Description of the ground and octupole bands in the symplectic extension of the interacting vector boson model

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In the framework of the symplectic extension of the interacting vector boson model a good description of the first excited positive and negative parity bands of the nuclei in the rare earth and the actinide region is achieved. The bands investigated in the model are extended to very high angular momenta as a result of their consideration as "yrast" bands with respect to the symplectic classification of the basis states. The analysis of the eigenvalues of the model Hamiltonian reveals the presence of an interaction between these bands. Due to this interaction the $\Delta L=1$ staggering effect between the energies of the states of two bands is also reproduced including the "beat" patterns.

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I. INTRODUCTION

The existence of nuclei with stable deformed shapes was realized early in the history of nuclear physics. The observation of large quadrupole moments led to the suggestion that some nuclei might have spheroidal shapes, which was confirmed by the observation of rotational band structures and measurements of their properties. For most deformed nuclei, a description as an axial- and reflection-symmetric spheroid is adequate to reproduce the band's spectroscopy. Because such a shape is symmetric under space inversion, all members of the rotational band have the same parity. However, with the first observation of negative parity states near the ground state, the possibility arose that some nuclei might have an asymmetric shape under reflection.

On the other hand, whenever symmetry breaking appears new behavior of the many-body system is expected. Reflection symmetry breaking is associated with a static octupole deformation which is expected to determine new collective features for the nuclear system.

Extensive investigations into the structure of nuclei with low-lying negative parity states have led to the conclusion that while reflection asymmetric shapes can play a role in the band structure, they are not as stable as the familiar quadrupole deformations. The rotational spectra of some even-even nuclei in the rare earth and light actinide region exhibit, next to the ground band, a negative parity band which consists of the states with $I^{\pi}=1^{-},3^{-},5^{-},\ldots$. These two bands are displaced from each other, which means that fluctuations back to space symmetric shapes must also be significant. Experimentally the presence of "octupole" bands for some isotopes from the light actinide and rare earth region [1] is firmly established.

There is a large variety of models that try to describe this behavior of the low-lying states of deformed nuclei. Particularly successful are algebraic models based on symmetry principles. The introduction of an additional octupole degrees of freedom is a common feature of most of those models.

The prescription for describing negative parity states by the addition of an f boson to the usual s and d ones of the interacting vector boson model (IBM) was first mentioned by Iachello and Arima [2]. It was suggested [3] that the inclusion of a p boson to the s,d, and f bosons may play an important role in the description of these collective states.

The coherent state method (CSM) was applied by Alonso *et al.* to the *spdf* SU(3) Hamiltonian with quadrupole and octupole interactions [4]. Recently Raduta and Ionescu [5] have used a generalization of the CSM. They suggested that both ground and octupole bands may be considered as being projected from a single deformed intrinsic state that exhibits both quadrupole and octupole deformations.

Another collective model based on point symmetry group considerations [6] has also been used very successfully for the description of the energy levels of the ground and octupole bands and reproduces odd-even staggering between these levels [7]. In this model the octupole field is parametrized by irreducible representations of the octahedron point symmetry group. A recent reflection asymmetric shell model [8] interprets and reproduces most of the general features of rotational octupole bands up to very high spins by means of a variational procedure, combined with a projection method. For completeness, the methods to describe the negative parity bands in the actinide region in terms of alpha clustering [9] or binary cluster-core model [10] should be mentioned. The variety of models and approaches created to investigate the observed low-lying negative parity bands illustrates the current high interest in understanding the cause for the appearance of these bands.

The introduction of an octupole degrees of freedom in the presence of comparatively large number of free parameters in all of these models allows for the reproduction of the experimental data on the energies of the negative parity states, at least in the low spin region.

In the beginning of the 1980s a phenomenological algebraic model called the interacting vector boson model (IVBM) was introduced [11]. This model is a generalization of the phenomenological broken-SU(3) symmetry model [12], which provided a good description of the low-lying ground and γ bands [13] of well deformed even-even nuclei. Its advantages were incorporated into the rotational limit of the IVBM [14], with a good description of all the positive parity bands of nuclei in the rare earth and the actinide region. Moreover, the U(6) extension of the model contains such sequences of SU(3) multiplets, some of which prove to be convenient for the description of the low-lying negative parity bands [15].

With the recent advance of the experimental technique the investigated collective bands were extended to very high angular momenta [1]. This motivated a new approach within the framework of the IVBM aimed at a description of the first positive and negative bands, up to very high spins. In this new application, we make use of the symplectic extension of the model [16]. This allows these bands to be considered as yrast bands in the sense that we take into account the states with a given L, which minimize the energy values with respect to N. N is the eigenvalue of the total number of bosons that build the basis states of the IVBM. Its eigenvalue changes as $\Delta N=2$ in the infinite spaces of the boson representation of $\mathrm{Sp}(12,R)$. When considering the dynamical symmetry of the symplectic extension of the model through the maximal compact subgroup $\mathrm{U}(6)\supset \mathrm{Sp}(12,R)$, we obtain the exactly solvable rotational limit with a Hamiltonian, diagonal in a basis defined by the irreducible representations of the corresponding chain of subgroups. The measured energies of the ground and octupole bands in even-even nuclei from the rare earth and actinide regions are reproduced in this framework with rather good accuracy. The analysis of the obtained results shows that this is due to the appearance of a vibrational-type term that influences the yrast energies. This term also plays the role of an interaction between the two considered bands, and is the reason for the correct reproduction of the odd-even staggering of their energies.

II. ALGEBRAIC BASIS OF THE IVBM

We start with a brief review of the model's assumptions and definitions. The IVBM is based on the introduction of two kinds of vector bosons (called p and n bosons), which "built up" the collective excitations in the nuclear system. The creation operators of these bosons are assumed to be SO(3) vectors and they transform according to two independent fundamental representations (1,0) of the group SU(3). These bosons form a "pseudospin" doublet of the U(2) group and differ in their "pseudospin" projection $\alpha=\pm\frac{1}{2}$. The introduction of this additional degree of freedom leads to the extension of the SU(3) symmetry to U(6). Then the operators

$$u_m^+(\alpha=\tfrac{1}{2})=p_m^+,\ \ u_m^+(\alpha=-\tfrac{1}{2})=n_m^+,\ \ m=0,\ \pm 1, \qquad (1)$$

transform according to the fundamental representation $[1]_6$ of the group U(6). The annihilation operators are obtained by the conjugation $(u_m^+(\alpha))^\dagger=u_m(\alpha)$ and transform according to the conjugate SU(3) representations (0,1). The so introduced boson creation and annihilation operators (1) do not have a related microscopic structure, such as the s and d bosons of the IBM [2], considered as fermion pairs coupled to a total angular momentum $L=0$ and $L=2$, respectively. The building blocks of the IVBM could be treated as "oscillator quarks" and are defined [17,11] in terms of the cyclic coordinates $x_m(\alpha)$, $m=0,\pm 1$, of a "quasiparticle" and their associated momenta $q_m(\alpha)=-i\,\partial/\partial x^m(\alpha)$ in the following way:

$$u_m^+(\alpha)=\frac{1}{\sqrt{2}}[x_m(\alpha)-iq_m(\alpha)],$$
$$u_m(\alpha)=\frac{1}{\sqrt{2}}[x^m(\alpha)+iq^m(\alpha)], \qquad (2)$$

where $x^m=\sum_n g^{mn}x_n$ and $g^{mn}=g_{mn}=(-1)^n\delta_{m,-n}$. The Schwinger representation of the angular momentum algebra so(3) is constructed by substituting the operators (2) in the standard definition of its generating operator:

$$L_M^1(\alpha,\alpha)=\sum_{\alpha,k,m}C_{1k1m}^{1M}x_k(\alpha)q_m(\alpha)=\sum_{\alpha,k,m}C_{1k1m}^{1M}u_k^+(\alpha)u_m(\alpha), \qquad (3)$$

which makes the introduced vector bosons a convenient mathematical tool for use in an algebraic model in the nuclear structure theory.

The bilinear products of the creation and annihilation operators of the two vector bosons generate the noncompact symplectic group $\mathrm{Sp}(12,R)$ [11]:

$$F_M^L(\alpha,\beta)=\sum_{k,m}C_{1k1m}^{LM}u_k^+(\alpha)u_m^+(\beta),$$
$$G_M^L(\alpha,\beta)=\sum_{k,m}C_{1k1m}^{LM}u_k(\alpha)u_m(\beta), \qquad (4)$$
$$A_M^L(\alpha,\beta)=\sum_{k,m}C_{1k1m}^{LM}u_k^+(\alpha)u_m(\beta),$$

where C_{1k1m}^{LM} are the usual Clebsch-Gordon coefficients and L and M define the transformational properties of Eq. (4) under rotations.

We consider $\mathrm{Sp}(12,R)$ to be the group of the dynamical symmetry of the model. Hence the most general one-and two-body Hamiltonian can be expressed in terms of its generators. Using commutation relations between the $F_M^L(\alpha,\beta)$ and $G_M^L(\alpha,\beta)$, the full range of the number of bosons preserving Hamiltonian can be expressed in terms of operators $A_M^L(\alpha,\beta)$:

$$H=\sum_{\alpha,\beta}h_0(\alpha,\beta)A^0(\alpha,\beta)+\sum_{M,L}\sum_{\alpha\beta\gamma\delta}(-1)^M V^L(\alpha\beta;\gamma\delta)A_M^L(\alpha,\gamma)A_{-M}^L(\beta,\delta), \qquad (5)$$

where $h_0(\alpha,\beta)$ and $V^L(\alpha\beta;\gamma\delta)$ are phenomenological constants.

Being a noncompact group, the representations of $\mathrm{Sp}(12,R)$ are of infinite dimension, which makes it impossible to diagonalize the most general Hamiltonian. The operators $A_M^L(\alpha,\beta)$ generate the maximal compact subgroup of $\mathrm{Sp}(12,R)$, namely, the group $\mathrm{U}(6)[\mathrm{Sp}(12,R)\supset \mathrm{U}(6)]$.

TABLE I. Classification scheme of the SU(3) irreps contained in the even U(6) irreps with $N=0,2,4,\ldots$ and $T=N/2, N/2-1,\ldots,0$ (first column). The columns of the table are labeled with the values of T_0.

N	$(T)\backslash T_0$	$\cdots$	±4	±3	±2	±1	0
0	0						(0,0)
	1						(2,0)
2						(2,0)	
	0						(0,1)
	2						(4,0)
						(4,0)	
4	1				(4,0)		(2,1)
						(2,1)	
	0						(0,2)
	3						(6,0)
						(6,0)	
	2				(6,0)		(4,1)
6				(6,0)		(4,1)	
	1				(4,1)		(2,2)
						(2,2)	
	0						(0,3)
	4						(8,0)
						(8,0)	
	3				(8,0)		(6,1)
				(8,0)		(6,1)	
8	2		(8,0)		(6,1)		(4,2)
				(6,1)		(4,2)	
	1				(4,2)		(2,3)
						(2,3)	
	0						(0,4)
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

So the even and odd unitary irreducible representations (UIR) of $\mathrm{Sp}(12,R)$ split into an infinite but countable number of symmetric UIR of U(6) of the type $[N,0,0,0,0,0]=[N]_6$, where $N=0,2,4,\ldots$ for the even set (see Table I) and $N=1,3,5,\ldots$ for the odd set [18]. These subspaces are of finite dimension, which simplifies the problem of diagonalization. Therefore the *complete* spectrum of the system can be calculated only through the diagonalization of the Hamiltonian in the subspaces of *all* the UIR of U(6), belonging to a given UIR of $\mathrm{Sp}(12,R)$.

The rotational limit [14] of the model is further defined by the chain as follows:

$$\mathrm{U}(6)\supset \mathrm{SU}(3)\otimes \mathrm{U}(2)\supset \mathrm{SO}(3)\otimes \mathrm{U}(1) \tag{6}$$

$$[N]\quad (\lambda,\mu)\quad (N,T)K\quad L\quad T_0 \tag{7}$$

where the labels below the subgroups are the quantum numbers (7) corresponding to their irreducible representations. Their values are obtained by means of standard reduction rules and are given in Ref. [14]. In this limit the operators of the physical observables are the angular momentum operator (3),

$$L_M=-\sqrt{2}\sum_{\alpha} A^1_M(\alpha,\alpha),$$

and the truncated ("Elliott") quadrupole operator,

$$Q_M=\sqrt{6}\sum_{\alpha} A^2_M(\alpha,\alpha),$$

which define the algebra of SU(3).

The operators of the "pseudospin" components and the number of bosons N,

$$T_{+1}=\sqrt{\frac{3}{2}}A^0(p,n),\qquad T_{-1}=-\sqrt{\frac{3}{2}}A^0(n,p),$$

$$T_0=-\sqrt{\frac{3}{2}}[A^0(p,p)-A^0(n,n)],$$

$$N=-\sqrt{3}[A^0(p,p)+A^0(n,n)],$$

define the algebra of U(2).

Since the reduction from U(6) to SO(3) is carried out by the mutually complementary groups SU(3) and U(2), their quantum numbers are related in the following way:

$$T=\frac{\lambda}{2},\qquad N=2\mu+\lambda. \tag{8}$$

The complementarity of the groups SU(3) and U(2) [19] makes it easy to construct the bases for the U(6) representations and to reduce it to the SO(3) representations [20] of the angular momentum. This provides for an elegant solution of the state labeling problem, so making use of the latter we can write the basis as

$$|[N]_6;(\lambda,\mu);K,L,M;T_0\rangle=|(N,T);K,L,M;T_0\rangle. \tag{9}$$

The ground state of the system is

$$|0\rangle=|(N=0,T=0);K=0,L=0,M=0;T_0=0\rangle, \tag{10}$$

which is the vacuum state for the $\mathrm{Sp}(12,R)$ group.

A. The symplectic extension of IVBM

The basis states associated with the even irreducible representation of the $\mathrm{Sp}(12,R)$ can be constructed by the application of powers of raising generators $F^L_M(\alpha,\beta)$ of the same group. Each raising operator will increase the number of bosons N by two. As a result we get a realization of the reduction scheme [18]:

$$\begin{array}{llll} & N & T^2 & T_0 \\ \mathrm{Sp}(12,R) & \to \mathrm{U}(6) & \to \mathrm{SU}(2)\times \mathrm{SU}(3) & \to \mathrm{SU}(3). \end{array}$$

The $\mathrm{Sp}(12,R)$ classification scheme for the SU(3) boson representations for even value of the number of bosons N is shown in Table I. Each row (fixed N) of the table corresponds to a given irreducible representation of the U(6). Then the possible values for the pseudospin are T

$=N/2, N/2-1, \ldots\ldots, 0$ and are given in the column next to the respective value of N. Thus when N and T are fixed, $2T+1$ equivalent representations of the group SU(3) arise. Each of them is labeled by the eigenvalues of the operator T_0:$-T, -T+1, \ldots, T$, defining the columns of Table I. The same SU(3) representations (λ, μ) arise for the positive and negative eigenvalues of T_0.

Hence, in the framework of the discussed boson representation of the Sp(12, R) algebra all possible irreducible representations of the group SU(3) are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators N and T^2. The equivalent use of the (λ, μ) labels facilitates the final reduction to the SO(3) representations, which define the angular momentum L and its projection M. The multiplicity index K appearing in this reduction is related to the projection of L in the body fixed frame and is used with the parity (π) to label the different bands (K^π) in the energy spectra of the nuclei. We define the parity of the states as $\pi=(-1)^T$. This allows us to describe both positive and negative bands.

B. The energy spectrum

The Hamiltonian, corresponding to the considered rotational limit of IVBM, is expressed in terms of the first-and second-order invariant operators of the different subgroups in the chain (6):

$$H = aN + \alpha_6 K_6 + \alpha_3 K_3 + \alpha_1 K_1 + \beta_3 \pi_3, \qquad (11)$$

where K_n are the quadratic invariant operators of the U(n) groups and π_3 is the SO(3) second-order Casimir operator. As a result of the connections (8) the Casimir operator K_3 with eigenvalue $(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)$ is expressed in terms of the operators N and T:

$$K_3 = 2Q^2 + \tfrac{3}{4}L^2 = \tfrac{1}{2}N^2 + N + T^2.$$

Making use of the above relation, Eq. (11) takes the form

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 \pi_3 + \alpha_1 T_0^2, \qquad (12)$$

and is obviously diagonal in the basis (9) labeled by the quantum numbers of the subgroups of the chain (6). Its eigenvalues are the energies of the basis states of the boson representations of Sp(12, R):

$$E((N,T);KLM;T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2. \qquad (13)$$

The energy of the ground state (10) of the system is obviously 0.

III. APPLICATION OF IVBM FOR THE DESCRIPTION OF THE GROUND STATE AND OCTUPOLE BANDS ENERGIES

In this paper we modify the earlier application of the IVBM [15] for the description of the first excited even and odd parity bands in order to reach much higher angular momentum states in both band types. We will apply the model to even- even deformed nuclei, which exhibit, next to the ground band, a low-lying negative parity band traditionally considered to be an octupole band [1]. In order to do this we first have to identify these experimentally observed bands with the sequences of basis states for the even representation of Sp(12, R) given in Table I. We choose the SU(3) multiplet $(0, \mu)$ for a description of the ground band, whereas for the octupole band the SU(3) multiplet $(2, \mu-1)$ is used. In terms of (N, T) this choice corresponds to $(N=2\mu, T=0)$ for the positive $(K^\pi=0^+)$ and $(N=2\mu+2, T=1)$ for the negative $(K^\pi=0^-)$ parity band, respectively.

A. Yrast bands

In this way, in the framework of the symplectic extension of boson representations of the number preserving U(6) symmetry, we are able to consider all even eigenvalues of the number of vector bosons N with the corresponding set of pseudospins T.

This approach is based on the fact that the energies (13) increase with the increasing of N. We define the energies of each state with given L as yrast energy with respect to N in the two considered bands. Hence their minimum values are obtained at $N=2L$ for the ground band, and at $N=2L+2$ for the octupole band, respectively. So for the description of the ground band our choice corresponds to the sequence of states with different numbers of bosons, $N=0, 4, 8, \ldots$, and pseudospin $T=0$ in the column labeled $T_0=0$ of Table I. Similarily for a description of the negative parity band, we choose the set of states with quantum numbers $N=8, 12, \ldots$ and $T=1$ from the same column $T_0=0$. Since these quantum numbers uniquely define the SU(3) multiplets, which reduce to the corresponding values of the angular momenta L, the ground band belongs to the SU(3) multiplet $(0, N/2)$ and the octupole band to $(2, N/2-1)$. In the so-defined SU(3) representations for each N the maximal values of L appear for the first time (see Table I).

According to the correspondence, identified above, between the basis states and the experimental data on the ground and octupole bands $T_0=0$, the last term in the energy formula (13) vanishes. The phenomenological model parameters a, b, α_3, and β_3 are evaluated by a fit to the experimental data [21]. Their values obtained for some even-even deformed nuclei belonging to light actinides and rare earth region are given in Table II. The second column gives the numbers of the experimental states used in the fitting procedure.

The comparison between the experimental spectra and our calculations using the values of the model parameters given in Table II for the ground and octupole bands of the nuclei Ra^{224}, Th^{226}, Sm^{152}, and Yb^{168} is illustrated in Fig. 1.

The agreement between the theoretical values obtained with only four model parameters and the experimental data for all the nuclei under consideration is rather good.

Applying the yrast conditions relating N and L the energies (13) for ground state band turn out to be given by

$$E(L) = \beta L(L+1) + \gamma L, \qquad (14)$$

while the energies of the negative parity band are

TABLE II. Values of the parameters in Eq. (13) determined by fitting the theoretical energies of the ground and octupole bands to their experimental values for nuclei in the first column.

Nucleus	n_s	a	b	α_3	β_3
Ra^{224}	13	0.0119	−0.0022	0.0789	0.0155
Ra^{226}	18	0.0269	−0.0005	0.0226	0.0060
Th^{222}	26	0.0558	0.0000	−0.0557	0.0030
Th^{224}	18	0.0242	−0.0011	0.0362	0.0100
Th^{226}	20	0.0194	−0.0009	0.0522	0.0094
Th^{228}	18	0.0092	−0.0020	0.1470	0.0138
Th^{232}	29	0.0155	−0.0021	0.3244	0.0128
U^{234}	19	0.0124	−0.0010	0.3608	0.0085
U^{236}	25	0.0154	−0.0010	0.2846	0.0086
U^{238}	27	0.0142	−0.0016	0.2851	0.0110
Yb^{168}	41	0.0235	−0.0056	0.6512	0.0295
Sm^{152}	15	0.0194	−0.0045	0.4290	0.0274

$$E(L) = \beta L(L+1) + (\gamma + \eta)L + \xi. \tag{15}$$

The new free parameters β, γ, η, and ξ are related to the previous ones given in Table II as follows:

$$\beta = 4b + \beta_3, \quad \gamma = 2a - 4b, \quad \eta = 8b, \quad \xi = 2a + 4b + 2\alpha_3. \tag{16}$$

As could be seen from Eqs. (14) and (15), the values of β and γ can be determined only from a fit to the positive band energies, while η and ξ are estimated from the negative ones, respectively. The values of the parameters (16) determine the behavior of the energies of the two bands and their positions with respect to each other. In some cases (^{232}Th, ^{234}U, ^{236}U, ^{238}U) the two bands are almost parallel. The shift between them depends on the parameter ξ. When they are very close they interact through the L-dependent interaction with a strength $\gamma+\eta$.

As a result of our theoretical assumptions we obtained simple formulas for the energy levels. Analyzing Eqs. (14) and (15) we can see that eigenstates of the first positive and negative bands consist of rotational $L(L+1)$ and vibrational L modes. The rotational interaction is with equal strength β in both of the bands. The obtained values of the parameter η are always negative, which means that the negative parity band is less vibrational than the positive one.

B. The staggering

In the collective rotational spectra of the considered deformed even-even nuclei in this mass region some fine structure effects as back-bending and staggering behavior are observed. Odd-even staggering patterns between the energies of states from the ground and octupole bands have been inves-

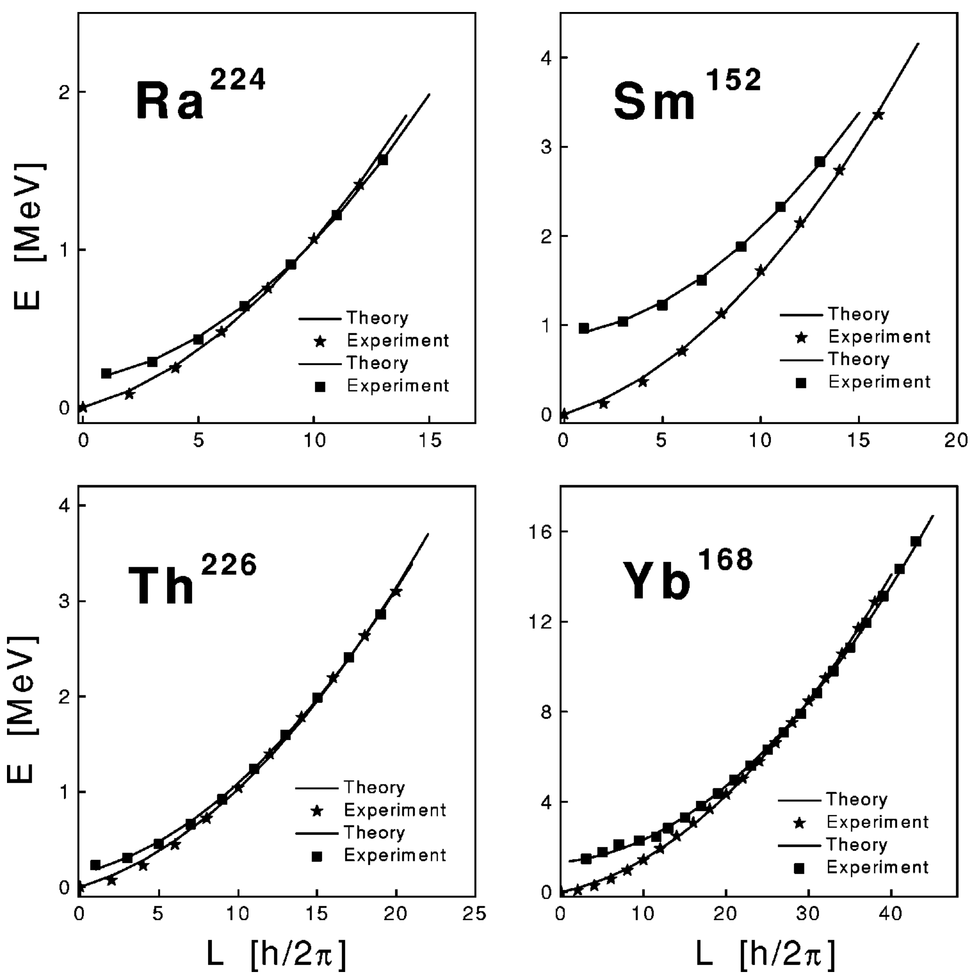

FIG. 1. Theoretical (‘Theory’) and experimental (‘Experiment’) energies of the ground (stars) and octupole (squares) bands of the ^{224}Ra, ^{152}Sm, ^{226}Th, and ^{168}Yb nuclei. The theoretical values (13) are calculated with the corresponding parameters, taken from Table II.

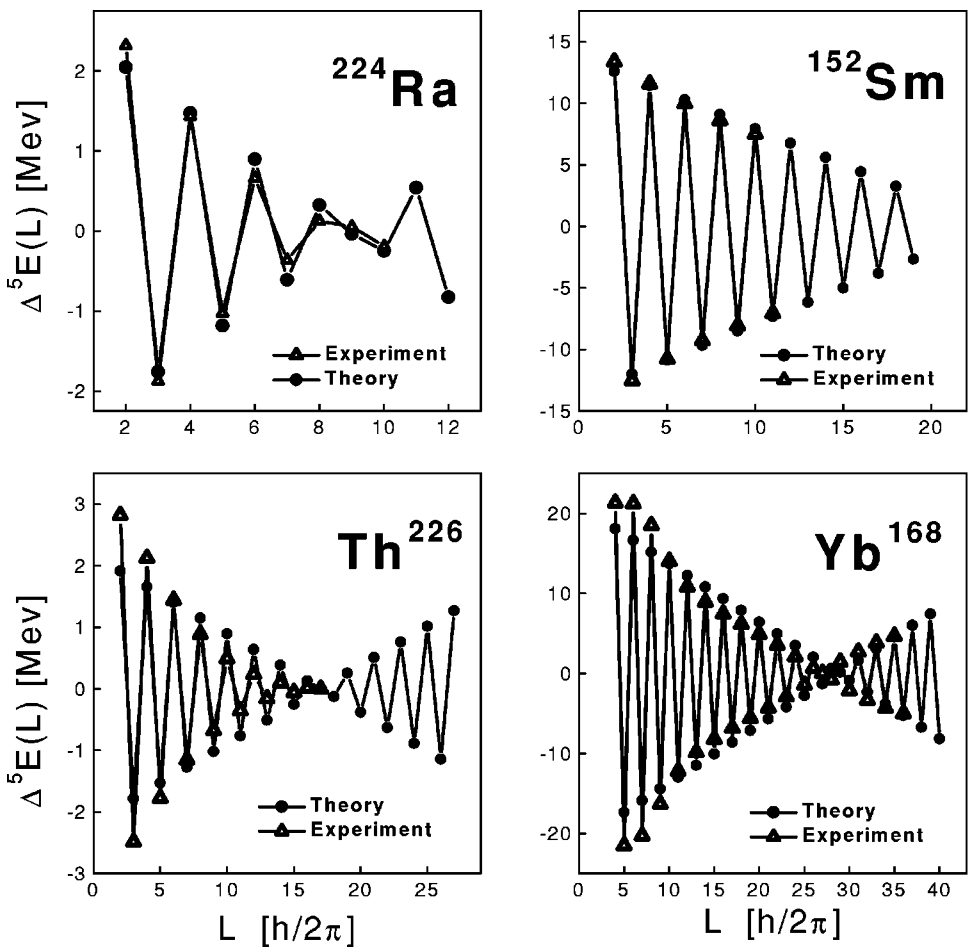

FIG. 2. Comparison of the theoretical ('Theory') and experimental ('Experiment') values of the staggering function $\Delta^5 E(L)$ (17) of the ground and octupole bands energies $\Delta L=1$ of the ^{224}Ra, ^{152}Sm, ^{126}Th, and ^{168}Yb nuclei.

tigated recently [7]. In order to test further our model we applied on the energies the staggering function defined as [22]

$$\Delta^5 E(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2), \quad (17)$$

where $\Delta E(L)=E(L)-E(L-1)$ gives the odd-even energy difference.. The function (17) is a finite difference of fourth order in respect to $\Delta E(L)$ or of fifth order in respect to the energy $E(L)$ (13), and is characteristic for the deviation of the rotational behavior from that of the rigid rotor. The calculated and experimental staggering patterns are illustrated in Fig. 2. One can see a good agreement with the experiment, as well as with the reproduction of the "beat" patterns of the staggering behavior. They occur in the region where the interaction between the two considered bands is most strong or they cross. Substituting the expressions (14) and (15) in the staggering function (17) it can be easily seen that it depends on ηL. The correct reproduction of the experimental staggering patterns is due to this term, which could be interpreted as interaction between the positive and negative parity bands. Hence the success of the model in describing the beat patterns in the $\Delta L=1$ staggering is a result of the introduced notion of yrast energies in the framework of the symplectic extension of the IVBM.

IV. CONCLUSIONS

We have applied the interacting vector boson model for the description of the ground and octupole bands in some even-even rare earth and actinide nuclei up to very high spins. In spite of the simplicity of the model without introducing additional degrees of freedom we are able to describe both positive and negative parity bands. This is due to the specific definition of the states parity depending on the pseudospin quantum number T.

The successful reproduction of the experimental energies and of their odd-even staggering was achieved as a result of their consideration as yrast energies in respect to the number of phonon excitation N that build the collective states. The introduction of this notion was possible, as we extended the IVBM to its symplectic dynamical symmetry $Sp(12,R)$, which allows the change of the number of bosons that are the building blocks of the model Hamiltonian. Nevertheless the Hamiltonian remains with only few phenomenological parameters and is still exactly solvable. Through the algebraic properties of the dynamical symmetry chain relations between SU(3) and U(2) quantum numbers are established. Combining these relations with the notion of yrast energies the physical meaning of each term of the Hamiltonian is clarified. In the rotational limit of the model in addition to the rotational character of the considered bands a purely vibrational mode is appearing, which introduces also some interaction between them. This is the reason for the reproduction also of the fine effect of the structure of these bands. The obtained physically meaningful results are also simple and easy for use, and they permit the application of the model to larger class of nuclei than the purely rotational ones.

The symplectic extension of the interacting vector boson model permits a richer classification of the states than its unitary version and gives the possibility for a further consideration of other collective bands. In general the model proves appropriate for the description of diverse nuclear structure problems.

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