

Soft-photon analysis of nucleon-nucleon bremsstrahlung: Anomalous magnetic moment effectsM. K. Liou,¹ T. D. Penninga,² R. G. E. Timmermans,² and B. F. Gibson³¹*Department of Physics and Institute for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210, USA*²*Theory Group, Kernfysisch Versneller Instituut, University of Groningen, Zernikelaan 25, NL-9747 AA Groningen, The Netherlands*³*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

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Two soft-photon amplitudes, the Low amplitude and the two- u -two- t special amplitude, agree with one another in the case of $np\gamma$ but disagree significantly in the case of $pp\gamma$. The two- u -two- t special amplitude describes well the available $pp\gamma$ cross section data. The relationship between the two amplitudes as well as the reason they agree and disagree is explored. Using these two soft-photon amplitudes plus a one-boson exchange amplitude, the contribution of the proton and neutron anomalous magnetic moments to $pp\gamma$ and $np\gamma$ has been investigated for projectile energies above 150 MeV and for laboratory scattering angles lying between 8° and 35° . The anomalous magnetic moment contribution was found to be significant in the $pp\gamma$ process but insignificant in the $np\gamma$ process. Additional aspects of the bremsstrahlung mechanism are discussed. Our findings play an important role in understanding the similarities and differences between the Low amplitude and the two- u -two- t special amplitude as well as why the two- u -two- t special amplitude calculation agrees well with the available $pp\gamma$ cross section data.

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During the past four decades, proton-proton and neutron-proton bremsstrahlung ($pp\gamma$ and $np\gamma$) have been investigated experimentally and theoretically. Most such theoretical investigations concentrated on nonrelativistic approaches using potential models. Here we utilize two relativistic approaches, the soft-photon approximation (SPA) and an analysis based upon one-boson exchange (OBE) amplitudes, to explore both $pp\gamma$ and $np\gamma$ in order to understand (the difference in) the fundamental photon emission mechanism governing these basic two-nucleon systems. The OBE approach has been previously discussed in Refs. [1,2]. Therefore, we focus our discussion here on the SPA approach. In our SPA calculations we have used the Low amplitude as derived by Nyman [3] plus a more recently developed amplitude, referred to as the two- u -two- t special (TuTts) amplitude [4–6]. Using these three relativistic amplitudes (Low, TuTts, and OBE) we have investigated the effect of the anomalous magnetic moments of the proton (κ_p) and the neutron (κ_n) in the $pp\gamma$ and $np\gamma$ processes. A primary purpose of this Rapid Communication is to report new results for laboratory nucleon-bombarding energies above 150 MeV and for laboratory scattering angles lying between 8° and 35° . In this region precision $pp\gamma$ cross sections, notably the high statistics $pp\gamma$ measurements from the Kernfysisch Versneller Instituut (KVI) experiment [7] are available for comparison. To the best of our knowledge the KVI cross section data have not been successfully described by any approach (relativistic model or nonrelativistic potential model) except for the TuTts SPA amplitude. Prior to the publication of the KVI data, the contribution of the anomalous magnetic moment was thought to be understood. These data show that is not the case. We demonstrate that, while the contribution from κ_p dominates the $pp\gamma$ cross sections, the contributions from κ_p and κ_n are negligible in the $np\gamma$ cross sections. This explains why all three amplitudes predict very similar $np\gamma$ cross sections. As will be shown below, it also

explains why the TuTts amplitude and the Low amplitude yield very different $pp\gamma$ cross sections in some cases. Understanding the difference between the Low and TuTts SPA amplitudes, which is due to κ_p in the $pp\gamma$ case, is one of our secondary objectives. Furthermore, we point out that the significant contribution from κ_p in the $pp\gamma$ case would suggest that the off-shell correction to the $p\gamma p$ vertex, which has been neglected in most relativistic model calculations, may be important at high energies for the $pp\gamma$ process. Given the lack of a potential model description of the KVI $pp\gamma$ data [7], our findings may also suggest that one should reexamine the electromagnetic operator employed in potential model calculations.

Historically, the soft-photon theorem was introduced by Low [8]. Based upon this theorem, a standard Low procedure has been used to construct Low soft-photon amplitudes for various hadron-hadron bremsstrahlung processes, including the $pp\gamma$ and $np\gamma$ processes [3]. Such amplitudes, which are valid through order K^0 (where K is the photon energy), can be calculated exactly in terms of the corresponding elastic scattering amplitude and electromagnetic constants of the particles involved. For a number of years the Low amplitude has been considered the standard soft-photon approximation; it has been used to predict cross sections for numerous bremsstrahlung processes in both nuclear and particle physics, even though its realm of validity has not been completely understood.

Systematic experimental measurements of different bremsstrahlung processes such as $pp\gamma$, $\pi^\pm p\gamma$, and $p^{12}C\gamma$ [9] have provided sensitive tests of the range of validity of bremsstrahlung amplitudes constructed from the SPA and other theoretical approximations or models. These tests have shown that the Low amplitude failed to adequately describe $\pi^\pm p\gamma$ and $p^{12}C\gamma$ cross section data near a strong resonance. Moreover, a large discrepancy was also found to exist between the Low amplitude and certain precision $pp\gamma$ data, in

particular, the KVI data. These telling experiments have played a crucial role in the development of alternative soft-photon amplitudes, one of which is the TuTts amplitude employed here. The TuTts amplitude has been tested by comparing with the KVI and other $pp\gamma$ data. Its validity in describing existing $pp\gamma$ cross sections has been well established. A discussion of important features of the TuTts amplitude can be found in the Appendix of Ref. [6].

The primary difference between the Low amplitude and the TuTts amplitude lies in the very different on-shell kinematic points at which the amplitudes are evaluated. We emphasize that this is the essential difference that leads to the dissimilar predictions for bremsstrahlung cross sections (or analyzing powers). That is, the different on-shell conditions specified for the Low amplitude and the TuTts amplitude lead to the TuTts SPA providing the better representation of $pp\gamma$ data. The reason for this is that four different external photon emission processes contribute, because a photon can be emitted from any one of the four proton legs in the process. That is, the external amplitude is a sum of four different half-off-shell amplitudes, and a different off-shell kinematic condition specifies each of the half-off-shell amplitudes. Each off-shell kinematic condition is described by a linear equation involving the three Mandelstam variables (s, t, u) and an off-shell factor; a detailed exposition can be found in Eqs. (47)–(50) of Ref. [6]. Therefore, four different off-shell kinematic conditions contribute to a given $pp\gamma$ (or $np\gamma$) process. The four on-shell conditions utilized in the TuTts amplitude are determined directly from the four off-shell kinematic conditions by introducing new Mandelstam variables “ s_{ij} ” ($i, j=1, 2$). Each new s_{ij} for each on-shell condition is obtained by combining “ s ” with the off-shell factor, so that there remain only two independent variables (u and t) specifying each on-shell condition. We emphasize that the four on-shell conditions used in the TuTts amplitude are actually equivalent to the original four off-shell kinematic conditions of the four half-off-shell bremsstrahlung amplitudes. Thus, the choice of the four on-shell points in the TuTts amplitude is both natural and physical.

In contrast, the Low amplitude imposes a common on-shell condition ($\bar{s} + \bar{t} + \bar{u} = 4m^2$, where \bar{s}, \bar{t} , and \bar{u} are the average s, t , and u , respectively, and m is the proton mass) to describe each of the four different photon emission processes. This on-shell condition is obtained by averaging the four off-shell kinematic conditions. Therefore, there is but a single on-shell point (\bar{s}, \bar{t}) determining the Low amplitude. Because only one on-shell point is used, each of the four half-off-shell external emission amplitudes must be expanded about (\bar{s}, \bar{t}) before the gauge invariance condition can be imposed to determine the corresponding internal amplitude. As a result, the Low amplitude depends only upon the elastic scattering amplitude and its derivatives evaluated at (\bar{s}, \bar{t}). This expansion about the on-shell point (\bar{s}, \bar{t}) is a standard process in the Low procedure. For the $pp\gamma$ process, a serious disadvantage arises in using this procedure: Part of the important contribution from the κ_p -dependent terms is lost, because the κ_p -dependent terms are separately gauge invariant. Such a loss can be avoided if the TuTts amplitude is used, because that amplitude depends upon four separate on-shell points, and no expansion about the point (\bar{s}, \bar{t}) is

required in the determination of the corresponding internal amplitude. The TuTts amplitude is free of any derivative of the pp elastic scattering amplitude with respect to s and/or t , and it retains most of the important contributions from all the κ_p -dependent terms. In fact, these κ_p terms dominate the $pp\gamma$ cross section for the kinematics investigated. Thus, one can understand why the TuTts amplitude has been found to be a better SPA than the Low amplitude for $pp\gamma$.

One way to understand the relationship between the Low amplitude and the TuTts amplitude is to expand the latter about the on-shell point (\bar{s}, \bar{t}) and to then compare the expanded TuTts amplitude with the Low amplitude. Consider the general case of nucleon-nucleon bremsstrahlung, where we follow the notation established in Ref. [6]. The process is

$$N_1(P_1^\mu) + N_2(P_2^\mu) \rightarrow N_1(P_1'^\mu) + N_2(P_2'^\mu) + \gamma(K^\mu),$$

where $N_1=N_2=p$ for the $pp\gamma$ process, while $N_1=n$ and $N_2=p$ for the $np\gamma$ process. The TuTts amplitude was derived in Ref. [5]; it was used in Ref. [6] to investigate noncoplanarity effects in $pp\gamma$. A TuTts amplitude for $np\gamma$ was recently reported in Ref. [10]. If these TuTts amplitudes are expanded about the on-shell point (\bar{s}, \bar{t}), then one obtains

$$M_\mu^{TuTts} = M_\mu^{Low} + M_\mu^{(3)}(K^1; \kappa) + O(K^2), \quad (1)$$

where, with $\eta=1$ for $pp\gamma$ and $\eta=0$ for $np\gamma$,

$$M_\mu^{Low} = M_\mu^{(1)}(K^{-1}; e) + M_\mu^{(2)}(K^0), \quad (2)$$

$$M_\mu^{(1)}(K^{-1}; e) = e \left[\eta \left(\frac{P'_{1\mu}}{P'_1 \cdot K} - \frac{P_{1\mu}}{P_1 \cdot K} \right) + \frac{P'_{2\mu}}{P'_2 \cdot K} - \frac{P_{2\mu}}{P_2 \cdot K} \right] F(\bar{s}, \bar{t}),$$

$$F(\bar{s}, \bar{t}) = \sum_{\alpha=1}^5 F_\alpha^e(\bar{s}, \bar{t}) U_\alpha(1) U^\alpha(2),$$

$$M_\mu^{(2)}(K^0) = M_\mu^{ex}(K^0) + M_\mu^{mag}(K^0; \kappa), \quad (3)$$

$$M_\mu^{mag}(K^0; \kappa) = e \sum_{\alpha=1}^5 F_\alpha^e(\bar{s}, \bar{t}) [\bar{u}(P'_1) X_{\mu\alpha}^{(1)} u(P_1) U^\alpha(2) + U^\alpha(1) \bar{u}(P'_2) X_{\mu\alpha}^{(2)} u(P_2)],$$

$$M_\mu^{(3)}(K^1; \kappa) = e \sum_{\alpha=1}^5 \frac{\partial F_\alpha^e(\bar{s}, \bar{t})}{\partial \bar{s}} [U^\alpha(1) \bar{u}(P'_2) Y_{\mu\alpha}^{(2)} u(P_2) - \bar{u}(P'_1) Y_{\mu\alpha}^{(1)} u(P_1) U^\alpha(2)] + e \sum_{\alpha=1}^5 \frac{\partial F_\alpha^e(\bar{s}, \bar{t})}{\partial \bar{t}} \times [\bar{u}(P'_1)(P'_1 - P_1) \cdot K X_{\mu\alpha}^{(1)} u(P_1) U^\alpha(2) - U^\alpha(1) \bar{u}(P'_2)(P_2 - P'_2) \cdot K X_{\mu\alpha}^{(2)} u(P_2)]. \quad (4)$$

Here we have defined

$$U^\alpha(j) = \bar{u}(P_j') \lambda^\alpha u(P_j),$$

$$X_{\mu\alpha}^{(j)} = \bar{R}_{\mu}^{P_j'} \lambda_\alpha - \lambda_\alpha \bar{R}_{\mu}^{P_j},$$

$$Y_{\mu\alpha}^{(j)} = (P_1' - P_2') \cdot K \bar{R}_{\mu}^{P_j'} \lambda_\alpha + (P_1 - P_2) \cdot K \lambda_\alpha \bar{R}_{\mu}^{P_j},$$

with $j=1, 2$, and

$$\bar{R}_{\mu}^{P_x} = \left(\frac{\delta}{4} [\gamma_\mu, \mathbf{K}] + \frac{\kappa_x}{8m} \{ [\gamma_\mu, \mathbf{K}], \mathbf{P}_x \} \right) / (P_x \cdot K)$$

for $P_x = P_1, P_2, P_1'$, or P_2' , and $(\delta, \kappa_x) = (1, \kappa_p)$ for a proton and $(\delta, \kappa_x) = (0, \kappa_n)$ for a neutron. The tensors λ_α are defined in Eq. (3) of Ref. [5].

In Eqs. (1)–(4), $M_\mu^{(1)}(K^{-1}; e)$, $M_\mu^{(2)}(K^0)$, and $M_\mu^{(3)}(K^1; \kappa)$ are the terms of order K^{-1} , K^0 , and K^1 , respectively, in the soft-photon expansion. $M_\mu^{(1)}(K^{-1}; e)$, which involves only the charge contribution, is the leading external amplitude. $F(\bar{s}, \bar{t})$ is the NN (that is, pp or np) elastic scattering amplitude. (The pp elastic amplitude is defined explicitly by Eq. (47) of Ref. [5].) $M_\mu^{ex}(K^0)$ includes both the external terms of order K^0 and the internal (gauge) exchange-current contribution of the same order. (The detailed expression can be obtained from Ref. [3].) Both $M_\mu^{mag}(K^0; \kappa)$ and $M_\mu^{(3)}(K^1; \kappa)$ belong to the external amplitude contribution, and they are functions of the anomalous magnetic moments of the proton and neutron.

Equation (1) demonstrates clearly that the TuTts amplitude reproduces the Low amplitude through order K^0 in the soft-photon expansion. Moreover, M_μ^{TuTts} includes the amplitude $M_\mu^{(3)}(K^1; \kappa)$ plus higher-order terms. The difference in cross sections calculated using the two amplitudes arises primarily from the additional amplitude $M_\mu^{(3)}(K^1; \kappa)$ in M_μ^{TuTts} . Such a difference is expected to be *small* if the contributions from κ_p and κ_n are negligible. In contrast, the difference will be *large* if the contributions from κ_p and κ_n are important; i.e., the difference will be large if $M_\mu^{(3)}(K^1; \kappa)$ is important.

We have investigated the size of the κ_p and κ_n effect in $pp\gamma$ and $np\gamma$ by calculating the cross sections using three different amplitudes: M_μ^{TuTts} , M_μ^{Low} , and M_μ^{OBE} . In this investigation we used an exact expression from Refs. [6,10] for M_μ^{TuTts} rather than the approximation given in Eq. (1). The expression for M_μ^{Low} is given by Eq. (2) for both the $pp\gamma$ and $np\gamma$ processes. In our SPA calculations we used state-of-the-art pp and np phase shifts [11]. For the OBE approximation the M_μ^{OBE} amplitudes can be found in Refs. [1,2]. We used two sets of magnetic moment values: (1) corresponding to $\kappa_p = 1.793$ and $\kappa_n = -1.913$ and (2) corresponding to $\kappa_p = \kappa_n = 0$. Different results calculated for these two sets were then compared. Sample results are shown in the figures.

In Figs. 1 and 2 we present coplanar $pp\gamma$ cross sections, calculated using M_μ^{TuTts} , as a function of photon angle ψ_γ : (Fig. 1) 190 MeV for proton angles $(\theta_1, \theta_2) = (8^\circ, 19^\circ)$ and (Fig. 2) 280 MeV for $(\theta_1, \theta_2) = (12^\circ, 12.4^\circ)$. The data are from the KVI [7] and TRIUMF [12] experiments, respectively. The solid curves correspond to $\kappa_p = 1.793$ or set (1), while the dashed curves correspond to $\kappa_p = 0$ or set (2). These curves are calculated using the M_μ^{TuTts} amplitude. The dotted

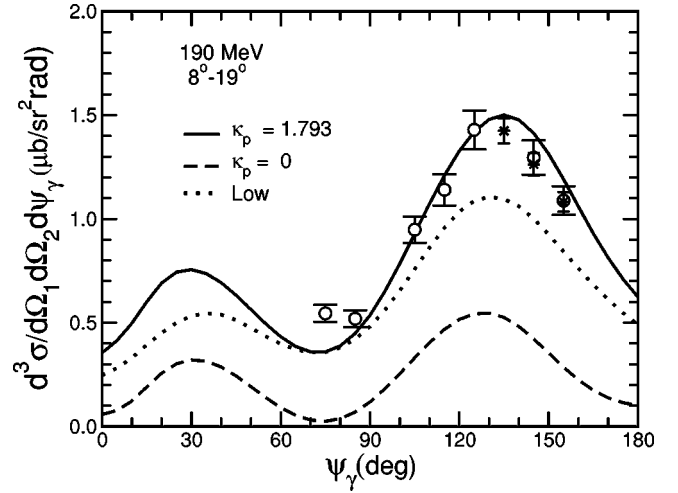


FIG. 1. Coplanar $pp\gamma$ cross sections calculated using M_μ^{TuTts} and M_μ^{Low} , as a function of the photon angle ψ_γ for the values of κ_p indicated. The data are from Ref. [7].

curve is calculated using the M_μ^{Low} amplitude with $\kappa_p = 1.793$. A comparison of the solid curves with the dashed curves enables one to establish that the contribution from κ_p dominates the $pp\gamma$ cross sections in the entire range of ψ_γ , especially for the higher energy 280 MeV case. Although not shown here, similar results hold for the M_μ^{OBE} amplitude. Thus, we have demonstrated that large magnetic moment effects in $pp\gamma$ are predicted by two relativistic (M_μ^{TuTts} and M_μ^{OBE}) amplitudes for the kinematics investigated, in agreement with a similar finding in the nonrelativistic approach based upon potential models [13]. Because the contribution from $M_\mu^{(3)}(K^1; \kappa)$ can be important in $pp\gamma$, the Low amplitude (which does not include $M_\mu^{(3)}(K^1; \kappa)$) fails to describe much of the KVI data.

Moreover, our results also suggest an explanation for the finding in Ref. [1] that the off-shell $p\gamma p$ vertex must be used in $pp\gamma$ calculations using the M_μ^{OBE} amplitude. That is, the

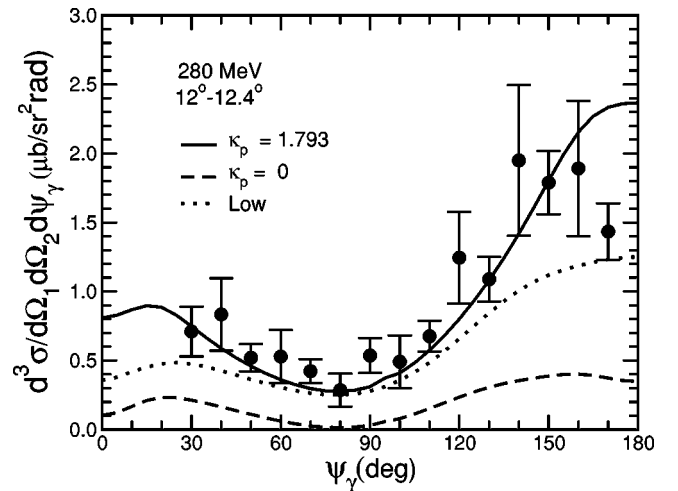


FIG. 2. Coplanar $pp\gamma$ cross sections calculated using M_μ^{TuTts} and M_μ^{Low} , as a function of the photon angle ψ_γ for the values of κ_p indicated. The data are from Ref. [12].

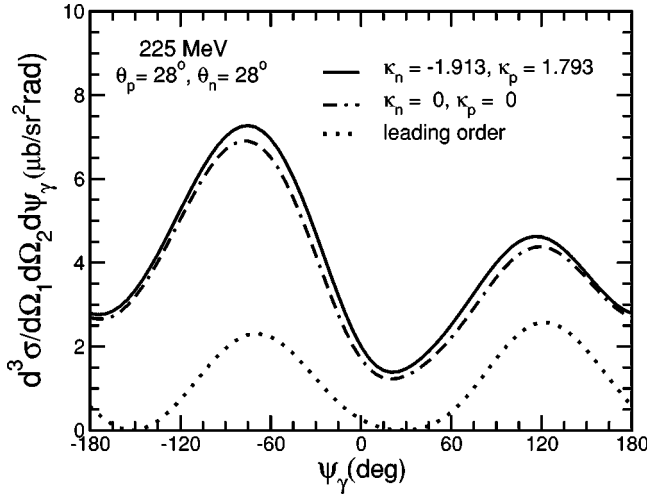


FIG. 3. Coplanar $np\gamma$ cross sections calculated using the M_{μ}^{TuTts} amplitudes, as a function of the photon angle ψ_{γ} , for the values of κ_p and κ_n indicated plus for just the leading $M_{\mu}^{(1)}(K^{-1};e)$ amplitude.

cross section and analyzing power data in the energy region between 157 MeV and 280 MeV can only be consistently described by the M_{μ}^{OBE} calculations when the off-shell $p\gamma p$ vertex is included. A possible explanation is that if the contribution from κ_p is so significant that the amplitude $M_{\mu}^{(3)}(K^1; \kappa)$ becomes important, then off-shell corrections to the $p\gamma p$ vertex may not be negligible, because such off-shell corrections are of the same order K^1 as $M_{\mu}^{(3)}(K^1; \kappa)$. In general the $pp\gamma$ cross sections calculated using the M_{μ}^{TuTts} amplitude are in good agreement with the KVI cross section data [7]. However, the same amplitude does not always enjoy similar success in describing the analyzing powers. The off-shell correction to the $p\gamma p$ vertex, which has not been included in the M_{μ}^{TuTts} amplitude, may be the missing element. The contribution of the off-shell correction to the $p\gamma p$ vertex may also be responsible for some of the large discrepancy between the KVI data and other relativistic model calculations. Effects such as explicit $N-\Delta$ coupling would not resolve the discrepancy, because they do not appear to provide a significant contribution below the threshold for pion production [14].

In Figs. 3 and 4 we exhibit the coplanar $np\gamma$ cross section as a function of ψ_{γ} at 225 MeV for $(\theta_n, \theta_p) = (28^{\circ}, 28^{\circ})$ and $(20^{\circ}, 20^{\circ})$. The solid curves were calculated using the amplitude M_{μ}^{TuTts} with $\kappa_p = 1.793$ and $\kappa_n = -1.913$ as input [from set (1)]. The dashed-dotted curves were also calculated using the same M_{μ}^{TuTts} amplitude but with $\kappa_p = \kappa_n = 0$ as the input [from set (2)]. The dotted curves were calculated using only the amplitude $M_{\mu}^{(1)}(K^{-1}; e)$ as specified by Eq. (3). If we compare the solid curve with the dashed-dotted curve, we observe that the difference between the two calculations is negligibly small over the entire range of ψ_{γ} , in contradistinction with the $pp\gamma$ process where the difference is found to be consistently large. We have also utilized the Low amplitude and the OBE amplitude M_{μ}^{OBE} to calculate $np\gamma$ cross sections using set (1) and set (2) as the input. Again, the difference between the $np\gamma$ cross sections calculated using the two sets of κ_p and κ_n values is negligibly small. Thus, we have used

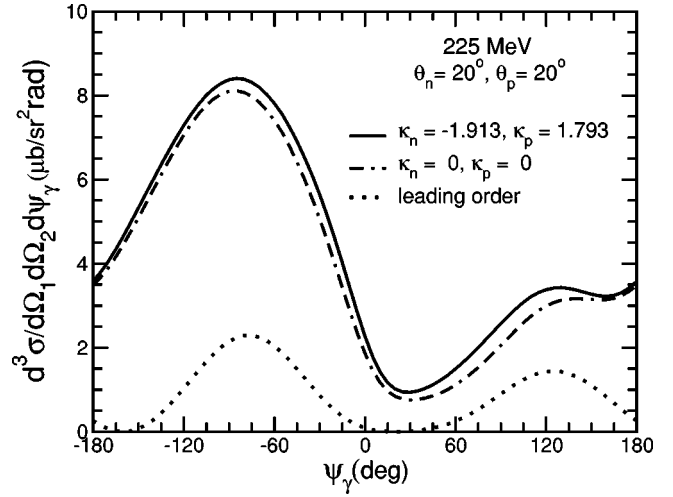


FIG. 4. See Fig. 3.

three different relativistic $np\gamma$ amplitudes (M_{μ}^{TuTts} , M_{μ}^{Low} , and M_{μ}^{OBE}) to demonstrate that the anomalous magnetic moments (κ_p and κ_n) play an insignificant role in the $np\gamma$ process for the kinematics investigated. We have also observed that the two amplitudes M_{μ}^{TuTts} and M_{μ}^{Low} predict quantitatively similar $np\gamma$ cross sections for most ψ_{γ} angles [10]. This implies that, due to the insignificant effect of κ_p and κ_n , the contribution from the amplitudes $M_{\mu}^{mag}(K^0; \kappa)$ and $M_{\mu}^{(3)}(K^1; \kappa)$ must be essentially negligible. Therefore, M_{μ}^{Low} is approximately equal to M_{μ}^{TuTts} for the $np\gamma$ case. This explains why the two amplitudes always yield very similar $np\gamma$ cross sections. We point out that the $np\gamma$ cross sections calculated with the M_{μ}^{OBE} amplitude are quite similar to those calculated using the M_{μ}^{TuTts} and M_{μ}^{Low} amplitudes for most cases which we studied.

As has already been mentioned, the dotted curves were calculated using the leading external amplitude $M_{\mu}^{(1)}(K^{-1}; e)$ as specified by Eq. (3), which is of order K^{-1} . If we compare these dotted curves with the solid curves or with the dashed-dotted curves, then we can see that the contribution from the leading order amplitude is small. Hence, the amplitude $M_{\mu}^{ex}(K^0)$, which is of order K^0 and involves the internal exchange current contribution, dominates the $np\gamma$ cross section. This is not surprising, because the meson-exchange currents have already been identified as the dominant source of bremsstrahlung emission in the $np\gamma$ process [15,16].

In conclusion, the relationship between the TuTts amplitude M_{μ}^{TuTts} and the Low amplitude M_{μ}^{Low} , for both the $pp\gamma$ and $np\gamma$ processes, has been analyzed. The main differences between these two amplitudes is that M_{μ}^{TuTts} includes an additional amplitude $M_{\mu}^{(3)}(K^1; \kappa)$ which is of order K and a function of the anomalous magnetic moments. The primary focus of this work has been to investigate the contribution of κ_p and κ_n to the $pp\gamma$ and $np\gamma$ processes. Using three different relativistic amplitudes (M_{μ}^{TuTts} , M_{μ}^{Low} , and M_{μ}^{OBE}), we have arrived at the same conclusion. For the $pp\gamma$ process in the kinematics investigated the κ_p contribution becomes so significant that the amplitude $M_{\mu}^{(3)}(K^1; \kappa)$ cannot be neglected in the calculation of $pp\gamma$ cross sections. Therefore, in the

$pp\gamma$ case the amplitude M_{μ}^{TuTts} is a better soft-photon approximation than is the amplitude M_{μ}^{Low} . On the other hand, for the $np\gamma$ process the κ_p and κ_n contribution is so small that the two magnetic moment dependent amplitudes $M_{\mu}^{mag}(K^0; \kappa)$ and $M_{\mu}^{(3)}(K^1; \kappa)$ are essentially negligible. Thus, M_{μ}^{Low} and M_{μ}^{TuTts} are approximately equivalent, and the two amplitudes predict quantitatively similar $np\gamma$ cross sections. Our investigation also illustrates that the amplitude $M_{\mu}^{ex}(K^0)$, which involves contributions from meson-exchange current effects, dominates the $np\gamma$ cross sections. Finally, additional exploration of anomalous magnetic moment effects, includ-

ing contributions from the off-shell $p\gamma p$ vertex, should shed useful light on the fundamental emission mechanism governing the $pp\gamma$ process.

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