Gamow-Teller states in relativistic nuclear models

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The Gamow-Teller (GT) states are investigated in relativistic models. The Landau-Migdal (LM) parameter is introduced in the Lagrangian as a contact term with the pseudovector coupling. In the relativistic model the total GT strength in the nucleon space is quenched by about 12% in nuclear matter and by about 6% in finite nuclei, compared with the Ikeda-Fujii-Fujita sum rule. The quenched amount is taken by nucleon-antinucleon excitations in the timelike region. Because of the quenching, the relativistic model requires a larger value of the LM parameter than nonrelativistic models in describing the GT excitation energy. On the other hand, the effect of the Pauli blocking terms is not important for the GT states.

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I. INTRODUCTION

For the last 30 years it has been shown that phenomenological relativistic models work very well to explain various nuclear phenomena [1]. Most of them assume that the nucleus is a relativistic system composed of Dirac particles moving in Lorentz scalar and vector potentials.

In the present paper, we study the excitation energy and strength of the Gamow-Teller (GT) states in these relativistic models. As far as we are aware of, the GT states have not been studied so far in detail in this framework [2]. We will discuss them mainly in nuclear matter, since we can obtain analytic expressions of the excitation energy and strength which make clear the structure of the results and the difference between the relativistic and nonrelativistic models.

In the following section we will present our relativistic framework to discuss the GT states. The Landau-Migdal (LM) parameter will be introduced in the Lagrangian as a contact term to take into account particle-hole correlations. In Sec. III the transverse correlation function will be calculated explicitly, from which an analytic expression of the excitation energy will be obtained in Sec. IV. In Sec. V, the GT strength will be calculated. We will show that the total GT strength is quenched by about 12% in nuclear matter and by about 6% in finite nuclei, compared with the nonrelativistic sum rule value. The quenched strength is taken by the nucleon-antinucleon excitations in the timelike region, which cannot be excited with charge-exchange reactions. Effects of the Pauli blocking terms on the excitation energy and strength will be shown to be negligible in Sec. IV and V. In Sec. VI, we will show that the way to add the LM parameter to the relativistic meson propagator, which has been frequently used in describing high-momentum transfer reactions [3], cannot describe the GT states. The last section will be devoted to a brief summary of the present work.

II. RELATIVISTIC MODEL

We assume that the mean field consists of Lorentz scalar and vector potentials. The random phase approximation (RPA) correlations are described using the basis given by this mean field, and they are assumed to be induced through the Lagrangian

$$\mathcal{L} = -g_\pi \bar{\psi}\Gamma_i^\mu \psi \partial_\mu \pi_i + \frac{g_5}{2}\bar{\psi}\Gamma_i^\mu \psi \bar{\psi}\Gamma_{\mu i}\psi \tag{1}$$

with

$$\Gamma_i^\mu = \gamma_5 \gamma^\mu \tau_i, \quad g_\pi = \frac{f_\pi}{m_\pi}, \quad g_5 = \left(\frac{f_\pi}{m_\pi}\right)^2 g'.$$

The first term stands for the usual pseudovector (PV) coupling between the pion and the nucleon, and the second term corresponds to a contact interaction whose strength is governed by the LM parameter g' [4]. Although the way to introduce g' in the relativistic model is not unique we will show that the above Lagrangian yields the known expression for the excitation energy of the GT state in the nonrelativistic limit. The first term, in fact, is not relevant for the GT states in nuclear matter. It is, however, kept in order to show later that if the LM parameter is put into the meson propagator, one cannot describe the GT state.

For the Lagrangian Eq. (1), the RPA correlation function $\Pi_{\rm RPA}$ is written in terms of the mean field one, Π [5],

$$\begin{aligned}\Pi_{\rm RPA}(\Gamma_A, \Gamma_B) = {} & \Pi(\Gamma_A, \Gamma_B) \\ & + \chi_\pi(q)\Pi(\Gamma_A, \Gamma_i \cdot q)\Pi_{\rm RPA}(\Gamma_i \cdot q, \Gamma_B) \\ & + \chi_5 \Pi(\Gamma_A, \Gamma_i^\mu)\Pi_{\rm RPA}(\Gamma_{\mu i}, \Gamma_B),\end{aligned} \tag{2}$$

where the following notations are employed:

$$\chi_\pi(q) = \frac{g_\pi^2}{(2\pi)^3}\frac{1}{m_\pi^2 - q^2 - i\varepsilon}, \quad \chi_5 = \frac{g_5}{(2\pi)^3}.$$

For isospin-dependent excitations, the mean field correlation function is given by

$$\Pi(\Gamma_\alpha,\Gamma_\beta) = -\frac{1}{2\pi i}\int d^4p \mathrm{Tr}_\sigma \mathrm{Tr}_\tau[\Gamma_\alpha G_\mathrm{F}(p+q)\Gamma_\beta G_\mathrm{D}(p) + \Gamma_\alpha G_\mathrm{D}(p+q)\Gamma_\beta G_\mathrm{F}(p) + \Gamma_\alpha G_\mathrm{D}(p+q)\Gamma_\beta G_\mathrm{D}(p) + \Gamma_\alpha G_\mathrm{F}(p+q)\Gamma_\beta G_\mathrm{F}(p)], \quad (3)$$

where we have defined the isospin operator τ_α,

$$\tau_\pm = \frac{\tau_x \pm i\tau_y}{\sqrt{2}}, \quad \tau_0 = \tau_z,$$

and the propagator,

$$G_\mathrm{H}(q) = G_\mathrm{F}(q) + G_\mathrm{D}(q),$$

$$G_\mathrm{D}(q) = G(k_\mathrm{p};q)\frac{1-\tau_z}{2} + G(k_\mathrm{n};q)\frac{1+\tau_z}{2},$$

with

$$G_\mathrm{F}(q) = \frac{\not{q} + M^*}{q^2 - M^{*2} + i\varepsilon},$$

$$G(k_i;q) = \frac{i\pi}{E_\mathbf{q}}(\not{q} + M^*)\,\delta(q_0 - E_\mathbf{q})\,\theta_\mathbf{q}^{(i)}, \quad (i = p, n).$$

Here, we have also used the abbreviation for the step function: $\theta_\mathbf{q}^{(i)} = \theta(k_i - |\mathbf{q}|)$, k_p and k_n being the Fermi momenta of the protons and neutrons, respectively. Moreover, $E_\mathbf{q}$ is equal to $\sqrt{M^{*2}+\mathbf{q}^2}$, where the Lorentz scalar potential is included in the nucleon effective mass M^*. The Lorentz vector potential does not show up explicitly in the present discussion of nuclear matter. In Eq. (3), the first three terms are density dependent, including the Pauli blocking terms, while the last one is density independent and divergent. The last term is usually neglected [6], but we keep it for later discussion. Effects of the Pauli blocking terms on the excitation energy and strength of the GT state will be also discussed later.

For the $\tau_\pm$ excitations, the RPA correlation function in Eq. (2) is described as

$$\Pi_\mathrm{RPA}(\Gamma_+^a, \Gamma_-^b) = [U^{-1}]^{ab'}\Pi(\Gamma_{b'+}, \Gamma_-^b), \quad (4)$$

where U denotes the dimesic function represented by the 5×5 matrix:

$$U^{ab} = g^{ab} - \chi_b \Pi(\Gamma_+^a, \Gamma_-^b)$$

with the notations for $a = -1, 0, \ldots, 3$,

$$\Gamma_\pm^a = \gamma_5\gamma^a\tau_\pm, \quad \gamma^a = \begin{cases}\gamma\cdot q \\ \gamma^\mu\end{cases}, \quad \chi_a = \begin{cases}\chi_\pi, & a=-1 \\ \chi_5, & a=\mu.\end{cases}$$

In the above equation, g^{ab} is defined as $g^{ab}=1(a=b=-1)$, $g^{\mu\nu}(a=\mu, b=\nu)$ and $g^{-1\mu}=g^{\mu-1}=0$.

The calculation of the mean field correlation function is straightforward. Separating $\Pi(\Gamma_+^a, \Gamma_-^b)$ into the density-dependent and density-independent parts,

TABLE I. The structure of the correlation function $\Pi(\Gamma_-^a, \Gamma_+^b)$. The first column and row indicate the values of a and b of $\Pi(\Gamma_-^a, \Gamma_+^b)$. The open boxes mean that $\Pi(\Gamma_-^a, \Gamma_+^b)$ has nonzero value. Table (a) is for $\mathbf{q}\neq\mathbf{0}$, while (b) for $\mathbf{q}=\mathbf{0}$.

	(a)						(b)				
	−1	0	1	2	3		−1	0	1	2	3
−1				0	0	−1			0	0	0
0				0	0	0			0	0	0
1				0	0	1	0	0		0	0
2	0	0	0		0	2	0	0	0		0
3	0	0	0	0		3	0	0	0	0	

$$\Pi(\Gamma_+^a, \Gamma_-^b) = \Pi_\mathrm{D}(\Gamma_+^a, \Gamma_-^b) + \Pi_\mathrm{F}(\Gamma_+^a, \Gamma_-^b), \quad (5)$$

they are obtained as

$$\Pi_\mathrm{D}(\Gamma_+^a, \Gamma_-^b) = \int d^4p \frac{\delta(p_0 - E_\mathbf{p})}{E_\mathbf{p}}\left(\frac{t^{ab}(p,q)}{(p+q)^2 - M^{*2} + i\varepsilon}\theta_\mathbf{p}^{(\mathrm{n})} + \frac{t^{ab}(p,-q)}{(p-q)^2 - M^{*2} + i\varepsilon}\theta_\mathbf{p}^{(\mathrm{p})}\right) + i\pi\int d^4p \frac{\delta(p_0 - E_\mathbf{p})\,\delta(p_0 + q_0 - E_{\mathbf{p}+\mathbf{q}})}{E_\mathbf{p}E_{\mathbf{p}+\mathbf{q}}} \times t^{ab}(p,q)\,\theta_\mathbf{p}^{(\mathrm{n})}\theta_{\mathbf{p}+\mathbf{q}}^{(\mathrm{p})}, \quad (6)$$

$$\Pi_\mathrm{F}(\Gamma_+^a, \Gamma_-^b) = \frac{1}{i\pi}\int d^4p \frac{t^{ab}(p,q)}{(p^2 - M^{*2} + i\varepsilon)((p+q)^2 - M^{*2} + i\varepsilon)}, \quad (7)$$

where $t^{ab}(p,q)$ is given by

$$t^{\mu\nu}(p,q) = 4[g^{\mu\nu}(M^{*2} + p^2 + p\cdot q) - 2p^\mu p^\nu - p^\mu q^\nu - p^\nu q^\mu], \quad (8)$$

$$t^{-1\nu}(p,q) = q_\mu t^{\mu\nu}(p,q) = 4[q^\nu(M^{*2} + p^2) - p^\nu(q^2 + 2p\cdot q)], \quad (9)$$

$$t^{-1-1}(p,q) = 4[q^2(M^{*2} + p^2) - p\cdot q(q^2 + 2p\cdot q)]. \quad (10)$$

If we choose the spatial axes such that $q^\mu = (q_0, q_x, 0, 0)$, we can show that $\Pi(\Gamma_-^a, \Gamma_+^b)$ has a structure as shown in Table I(a), where open boxes mean $\Pi(\Gamma_-^a, \Gamma_+^b)$ to be nonzero. Thus, the transverse parts of $\Pi(\Gamma_-^a, \Gamma_+^b)$ are decoupled from the pion ($a=-1$)–, time ($a=0$)–, and longitudinal ($a=1$) ones. In the present paper, we are interested in the GT states excited at $\mathbf{q}=\mathbf{0}$. In this case, the longitudinal part is also decoupled from the pion and time-component (PT), as in Table I(b). Consequently, the determinant of the dimesic function is factorized into four parts,

$$\det U = -(D_\mathrm{T})^2 D_\mathrm{L} D_\mathrm{PT}, \quad (11)$$

where the transverse, longitudinal, and PT dimesic functions are written as

$$D_\mathrm{T} = 1 + \chi_5 \Pi(\Gamma_+^2, \Gamma_-^2), \quad D_\mathrm{L} = D_\mathrm{T}, \quad (12)$$

$$D_{\rm PT}=[1-\chi_\pi\Pi(\Gamma_+^{-1},\Gamma_-^{-1})][1-\chi_5\Pi(\Gamma_+^0,\Gamma_-^0)] - \chi_5\chi_\pi\Pi(\Gamma_+^{-1},\Gamma_-^0)\Pi(\Gamma_+^0,\Gamma_-^{-1}). \quad (13)$$

III. THE TRANSVERSE CORRELATION FUNCTION

In this section we derive a more explicit form of the transverse correlation function at $\mathbf{q}=\mathbf{0}$ which is used for describing the GT states. First, we will calculate the real and imaginary parts of the density-dependent transverse correlation function separately, and next the density-independent part.

According to Eq. (6), the real part of the density-dependent transverse correlation function is written as

$$\text{Re }\Pi_{\rm D}(\Gamma_+^2,\Gamma_-^2)=J^{22}(k_{\rm n},q_0)+J^{22}(k_{\rm p},-q_0), \quad (14)$$

where $J^{22}(k_i,q)$ represents

$$J^{22}(k_i,q)=\int d^4p\frac{\delta(p_0-E_{\mathbf p})}{E_{\mathbf p}}\frac{t^{22}(p,q)}{(p+q)^2-M^{*2}}\theta_{\mathbf p}^{(i)}. \quad (15)$$

Since t^{22} at $\mathbf{q}=0$ is given by

$$t^{22}(p,q)=-4(2M^{*2}+E_{\mathbf p}q_0+2p_y^2), \quad (16)$$

the real part Eq. (14) becomes

$$\text{Re}\Pi_{\rm D}(\Gamma_+^2,\Gamma_-^2)=-\frac{4}{q_0}\int d^3p\frac{M^{*2}+p_y^2}{E_{\mathbf p}^2}(\theta_{\mathbf p}^{(\rm n)}-\theta_{\mathbf p}^{(\rm p)}) -4\int d^3p\frac{\mathbf{p}^2-p_y^2}{E_{\mathbf p}^2}\left(\frac{\theta_{\mathbf p}^{(\rm n)}}{2E_{\mathbf p}+q_0}+\frac{\theta_{\mathbf p}^{(\rm p)}}{2E_{\mathbf p}-q_0}\right). \quad (17)$$

The imaginary part of the density-dependent correlation function at $\mathbf{q}=\mathbf{0}$ is given by Eq. (6) as

$$\text{Im }\Pi_{\rm D}(\Gamma_+^a,\Gamma_-^b)=-\pi\int d^4p\frac{\delta(p_0-E_{\mathbf p})}{E_{\mathbf p}}\Bigg[t^{ab}(p,q)\,\delta(q_0^2+2p_0q_0)\,\theta_{\mathbf p}^{(\rm n)}+t^{ab}(p,-q)\,\delta(q_0^2-2p_0q_0)\,\theta_{\mathbf p}^{(\rm p)} -\frac{\delta(q_0)}{E_{\mathbf p}}t^{ab}(p,q)\,\theta_{\mathbf p}^{(\rm n)}\theta_{\mathbf p}^{(\rm p)}\Bigg].$$

Using

$$\delta(q_0^2\pm 2p_0q_0)=\frac{1}{2p_0}[\delta(q_0)+\delta(q_0\pm 2p_0)],$$

the above equation is rewritten as

$$\text{Im }\Pi_{\rm D}(\Gamma_+^a,\Gamma_-^b)=-\pi\delta(q_0)\int d^4p\frac{\delta(p_0-E_{\mathbf p})}{2E_{\mathbf p}^2}[t^{ab}(p,q)\,\theta_{\mathbf p}^{(\rm n)} +t^{ab}(p,-q)\,\theta_{\mathbf p}^{(\rm p)}-2t^{ab}(p,q)\,\theta_{\mathbf p}^{(\rm n)}\theta_{\mathbf p}^{(\rm p)}]+R_{N\bar N}.$$

The last term $R_{N\bar N}$ comes from the N-$\bar N$ excitations,

$$R_{N\bar N}(q_0)=-\pi\int d^4p\frac{\delta(p_0-E_{\mathbf p})}{2E_{\mathbf p}^2}[t^{ab}(p,q)\,\delta(q_0+2E_{\mathbf p})\,\theta_{\mathbf p}^{(\rm n)} +t^{ab}(p,-q)\,\delta(q_0-2E_{\mathbf p})\,\theta_{\mathbf p}^{(\rm p)}].$$

Inserting Eq. (16) into the above equations, the imaginary part of the density-dependent transverse correlation function is obtained as

$$\text{Im }\Pi_{\rm D}(\Gamma_+^2,\Gamma_-^2)=4\pi\delta(q_0)\int d^3p\frac{M^{*2}+p_y^2}{E_{\mathbf p}^2}(\theta_{\mathbf p}^{(\rm n)}+\theta_{\mathbf p}^{(\rm p)} -2\theta_{\mathbf p}^{(\rm n)}\theta_{\mathbf p}^{(\rm p)})+R_{N\bar N}, \quad (18)$$

where $R_{N\bar N}$ is given by

$$R_{N\bar N}=-4\pi\int d^3p\frac{\mathbf{p}^2-p_y^2}{E_{\mathbf p}^2}[\delta(q_0+2E_{\mathbf p})\,\theta_{\mathbf p}^{(\rm n)}+\delta(q_0-2E_{\mathbf p})\,\theta_{\mathbf p}^{(\rm p)}]. \quad (19)$$

From Eqs. (17), (18), and (19), the density-dependent part of the transverse correlation function is described as

$$\Pi_{\rm D}(\Gamma_+^2,\Gamma_-^2)=-4\int d^3p\frac{M^{*2}+p_y^2}{E_{\mathbf p}^2}\left(\frac{\theta_{\mathbf p}^{(\rm n)}(1-\theta_{\mathbf p}^{(\rm p)})}{q_0+i\varepsilon} -\frac{\theta_{\mathbf p}^{(\rm p)}(1-\theta_{\mathbf p}^{(\rm n)})}{q_0-i\varepsilon}\right) -4\int d^3p\frac{\mathbf{p}^2-p_y^2}{E_{\mathbf p}^2}\left(\frac{\theta_{\mathbf p}^{(\rm p)}}{2E_{\mathbf p}-q_0-i\varepsilon} +\frac{\theta_{\mathbf p}^{(\rm n)}}{2E_{\mathbf p}+q_0-i\varepsilon}\right). \quad (20)$$

The density-independent part of the transverse correlation function is calculated in the same way. From Eqs. (7) and (16), we obtain

$$\Pi_{\rm F}(\Gamma_+^2,\Gamma_-^2)=4\int d^3p\frac{\mathbf{p}^2-p_y^2}{E_p^2}\left(\frac{1}{2E_p-q_0-i\varepsilon} +\frac{1}{2E_p+q_0-i\varepsilon}\right). \quad (21)$$

The sum of Eqs. (20) and (21) provides us with the full transverse correlation function $\Pi(\Gamma_-^2,\Gamma_+^2)$. It is also expressed as a sum of contributions from particle-hole and N-$\bar N$ excitations,

$$\Pi(\Gamma_+^2,\Gamma_-^2)=\Pi_{\rm ph}(\Gamma_+^2,\Gamma_-^2)+\Pi_{\rm N\bar N}(\Gamma_+^2,\Gamma_-^2), \quad (22)$$

where each term is described as

$$\Pi_{\rm ph}(\Gamma_+^2,\Gamma_-^2)=-4\int d^3p\frac{M^{*2}+p_y^2}{E_{\mathbf p}^2}\left(\frac{\theta_{\mathbf p}^{(\rm n)}(1-\theta_{\mathbf p}^{(\rm p)})}{q_0+i\varepsilon} -\frac{\theta_{\mathbf p}^{(\rm p)}(1-\theta_{\mathbf p}^{(\rm n)})}{q_0-i\varepsilon}\right), \quad (23)$$

$$\Pi_{N\bar{N}}(\Gamma_+^2,\Gamma_-^2) = 4\int d^3p \frac{\mathbf{p}^2 - p_y^2}{E_{\mathbf{p}}^2}\left(\frac{1-\theta_{\mathbf{p}}^{(\mathrm{p})}}{2E_{\mathbf{p}} - q_0 - i\varepsilon} + \frac{1-\theta_{\mathbf{p}}^{(\mathrm{n})}}{2E_{\mathbf{p}} + q_0 - i\varepsilon}\right). \tag{24}$$

IV. THE EXCITATION ENERGY OF THE GT STATE

The eigenvalues of the excitation energies are given by the real part of the dimesic function,

$$\det \mathrm{Re}\, U = 0. \tag{25}$$

The excitation energy of the GT state is evaluated by using the transverse part of the dimesic function in Eq. (12). The real part of the transverse correlation function, which we need in the dimesic function, is obtained from Eqs. (22)–(24).

A. The excitation energy in the nucleon space

In this section, we will calculate the excitation energy of the GT state, neglecting perfectly the antinucleon degrees of freedom. In this case, according to Eqs. (12) and (23), the real part of the transverse dimesic function is given by

$$\mathrm{Re}\, D_{\mathrm{T}} = 1 + \chi_5\, \mathrm{Re}\, \Pi_{\mathrm{ph}}(\Gamma_+^2,\Gamma_-^2),$$

$$\mathrm{Re}\, \Pi_{\mathrm{ph}}(\Gamma_+^2,\Gamma_-^2) = -\frac{16\pi}{3}\frac{Q(k_{\mathrm{n}}) - Q(k_{\mathrm{p}})}{q_0}, \tag{26}$$

where $Q(k_i)$ is given by

$$Q(k_i) = \frac{3}{4\pi}\int_0^{k_i} d^3p \frac{M^{*2} + p_y^2}{E_{\mathbf{p}}^2} = \frac{k_i^3}{3} + 2k_i M^{*2} - 2M^{*3}\tan^{-1}\frac{k_i}{M^*}. \tag{27}$$

From $\mathrm{Re}\, D_{\mathrm{T}}=0$, we obtain the relativistic expression of the excitation energy in nuclei with $k_{\mathrm{n}} > k_{\mathrm{p}}$,

$$\omega_0 = \frac{2g_5}{3\pi^2}[Q(k_{\mathrm{n}}) - Q(k_{\mathrm{p}})]. \tag{28}$$

Relativistic effects on Eq. (28) can be seen more transparently by defining the Fermi momentum k_{F} as usual,

$$k_{\mathrm{n}}^3 = \frac{2N}{A}k_{\mathrm{F}}^3, \quad k_{\mathrm{p}}^3 = \frac{2Z}{A}k_{\mathrm{F}}^3. \tag{29}$$

This yields a relation for $(N-Z)/A \ll 1$,

$$k_{\mathrm{n}} - k_{\mathrm{p}} \approx \frac{2}{3}k_{\mathrm{F}}\frac{N-Z}{A}. \tag{30}$$

By using the equation

$$Q'(k_{\mathrm{F}}) = \frac{dQ(k_{\mathrm{F}})}{dk_{\mathrm{F}}} = \frac{k_{\mathrm{F}}^2(3M^{*2} + k_{\mathrm{F}}^2)}{M^{*2} + k_{\mathrm{F}}^2} = 3k_{\mathrm{F}}^2\left(1 - \frac{2}{3}v_{\mathrm{F}}^2\right), \tag{31}$$

$$v_{\mathrm{F}} = \frac{k_{\mathrm{F}}}{\sqrt{M^{*2} + k_{\mathrm{F}}^2}},$$

we can expand $[Q(k_{\mathrm{n}}) - Q(k_{\mathrm{p}})]$ in Eq. (28) up to first order in $(k_{\mathrm{n}} - k_{\mathrm{p}})$. Then, replacing $(k_{\mathrm{n}} - k_{\mathrm{p}})$ by Eq. (30) we obtain the relativistic expression of the GT energy:

$$\omega_0 \approx \left(1 - \frac{2}{3}v_{\mathrm{F}}^2\right)g_5\frac{8k_{\mathrm{F}}^3}{3\pi^2}\frac{N-Z}{2A}. \tag{32}$$

The first factor depending on the Fermi velocity v_{F} shows relativistic effects on the excitation energy.

In the nonrelativistic limit $v_{\mathrm{F}}^2 \ll 1$ the GT energy is

$$\omega_0 = g_5\frac{8k_{\mathrm{F}}^3}{3\pi^2}\frac{N-Z}{2A}. \tag{33}$$

This result can be also obtained without using the approximation Eq. (30). In the non-relativistic limit $\mathbf{p}^2 \ll M^{*2}$, Eq. (27) becomes

$$Q(k_i) \approx k_i^3. \tag{34}$$

This, together with Eqs. (28) and (29), yields the same result as Eq. (33).

Equation (33) is just the result obtained previously in nonrelativistic models with $g_5 = g'(f_\pi/m_\pi)^2$ [7]. In relativistic models, the excitation energy of the GT state in nuclear matter is thus given by the transverse part of the dimesic function, and it is independent of the pion exchange, even when its energy dependence is taken into account.

We will show later that the relativistic factor $(1 - 2v_{\mathrm{F}}^2/3)$ in Eq. (32) stems from the quenching of the GT strength in the nucleon space. In most of the relativistic models the nucleon effective mass is about $0.6M$ [8], which yields $v_{\mathrm{F}}=0.43$ for $k_{\mathrm{F}}=1.36\ \mathrm{fm}^{-1}$. This value implies that we must use a value of g' larger by 14% in the relativistic model than that in nonrelativistic models. Nonrelativistic models require the value of g' to be about 0.6 in order to reproduce experimental data [9]. In this case the relativistic model needs $g'=0.68$.

B. Effects of the Pauli blocking term

It is known in the relativistic model that the antinucleon degrees of freedom play an important role for some physical quantities [10,11]. Even in the RPA based on the mean field approximation, a part of the antinucleon excitations should be taken into account in order to satisfy the continuity equation [5]. It is the density-dependent part in the antinucleon excitations which is usually called the Pauli blocking term. Without the Pauli blocking term, for example, the orbital part of the magnetic moment and the multipole giant resonances are not described correctly [10,11]. In the case of the GT state at $\mathbf{q}=\mathbf{0}$, it is not clear whether or not the Pauli blocking term does play an important role. Here, we examine its effects on the excitation energy of the GT state.

The Pauli blocking term in the present case is given by the density-dependent parts of Eq. (24):

$$\Pi_{\rm Pauli}(\Gamma_+^2,\Gamma_-^2) = -4\int d^3p \frac{\mathbf{p}^2-p_y^2}{E_{\mathbf{p}}^2}\left(\frac{\theta_{\mathbf{p}}^{(\rm p)}}{2E_{\mathbf{p}}-q_0-i\varepsilon} + \frac{\theta_{\mathbf{p}}^{(\rm n)}}{2E_{\mathbf{p}}+q_0-i\varepsilon}\right).$$

Its real part is written as

$$\mathrm{Re}\,\Pi_{\rm Pauli}(\Gamma_+^2,\Gamma_-^2) = \frac{16\pi}{3}\kappa, \qquad \kappa = -P_{\overline{\rm N}}(k_{\rm n},q_0) - P_{\overline{\rm N}}(k_{\rm p},-q_0), \tag{35}$$

where we have defined

$$P_{\overline{N}}(k_{\rm F},q_0) = \frac{3}{4\pi}\int_0^{k_{\rm F}} \frac{d^3p}{E_{\mathbf{p}}^2}\frac{\mathbf{p}^2-p_y^2}{2E_{\mathbf{p}}+q_0}. \tag{36}$$

For $q_0 \ll M^*$, as in the GT state, $P_{\overline{\rm N}}$ is approximately given by

$$\begin{aligned} P_{\overline{\rm N}}(k_{\rm F},q_0) &\approx P(k_{\rm F}) \\ &= \frac{3}{4\pi}\int_0^{k_{\rm F}} \frac{d^3p}{E_{\mathbf{p}}^2}\frac{\mathbf{p}^2-p_y^2}{2E_{\mathbf{p}}} \\ &= E_{\rm F}^2\left(\frac{3}{2}v_{\rm F} - v_{\rm F}^3 - \frac{3}{4}(1-v_{\rm F}^2)\ln\frac{1+v_{\rm F}}{1-v_{\rm F}}\right) \\ &= k_{\rm F}^2\frac{v_{\rm F}^3}{5}\left(1+\frac{3}{7}v_{\rm F}^2+\cdots\right), \end{aligned} \tag{37}$$

where $E_{\rm F}$ denotes $\sqrt{M^{*2}+k_{\rm F}^2}$. Using this result to evaluate Eq. (26), the excitation energy of the GT state is obtained as

$$\omega_0 \approx \frac{1-\frac{2}{3}v_{\rm F}^2}{1+\frac{2g_5}{3\pi^2}\kappa} g_5 \frac{8k_{\rm F}^3}{3\pi^2}\frac{N-Z}{2A}. \tag{38}$$

This result shows that, when we use the parameter values as mentioned at the end of the preceding section the effect of the Pauli blocking terms is negligible, being less than 0.5% of the excitation energy.

V. THE GT STRENGTH

In this section, first we will discuss the total GT strength in nuclear matter where we can obtain its analytic expression and understand the structure of the relativistic results. Next we will investigate the finite size effects by studying the case of nuclei.

A. The GT strength in nuclear matter

The total GT strength is calculated by integrating the response function R over the excitation energy. The relationship of the response function to the correlation function Π is given by [5]

$$R = \frac{3}{16\pi^2}\frac{A}{k_{\rm F}^3}\mathrm{Im}\,\Pi.$$

First, we investigate the total GT strength in the mean field approximation without the RPA correlations. For this purpose we can employ the imaginary parts of Eqs. (22)–(24). The total strength for the β^- transitions in the nucleon space is given by the first term in the parentheses of Eq. (23),

$$S_{\rm ph}^- = \frac{3}{4\pi}\frac{A}{k_{\rm F}^3}\int d^3p \frac{M^{*2}+p_y^2}{E_{\mathbf{p}}^2}(\theta_{\mathbf{p}}^{(\rm n)}-\theta_{\mathbf{p}}^{(\rm p)}) = \frac{A}{k_{\rm F}^3}[Q(k_{\rm n})-Q(k_{\rm p})]. \tag{39}$$

When we expand Q in terms of $(k_{\rm n}-k_{\rm p})$ as before, we obtain the value of the total strength in the nucleon space,

$$S_{\rm ph}^- \approx \left(1-\frac{2}{3}v_{\rm F}^2\right)2(N-Z). \tag{40}$$

In the present definition of the GT operators,

$$F_\pm = \sum_i^A (\tau_\pm\sigma_y)_i,$$

the Ikeda-Fujii-Fujita sum rule in nonrelativistic models [12] becomes

$$\langle|F_+F_-|\rangle - \langle|F_-F_+|\rangle = 2(N-Z). \tag{41}$$

This is nothing but the result of the commutation relation:

$$[\tau_+\sigma_y, \tau_-\sigma_y] = 2\tau_z.$$

If we assume that there is no ground-state correlation,

$$F_+|\rangle = 0, \tag{42}$$

we have simply from Eq. (41)

$$\langle|F_+F_-|\rangle = 2(N-Z) \tag{43}$$

in nonrelativistic models. Comparing Eq. (40) with the above equation, it is seen that the relativistic sum value is quenched by the factor $(1-2v_{\rm F}^2/3)$, which is about 0.88 for the previous value $v_{\rm F}=0.43$.

The strength of the β^+ transition in the nucleon space is given by the second term in the parentheses of Eq. (23) with replacing q_0 by $-q_0$, but its value is zero for $k_{\rm n}>k_{\rm p}$ as in Eq. (42). The quenched strength in the nucleon space in Eq. (40) is not taken by the β^+ transition, but by the antinucleon degrees of freedom. This fact is shown as follows. According to the first term of Eq. (24), the strength of the β^- transition in the nucleon-antinucleon excitations is given by

$$S^{-}_{N\bar{N}} = \frac{3}{4\pi}\frac{A}{k_F^3}\int d^3p \frac{\mathbf{p}^2 - p_y^2}{E_{\mathbf{p}}^2}(1-\theta^{(\mathrm{p})}_{\mathbf{p}}), \tag{44}$$

while that of the β^+ transition is provided by the second term with replacing q_0 by $-q_0$,

$$S^{+}_{N\bar{N}} = \frac{3}{4\pi}\frac{A}{k_F^3}\int d^3p \frac{\mathbf{p}^2 - p_y^2}{E_{\mathbf{p}}^2}(1-\theta^{(\mathrm{n})}_{\mathbf{p}}). \tag{45}$$

The above two equations are both divergent, but their difference is finite:

$$S^{-}_{N\bar{N}} - S^{+}_{N\bar{N}} = \frac{3}{4\pi}\frac{A}{k_F^3}\int d^3p \frac{\mathbf{p}^2 - p_y^2}{E_{\mathbf{p}}^2}(\theta^{(\mathrm{n})}_{\mathbf{p}} - \theta^{(\mathrm{p})}_{\mathbf{p}}). \tag{46}$$

The sum of the above equation and Eq. (39) provides us with sum rule corresponding to Eq. (41),

$$S^{-}_{\mathrm{ph}} + S^{-}_{N\bar{N}} - S^{+}_{N\bar{N}} = 2(N-Z). \tag{47}$$

The interpretation of this result is the following. In order to obtain the sum rule value $2(N-Z)$, we need a complete set of the nuclear wave functions. This fact requires both the nucleon and the antinucleon space in relativistic models. Since the nucleon-antinucleon states are in the timelike region, the GT strength for charge-exchange reactions which excite nuclear states in the spacelike region is quenched by the amount of Eq. (46). This quenching can be also discussed by calculating GT matrix elements directly, as we have done in Ref. [13].

Next, we calculate the RPA strength of the GT state, using the RPA correlation function $\Pi_{\mathrm{RPA}}(\Gamma_+^2,\Gamma_-^2)$. Using the shorthand notations $\Pi_{\mathrm{RPA}}(\Gamma_+^2,\Gamma_-^2)=\Pi_{\mathrm{RPA}}(q_0)$ and $\Pi(\Gamma_+^2,\Gamma_-^2)=\Pi(q_0)$, $\Pi_{\mathrm{RPA}}(q_0)$ is written as

$$\Pi_{\mathrm{RPA}}(q_0) = \frac{\Pi(q_0)}{D_{\mathrm{T}}(q_0)}, \quad D_{\mathrm{T}}(q_0) = 1 + \chi_5\Pi(q_0).$$

Expanding $D_{\mathrm{T}}(q_0)$ at $q_0=\omega_0$, we have

$$\Pi_{\mathrm{RPA}}(q_0) = \left(\frac{dD_{\mathrm{T}}}{d\omega_0}\right)^{-1}\frac{\Pi(q_0)}{q_0 - \omega_0 + i\varepsilon}.$$

Keeping only the density-dependent part of the correlation function, the imaginary part of the above equation gives the strength of the GT state,

$$S_{\mathrm{GT}} = \frac{A}{k_F^3}\frac{Q(k_{\mathrm{n}}) - Q(k_{\mathrm{p}})}{[1+2g_5\kappa/(3\pi^2)]^2} \approx \frac{1-2v_F^2/3}{[1+2g_5\kappa/(3\pi^2)]^2}2(N-Z). \tag{48}$$

The κ-dependent term stems from the Pauli blocking effects, and it is negligible as mentioned before. Thus, all the strength in the nucleon space is contained in the GT state. Comparing Eq. (28) with the above equation, we can see that the factor $(1-2v_F^2/3)$ in the expression of the excitation energy Eq. (32) is due to the quenching of the GT strength in the nucleon space, but not from the relativistic kinematics.

B. The GT strength in finite nuclei

We have shown analytically that the GT strength is quenched by about 12% in nuclear matter. The quenched amount, however, depends on the momentum distribution and the value of the nucleon effective mass near the nuclear surface, as seen in Eq. (39). Therefore, we now proceed to estimate numerically the GT strength in finite nuclei, in the mean field approximation without RPA correlations.

We write the four-component nucleon spinor as

$$\psi_{am} = \begin{pmatrix} \dfrac{iG_a(r)}{r}|\ell jm\rangle \\ -\dfrac{F_a(r)}{r}|\bar{\ell}jm\rangle \end{pmatrix},$$

where a stands for the quantum numbers $\{n\ell j\}$, and $\bar{\ell}$ is given by $\bar{\ell}=j\pm 1/2=\ell\pm 1$ for $j=\ell\pm 1/2$. We define the GT strength as

$$T_{aa'}(\sigma_\mu) = 2\sum_{mm'}|\langle a'm'|\sigma_\mu|am\rangle|^2 = \frac{2}{3}|\langle \ell'j'||\sigma||\ell j\rangle_{\mathrm{rel}}|^2,$$

using the notations

$$\langle \ell'j'||\sigma||\ell j\rangle_{\mathrm{rel}} = \delta_{\ell\ell'}\langle \ell j'||\sigma||\ell j\rangle g(a,a') + \delta_{\bar{\ell}\bar{\ell}'}\langle \bar{\ell}j'||\sigma||\bar{\ell}j\rangle f(a,a'),$$

$$g(a{:}a') = \int_0^\infty dr G_a(r)G_{a'}(r), \quad f(a{:}a') = \int_0^\infty dr F_a(r)F_{a'}(r).$$

If we calculate the strengths for the transition from $j=\ell+1/2$ to $j'=\ell\pm 1/2(n'=n)$ only, as in nonrelativistic models for closed subshell nuclei, the sum of the GT strengths is given by

$$\sum_{a'} T_{aa'}(\sigma_\mu) = \frac{4(\ell+1)(2\ell+3)}{3(2\ell+1)}\left(g_+ - \frac{2\ell+1}{2\ell+3}f_+\right)^2 + \frac{16\ell(\ell+1)}{3(2\ell+1)}g_-^2,$$

with

$$g_\pm = g(n,\ell,\ell+1/2{:}n,\ell,\ell\pm 1/2),$$

$$f_\pm = f(n,\ell,\ell+1/2{:}n,\ell,\ell\pm 1/2).$$

Assuming the same wave functions for neutrons and protons we have $g_+ + f_+ = 1$ from the normalization of the wave functions. Moreover, it seems reasonable to assume that $g_- \approx 1 - f_+$. Then, the sum of the GT strengths is approximately given by

$$\sum_{a'} T_{aa'}(\sigma_\mu) \approx 2(2j+1)\left(1 - \frac{8}{3}f_+\right).$$

Since most of the relativistic models provides us with $f_+ \sim 0.02$, the above equation shows that the GT strength is

TABLE II. The GT strength of the single-particle transitions in ^{48}Ca. The top table shows the results obtained using neutron wave functions for the initial and final states, while the bottom one those using the proton wave functions for final states. The value in the parentheses following the single-particle quantum number is the single-particle energy in MeV. $T_{aa'}$ is the value of the GT strength, and $g(a{:}a')$ and $f(a{:}a')$ show the overlap of the radial wave functions, as defined in the text. The VALUES underlined do not contribute to the GT strength.

$n\ell j$	$n'\ell' j'$	$T_{aa'}$	$g(a{:}a')$	$f(a{:}a')$
	$n \rightarrow n$			
$1p_{3/2}(-39.41)$	$2p_{3/2}(-3.97)$	0.001	0.0110	−0.0110
$1p_{3/2}(-39.41)$	$1f_{5/2}(-2.09)$	0.002	$\underline{0.7765}$	−0.0175
$1p_{3/2}(-39.41)$	$2p_{1/2}(-2.74)$	0.004	0.0323	$-\underline{0.0091}$
$1p_{1/2}(-36.23)$	$2p_{3/2}(-3.97)$	0.001	−0.0128	$\underline{0.0116}$
$1p_{1/2}(-36.23)$	$2p_{1/2}(-2.74)$	0.001	0.0102	−0.0102
$1f_{7/2}(-10.00)$	$1f_{5/2}(-2.09)$	8.411	0.9592	$-\underline{0.0150}$
$1f_{7/2}(-10.00)$	$1f_{7/2}(-10.00)$	6.390	0.9805	0.0195
Total		14.810		
	$n \rightarrow p$			
$1p_{3/2}(-39.41)$	$2p_{3/2}(-1.09)$	0.000	−0.0113	−0.0116
$1p_{3/2}(-39.41)$	$1f_{5/2}(-1.16)$	0.002	$\underline{0.8084}$	−0.0180
$1p_{1/2}(-36.23)$	$2p_{3/2}(-1.09)$	0.005	−0.0360	$\underline{0.0117}$
$1f_{7/2}(-10.00)$	$1f_{5/2}(-1.16)$	8.629	0.9715	$-\underline{0.0148}$
$1f_{7/2}(-10.00)$	$1f_{7/2}(-9.59)$	6.361	0.9787	0.0200
Total		14.997		

quenched by about 5%, compared with the nonrelativistic sum value $2(2j+1)$.

In realistic situations there are other transitions even in closed subshell nuclei such as ^{48}Ca. Table II shows their contributions to the total GT strength. The calculations are done with the effective Lagrangian NL-SH [14] chosen as an example. We calculate only the transitions between bound states. Contributions from the continuum states are expected to be small. In the table, the top one shows the results in using the neutron wave functions for the initial and final states, and the bottom one those obtained using the proton wave functions for the final states. These calculations are performed to see the effects of the Coulomb force. The nonrelativistic sum value for ^{48}Ca is 16 in the present definition. The table shows that the relativistic sum value is quenched by about 6%, compared with the nonrelativistic one. This reduction of the quenched amount, compared with the one in nuclear matter, was expected from the value of the nucleon effective mass near the nuclear surface, as mentioned before. Since the total GT strength in the nucleon space is quenched in the mean field approximation, we expect that the sum of the RPA strengths in finite nuclei is also quenched, as in the case of nuclear matter.

VI. THE PION AND THE TIME PART OF THE CORRELATION FUNCTION

In the preceding sections we have discussed the problems related to the transverse part of the correlation function. Let us now briefly discuss the structure of the pion and the time component, mainly in order to study the way to use g' in relativistic models.

Since at $\mathbf{q}=\mathbf{0}$, the correlation functions satisfy

$$\Pi(\Gamma_+^{-1},\Gamma_-^{-1}) = q_0^2 \Pi(\Gamma_+^0,\Gamma_-^0), \quad \Pi(\Gamma_+^{-1},\Gamma_-^0) = q_0 \Pi(\Gamma_+^0,\Gamma_-^0),$$

the pion and the time component of the dimesic function Eq. (13) can be rewritten in terms of $\Pi(\Gamma_+^0,\Gamma_-^0)$,

$$D_{\mathrm{PT}} = 1 - \frac{1}{(2\pi)^3}\left(g_5 + g_\pi^2 \frac{q_0^2}{m_\pi^2 - q_0^2}\right)\Pi(\Gamma_+^0,\Gamma_-^0).$$

The function $t^{00}(p,q)$ at $\mathbf{q}=\mathbf{0}$ in $\Pi(\Gamma_+^0,\Gamma_-^0)$ is calculated according to Eq. (8) as

$$t^{00}(p,q) = 4[2M^{*2} - E_{\mathbf{p}}(2E_{\mathbf{p}} + q_0)].$$

Taking into account the density-dependent parts only, we have the real part of the time component as

$$\mathrm{Re}\,\Pi(\Gamma_+^0,\Gamma_-^0) = J^{00}(k_{\mathrm{n}},q_0) + J^{00}(k_{\mathrm{p}},-q_0), \tag{49}$$

where J^{00} is given by

$$J^{00}(k_i,q_0) = -\frac{4}{q_0}\int d^3p \frac{\mathbf{p}^2}{E_{\mathbf{p}}^2}\theta_{\mathbf{p}}^{(\mathrm{i})} - 4M^{*2}\int d^3p \frac{\theta_{\mathbf{p}}^{(\mathrm{i})}}{E_{\mathbf{p}}^2(2E_{\mathbf{p}}+q_0)}. \tag{50}$$

For $q_0 \ll M^*$, neglecting the q_0 dependence of the second term, we obtain

$$J^{00}(k_i,q_0) \approx 8\pi M^{*2}\left(\frac{Q_0(k_i)}{q_0} + P_0(k_i)\right), \tag{51}$$

with

$$\begin{aligned} P_0(k_{\mathrm{F}}) &= -\frac{1}{2\pi}\int_0^{k_{\mathrm{F}}} d^3p \frac{1}{2E_{\mathbf{p}}^3} \\ &= \frac{k_{\mathrm{F}}}{E_{\mathrm{F}}} - \ln\frac{E_{\mathrm{F}} + k_{\mathrm{F}}}{M^*} \\ &= -\frac{v_{\mathrm{F}}^3}{3}\left(1 + \frac{3}{5}v_{\mathrm{F}}^2 + \cdots\right), \end{aligned} \tag{52}$$

$$Q_0(k_{\mathrm{F}}) = -\frac{1}{2\pi M^{*2}}\int_0^{k_{\mathrm{F}}} d^3p \frac{\mathbf{p}^2}{E_{\mathbf{p}}^2} = 2k_{\mathrm{F}} - 2M^*\tan^{-1}\frac{k_{\mathrm{F}}}{M^*} - \frac{2k_{\mathrm{F}}^3}{3M^{*2}}. \tag{53}$$

From the above equations, the real part of $\Pi(\Gamma_+^0,\Gamma_-^0)$ is described as

$$\mathrm{Re}\,\Pi(\Gamma_+^0,\Gamma_-^0) \approx 16\pi M^{*2}\left(P_0(k_{\mathrm{F}}) + \frac{k_{\mathrm{n}} - k_{\mathrm{p}}}{2q_0}Q_0'(k_{\mathrm{F}})\right), \tag{54}$$

where $Q_0'(k_{\mathrm{F}})$ denotes the derivative of $Q_0(k_{\mathrm{F}})$ with respect to k_{F},

$$Q_0'(k_{\rm F}) = -\frac{2k_{\rm F}^4}{M^{*2}E_{\rm F}^2} = -\frac{2v_{\rm F}^4}{1-v_{\rm F}^2}. \tag{55}$$

Finally, the real part of the PT dimesic function is given by

$$\text{Re}\, D_{\rm PT} \approx 1 - \frac{2M^{*2}}{\pi^2}\left(g_5 + g_\pi^2 \frac{q_0^2}{m_\pi^2 - q_0^2}\right) \times \left(P_0(k_{\rm F}) + \frac{k_{\rm n}-k_{\rm p}}{2q_0}Q_0'(k_{\rm F})\right). \tag{56}$$

The structure of Re $D_{\rm PT}$ is similar to Re $D_{\rm T}$ in Eq. (26) to which Eq. (35) is added. Equations (52) and (55), however, show that the second parentheses of Eq. (56) are negative. Therefore, the excitation energy given by Re $D_{\rm PT}=0$ should be higher than the pion mass, $q_0 > m_\pi$.

The first parenthesis of Eq. (56) may be obtained by the insertion of g' into the pion propagator as

$$\frac{1}{m_\pi^2 - q^2} \to \frac{1}{m_\pi^2 - q^2} + \frac{g'}{q^2}, \tag{57}$$

which was frequently used in relativistic description of high-momentum transfer reactions [3]. Equation (56), however, shows that this way to put g' in the meson propagator cannot describe the GT states. In order to show this fact, we have used the Lagrangian Eq. (1), although the GT state can be described just with the contact term in nuclear matter. The above modification of the meson propagator in Eq. (57) was introduced from nonrelativistic models. Those models use a static potential and modify the meson propagator so as to cancel the short range part of the interaction as

$$\frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \to \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} - g'. \tag{58}$$

Equation (57), however, is not a reasonable extension of Eq. (58) for the GT state.

The latter statement, of course, does not mean that the Lagrangian form in Eq. (1) provides us with a correct four-momentum dependence of g'. In nonrelativistic models also we do not know the momentum dependence so well [15]. The Lagrangian form in Eq. (1) can describe the GT state at $\mathbf{q}=\mathbf{0}$, and cancel the short range part of the interaction, but it yields an additional four-momentum transfer dependence of the dimesic function. In fact, the dimesic function, except for the transverse part, is written at the static limit $q_0=0$ as

$$-D_{\rm PTL} = (1-\chi_5\Pi^{00})[1 + (\chi_5 - \chi_\pi q_x^2)\Pi^{11}] + \chi_5(\chi_5 - \chi_\pi q_x^2)(\Pi^{10})^2, \tag{59}$$

where we have used the abbreviation $\Pi^{ab}=\Pi(\Gamma_+^a, \Gamma_-^b)$. More detailed investigations on g' are necessary for discussions of high-momentum transfer phenomena.

Finally we mention the effects of the Pauli blocking terms. When we take into account the particle-hole excitations only, we have

$$\text{Re}\,\Pi(\Gamma_+^0, \Gamma_-^0) = \frac{1}{q_0 + i\varepsilon}\int d^3p \frac{t^{00}}{2E_{\mathbf{p}}^2}(\theta_{\mathbf{p}}^{(\rm n)} - \theta_{\mathbf{p}}^{(\rm p)}) = \frac{8\pi M^{*2}}{q_0 + i\varepsilon}[Q_0(k_{\rm n}) - Q_0(k_{\rm p})].$$

This shows that the term $P_0(k_{\rm F})$ in Eq. (54) comes from the Pauli blocking terms. The effects of the Pauli blocking terms are not small in the present case, compared with those in the transverse mode. In fact, the relationship between the contributions from the particle-hole terms to the Pauli blocking one is given by

$$Q_0'(k_{\rm F}) \approx -2v_{\rm F}^4, \quad P_0(k_{\rm F}) \approx -\frac{v_{\rm F}^3}{3} \approx \frac{1}{6v_{\rm F}}Q_0'(k_{\rm F})$$

in the present case, while in the transverse mode, we have from Eqs. (31) and (37)

$$Q'(k_{\rm F}) \approx 3k_{\rm F}^2, \quad P(k_{\rm F}) \approx k_{\rm F}^2\frac{v_{\rm F}^3}{5} \approx \frac{v_{\rm F}^3}{15}Q'(k_{\rm F}).$$

VII. SUMMARY

About 15 years ago, analytic expressions of the excitation energies for the giant monopole and quadrupole resonance states were derived in the relativistic model [16]. When they are expressed in terms of the Landau-Migdal (LM) parameters, they are formally equal to the nonrelativistic expressions, in spite of the fact that the LM parameters are strongly dominated by relativistic effects. In this paper, we have obtained the relativistic expression of the excitation energy for the Gamow-Teller (GT) state in nuclear matter. It is described in terms of the LM parameter g' which is introduced in the Lagrangian as a contact term. Compared with the corresponding nonrelativistic one, the relativistic expression has an additional factor of $(1-2v_{\rm F}^2/3)$, $v_{\rm F}$ being the Fermi velocity. This means that in order to reproduce the same excitation energy as in nonrelativistic models, the present relativistic model requires a value of g' to be larger by this factor.

The above relativistic factor comes from the quenching of the GT strength in the nucleon space. A part of the GT strength is taken by the nucleon-antinucleon states in the timelike region which are not excited in usual charge-exchange reactions. This quenching is thus peculiar to the relativistic models. The quenched amount is estimated to be 12% of the classical Ikeda-Fujii-Fujita sum rule value in nuclear matter, and 6% in finite nuclei. We have found that the Pauli blocking terms are not important for discussions of the GT states.

We have also discussed whether or not it is appropriate for the relativistic model to insert the LM parameter g' into the meson propagator. This method was frequently employed for the study of high-momentum reactions, but we have shown that this method cannot describe the GT states.

Finally it may be worthwhile noting future problems. The quenching phenomena of the GT strength have been discussed for a long time in nonrelativistic models [17]. Most recent experiments have observed 90% of the classical sum rule value in ^{90}Zr [18] with nonrelativistic analysis. So far all the quenching of 10% has been considered to be due to the coupling of the particle-hole states with Δ-hole states. Under this assumption, the LM parameter $g'_{\Delta N}$ for the coupling is estimated to be about 0.2–0.3, depending on the nonrelativistic models [9]. The determination of the value of $g'_{\Delta N}$ is very important for studies of nuclear magnetic moments, spin-dependent response functions, and pion condensation. In particular, the critical density of the pion condensation is dominated by the values of $g'_{\Delta N}$ as well as g'. It has been shown that if the values of $g'_{\Delta N}$ and g' are about 0.2 and 0.6, respectively, a rough nonrelativistic calculation yields the critical density to be about two times of the normal density [19].

In the present relativistic model, it has been shown that the nucleon-antinucleon excitations are also responsible for some quenching of the GT strength. This fact may imply that the value of $g'_{\Delta N}$ would become smaller than the above quoted value, and consequently the critical density would be lower. For more detailed investigations, on the one hand, experimental data should be analyzed in the relativistic framework, for example, by using a multipole decomposition method with relativistic wave functions. On the other hand, a consistent relativistic model is required to discuss the pion condensation including Δ degrees of freedom, although a few attempts are found in the literature [3,20].

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