eq. (4.2) by the ensemble average defined in Eq. (3.1), which is denoted now by the angular bracket,

$$-i(2\pi)^4\delta^4(k-k')\delta_{i'i}\Pi(k) = \langle a(k',i')(S-1)a^\dagger(k,i)\rangle. \tag{4.3}$$

It is here that we make use of the virial expansion in powers of the distribution function. We expand the ensemble average of any operator $\mathcal{O}$ as

$$\langle \mathcal{O} \rangle = \langle 0|\mathcal{O}|0\rangle + \sum_N \int \frac{d^3p}{(2\pi)^3 2E_p} n^+(p)\langle p,s|\mathcal{O}|p,s\rangle + \cdots, \tag{4.4}$$

where the sum is over the nucleon spin and isospin states. Applying this expansion to the right-hand side of Eq. (4.3), we get

$$-i(2\pi)^4\delta^4(k-k')\Pi^{(n)}(k) = \sum_N \int \frac{d^3p}{(2\pi)^3 2E_p} n^+(p)\langle p,s|a(k',i)(S-1)a^\dagger(k,i)|p,s\rangle, \tag{4.5}$$

where $\Pi^{(n)}(k)$ stands as before for the difference $\Pi(k) - \Pi^{(0)}(k)$. Note that there is no sum over the pion isospin index i. We now use Eq. (4.1) to express the self-energy in terms of the forward scattering amplitude $T_f(p,k)$,

$$\Pi^{(n)}(k) = -\int \frac{d^3p}{(2\pi)^3 2E_p} n^+(p)\sum_N T_f(p,k), \tag{4.6}$$

taking the summation inside the integral, as the distribution function is the same for all the four nucleon states.

Equation (4.6) is the desired first order virial expansion formula for the pion self-energy. Although such formulas have been used to find the mass shifts in different cases, its application to pion in nuclear medium does not exist in the literature. What has been utilized earlier is an approximation to the above formula [14]

$$\overline{\Pi}^{(n)}(k) = -\frac{\bar{n}}{8m_N}\sum_N T_f(k). \tag{4.7}$$

But if the amplitude is not constant, even approximately, within the interval of integration in Eq. (4.6), this formula cannot clearly be trusted. As we shall see below, this is indeed the situation for the case at hand.

To evaluate the self-energy (at threshold) using the experimental data [23], we first carry out the indicated sum in a general kinematic situation. The matrix structure of T in nucleon isospin space may be written as

$$T_{i'i;s's} = T^+_{s's}\delta_{i'i} + T^-_{s's}\tfrac{1}{2}[\tau_{i'},\tau_i], \tag{4.8}$$

where each of the $T^\pm$ has the invariant spin decomposition,

$$T^\pm_{s's} = \bar{u}(p',s')\left\{A^\pm + \tfrac{1}{2}(\not{k}+\not{k}')B^\pm\right\}u(p,s). \tag{4.9}$$

Taking the amplitude for any one of the charged states $\pi^{\pm,0}$ for the pion, we can readily carry out the sum over the nucleon states,

$$\sum_N T_f(p,k) = 8(m_N A^+ + p\cdot k B^+), \tag{4.10}$$

in terms of the isospin even amplitude only.

As given by Eq. (4.6), $\Pi^{(n)}(m_\pi)$ is an integral over the three-momentum of the nucleon in the pion rest frame ($\vec{k}=0$), while the experimental data is given as a function of the pion energy $\omega_k = \sqrt{\vec{k}^2 + m_\pi^2}$ in the nucleon rest (lab) frame. The two variables are related by the equation, $m_\pi E_p = m_N\omega_k$, $E_p = \sqrt{\vec{p}^2 + m_N^2}$. We thus have finally the complex pole position in the pion propagator as

$$m_\pi^{(n)} - \frac{i}{2}\gamma_\pi^{(n)} = m_\pi + \frac{\Pi^{(n)}(m_\pi)}{2m_\pi} = m_\pi - \frac{1}{\pi^2}\left(\frac{m_N}{m_\pi}\right)^3 \int_1^{\bar{\omega}_k} d\omega_k\sqrt{\omega_k^2 - m_\pi^2}\, D^+(\omega_k), \tag{4.11}$$

where $D^+(\omega_k) = A^+ + \omega_k B^+$, is the isospin even forward πN scattering amplitude. The upper limit $\bar{\omega}_k$ is determined by the nucleon number density

$$\bar{\omega}_k = m_\pi\sqrt{1 + \left(\frac{3\pi^2\bar{n}}{2m_N^3}\right)^{2/3}}. \tag{4.12}$$

The imaginary part of the pole position represents the damping rate of pionic excitations in the medium.

The numerical evaluations are shown in Figs. 2 and 3, where the pion mass shift and its imaginary part are plotted as a function of the nucleon number density in units of the

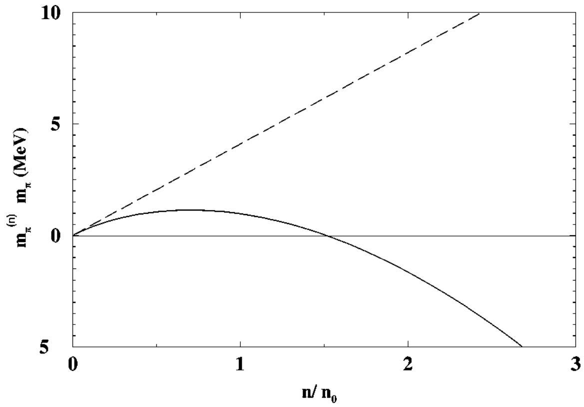

FIG. 2. Shift in pion mass in nuclear medium. The solid curve results from the exact, first order virial formula, while the dashed line follows from the approximate virial formula.

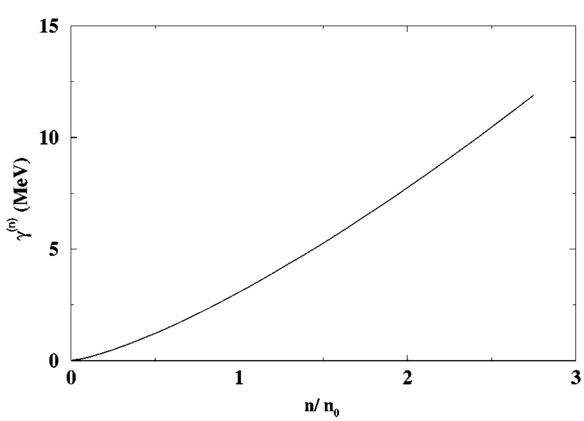

FIG. 3. Damping rate of pionic excitations in nuclear medium.

normal density $\bar{n}_0=(110\text{ MeV})^3$. Our result for the pion mass shift in nuclear medium may be compared with that for the nucleon mass shift in pionic medium, calculated in Ref. [4]. It will be observed that while both the shifts are given essentially by integrals over the same πN amplitude times the corresponding distribution functions, the curves bend in opposite directions. The reason is that as the pion energy increases, there is a change in sign in the real part of the amplitude, which is weighted differently by the distribution functions in the two cases.

For comparison we also show in Fig. 2 the mass shift following from the approximate virial formula (4.7), which at threshold is simply given by

$$\bar{\Pi}^{(n)}(m_\pi) = -\bar{n}D^{+}(m_\pi). \tag{4.13}$$

Our exact, first order formula (4.6) for the pion self-energy function may find another application to the long-standing problem of "missing repulsion" in the potential [24], needed to reproduce the accurate data on the energy levels of negatively charged pions bound to heavy nuclei [25]. As may be seen from Fig. 2, the difference at threshold $\Pi^{(n)}(m_\pi)-\bar{\Pi}^{(n)}(m_\pi)$, has indeed a large negative value at normal nuclear density (and beyond). It is thus of interest to see if this is so also at higher energies, when one may hope to find the required repulsion, at least in part, in the potential given by the exact formula (4.6), which appears missing in its approximate version (4.7) used in the literature so far.

Finally a comparison with the mass shift obtained from χPT in the preceding section is in order. Instead of using the values of the constants $c_{1,2,3}$ as given by Eqs. (2.17) to evaluate this shift from Eq. (3.18), we may calculate the amplitude sum $\Sigma_N T_f(m_\pi)$ in χPT, when the latter will be seen to contain exactly the combination of constants, $c_{1,2,3}$ and g_A as in Eq. (3.18). Eliminating this combination in terms of the amplitude sum, we get the same formula as given by Eq. (4.7) at threshold.

V. DISCUSSION

Having derived the effective parameters of the pion in nuclear medium by two different methods, we first discuss the validity of the results. Considering χPT, the region in which the leading correction term may represent a meaningful approximation to a quantity depends on the proximity of the resonances in the relevant channel. In the present case, a number of resonances, particularly the $\Delta(1232)$, lie close to the threshold, making the values of the coupling constants c_1, c_2, and c_3 in the effective Lagrangian rather large. Then the region of nuclear density in which the results may be valid, is expected to shrink considerably.

The calculated results follow this expectation. In the expression (3.18) for the effective mass, there is, however, a large cancellation among the contributions of the different vertices, making the mass shift to be only a few MeV at $\bar{n}=\bar{n}_0$, the normal nuclear density. But in the expression (3.20) for F_π^s the contributions of the vertices add up, making the "correction" at this density overwhelm the unit term. Clearly, the first order χPT results for the pion traversing nuclear matter at normal density are unacceptable.

On the other hand, in the virial expansion formula we may avoid any inaccuracy in calculating the scattering amplitude by taking it from experiment. We point out that we use the exact formula, rather than its approximate version used so long by different authors. However, the nuclear medium must be dilute enough for an expansion of the self-energy function in powers of nuclear density to be valid. At normal nuclear density $\bar{n}_0=(110\text{ MeV})^3$, the mean distance between the nucleons is about 2 fm. Thus our first order virial formula, where the pion propagation is perturbed by a single interaction with one of the nucleons in the medium, should be a reasonable approximation up to about this density. Of course, we are treating the medium as a Fermi gas of noninteracting nucleons. The many-body effects in real nuclear matter and the absorption channels may give rise to important contributions to the self-energy at higher order.

At normal nuclear density, the prediction of our first order virial formula for the mass-shift and the decay width of the pion is that both are small, being only a few MeV. (The virial expansion does not say anything about the residues, F_π^t and F_π^s.) We also point out a possible important application of this formula to the calculation of the energy levels of pionic atoms.

We conclude with a comment on determining the mass shift in the more interesting case of the nucleon in nuclear medium. Here the simple chiral Lagrangian for the *NN* system [26] is not expected to apply, as it does not take properly into account the presence of bound or virtual two nucleon states close to the *NN* threshold. In fact, it produces an absurdly large value for the nucleon mass shift [27]. But the virial expansion formula for the nucleon self-energy should apply here, at least in the neighborhood of the nuclear saturation density [28].

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