Inadequacy of scaling arguments for neutrino cross sections

A. C. Hayes
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
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The problem with the use of scaling arguments for simultaneous studies of different weak interaction processes is discussed. When different neutrino scattering cross sections involving quite different momentum transfers are being compared it is difficult to define a meaningful single scaling factor to renormalize calculated cross sections. It has been suggested that the use of such scaling can be used to estimate high-energy neutrino cross sections from low-energy neutrino cross sections. This argument has lead to questions on the consistency of the magnitude of the Liquid Scintillating Neutrino Detector (LSND) muon neutrino cross sections on ^{12}C relative to other lower-energy weak processes. The issue is revisited here and from inspection of the structure of the form factors involved it is seen that the problem arises from a poor description of the transition form factors at high-momentum transfer. When wave functions that reproduce the transverse magnetic inelastic (e,e') scattering form factor for the 15.11 MeV state in ^{12}C are used there is no longer a need for scaling the axial current, and the different weak interactions rates involving the T=1 1^+ triplet in mass 12 are consistent with one another.

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Quenching of spin matrix elements relative to model predictions is a common phenomenon in nuclear structure calculations [1]. There are a number of reasons that this occurs, including inadequate model wave functions and the need to include meson exchange and relativistic corrections to the operators involved. For low-energy $M1$ γ transitions, magnetic moments, and Gamow-Teller β decays effective one-body operator have been derived from calculations [1,2] of the higher order corrections to both the wave functions and the bare operators, and from empirical fits [3] to a large body of data. The simplest effective one-body operators involve a scaling of the orbital and spin gyromagnetic ratios appearing in the magnetic operators and a scaling of the axial vector coupling constant for the weak interaction Gamow-Teller strength.

For weak interaction processes involving higher-momentum transfers, such as energetic neutrino scattering and muon capture, the issue of effective operators becomes more complicated because of the possible need for a momentum-dependent effective coupling constants. Siiskonen *et al.* [4] have examined the quenching of the axial vector coupling by calculating corrections to the bare operator within the shell model. They found that the quenching factor remains approximately constant up to about 60 MeV/c, above which it becomes momentum dependent. Cowell and Pandharipande [5] have calculated the momentum-dependent quenching of the weak charge current in nucleon matter and found that most of the quenching is due to spin-isospin correlations induced by one pion exchange interactions.

A momentum-independent scaling to estimate neutrino cross sections has been used by Engel *et al.* [6] and Auerbach and Brown [7]. Engel *et al.* [6] examined the transition to the 1^+ T=1 isospin triplet in mass 12 and determined the axial quenching factor from β decay. Different models that had been fitted to various combinations of the electron scattering transverse magnetic form factor, β decay, and μ capture were found to give very similar predictions for (ν_e, e^-) and (ν_μ, μ^-) scattering up to momentum transfers of about 100–200 MeV/c, provided the axial vector coupling constant was scaled separately for each model. All of the scaled models examined were in reasonable agreement with the measured Liquid Scintillating Neutrino Detector (LSND) cross sections. More recently, Auerbach and Brown [7] have examined quenching of the weak axial isovector strength in p-shell nuclei within the shell model using a single momentum-independent quenching factor. They found that a single quenching factor worked reasonably well for the lower-momentum transfer processes but not for the LSND (ν_μ, μ^-) neutrino cross section on ^{12}C from pion decay-in-flight neutrinos (DIF) [8], where the average momentum transfer is ~200 MeV/c. They concluded that the failure of the model to reproduce the DIF cross section with the same quenching factor needed for the low-momentum transfer weak processes was evidence for a systematic problem with the LSND DIF data.

In this Brief Report I discuss the difficulties that arise from the use of a momentum-independent scaling factor or effective axial coupling constant. In general, such scaling cannot be used to describe weak interaction processes involving quite different momentum transfers. Unless the model used is known to provide a reasonable description of the semileptonic (or electromagnetic) form factor at the momentum transfers of interest (as was the case in Ref. [6]), estimates of high-energy neutrino cross sections from experimentally determined cross sections for lower-energy neutrino spectra (or μ capture) are unlikely to be reliable. As discussed below, such arguments provide no evidence for a problem with the LSND DIF cross sections. Rather they suggest a shortcoming in the model used.

The problem with simple scaling arguments can be demonstrated by examining model predictions for the transition to the T=1 1^+ isospin triplet in mass 12, for which there are extensive experimental data. Within a p-shell model the one-body transition is completely described by specifying four one-body density matrix elements (OBDMEs) and the oscillator parameter b. Several authors [6,9–11] have obtained fits to the OBDMEs and the oscillator parameter from different combinations of the available experimental data. Comparisons of the predictions of these fitted models for neutrino scattering have been presented in Ref. [6].

There are four basic single particle operators that contribute to the $T=1$ 1^+ transition in mass 12. In general, the $J=1^+$ electron scattering form factor can be expressed as

$$F(q) = \langle J_f \| T^{mag}(q) \| J_i \rangle = \frac{2\sqrt{2}q}{ZM_n b}(A + By + Cy^2 + \ldots)e^{-y} f_{sn}(q) f_{\mathrm{c.m.}}(q), \quad (1)$$

where $y=(bq/2)^2$, $f_{sn}(q)$ is the single nucleon form factor, and $f_{\mathrm{c.m.}}(q)$ is the c.m. correction. The coefficients A,B,C,D,... are determined by both the structure of the operator T^{mag} and the OBDMEs describing the transition. A similar expression holds for the axial form factor contributing to neutrino scattering.

If the coefficient A can be determined from β decay or some other low-q observable (as is the case for the 1^+ transition in mass 12), then those weak interaction processes for which $A \gg By$ are likely to be well described by simply scaling the model form factors to A. However, for processes involving momentum transfers approaching $q^2 \sim (4A/b^2B)$ q-independent scaling arguments are invalid. To correct for an inadequate model description of the higher terms in $y, y^2, \ldots$ in the form factors one has to calculate explicitly a momentum-dependent effective scaling. Alternatively, the model can be adjusted (or fitted) to provide a good description of the (e,e') form factor up to momentum transfers of interest, see Refs. [6,9–11]. In the case of the 1^+ transition in mass 12, the magnitude of the second term in the (e,e') form factor becomes equal to the first at about 250 MeV/c. Thus, the term By is an important contribution to the LSND DIF cross section, and a simple scaling of the model to reproduce the term A is not sufficient.

A simple demonstration of the problem is obtained by considering what happens if one simply changes the oscillator parameter for a given model calculation. This clearly has the effect of changing the relative magnitudes of the terms A, By, Cy^2, and therefore of changing the predicted ratio of the different neutrino cross sections. For the CK(8-16) OBDMEs $A=-0.468$ and $B=0.226$. Increasing the oscillator parameter shifts the position of the first minimum in the form factor and generally shifts the form factor to lower q. An oscillator parameter of $b=1.64$ fm is needed to fit the ^{12}C ground state rms radius, while a larger value $b=1.82$–1.888 fm [10,11] is needed for the transverse magnetic (e,e') form factor. The difference in the predicted shape of the (e,e') form factor for these two oscillator parameters for the CK wave functions is shown in Fig. 1. Also shown are the average momentum transfers involved in muon capture, the (ν_e, e^-) cross section from neutrino produced from the decay of the pion at rest (DAR), and the LSND DIF cross sections. The large difference between the predicted and measured shape for the (e,e') form factor makes it impossible to find a momentum-independent scaling correction to the CK prediction, especially for $b=164$ fm. It is also clear from Fig. 1 that of the three weak processes considered the LSND DIF cross section is the most difficult to reproduce by simply quenching the axial vector coupling.

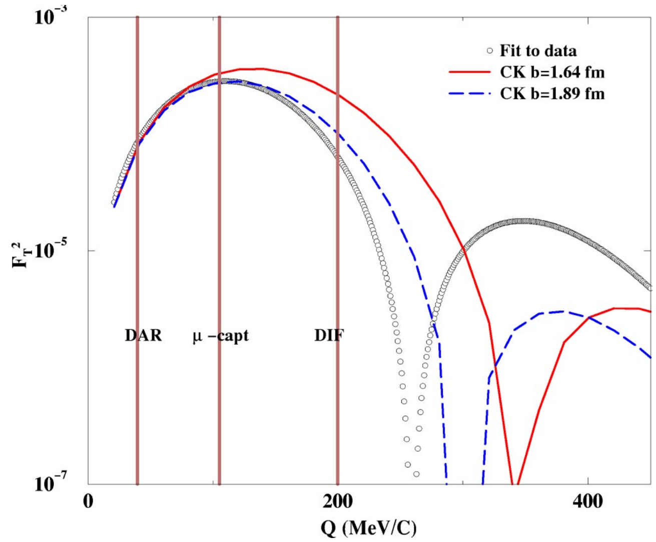

FIG. 1. The transverse magnetic electron scattering form factor for the 15.11 MeV $T=1$ 1^+ state in ^{12}C. The solid (dashed) curve is the Cohen-Kurath prediction using $b=1.64$ fm (1.89 fm). The vertical lines show the average momentum transfer for DAR, μ capture, and DIF.

Table I lists the predicted weak interaction rates for the CK wave functions using the two different oscillator parameters. For $b=1.64$ fm, the muon capture [13] and (ν_e, e^-) neutrino cross sections [12] are in reasonable agreement with experiment, being about 10% too high. However, the (ν_μ, μ^-) cross section is 50% too high. This is consistent with the calculations of Auerbach and Brown who use the same size model space, $b=164$ fm, but a different p-shell interaction. These latter calculations predict the muon capture rate and the (ν_e, e^-) cross section about 60% too high and the (ν_μ, μ^-) cross section about 212% too high. Although, the CK wave functions are considerably closer to experiment, the two cal-

TABLE I. Predicted weak interaction rates for the $^{12}C \rightarrow T=1$ 1^+ transitions. The units are 10^{-42} cm^2 for the (ν_e, e^-) DAR cross section, 10^{-40} cm^2 for (ν_μ, μ^-) DIF cross section and 10^3 sec^{-1} for muon capture.

Interaction	CK $b=1.64$ fm	CK $b=1.888$ fm	Auerbach+Brown	Experiment
(ν_e, e^-)	9.93	9.5	14.6	8.9±0.3±0.9 [12]
(ν_μ, μ^-)	0.922	0.66	1.4	0.56±0.08±0.01 [8]
μ capture	6.41	5.6	9.4	6.0±0.4 [13]

culations show about the same level of discrepancy between the low q and high q weak processes. In strong contrast, the CK model predictions for $b = 1.888$ fm provide a reasonable description of each of the three weak interaction processes and the predictions agree with experiment within the quoted experimental uncertainties. This is not surprising since the model fits the (e,e') form factor reasonably well up to the average momentum transfer involved in all three weak processes. It does, however, simply demonstrate the increased importance of the higher order terms in Eq. (1) for the DIF cross section and hence the danger in approximating the q-dependent core-polarization corrections to weak interaction form factors by a q-independent scaling.

A single effective coupling constant or scaling factor works only for processes involving momentum transfers where the shape of the form factors involved is correctly predicted. Otherwise, the use of such a scaling factor can produce quite unreliable results, particularly for higher momentum transfer processes where the shell model calculations restricted one-body weak currents have traditionally had difficulty. In contrast to the conclusions of Auerbach and Brown, we find no evidence for a problem with the LSND DIF exclusive cross section. This is in agreement with the findings of Refs. [6,14]. The shortcomings of a momentum-independent quenching of the axial vector coupling constant is likely to be even worse for the LSND inclusive DIF cross section. There the final states are unbound and cannot be described with harmonic oscillator wave functions, and the cross section is difficult to predict accurately [15].

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