Potential energy surfaces and spectra of superheavy elements

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The potential energy surfaces of some superheavy nuclei are determined, using a mapping from the microscopic shell model space to a geometrical model. The content of the shell model space is determined through the knowledge of the *absolute* deformation and a single-particle spectrum as a function of deformation. Both have to be extracted from a microscopic model. We show that one cannot restrict to only prolate or oblate deformations because the content of the microscopic space already implies triaxiality. Also cases of γ instability occur. The mass parameter of ^{254}No is determined through the knowledge of at least one rotational state of the ground-state band. Assuming the same mass parameter for the other superheavy elements, the spectrum of each of them is determined. The following superheavy nuclei are considered: ^{254}No, ^{260}Rf, ^{262}Sg, ^{270}Hs, ^{274}110(Ds), ^{276}112, and ^{290}114, where ^{260}Rf and ^{262}Sg turn out to be γ unstable.

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I. INTRODUCTION

Since almost five decades transfermium nuclei ($Z>100$) have been produced [1]. The most spectacular recent discovery was the superheavy element with atomic charge $Z=116$ in Dubna [2]. However, little is known about their structure. Except for two isotopes of the element $Z=102$, no further experimental information is available. Very recently, the isotope ^{254}No has been measured after the ^{208}Pb(^{48}Ca, $2n$) reaction at 215 and 219 MeV and the ground-state band populated up to spin $22\hbar$ and excitation energy $\geqslant 6$ MeV [3]. Shortly after, the production cross section for the isotope ^{252}No was measured in the reaction ^{206}Pb(^{48}Ca, $2n$) with a magnitude approximately ten times smaller than the one for ^{254}No and levels up to spin 20 were populated [4]. The experimental data showed that both isotopes are well deformed with a well defined rotational level sequence.

There exist various theoretical models which attempted to describe the collective structure of those nuclei and thus provide some information about physical observables, such as deformation and the lowest states in the spectrum.

The first calculation in the framework of the Hartree-Fock-Bogoliubov approximation was carried out in Ref. [5] for ^{254}No and the properties of the ground-state rotational band discussed. This fully self-consistent approach showed that apart from the first barrier, the nucleus is axially symmetric at all deformations.

The ground-state rotational energies of several nuclei in the range $Z=88-112$ were calculated in the macroscopic-microscopic approach for axial-symmetric nuclei by the Warsaw group [6].

Further away from $Z=102$, the β-vibrational spectrum of the putative double magic superheavy nucleus ^{292}120, as predicted by the relativistic mean field, was studied in a phenomenological collective approach [7]. It turned out that the spectrum of this nucleus, in the absence of γ deformations does not exhibit an equidistant spacing between the vibrational states as it is the case of the heaviest known double magic nucleus, ^{208}Pb.

In Ref. [8] a method of how to obtain, without any parameter, the potential energy surface (PES) starting from a known content of the shell model space was proposed. The method uses as one possible input the deformation and the Nilsson diagram as a function of the deformation (any other single-particle spectrum as a function in the deformation can also be used), data which have to be obtained from somewhere else. It is based on the symplectic description of the shell model space and it was able to reproduce the structure of ^{190}Pt, ^{238}U, and Sm isotopes, and qualitatively the spectra and $B(E2)$-transition values.

The objective of this paper is to investigate the quadrupole structure of some superheavy elements, using the method proposed in Ref. [8]. It will give some important hints on the collective structure of these nuclei. We do not investigate all superheavy elements identified, but rather will use an isotope from each even chemical element from $Z=102$ to 114. Also the results have to be seen as an estimation of the structure of these nuclei.

We will show that the restriction to axial-symmetric nuclei is not justified by the content of the shell model space. We encountered throughout our investigation examples of γ-unstable nuclei and/or of a soft γ behavior.

The paper is organized as follows. In Sec. II the basic ideas of the mapping to the geometrical model is outlined. In Sec. III the PES, spectra, and some $B(E2)$-transition rates are calculated. Finally, in Sec. IV conclusions are drawn. Only

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even-even nuclei will be considered because any odd proton or neutron changes the structure of the low lying spectrum significantly. However, the PES deduced can be used to identify the PES of the core of neighboring even-odd, odd-even, and odd-odd nuclei.

II. THE MAPPING OF THE PES

In Ref. [8] a mapping to the potential of a geometrical model [9] is presented, starting from the microscopic shell model space and using the Hamiltonian of the pseudosymplectic model (PSM) [10]. This Hamiltonian provides a measure for the relative distances in energy of the SU(3) irreducible representations (irreps). Here we outline briefly the procedure exposed in a more detailed form in Refs. [8,11,12].

The PSM is an extension of the pseudo-SU(3) model [13] taking into account $2n\hbar\omega$ shell excitations. In this model the orbitals are divided into intruders and the normal orbitals. The intruders are the orbitals with maximal spin in each oscillator shell. By inspecting the normal orbitals and redefining orbital angular momentum into pseudo-orbital angular momentum and spin into pseudospin, a new degeneration is observed which allows one to introduce a pseudo-SU(3) model [denoted from here on as $\widetilde{\mathrm{SU}(3)}$]. The PSM adds to the $\widetilde{\mathrm{SU}(3)}$ intershell excitations of the type $2n\hbar\omega$. It is equivalent to the extension of the Elliott model of SU(3) [14] to the symplectic model [15], including an approximation called contraction [16]. In the PSM the nucleons in the intruder orbitals are treated as spectators. The model describes without any parameter the polarization of the closed shells. In order to take into account the nucleons in the intruder orbitals, as an effective charge, a definite scaling is used in terms of Z and A, the number of charge and nucleons, and Z_n and A_n are the same for the particles in the normal orbitals.

In order to determine Z_n and the number of neutrons N_n in the normal orbitals ($A_n=Z_n+N_n$), one needs the external information of the deformation of the nucleus and a single-particle spectrum (e.g., a Nilsson diagram or the ones given in Ref. [17]) for either $\beta<0$ (or equivalently $\beta>0$ with $\gamma=60°$) or $\beta>0$. The deformation can be deduced experimentally [18] or by a microscopic model, which is indispensable for obtaining a single-particle spectrum, e.g., a Nilsson diagram. In Ref. [17] the single-particle spectra of different light and heavy nuclei are given, also taking into account higher multipolarities. For a particular nucleus we use a single-particle spectrum of the next possible one, which is listed in Ref. [17]. This should give for the neighboring nuclei a fairly good first approximation of the single-particle diagram. The main objective is to get the approximate occupation of a particular nucleus in terms of nucleons in normal and unique orbitals. We also checked what happens when the nucleons are filled into the orbitals at a prolate or oblate deformation. As a result it changes maximally by 2, for the cases considered. From this we obtained a fairly good idea of the absolute changes going from prolate to oblate nuclei. This is important because, as we will see, the restriction to axial-symmetric nuclei is too strong and the microscopic shell model space dictates in many cases triaxiality or even γ instability. The result, of course, will depend on this model. Here, we use the macroscopic-microscopic calculations of Möller *et al.* for both the quadrupole deformation [19] (using $\epsilon_2\approx\beta$ [20]) and the single-particle spectrum for the folded Yukawa potential as a function in the deformation [17]. In this sense, our result will contain the effects of relevant microscopic interactions. For the excited states we restrict to quadrupole *excitations* only. It was shown in Ref. [9] that they are sufficient in order to describe low lying, positive parity states and their $B(E2)$ transitions. This should not be confused with the argument that higher multipolarities are important for the description of the ground-state properties. These effects were taken into account through the use of the listed deformation value and the single-particle spectrum in order to obtain the distribution of the nucleons into the normal and unique orbitals. Once the numbers β, Z_n, and N_n are determined, the content of the shell model space in the valence shells (protons and neutrons) is fixed using the tables of Ref. [21] or the SU(3)-package routines of Refs. [22,23]. This gives a list of SU(3) irreps (λ_π, μ_π) for protons and (λ_ν, μ_ν) for neutrons. The pseudosymplectic space is then obtained by allowing $2n\hbar\omega$ excitations on top of $(\lambda, \mu)=(\lambda_\pi+\lambda_\nu, \mu_\pi+\mu_\nu)$, i.e., the stretched irreps of SU(3) (the others, e.g., scissors modes, are higher in energy and do not contribute to the envelope of the PES). Because the nucleons in the unique orbitals are treated as spectators and their influence is only taken into account via the effective charge, a PES of two successive isotopes will be similar when the next orbitals filled are unique.

In order to estimate the relative positions in energy of the different symplectic irreps, the following Hamiltonian is used:

$$H=\hbar\omega\tilde{N}-\tfrac{1}{2}\chi[Q^c\cdot Q^c-(Q^c\cdot Q^c)_{shell}]+H_r, \qquad (1)$$

where $\tilde{N}$ is the phonon number operator of the protons plus the neutrons, $\hbar\omega$ is $45A^{1/3}-25A^{2/3}$ for light and $41A^{1/3}$ for heavy nuclei. The Q^c_m is the physical quadrupole operator and $(Q^c\cdot Q^c)_{shell}$ is the *trace equivalent part* of the quadrupole-quadrupole interaction, which guarantees that even for large deformation, on an average, the shell structure is conserved [24]. H_r contains residual interactions for the rotor Hamiltonian [10]. The term $\hbar\omega\tilde{N}$ guarantees that states of a higher shell are shifted to larger energies.

Following Ref. [8] the PES is defined as the expectation value of the microscopic Hamiltonian H with respect to a trial state

$$V=\langle\Phi(\alpha_{2\mu})|H|\Phi(\alpha_{2\mu})\rangle. \qquad (2)$$

The collective coordinates $\alpha_{2\mu}$ are nothing else but the geometrical quadrupole variables [9]. The trial state is the product of a coherent state, describing intershell mixtures, and the Elliott $\widetilde{\mathrm{SU}(3)}$ state [14], describing the irreps within the valence shell. The coherent state $|\Phi(\alpha_{2\mu})\rangle$ is given by an operator, which is an exponential function in the symplectic operators which raise the number of phonons by two, applied to a lowest weight state of an

$\widetilde{\mathrm{SU}(3)}$ irrep in the valence shell. In this way a symplectic irrep is defined.

Restricting to *only one* symplectic irrep, in the lowest order, the potential is given by

$$V \approx \frac{65}{16\pi}(\chi N_0^2)\beta_s^2 + \frac{5}{4\pi}N_0\left(\hbar\omega - \frac{7}{6}\chi N_0\right)(\tilde{\beta}-\beta_s)^2 - (\chi N_0^2)\frac{5}{\pi}\beta_s\tilde{\beta} + \frac{5}{2\pi}N_0\left(\hbar\omega + \frac{5}{6}\chi N_0\right)\beta_s\tilde{\beta} \times(1-\cos(\gamma-\gamma_s)), \quad (3)$$

with N_0 the total number of phonons in the lowest SU(3)state plus $3(A-1)/2$ and $\tilde{\beta}=\beta/\kappa$. The value κ is approximately given by

$$\frac{1}{2}\left(\frac{\eta_\pi - 1}{\eta_\pi} + \frac{\eta_\nu - 1}{\eta_\nu}\right),$$

with η_π and η_ν the valence shell number of protons and neutrons, respectively. (β_s, γ_s) in Eq. (3) is related to the deformation of the $0\hbar\omega$ space in the strong coupled formalism, without multi-$\hbar\omega$ excitations. For large β the system is dominated by the monopole and quadrupole intershell excitations acting on the leading irrep. However, for lower values of β a different Sp(3, *R*) irrep minimizes the expectation value in Eq. (2), leading to a different β_s and therefore to a dependence of the form $\beta_s=\beta\tan\phi$, with ϕ a function in χ, N_0, and $\hbar\omega$ (see for details Ref. [8]). The total PES is then obtained by the envelope of all potentials related to different symplectic irreps.

The expression in Eq. (3) depends on the parameters χ, N_0, and $\hbar\omega$. $\hbar\omega$ and N_0 are fixed as explained above. The χ value is obtained by requiring that the total potential has its minimum either at the experimentally deduced position $\beta_{\mathrm{expt.}}$ or one can use the predictions of the macroscopic-microscopic model [19], or any other source of information and theoretically calculated values of the deformation from a microscopic model.

The only parameter which cannot be deduced is the collective mass B_2 of the geometrical model [9]. This parameter has to be adjusted to, e.g., a state in the ground-state band. Another possibility is to use a cranking model to obtain the mass parameter. However, this method is quite insecure and will not be used here. Fitting B_2 to the experimental data automatically gives a good estimate. Also assuming for neighboring nuclei the same value of B_2 is in general far more accurate than using the cranking model. This conjecture can be tested in lighter isotopes. In Ref. [25] the Pt, Os, and W isotopes were investigated within the generalized collective model (GCM). The adjusted mass parameter B_2 is of the same order over a wide range of nuclei.

The microscopic shell model space in the pseudosymplectic model can be obtained by first filling the single-particle orbitals from below at a given deformation, counting the number of nucleons in the normal orbitals, and finally use the tables of the reduction $\mathrm{U}(n)\times\mathrm{U}(2)\supset\mathrm{SU}(3)$ [21], with U(2) as the spin group. An alternative is to use the SU(3) package of the Louisiana group [22,23]. In the reduction of $\mathrm{U}(n)\times\mathrm{U}(2)\supset\mathrm{SU}(3)$ only spin $S=0$ states were considered, because states with larger *S* are lying higher in energy and do not contribute to the construction of the PES.

This method was applied in Ref. [11] to ^{238}U and ^{190}Pt. In Ref. [12] several Sm isotopes were investigated. The PESs obtained were in agreement with the knowledge of their structure. For example, in ^{190}Pt a γ-unstable potential was obtained [11] due to many similar $\widetilde{\mathrm{SU}(3)}$ irreps, i.e., with a similar eigenvalue of the second-order Casimir operator. In all cases, the agreement of the spectrum and $B(E2)$transitions with experiment was remarkably well, considering the simple approach. The energies came out more or less right, while the $B(E2)$ transitions deviated about a factor of 2. The $B(E2)$ systematics, however, were excellently reproduced.

We emphasize that the method presented in Ref. [8] gives a procedure to obtain the PES for a given nucleus, *if* the deformation of a nucleus is known (or proposed by a theoretical model) and *if* the distribution of nucleons in the normal and unique orbitals can be obtained from a single-particle spectrum diagram, deduced by a microscopic model as the one in Ref. [17]. In this sense, our approach depends on the underlying microscopic model and may vary from one model to the other. Thus, trusting one of these models, our method gives the first estimate to the PES. As we will see, however, sometimes the PES obtained is in contradiction to an initial assumed axial symmetry and sheds some doubt on the used microscopic model.

In Ref. [11] the spectrum of a nucleus could be determined by solving numerically the Schrödinger equation using the finite element method. Unfortunately, the program is not available anymore. An alternative is to estimate the stiffness of the potential (as obtained by our procedure) in β and γ and use it as an input to the routine, published in Ref. [26], which calculates the spectrum and the $B(E2)$ values within the GCM [9], using a potential expanded up to the sixth power in β. The program also estimates the potential parameters, once the depth and the stiffness of the potential is known.

In the application of the following section we will see that the procedure works more or less well. Care has to be taken when the potential is spherical or its minimum is at $\gamma=0°$ or 60° or very near to one of these values. The estimation method then has a singularity [11]. The position of the potential minimum in β and γ will be reproduced as well as the depth. The stiffness, however, is not always well reproduced. The error is, however, not large because the frequency, i.e., $\hbar\omega_\beta$, is proportional to the square root of the stiffness.

III. APPLICATIONS TO SUPERHEAVY NUCLEI

The method of obtaining the PES, shortly resumed in the preceding section, will be applied to the nuclei ^{254}No, ^{260}Rf, ^{262}Sg, ^{270}Hs, ^{274}110 (Ds), ^{276}112, and ^{290}114.

The first nucleus to be considered is ^{254}No for which experimental data on excited states in the ground-state band are available [3]. This permits us to adjust the collective mass parameter and to deduce with more confidence the spectrum, apart from the PES which does not need this information. For the other nuclei, after having determined the PES in each

case, the same mass parameter will be assumed. However, care has to be taken because this mass parameter might vary from nucleus to nucleus. A fixed mass parameter already gives us a rough estimate about the spectrum (see the discussion below). Changing the mass parameter by a given factor more or less raises the energies, for smaller B_2, or lowers, for greater B_2, by the same factor. It is better realized for states within the ground-state band than for the vibrational band heads. This way of estimating the mass parameter is in general more accurate than calculating it from a microscopic model, which normally has a bad performance in determining B_2.

A. ^{254}No

The physical proton (neutron) shell has $\eta_\pi=5$ ($\eta_\nu=6$), which implies the pseudoshells $\tilde{\eta}_\pi=4$ ($\tilde{\eta}_\nu=5$). It will be the same for all cases studied here. The occupation in the pseudoproton (neutron) shell turns out to be 12 protons (16 neutron), having before filled the nucleons in the single-particle scheme of Ref. [17], with $\beta=0.246$ [19], and counted the nucleons in the normal orbitals. The 12 protons (16 neutrons) in the pseudoshell $\tilde{\eta}_\pi=4$ ($\tilde{\eta}_\nu=5$) correspond to a nearly half filled shell. Because it is less than half filled, we expect a prolate deformation. The spin representations used are $[6^2]$ for protons and $[8^2]$ for neutrons.

The largest $\widetilde{\mathrm{SU}(3)}$ irrep formed by the product of proton and neutron irreps is in the sector between 5° and 10° and is given by (58,8) with the eigenvalue of the second-order Casimir operator 4090 and the $\gamma=7°$. [The eigenvalue of the second-order Casimir operator $\mathcal{C}_2$ for the irrep (λ, μ) is given by $(\lambda^2+\lambda\mu+\mu^2+3\lambda+3\mu)$.] The largest irrep in the sector of 25°–30° is (38,30) at $\gamma=26.2°$ with the eigenvalue of the second-order Casimir operator of 3688, which is a difference of 402 to the former one. The separation is well pronounced, explaining the rise in γ. The κ value is 1.168 and $N_0=633.5$.

In upper left corner of Fig. 1 the deduced potential energy surface is given. For ^{254}No two potentials are drawn, the upper one is the potential as derived by our procedure while the lower one gives the approximation of the potential within the GCM. Note that the GCM potential is used finally in the calculation of the energies and the $B(E2)$ transitions. As ground-state deformation we use the value of $\beta_0=2.046$ from Ref. [19]. In the lower panel of ^{254}No we depicted the function which approximates the deduced potential within a finite power expansion up to sixth order in β. The potential exhibits a near prolate minimum and a stiff behavior towards large γ values, as can be seen in Fig. 2. Even for this prolate nucleus a slight triaxiality is produced by the content of the shell model space, as can be appreciated by the value μ different from zero in $(\lambda, \mu)=(58, 8)$ of the lowest SU(3) irrep.

In Ref. [3] the states in the ground-state band up to spin 20 are given. We adjust the mass parameter such that the 2^+ and 6^+ states at 0.044 MeV and 0.305 MeV are well reproduced. For this we use the formula $(\hbar^2/4B_2\beta_0^2)L(L+1)$ of the rotor [9], where L is the spin, B_2 the mass parameter of the model, and β_0 the ground-state deformation of the nucleus (fitted using the routine of Ref. [26]). Our fit gives 0.045 MeV for the 2^+ state and 0.301 MeV for the 6^+ state and the mass parameter acquires the value of $B_2=230\times10^{-42}$ MeV s^2 In Fig. 3 the corresponding spectrum is plotted. The spin of each level is indicated to the right. The spectrum shows the typical structure of an axially symmetric rotor with its β and γ bands.

In Table I some $B(E2)$-transition values in units of e^2 b^2 are given. It can be used as an estimate for expected transition strengths in experiment. The $B(E2)$ values indicate a clear identification of the ground state, the β, and the γ band. For higher states the identification is not clear because the $B(E2)$ values from one given spin to higher ones are of the same order for more than one transition.

B. ^{260}Rf

In the physical proton (neutron) shell the occupation is 22 (30), which indicates that the shell is filled more than half. This suggests a tendency to an oblate deformation. We applied the same steps as in the former case, using a deformation value of $\beta=0.228$ [19] (prolate). The resulting PES turned out to be γ unstable, which is a consequence of many $\widetilde{\mathrm{SU}(3)}$ irreps lying near together in energy. In order to be consistent with the shell model space we then used a negative deformation (oblate) with the same absolute value as reported in Ref. [19] (note that we have changed the sign of $\beta\approx\epsilon_2$ as given in Ref. [19]). The result is given in the following paragraph. Compared to the prolate filling of the single-particle diagram [17] the occupation in the normal orbitals is a little bit different, but the PES obtained is nearly indistinguishable and the shell model space also favors $\widetilde{\mathrm{SU}(3)}$ irreps which are oblate.

Changing the sign of the deformation (i.e., $\beta=-0.228$ and assuming that at least the absolute deformation is correct), the occupation in the pseudoproton (neutron) shell turns out to be 14 protons (22 neutron), having before filled the nucleons in the Nilsson scheme of Ref. [17], with Ref. [19], and counted the nucleons in the normal orbitals. The 14 protons (22 neutrons) in the pseudoshell $\tilde{\eta}_\pi=4(\tilde{\eta}_\nu=5)$ correspond to shells filled more than half. The spin representations used are $[7^2]$ for protons and $[11^2]$ for neutrons.

The largest $\widetilde{\mathrm{SU}(3)}$ irrep is in the sector of 50°–55° and is given by (8,58) with the eigenvalue of the second-order Casimir operator 4090 and the γ angle 53°. The highest irrep in the sector of 30°–35° is (31,39) at $\gamma=33.7°$ with the eigenvalue of the second-order Casimir operator of 3901, where the difference in its value to the former one is 189, which indicates a softer behavior in γ as in the former case. Another information is that the largest irreps indicate an *oblate deformation*, though in the tables of Ref. [19] a prolate deformation (positive β) is reported. This hints already to a problem between the shell model space and the supposed deformation. The κ value is 1.168 and $N_0=683.5$.

The deduced potential energy surface is given in the upper right part of Fig. 1. Again, two PESs are plotted, the upper part referring to the PESs as deduced by our procedure and the lower one its approximation within the GCM potential.

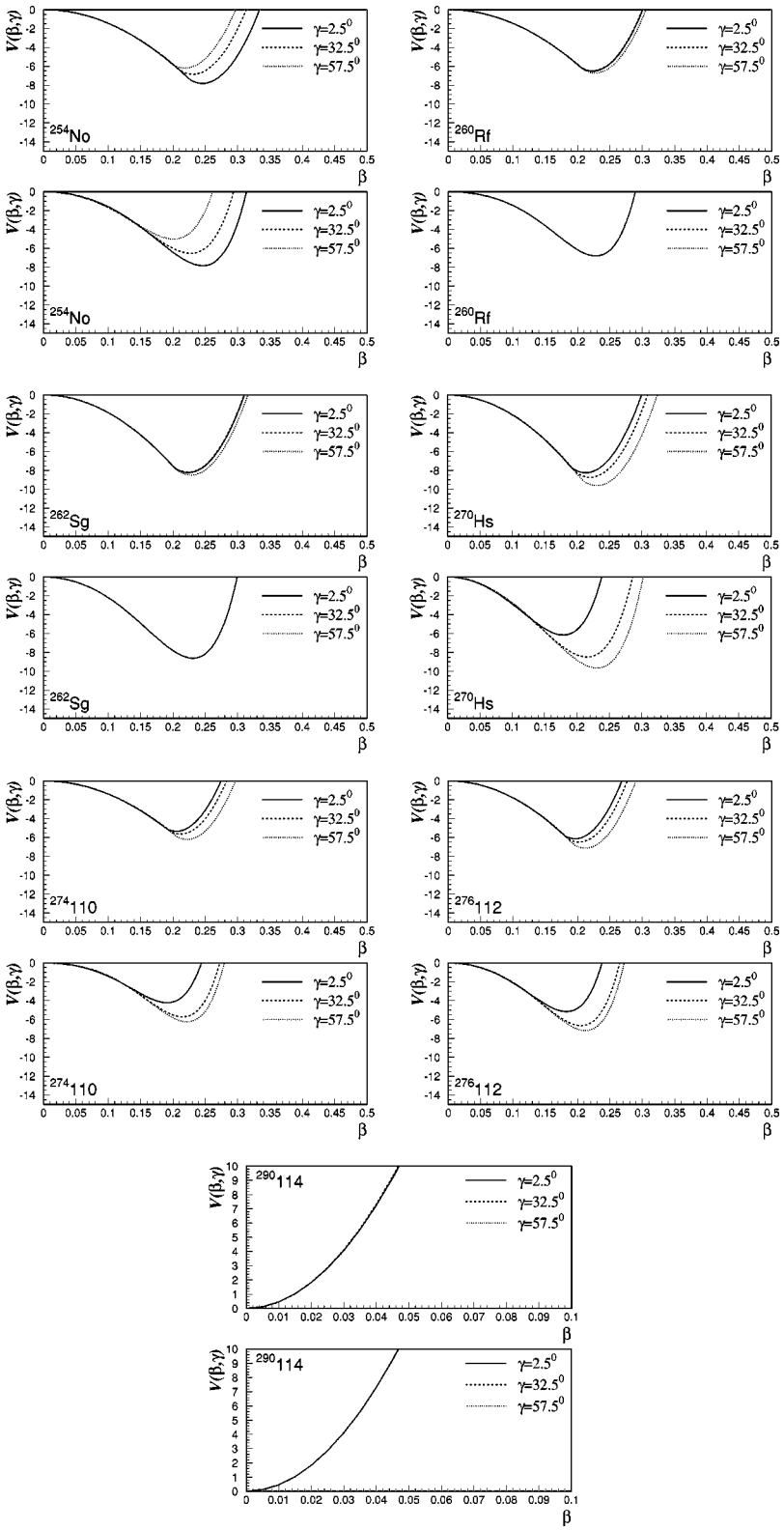

FIG. 1. The potential energy surface of the isotopes considered in the text. Each isotope is represented by two figures. The one on top of each sequence gives the result of the deduced PES with the method explained in Sec. II. In the bottom part of each sequence the approximation via the potential within the generalized collective model (GCM) is given.

Contrary to the previous case, this Rf isotope turns out to be γ unstable (see Fig. 2). Note that the microscopic shell model space is purely determined by the number of valence protons and valence neutrons in the normal orbitals at the measured deformation, given by a microscopic calculation [17]. This is the first example that the restriction to only prolate or oblate deformation creates problems of consistency. According to the microscopic shell model space a γ-unstable potential is preferred.

We use for this isotope of Rf, like for all other nuclei under study in this paper, the same mass parameter as for ^{254}No. The corresponding spectrum is plotted in Fig. 3 (first line, right). On the right-hand side of each level the spin and parity are indicated. Please note the near degeneracy of the

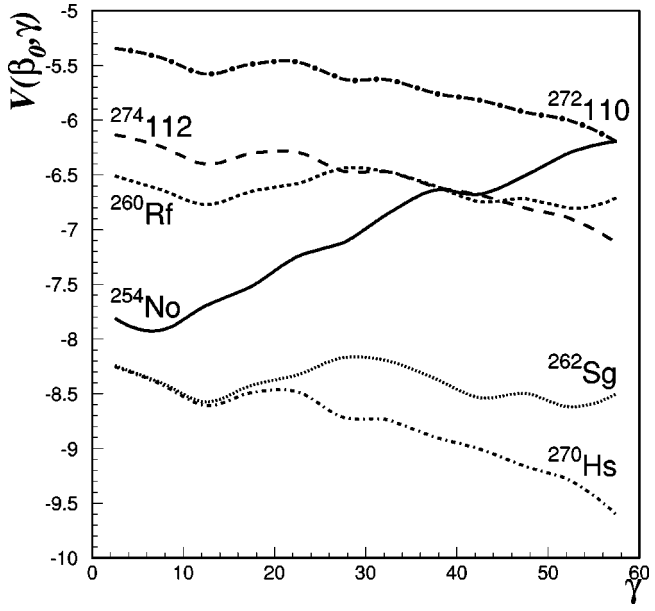

FIG. 2. Sections of the PES along the γ direction for the following superheavy nuclei ^{254}No, ^{260}Rf, ^{262}Sg, ^{270}Hs, ^{274}110 (Ds), and ^{276}112.

first 4^+ with the second 2^+. Also, the first 3^+, the second 4^+, and the first excited 0^+ are degenerate. This kind of degeneracy can also be observed approximately in the IBA-I within the $O(6)$ limit [27–29]. This particular structure should be observed in experiment. In experiment the degeneracy in the γ band might be seen as an extreme even-odd staggering.

Some $B(E2)$-transition values in units of e^2 b^2 are given in Table I. It can be used as an estimate for expected transition strengths in experiment. As in the previous case, the $B(E2)$ values indicate a clear identification of the ground-state, the β, and the γ band.

C. ^{262}Sg

The physical proton (neutron) shell is as in the previous case more than half filled, and the number of protons (neutrons) in the normal orbitals is 24 (30). This occupation suggests again a tendency towards an oblate deformation. We applied the same steps as in the former case, using a deformation value of β=0.229 [19] (prolate). The resulting PES turned out to be nearly γunstable, which is a consequence of many $\widetilde{\mathrm{SU}(3)}$ irreps lying near together in energy. In order to be consistent with the shell model space we used again a

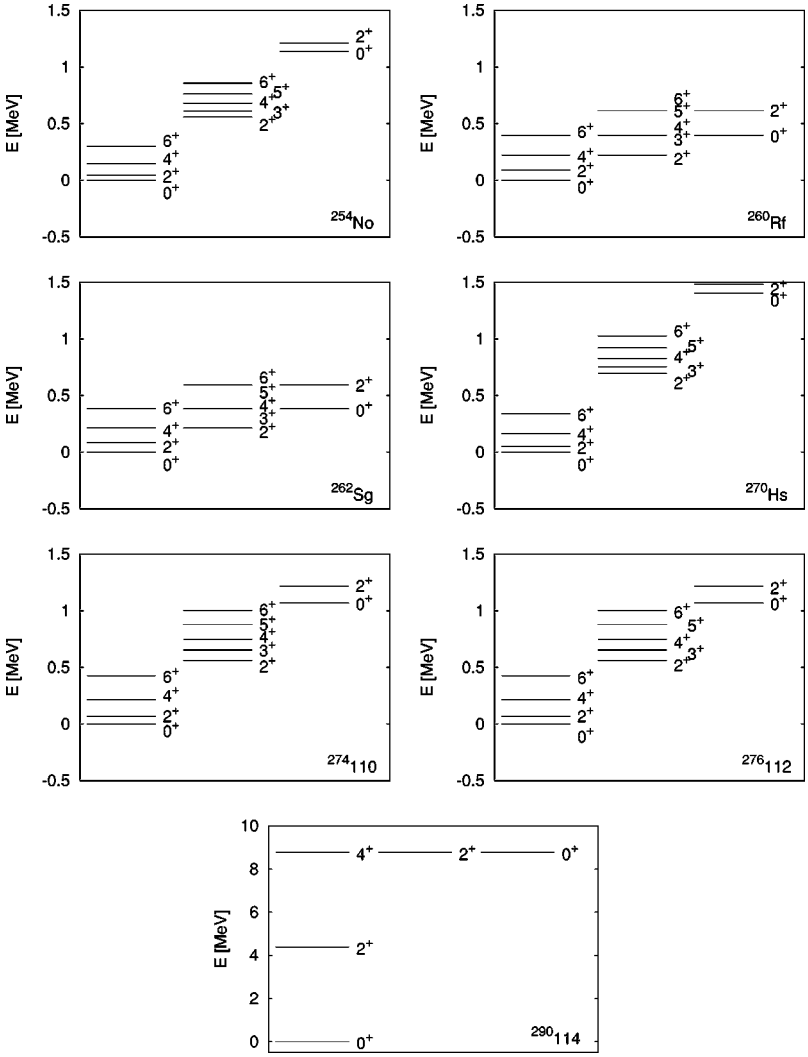

FIG. 3. Spectra of the isotopes discussed in the text.

TABLE I. Some $B(E2)$ transitions, in units of e^2 b^2. 0_1^+, 2_1^+, 4_1^+, and 6_1^+ from the ground-state band, while 2_2^+, 3_1^+, 4_2^+, 5_1^+, and 6_2^+ from the γ-vibrational and 0_2^+ and 2_3^+ from the β-vibrational band. This association was done, except for ^{290}114, looking for the largest $B(E2)$ values connecting a band. For higher states the identification is not easy because there are more than one $B(E2)$ transitions of the same order connecting a given spin with a higher one. For ^{290}114 the states are ordered according to a harmonic spectrum.

	^{254}No	^{260}Rf	^{262}Sg	^{270}Hs	^{274}110	^{276}112	^{290}114
$0_1^+\rightarrow 2_1^+$	8.679	7.346	8.626	6.945	8.455	6.410	0.250
$2_1^+\rightarrow 4_1^+$	4.528	3.797	4.456	3.633	4.370	3.410	0.180
$4_1^+\rightarrow 6_1^+$	4.093	3.579	4.197	3.173	4.119	3.168	0.217
$2_2^+\rightarrow 3_1^+$	3.976	2.478	2.906	3.282	2.852	2.876	0.150
$2_2^+\rightarrow 4_2^+$	1.767	2.337	2.740	1.464	2.689	1.452	0.141
$4_2^+\rightarrow 5_1^+$	1.810	0.795	0.932	1.485	0.915	1.210	0.058
$4_2^+\rightarrow 6_2^+$	2.848	2.685	3.145	2.362	3.089	2.299	0.197
$0_2^+\rightarrow 2_3^+$	7.328	4.998	5.855	3.300	5.750	5.758	0.350
$0_1^+\rightarrow 2_2^+$	0.305	0.048	0.058	0.617	0.053	0.736	0.000
$0_1^+\rightarrow 2_3^+$	0.039	0.000	0.000	0.024	0.000	0.025	0.000
$2_1^+\rightarrow 4_2^+$	2.461	0.015	0.019	0.084	0.017	0.066	0.000
$0_2^+\rightarrow 2_1^+$	0.054	0.016	0.020	0.031	0.018	0.037	0.100

negative deformation (oblate) with the same absolute value as reported in Ref. [19]. The result is given in the following paragraph.

After having filled the nucleons at deformation $\beta=-0.229$ the occupation of protons (neutrons) in the normal orbitals is 14 (22). The spin representations used are $[7^2]$ ($[11^2]$). The proton pseudoshell is less than half filled, while the neutron pseudoshell is more than half filled. This is the identical occupation of the proton and neutron pseudoshells as for the case ^{260}Rf. The reason is that the additional pair of protons are in a unique orbital which does not contribute to the PES within the pseudosymplectic description. The nucleons in the normal orbitals already trace the deformation of the system. The number of neutrons is the same. Due to the same occupation, the PES and the spectrum are expected to be similar. Differences appear due to the different deformation reported in Ref. [19]. The κ value is 1.168, $N_0=683.5$.

In Fig. 1 the deduced potential energy surface is given. In the lower panel of ^{262}Sg the potential is depicted which approximates the deduced potential within a finite power expansion up to sixth order in β. As in ^{260}Rf, this isotope turns out to be γunstable (see Fig. 2).

The corresponding spectrum is plotted in Fig. 3, which is similar to the spectrum of ^{260}Rf.

In Table I some $B(E2)$-transition values in units of e^2 b^2 are given. The $B(E2)$values indicate a clear identification of the ground-state, the β, and the γ band.

D. ^{270}Hs

In the physical proton (neutron) shell the occupation is 26 (36) which is more than half filled. This suggests again a tendency to an oblate deformation. We applied the same steps as in the former case, using a deformation value of $\beta=0.231$ [19] (prolate). The resulting PES turned out to be oblate, in contrast to the initially assumed prolate deformation. In order to be consistent with the shell model space, we used again a negative deformation (oblate) with the same absolute value as reported in Ref. [19]. The result is given in the follwing paragraph.

The occupation of the normal orbitals in the pseudoproton shell is 18, while for the pseudoneutron shell it is 22. The corresponding spin-zero representations are $[9^2]$ and $[11^2]$, respectively.

For this isotope of Hs the largest $\widetilde{\mathrm{SU}(3)}$ irrep is in the sector of 55°–60° and it is (0,64) with the eigenvalue of the second-order Casimir operator 4288 and the γ angle 59.2°. The largest highest irrep in the sector 30°–35° is (30,40) at $\gamma=34.6°$ with the eigenvalue of the second-order Casimir operator of 3910, where the difference in its value with the former one is 378, which hints to a well deformed oblate nucleus. The κ value is 1.168, $N_0=705.5$.

In Fig. 1 the deduced potential energy surface is given. As the deformation of the system the value of $\beta_0=-0.231$ from Ref. [19] was used. In the lower panel of ^{270}Hs the potential is depicted, which approximates the deduced potential within a finite power expansion up to sixth order in β. The PES has a minimum near 60°, i.e., it is an oblate nucleus as can be clearly noticed from Fig. 2. In the approximated PES the behavior near $\gamma=60°$ and 30° is well reproduced, while at 0° the potential is too high. The error in the spectrum is not large because the stiffness in γ at 60° is well reproduced. The potential hints to an oblate rotor. This is in contrast to the proposed prolate structure of Ref. [19].

In Fig. 3 the corresponding spectrum is plotted. There is now a clear distinction of the rotational bands. No odd-even staggering in the γ band is observed.

In Table I some $B(E2)$-transition values in units of e^2 b^2 are given.

E. ^{274}110 (Ds)

For this nucleus the physical proton (neutron) shell is as well filled more than half and the occupation is 28 (38). This

suggests again a tendency to an oblate deformation. Following the same steps as in the previous cases, we use a deformation value of β=0.222 Ref. [19] (prolate). The resulting PES turned out to be oblate. In order to be consistent with the shell model space we used again a negative deformation (oblate) with the same absolute value as reported in Ref. [19]. The result is given in the following paragraph.

There are 18 protons (22 neutrons) in the pseudovalence shell and the spin representations used are $[9^2]$ for protons and $[12^2]$ for neutrons. It is the same as for ^{270}Hs, which then results into the same $\widetilde{\mathrm{SU}}(3)$ irreps. The reason is that the additional nucleons occupy unique orbitals and thus the occupation in the normal orbitals is the same. The κ value is 1.168, N_0=692.5.

In Fig. 1 the deduced potential energy surface is given. As the deformation of the system the value of β_0=−0.222 from Ref. [19] was used. In the lower panel of ^{274}110 the potential is depicted, which approximates the deduced potential within a finite power expansion up to sixth order in β. The PES has an oblate minimum, which is less pronounced than the one for ^{270}Hs (see Fig. 2).

In Fig. 3 the corresponding spectrum is plotted. To the right of each level the spin and parity are indicated. The same degeneration is observed as in the other cases of γ instability, presented so far.

Table I lists some $B(E2)$-transition values in units of e^2 b^2.

F. ^{276}112

In the physical proton (neutron) shell the occupation is 30 (38), which is more than half filled. This again suggests a tendency to an oblate deformation. We applied the same steps as in the former case, using a deformation value of β =0.212 [19] (prolate). The resulting PES turned out to be oblate. In order to be consistent with the shell model space we used again a negative deformation (oblate) with the same absolute value as reported in Ref. [19]. The result is given in the following paragraph.

The occupation of the protons and neutrons in the normal orbitals is the same as for ^{270}Hs and ^{274}110. The additional protons and neutrons all fall into unique orbitals, which do not contribute to the PES. The largest $\widetilde{\mathrm{SU}}(3)$ irrep is again (0,64) with the eigenvalue of the second-order Casimir operator 4288 and the γ angle 59.2°. The largest, highest irrep in the sector of 30°−35° is (30,40) at γ=34.6° with the eigenvalue of the second-order Casimir operator of 3910, where the difference in its value with the former one is only 378, which hints to a well deformed oblate nucleus. The κ value is 1.168, N_0=705.5. As before the irreps hint to an *oblate deformation*. Because of the same occupation as for ^{270}Hs we expect a similar spectrum and PES.

In Fig. 1 the deduced potential energy surface is given. As the deformation of the system the value of β_0=−0.212 from Ref. [19] was used. In the lower panel of ^{276}112 the potential is depicted, which approximates the deduced potential within a finite power expansion up to sixth order in β. The PES has a minimum near 60° (Fig. 2), i.e., it has an oblate shape like the two previous investigated nuclei. The stiffness towards lower γ has increased further. Note again that we started from a prolate deformation, but the shell model space suggests an oblate deformation.

In Fig. 3 the corresponding spectrum is plotted.

In Table I some $B(E2)$-transition values in units of e^2 b^2 are given. The $B(E2)$values indicate again a clear identification of the ground-state, the β and the γ band.

G. ^{290}114

The occupation of the pseudoshells is 18 for protons and 50 for neutrons. The largest $\widetilde{\mathrm{SU}}(3)$ irrep is (0,60). Because the microscopically deduced deformation is only −0.026 (slightly oblate) the resulting potential should be nearly spherical. In this case a comparison of the different $\widetilde{\mathrm{SU}}(3)$ irreps does not give much insight into the γ dependence of the potential, which is washed out by the small deformation.

In Fig. 1 the potential energy surface is given. It is identical for the deduced potential as well for the potential within the GCM, because it is just an oscillator which can be given by the term β^2 of the potential.

In Fig. 3 the corresponding spectrum is plotted. Towards the right of each level the spin and parity are indicated. The spectrum is typical of an oscillator.

In Table I some $B(E2)$-transition values in units of e^2 b^2 are given.

IV. CONCLUSIONS

In this paper we estimated the collective quadrupole structure of some superheavy nuclei, with atomic charge from 102 to 114. The PES, the low lying spectrum, and some $B(E2)$-transition rates were calculated for each nucleus. The method used [8] is based on the pseudosymplectic description of nuclei and is quite robust, i.e., it depends only on the external information of the deformation of a nucleus and the calculated single-particle diagram. All the shell structure inputs are determined by microscopic models, such as in Refs. [17,19]. Changing slightly the occupation and/or the deformation has similar effects on the spectrum, i.e., small changes do not effect the core of the results.

We showed that ^{254}No is a prolate rotor in accordance with Ref. [19]. However, ^{260}Rf and ^{262}Sg are typical γ-unstable nuclei. For example, i.e., ^{270}Hs, ^{274}110, and ^{276}112, the nuclei turn out to be oblate, contrary to the reported positive value of β in Ref. [19] or [6]. Finally, ^{290}114 is a spherical nucleus.

Our study indicates the necessity to include dynamically the γ-vibrational degree of freedom. Though, assuming an axial-symmetric shape the microscopic shell model content dictates γ values different from zero and the PES deduced even contains γ-unstable cases. We assume that at least the absolute deformation is well reproduced by the microscopic model. Our results then suggest the correct γ triaxiality (or instability).

Only even-even nuclei were considered. The reason is that an odd proton and/or neutron changes the spectrum significantly at low energies. However, the PES can be used to deduce the PES of the core of neighboring even-odd, odd-even, and odd-odd nuclei.

We did not consider in the present paper the nucleus ^{252}No, because from the point of view of the microscopical shell-model space it is identical with ^{254}No. This is due to the fact that the only orbital which is removed because it has two neutrons less is an intruder. It is possible that the mass parameter changes slightly, but the spectrum is more or less the same as for ^{254}No.

The present compilation can be used to look into experiments for the structure of low lying states. Some typical signatures were discussed in the text, though care has to be taken with respect to the absolute scale. Using only relative scales is more secure. Also, the reported $B(E2)$ values have to be interpreted as depicting tendencies and first estimates of their absolute value.

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