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Momentum dependence of the symmetry potential and nuclear reactions induced by neutron-rich nuclei at RIA

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Effects of the momentum dependence of the symmetry potential in nuclear reactions induced by neutron-rich nuclei at Rare Isotope Accelerator (RIA) energies are studied using an isospin- and momentum-dependent transport model. It is found that symmetry potentials with and without the momentum-dependence but corresponding to the same density-dependent symmetry energy $E_{sym}(\rho)$ lead to significantly different predictions on several $E_{sym}(\rho)$-sensitive experimental observables. The momentum dependence of the symmetry potential is thus critically important for investigating accurately the equation of state and novel properties of dense neutron-rich matter at RIA.

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The rapid advance in technologies to accelerate radioactive beams has opened up several new frontiers in nuclear sciences [1,2]. Particularly, the high-energy radioactive beams to be available at the planned Rare Isotope Accelerator (RIA) and the accelerator facility at GSI provide a great opportunity to explore the equation of state (EOS) and properties of dense neutron-rich matter [3–6]. The energy per particle $E(\rho,\delta)$ in asymmetric nuclear matter of density ρ and relative neutron excess $\delta=(\rho_n-\rho_p)/(\rho_p+\rho_n)$ is usually expressed as $E(\rho,\delta)=E(\rho,\delta=0)+E_{\rm sym}(\rho)\delta^2+\mathcal{O}(\delta^4)$, where $E_{\rm sym}(\rho)$ is the density-dependent nuclear symmetry energy. The latter is among the most important and yet very poorly known properties of dense neutron-rich matter [7,8]. For instance, it is important for Type II supernova explosions, for neutron-star mergers, and for the stability of neutron stars. It also determines the proton fraction and electron chemical potential in neutron stars at β equilibrium. These quantities consequently determine the cooling rate and neutrino emission flux of protoneutron stars and the possibility of kaon condensation in dense stellar matter [7,8]. In nuclear reactions induced by neutron-rich nuclei, the $E_{sym}(\rho)$ reveals itself through dynamical effects of the corresponding symmetry potentials acting differently on neutrons and protons. Based on isospin-dependent transport model calculations, several experimental observables have been identified as promising probes of the $E_{sym}(\rho)$, such as, the neutron/proton ratio [9–11], the neutron-proton differential flow [12,13], the neutron-proton correlation function [14], and the isobaric yield ratios of light clusters [15]. However, in all existing transport models the momentum dependence of the symmetry (isovector) potential has never been taken into account. Effects of the momentum dependence of the symmetry potential in nuclear reactions are thus completely unknown, although effects of the momentum dependence of the isoscalar potential are well known [16–19]. This is mainly because only very recently was the momentum dependence of the symmetry potential given in a form practically usable in transport model calculations [20]. In this work, we study the effects of the momentum dependence of the symmetry potential within an isospin- and momentum-dependent transport model for nuclear reactions induced by neutron-rich nuclei at RIA energies. It is found that symmetry potentials with and without the momentum dependence but corresponding to the same symmetry energy $E_{sym}(\rho)$ lead to significantly different predictions on several sensitive probes of the $E_{sym}(\rho)$. Moreover, these observables are more sensitive to the variation of $E_{sym}(\rho)$ in calculations with the momentum-dependent symmetry potentials.

That the momentum dependence of the single particle potential is different for neutrons and protons in asymmetric nuclear matter is well known and has been a subject of intensive research based on various many-body theories using nonlocal interactions, see, e.g., Ref. [21] for a review. Guided by a Hartree-Fock calculation using the Gogny effective interaction, the single nucleon potential was recently parametrized as [20]

$$\begin{aligned} U(\rho,\delta,\vec{p},\tau) &= A_u\frac{\rho_{\tau'}}{\rho_0}+A_l\frac{\rho_\tau}{\rho_0}+B\left(\frac{\rho}{\rho_0}\right)^\sigma(1-x\delta^2) \\ &\quad - 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'} \\ &\quad + \frac{2C_{\tau,\tau}}{\rho_0}\int d^3p'\frac{f_\tau(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2} \\ &\quad + \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3p'\frac{f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}. \end{aligned} \quad (1)$$

In the above $\tau=1/2(-1/2)$ for neutrons (protons) and $\tau\neq\tau'$; $\sigma=4/3$; $f_\tau(\vec{r},\vec{p})$ is the phase space distribution function at coordinate $\vec{r}$ and momentum $\vec{p}$. The parameters $A_u, A_l, B, C_{\tau,\tau}, C_{\tau,\tau'}$, and Λ listed in Ref. [20] were obtained by fitting the momentum dependence of the $U(\rho,\delta,\vec{p},\tau)$ predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock calculations, saturation properties of symmet-

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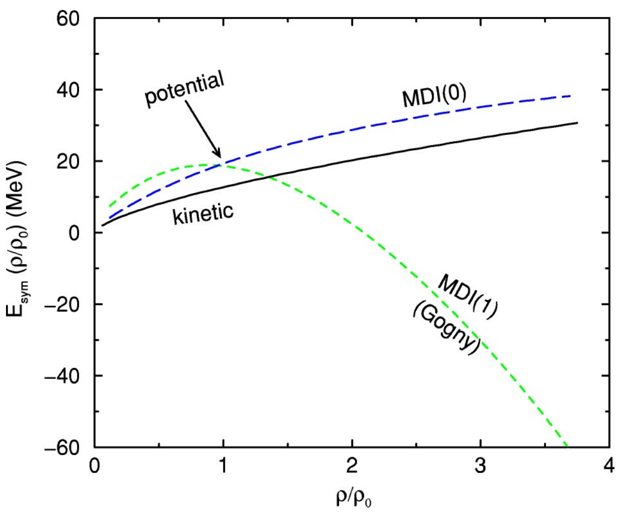

FIG. 1. (Color online) The density dependence of the potential and kinetic parts of nuclear symmetry energy.

ric nuclear matter and the symmetry energy of 30 MeV at normal nuclear matter density $\rho_0=0.16/\mathrm{fm}^3$. The compressibility of symmetric nuclear matter K_0 is set to be 211 MeV. The momentum dependence of the symmetry potential stems from the different interaction strength parameters $C_{\tau,\tau'}$ and $C_{\tau,\tau}$ for a nucleon of isospin τ interacting, respectively, with unlike and like nucleons in the background fields. More specifically, $C_{unlike}=-103.4$ MeV, while $C_{like}=-11.7$ MeV.

The parameter x was introduced to reflect the largely uncertain behavior of $E_{sym}(\rho)$, specifically its potential part $E^p_{sym}(\rho)$. The latter corresponding to $x=1$ [denoted by MDI(1) which gives the same $E_{sym}(\rho)$ as the default Gogny interaction] and $x=0$ [MDI(0)] together with the kinetic contribution $E^k_{sym}(\rho)=\hbar^2/6m_n(3\pi^2\rho/2)^{2/3}$ are shown in Fig. 1. The variation of $E_{sym}(\rho)$ resulted by changing from the MDI(1) to the MDI(0) parameter set is well within the uncertain range predicted by various many-body theories, see, e.g., Refs. [22–24]. The potential contribution to the symmetry energy can be parametrized as

$$E^p_{sym}(\rho) = 3.08 + 39.6u - 29.2u^2 + 5.68u^3 - 0.523u^4 \quad (2)$$

for the MDI(1) (Gogny) and

$$E^p_{sym}(\rho) = 1.27 + 25.4u - 9.31u^2 + 2.17u^3 - 0.21u^4 \quad (3)$$

for the MDI(0), where $u \equiv \rho/\rho_0$ is the reduced nucleon density.

We implemented the single particle potential in Eq. (1) in an isospin-dependent transport model [9]. Since the C terms in the single particle potential depend inseparably on the density, momentum, and isospin, to investigate effects of the momentum dependence of the symmetry potential we shall compare results obtained using Eq. (1) with those obtained using a single nucleon potential $U_{noms}(\rho,\delta,\vec{p},\tau) \equiv U_0(\rho,\vec{p}) + U_{sym}(\rho,\delta,\tau)$ that has almost the same momentum-dependent isoscalar part $U_0(\rho,\vec{p})$ as that embedded in Eq. (1) and a momentum-independent symmetry potential $U_{sym}(\rho,\delta,\tau)$ that gives the same $E_{sym}(\rho)$ as Eq. (1). The $U_{sym}(\rho,\delta,\tau)$ can be obtained from $U_{sym}(\rho,\delta,\tau)=\partial W_{sym}/\partial\rho_\tau$, where W_{sym} is the isospin-dependent part of the potential energy density $W_{sym}=E^p_{sym}(\rho)\rho\delta^2$. Using the parametrizations for $E^p_{sym}(\rho)$ in Eqs. (2) and (3), we obtain

$$U^{MDI1}_{sym}(\rho,\delta,\tau) = 4\tau\delta(3.08 + 39.6u - 29.2u^2 + 5.68u^3 - 0.52u^4) - \delta^2(3.08 + 29.2u^2 - 11.4u^3 + 1.57u^4) \quad (4)$$

for the MDI(1) parameter set and

$$U^{MDI0}_{sym}(\rho,\delta,\tau) = 4\tau\delta(1.27 + 25.4u - 9.31u^2 + 2.17u^3 - 0.21u^4) - \delta^2(1.27 + 9.31u^2 - 4.33u^3 + 0.63u^4) \quad (5)$$

for the MDI(0) parameter set. To identify reliably effects of the momentum dependence of the symmetry potential without much interference from density effects the $U_{noms}(\rho,\delta,\vec{p},\tau)$ should lead to almost the same reaction dynamics and evolution of density profiles as the single particle potential in Eq. (1). Both of them are mainly determined by the isoscalar potential for which we select the original MDYI interaction [17]

$$U_0(\rho,\vec{p}) = -110.44u + 140.9u^{1.24} - \frac{130}{\rho_0}\int d^3p' \frac{f(\vec{r},\vec{p}\,')}{1 + (\vec{p}-\vec{p}\,')^2/(1.58p^0_F)^2}, \quad (6)$$

where p^0_F is the Fermi momentum. The compressibility K_0 for this interaction is 215 MeV. We compared numerically $U_0(\rho,\vec{p})$ in Eq. (6) with $[U_n(\rho,\delta,\vec{p})+U_p(\rho,\delta,\vec{p})]/2$ obtained from Eq. (1). They are indeed very close and both agree with the nucleon optical potential data. We also verified numerically that the potential $U_{noms}(\rho,\delta,\vec{p},\tau)$ constructed this way leads to about the same reaction dynamics and the evolution of density profiles as the potential in Eq. (1).

The strengths of the symmetry potentials with and without the momentum dependence but corresponding to the same $E_{sym}(\rho)$ are compared in Fig. 2 by examining the difference between neutron and proton potentials $(U_n-U_p)/2$. The symmetry potential without the momentum dependence is higher in magnitude and has generally steeper slopes than the momentum-dependent one for $\rho/\rho_0 \leqslant 2.3$ with the MDI(1) parameter set and at all densities for the MDI(0) parameter set. Moreover, the strength of momentum-dependent symmetry potentials decreases with the increasing momentum. Thus the difference between the symmetry potentials with and without the momentum dependence is larger for nucleons with higher momenta. Several experimental observables are known to be sensitive only to the symmetry potential but not to the isoscalar potential. These observables are mainly neutron-proton differential or relative quantities [9,12] where effects of the isoscalar potential with or without the momentum dependence are largely canceled out. These observables are also insensitive to the in-medium nucleon-nucleon (NN) cross sections [3,9,14]. We use here the isospin-dependent

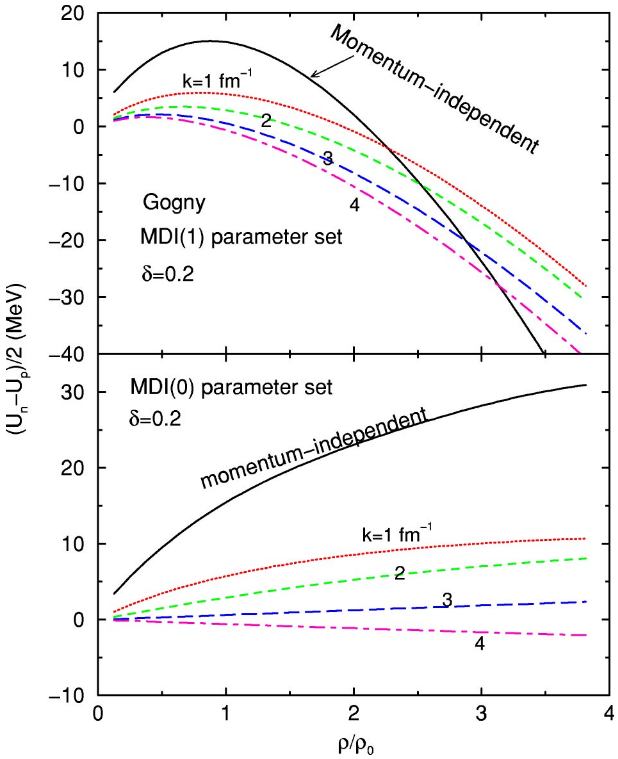

FIG. 2. (Color online) Strengths of the symmetry potentials with and without (solid) the momentum-dependence as a function of density as measured by the difference between neutron and proton potentials.

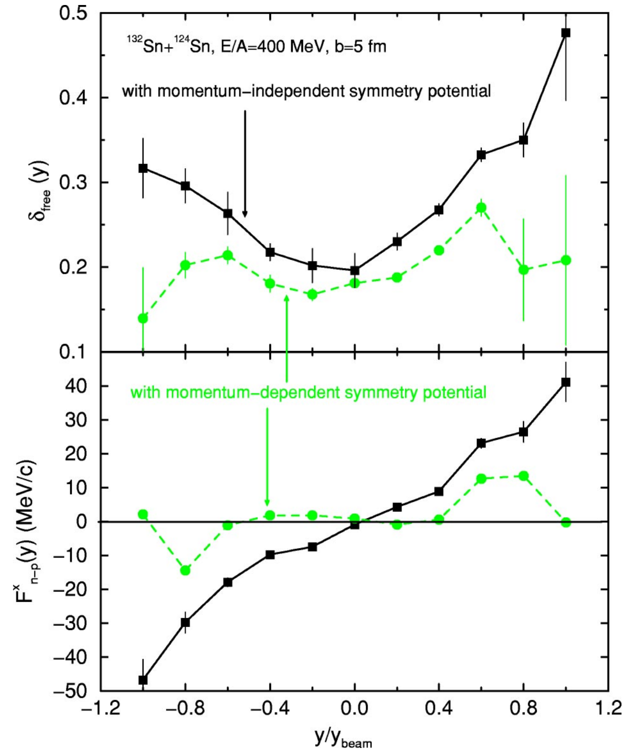

FIG. 3. (Color online) Isospin asymmetry (upper window) and neutron-proton differential flow (lower window) of free nucleons as a function of rapidity. The solid (dashed) lines are calculated with the momentum-independent(-dependent) symmetry potential.

experimental *NN* cross sections. Assuming the symmetry potential is momentum independent, these observables were previously proposed as promising probes of the density dependence of the symmetry energy. We compare in the following several such observables calculated with the $U(\rho,\delta,\vec{p},\tau)$ in Eq. (1) and the $U_{noms}(\rho,\delta,\vec{p},\tau)$ corresponding to the MDI(1) parameter set. Conclusions based on calculations using the MDI(0) parameter set are the same [25].

Shown in the upper window of Fig. 3 is the average isospin asymmetry $\delta_{free}(y)$ of free nucleons as a function of rapidity y from 1400 events of ^{132}Sn+^{124}Sn reactions at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm. We define here free nucleons as those with local baryon densities less than $\rho_0/8$. The value of $\delta_{free}(y)$ reflects mainly the degree of isospin fractionation between the free nucleons and the bound ones at freeze-out. It is also influenced slightly by the production of more π^- than π^+ mesons in the reaction. The momentum-independent symmetry potential leads to significantly higher value of $\delta_{free}(y)$ than the momentum-dependent one. Moreover, the difference tends to increase with rapidity. At midrapidity the predicted $\delta_{free}(y)$ values are close to the value expected when a complete isospin equilibrium is established among all target and projectile nucleons. These features are what one expects from the strength of the symmetry potentials as shown in the upper window of Fig. 2. The generally repulsive (attractive) symmetry potential for neutrons (protons) around ρ_0 causes more neutrons (protons) to be free (bound). The momentum independent symmetry potential is higher and steeper than the momentum-dependent one and the difference between them increases with momentum, it thus leads to the higher $\delta_{free}(y)$ values especially at higher rapidities.

Shown in the lower window of Fig. 3 is the neutron-proton differential flow $F^x_{n-p}(y)\equiv\Sigma_{i=1}^{N(y)}(p_i^x w_i)/N(y)$, where $w_i=1(-1)$ for neutrons (protons) and $N(y)$ is the total number of free nucleons at rapidity y. The differential flow combines constructively effects of the symmetry potential on the isospin fractionation and the collective flow. It has the advantage of maximizing effects of the symmetry potential while minimizing effects of the isoscalar potential [12]. Compared to the momentum-dependent symmetry potential embedded in Eq. (1), the momentum-independent symmetry potential $U_{sym}^{MDI1}(\rho,\delta,\tau)$ makes not only more neutrons free but also gives them higher transverse momenta in the reaction plane because of its higher magnitude and steeper density slopes. As a result, the differential flow with the momentum-independent symmetry potential is significantly higher.

Complementary information about how the symmetry potential depends on the momentum can be obtained by studying the ratio of neutrons to protons as a function of their transverse momentum p_t. This ratio around the midrapidity within $|y_{c.m.s.}/y_{beam}|\leqslant 0.3$ is shown in Fig. 4. The overall rise of the ratio at low p_t is due to the Coulomb force which shifts protons from lower to higher energies. It is seen that the difference between the predicted ratios increases with p_t, reflecting the increasingly larger difference between the symmetry potentials with and without the momentum dependence for nucleons with higher momenta as shown in Fig. 2.

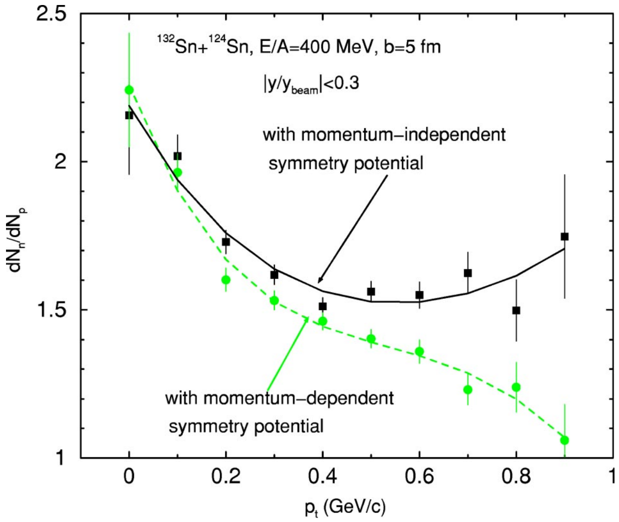

FIG. 4. (Color online) The ratio of free neutron to proton multiplicity as a function of transverse momentum at midrapidity. The solid (dashed) line is calculated with the momentum-independent (-dependent) symmetry potential.

The high p_t nucleons are thus more useful for studying the momentum dependence of the symmetry potential.

Are the observables studied above still sensitive to the variation of $E_{sym}(\rho)$ when the momentum-dependent symmetry potential is used? To answer this question we have compared calculations using the MDI(1) and MDI(0) parameter sets with the momentum-dependent and -independent symmetry potentials. Using the potential in Eq. (1) these observables are actually more sensitive to the variation of $E_{sym}(\rho)$ than the calculations with the momentum-independent symmetry potentials. For instance, the slope of the differential flow $F^x_{n-p}(y)$ at $y=0$ changes from −1.9 to 21.4 MeV/c using Eq. (1) by changing from the MDI(1) to the MDI(0) parameter set, while it changes from 26 to 39 MeV/c with the momentum-independent symmetry potentials. The net change due to the variation of the $E_{sym}(\rho)$ is thus larger with the momentum-dependent symmetry potentials.

In conclusion, the momentum dependence of the symmetry potential is found to play an important role in heavy-ion collisions induced by neutron-rich nuclei at RIA energies. Symmetry potentials with and without the momentum dependence but corresponding to the same symmetry energy $E_{sym}(\rho)$ lead to significantly different predictions on several experimental observables that were previously identified as promising probes of the $E_{sym}(\rho)$. With the momentum-dependent symmetry potential these observables are more sensitive to the change of $E_{sym}(\rho)$. The momentum dependence of the symmetry potential is thus critical for investigating accurately the EOS of dense neutron-rich matter at RIA energies.

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