# Photoproduction of mesons from the nucleon

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A general formalism is established for constructing models for the photoproduction of mesons from the nucleon. The essential ingredient is a mass operator which describes the coupling between meson-baryon, photon-baryon, and single-baryon channels. The most general forms for the mass operator interactions which produce these couplings are derived. These forms also provide generalizations of the Chew-Goldberger-Low-Nambu amplitudes for pion-nucleon photoproduction to any meson-baryon final state. The models lead to $S$-matrix elements that transform properly under inhomogeneous Lorentz transformations and are gauge invariant. The photoproduction amplitudes include final state interactions and satisfy Watson's theorem. A specific model is constructed by deriving the mass operator interactions from effective Lagrangians that describe the couplings of mesons, photons, and baryons. The electromagnetic interactions include direct and crossed nucleon contributions, as well as direct contributions from the $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, and $S_{11}(1535)$ resonances. A contact term and exchange terms due to the $\pi$, $\rho$, and $\omega$ mesons are also included. The model gives a good fit to the significant multipoles in the energy range from the single-pion, photoproduction threshold up to a center-of-momentum energy of $W=1550$ MeV, which corresponds to a photon lab energy of 810 MeV.

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## I. INTRODUCTION

The first reasonably successful calculation of the photoproduction of pions from the nucleon was carried out by Chew and Low [1], based on a straightfoward extension of their static cutoff model for pion-nucleon scattering. The lack of Lorentz and gauge invariance in the Chew-Low model was remedied in a seminal paper by Chew, Goldberger, Low, and Nambu [2] who formulated relativistic dispersion relations for the photoproduction amplitudes. As an important by-product of their analysis, they established the general form of the Lorentz and gauge invariant amplitudes for single-pion photoproduction from the nucleon, i.e., the by now well-known Chew-Goldberger-Low-Nambu (CGLN) amplitudes. A detailed study based on the CGLN approach was carried out by Berends, Donnachie, and Weaver [3], who showed that dispersion relations can account for the main features of the data up through the energy of the $\Delta(1232)$ resonance. The early work on $\pi$, $\eta$, $K$, and vector-meson photoproduction has been thoroughly reviewed by Donnachie [4]. A more recent review has been presented by Dieter and Tiator [5].

Dispersion relations continue to be used in the analysis of photoproduction and electroproduction of mesons from the nucleon. A number of workers have used them to carry out multipole analyses of the data [6–8]. They have also been used to extract the $\Delta(1232)$ contribution from the $M_{1+}^{3/2}$, $E_{1+}^{3/2}$, and $S_{1+}^{3/2}$ electroproduction multipoles [9].

One of the most popular approaches for constructing photoproduction amplitudes is through the use of effective Lagrangians [10–18]. Typically, the tree level amplitudes obtained from these Lagrangians are unitarized following some prescription so as to satisfy Watson's theorem [19]. According to this theorem, below the threshold for two-pion photoproduction, the phase of a photoproduction amplitude which leads to a partial wave, pion-nucleon state is the same as the elastic scattering phase shift for that partial wave.

An interesting variation on the effective Lagrangian approach involves the use of an effective Lagrangian for a quark-pseudoscalar-meson coupling in a constituent quark model [20–23]. The quark-model wave functions for the nucleon and the baryon resonances provide a form factor for each interaction vertex, and the s- and u-channel resonances can be consistently included in calculating the meson production amplitudes.

One of the most elaborate and complete models for the photoproduction and electroproduction amplitudes is a unitary isobar model by Drechsel *et al.* [24]. This model contains the standard Born terms, along with five resonances, and vector-meson exchanges. It succeeds in describing the data up to 1 GeV.

A dynamical model of pion photoproduction from the nucleon has been developed by Surya and Gross [25] based on a three-dimensional reduction of the Bethe-Salpeter equation. This model satisfies unitarity and is gauge invariant. The Born terms and kernels of their integral equations include nucleon ($N$), delta ($\Delta$), Roper ($R$), and $D_{13}$ ($D$) direct poles, crossed $N$ and $R$ poles, as well as a contact term and $\pi$, $\rho$, and $\omega$ exchange terms. The model gives a good fit to all $L \leqslant 2$ multipoles up to a photon lab energy of 770 MeV.

Neutral pion photoproduction off protons and deuterons provides an important test of chiral pion-nucleon dynamics. Such a test has been provided by an investigation of near-threshold neutral pion photoproduction off protons to fourth order in heavy-baryon chiral perturbation theory [26]. This work solidifies the parameter-free third-order predictions, which are in good agreement with the data.

Hamiltonian models have provided a generally successful framework for carrying out photoproduction and electroproduction calculations [27–36]. In these models a Hamiltonian acts in a limited Hilbert space such as $H=N\oplus\Delta\oplus\pi N\oplus\gamma N$. Since these models are essentially coupled-channel potential models, they satisfy two-particle unitarity exactly and therefore lead to photoproduction amplitudes that satisfy Watson's theorem [19]. The electromagnetic parts of the Hamiltonian are defined in terms of matrix elements which describe transitions, such as $\gamma N\leftrightarrow\pi N$ and $\gamma N\leftrightarrow\Delta$. These matrix elements are calculated from effective Lagrangians in lowest order perturbation theory. The strong interaction part of the Hamiltonian provides a model for $\pi N$ scattering in the absence of electromagnetic couplings, and accounts for the rescattering that occurs after the photon has interacted with the nucleon. It contains potentials which couple meson-baryon channels directly to meson-baryon channels as well as vertex interactions. The potentials are either purely phenomenological separable potentials [27–29], or are taken from a meson-exchange model [30–36]. The Hamiltonian models are three dimensional in character, with the total three-momentum conserved in intermediate states, but not the four-momentum. This can create problems with ensuring gauge invariance. Nozawa *et al.* [29] were able to maintain gauge invariance by requiring that in the second-order matrix elements that describe the transition $\gamma N\leftrightarrow\pi N$, the four-momentum is conserved at the $\gamma NN$ vertex, but not necessarily at the $\pi NN$ vertex. This approach is rather limited in that it is necessary to use a common form factor for the Born term interactions.

Maintaining gauge invariance is a problem that has attracted the attention of many workers. In dealing with this problem several authors have focused on the Ward-Takahashi (WT) identities [37]. Ohta [38] has derived an electromagnetic current operator from the most general form of the extended pion-nucleon vertex function using the minimal substitution prescription, and has shown that the resulting current operator and the isolated pole contribution satisfy the WT identities. He has also shown [39] that it is possible to derive electromagnetic interactions that are nonlocal and at the same time maintain local gauge invariance. Naus *et al.* [40] have used the WT identities to enforce gauge invariance at the operator level, rather than on just the amplitude level. Van Antwerpen and Afnan [41] have derived coupled-channel integral equations that lead to photoproduction amplitudes that satisfy both two-body unitarity and generalized WT identities. Gross and Riska [42] have shown that the WT identities play a central role in ensuring that the electromagnetic coupling to a two-body system described by the Bethe-Salpeter equation [43] or one of its three-dimensional reductions [44,45] leads to a conserved current and thereby to gauge invariance.

Haberzettl [46] has developed a gauge invariant model of pion photoproduction starting with an effective field theory of hadrons. His equations are nonlinear integral equations which can be difficult to solve in practice. He has discussed approximations that make the nonlinear formalism manageable and yet preserve gauge invariance. He and his collaborators [47] have shown how to implement his formulation at tree level with form factors describing composite nucleons, and he has also shown how to preserve the gauge invariance of meson production currents in the presence of explicit final-state interactions [48].

A general method for incorporating an external electromagnetic field into descriptions of few body systems whose strong interactions are described by integral equations has been developed by Kvinikhidze and Blankleider [49]. Their method involves the idea of gauging the integral equations themselves and leads to conserved currents which in turn ensures gauge invariance.

Here we develop a model for meson photoproduction from the nucleon that leads to amplitudes that satisfy both unitarity and gauge invariance. Our model is closest in spirit to the Hamiltonian models [27–36]. It differs in two important aspects. There is a more careful treatment of relativity, and gauge invariance is implemented in a very general way. Some features of the model have already been presented [50,51]. The model is developed within the framework of relativistic quantum mechanics, where by relativistic quantum mechanics is meant a theory in which the quantum mechanical state vectors of a system transform according to a unitary representation of the Poincaré group [52]. The continuous transformations, which form the proper subgroup, can be expressed in terms of ten generators; four of which generate translations in space time $(x'=x+b)$, while the other six generate the homogeneous Lorentz transformations $(x'=ax)$. These ten generators satisfy a set of commutation relations known as the Poincaré algebra. Several subsets of these generators satisfy a closed subset of these commutation relations and thereby generate a subgroup of the proper Poincaré transformations. Some of these subgroups are associated with three-dimensional hypersurfaces in Minkowski space that do not contain timelike directions. Each form of relativistic quantum mechanics is associated with such a hypersurface and its corresponding subgroup [52,53]. In relativistic quantum mechanics the generators are Hermitian operators in the Hilbert space of the system. In each form the generators of the subgroup of transformations that map the form's hypersurface into itself are chosen to be noninteracting. The remaining generators contain interactions. The instant form is based on the hypersurface $t$=const., the front form uses the null plane $ct+z=0$, and the point form is based on the hypersurface $c^2t^2-\mathbf{x}^2$=const.

Here we will use the instant form for which the three-momentum operator $\mathbf{P}$ and the angular momentum operator $\mathbf{J}$ are noninteracting, while the Hamiltonian $H$ and the generator of rotationless boosts $\mathbf{K}$ are interacting. The operators $\mathbf{P}$ and $\mathbf{J}$ generate translations and rotations respectively in ordinary three-dimensional space, which of course is the hypersurface $t$=const. Here we will use the Bakamjian-Thomas procedure [54] for constructing the generators. In this procedure the generators are expressed in terms of the set of ten operators $\{M,\mathbf{P},\mathbf{S},\mathbf{X}\}$ where $M$, $\mathbf{S}$, and $\mathbf{X}$ are the mass operator, the spin operator, and the Newton-Wigner position operator [55], respectively. The set $\{M,\mathbf{P},\mathbf{S},\mathbf{X}\}$ satisfies simpler commutation rules than the generators. In the Bakamjian-Thomas construction $\mathbf{P}$, $\mathbf{S}$, and $\mathbf{X}$ are noninteracting and only the mass operator $M$ contains an interaction. As a result of this, in order to ensure Poincaré invariance it is only necessary to choose the interaction in $M$ so that $M$ commutes with $\mathbf{P}$, $\mathbf{S}$, and $\mathbf{X}$.

The outline of the paper is as follows. In Sec. II the fundamentals of the Poincaré group are summarized and the Bakamjian-Thomas construction is outlined. In Sec. III we define our single-particle states both for massive and massless particles and describe how they transform under rotations and space inversion. For the photon we work with both helicity states and special combinations of them that transform similarly to massive particle states. Section IV introduces our photon-baryon ($\gamma B$) and meson-baryon ($\mu B$) states and shows how they transform under rotations and spatial inversions. We note at this point that throughout, mesons are indicated by $\mu$'s and baryons are indicated by $B$'s or $b$'s. In Sec. V we write our mass operator in the form $M=M_0+U$, where $M_0$ is the mass operator for the system without interactions and $U$ is the interaction. We deduce the most general form for the matrix elements of $U$ consistent with Poincaré invariance. Our interaction $U$ describes the couplings; $\gamma B \leftrightarrow b$, $\mu B \leftrightarrow b$, $\gamma B \leftrightarrow \mu b$, and $\mu B \leftrightarrow \mu' B'$. We consider matrix elements in both an angular momentum basis and a "plane wave" basis. Section V also introduces a set of vector spherical harmonics that are used to describe the photon-baryon states and which make it possible to satisfy gauge invariance in a particularly simple way. The analysis of Sec. V also gives the general forms for the $\gamma B \rightarrow b$ and $\gamma B \rightarrow \mu b$ photoproduction amplitudes. In Sec. VI we give the relations we use for calculating the photoproduction amplitudes from the electromagnetic interactions and the off-shell, pion-nucleon, strong interaction $T$ matrix. Section VII gives our method for constructing mass operator interactions from effective Lagrangians using the Okubo method [56]. Here we construct the vertex interactions that describe the processes $\gamma N \leftrightarrow N$ and $\pi N \leftrightarrow N$. These interactions give rise to an electromagnetic potential through the direct process $\gamma N \rightarrow N \rightarrow \pi N$. We also derive an electromagnetic potential from a contact interaction and from the crossed process $\gamma N \rightarrow \gamma \pi N, N N \bar{N} \rightarrow \pi N$. We also use the process $\gamma N \rightarrow \gamma \pi N, \pi \pi N \rightarrow \pi N$, in which the photon couples directly to a pion, to construct an electromagnetic potential. The vector mesons $\mu=\rho$, $\omega$ also contribute potentials through the processes $\gamma N \rightarrow \gamma \mu N, \pi \mu N \rightarrow \pi N$. The resonances $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, and $S_{11}(1535)$ contribute electromagnetic potentials through direct processes in which they provide the intermediate states. Besides coupling to a $\pi N$ final state, the nucleon, the $P_{11}(1440)$, and the $D_{13}(1520)$ also couple to a $\pi\Delta$ final state. The $S_{11}(1535)$ couples to both a $\pi N$ and a $\eta N$ final state. In Sec. VIII we present the results of our multipole calculations with these electromagnetic potentials and the Elmessiri-Fuda model [57] for the off-shell, pion-nucleon, strong interaction $T$-matrix. A discussion of our results and suggestions for future extensions and improvements of our model are given in Sec. IX.

## II. GENERAL BACKGROUND

A Poincaré transformation is a linear, inhomogeneous transformation that maps the components of a space-time four-vector $x$ associated with one inertial frame to the components of a four-vector $x'$ associated with another inertial frame according to the relation

$$x' = ax + b. \tag{2.1}$$

Here $b$ is a four-vector and $a$ is a Lorentz transformation. For proper transformations $a$ can be parametrized in the form [52,58]

$$a = \exp[i(\boldsymbol{\omega}\cdot\mathbf{k} + \boldsymbol{\theta}\cdot\mathbf{j})]. \tag{2.2}$$

Here $\mathbf{j}$ is the generator of three-rotations, $\mathbf{k}$ is the generator of rotationless boost, and $\boldsymbol{\omega}$ and $\boldsymbol{\theta}$ are three-vectors whose components provide the necessary six parameters. In a satisfactory relativistic model, there exists a unitary operator $U(a,b)$, corresponding to the Poincaré transformation $(a,b)$ that maps a quantum mechanical state vector $|\psi\rangle$ associated with the $x$-frame to the vector $|\psi'\rangle$ associated with the $x'$-frame according to

$$|\psi'\rangle = U(a,b)|\psi\rangle. \tag{2.3}$$

For proper transformations the unitary operator can be parametrized in the form

$$U(a,b) = \exp(ib\cdot P)\exp[i(\boldsymbol{\omega}\cdot\mathbf{K} + \boldsymbol{\theta}\cdot\mathbf{J})], \tag{2.4}$$

with

$$P = (H, \mathbf{P}). \tag{2.5}$$

Here $\mathbf{K}$ is a boost operator, $\mathbf{J}$ is the angular momentum operator, $H$ is the Hamiltonian of the system, and $\mathbf{P}$ is the three-momentum operator. Since the law of combination for the Poincaré transformations is $(a',b')\circ(a,b) =(a'a, a'b+b')$, the unitary operators must combine according to

$$U(a',b')U(a,b) = U(a'a, a'b+b') \tag{2.6}$$

so as to provide a representation of the Poincaré group. This implies a set of commutation relations for the generators $\{H,\mathbf{P},\mathbf{K},\mathbf{J}\}$, which is commonly known as the Poincaré algebra [58].

In constructing the ten generators $\{H,\mathbf{P},\mathbf{K},\mathbf{J}\}$, it is convenient to work with another set of ten hermitian operators, i.e., $\{M,\mathbf{P},\mathbf{S},\mathbf{X}\}$ where $M$ is the mass operator, $\mathbf{S}$ is a spin operator, and $\mathbf{X}$ is the so-called Newton-Wigner position operator [52,55]. This second set of operators satisfies a much simpler set of commutation rules than the Poincaré algebra; in fact, the only nonzero commutators are

$$[P^m, X_n] = -i\delta_{mn}, [S^l, S^m] = i\varepsilon_{lmn}S^n, \tag{2.7}$$

which are familiar from nonrelativistic quantum mechanics. The three-momentum operator $\mathbf{P}$ is common to both sets, while the other generators can be expressed in terms of the operators of the second set by the relations [52]

$$H = (\mathbf{P}^2 + M^2)^{1/2}, \tag{2.8a}$$

$$\mathbf{J} = \mathbf{X}\times\mathbf{P} + \mathbf{S}, \tag{2.8b}$$

$$\mathbf{K} = -\frac{1}{2}(\mathbf{X}H + H\mathbf{X}) - \frac{\mathbf{P}\times\mathbf{S}}{M+H}. \tag{2.8c}$$

It can be shown that if the commutators of the set $\{M,\mathbf{P},\mathbf{S},\mathbf{X}\}$ are zero, except for those given by Eq. (2.7), then the generators given by Eq. (2.8), in combination with $\mathbf{P}$, satisfy the Poincaré algebra.

In the Bakamjian-Thomas construction [54] of the set $\{M,\mathbf{P},\mathbf{S},\mathbf{X}\}$, the operators $\mathbf{P}$, $\mathbf{S}$, and $\mathbf{X}$ are chosen to be the same as those for the system of particles without interactions, while the mass operator $M$ contains interactions. The mass operator can be written in the form

$$M = M_0 + U, \tag{2.9}$$

where $M_0$ is the non interacting mass operator and $U$ is an interaction. The commutation rules for $\mathbf{P}$, $\mathbf{S}$, and $\mathbf{X}$ are then automatically satisfied, and it is only necessary to ensure that

$$[U,\mathbf{P}] = [U,\mathbf{S}] = [U,\mathbf{X}] = 0. \tag{2.10}$$

It is important to note that it has been proven [59–62] that the $S$-matrix elements that arise in models based on the Bakamjian-Thomas construction transform properly in going from one inertial frame to another. The probability of a physical process is Poincaré invariant.

## III. SINGLE-PARTICLE STATES

For massive particles of mass $M_x$, states can be constructed by boosting a rest frame state $|sm\rangle$ according to

$$|\mathbf{p}sm\rangle = U[l_c(p)]|sm\rangle, \tag{3.1}$$

where $s$ and $m$ are the particle's spin and three-component, respectively; and the so-called canonical boost is given by

$$U[l_c(p)] = \exp(-i\rho\hat{\mathbf{p}}\cdot\mathbf{K}),\ \rho = \tanh^{-1}[|\mathbf{p}|/\varepsilon_x(\mathbf{p})], \tag{3.2a}$$

$$\varepsilon_x(\mathbf{p}) = (\mathbf{p}^2 + M_x^2)^{1/2}. \tag{3.2b}$$

Assuming that $|sm\rangle$ is an SU(2) basis state and using the fact that $\mathbf{K}$ is a three-vector operator, which implies that $U(r)\hat{\mathbf{p}}\cdot\mathbf{K}U^{-1}(r) = (r\hat{\mathbf{p}})\cdot\mathbf{K}$, it follows that the state (3.1) rotates according to

$$U(r)|\mathbf{p}sm\rangle = \sum_{m'=-s}^{s} |r\mathbf{p},sm'\rangle D^{(s)}_{m'm}(r). \tag{3.3}$$

Here $D^{(s)}$ is a standard matrix representation of SU(2). For the state of a massless particle with helicity λ we can write

$$|\mathbf{p}\lambda\rangle = U[r(\hat{\mathbf{p}})]\,||\mathbf{p}|\mathbf{e}_3,\lambda\rangle, \tag{3.4}$$

where $||\mathbf{p}|\mathbf{e}_3,\lambda\rangle$ describes a massless particle with three-momentum $|\mathbf{p}|\mathbf{e}_3$, where $\mathbf{e}_3 = (0,0,1)$, and λ is an eigenvalue of $\mathbf{J}_3$. The rotation $r(\hat{\mathbf{p}})$ has the property

$$r(\hat{\mathbf{p}})\mathbf{e}_3 = \hat{\mathbf{p}}. \tag{3.5}$$

We follow the Jacob-Wick convention [63] and choose

$$r(\hat{\mathbf{p}}) = \exp(-i\phi j_3)\exp(-i\theta j_2)\exp(i\phi j_3), \tag{3.6a}$$

$$\hat{\mathbf{p}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta),$$
$$0 \leqslant \theta \leqslant \pi,\ 0 \leqslant \phi < 2\pi. \tag{3.6b}$$

In order to see how the states (3.4) rotate, we can write $rr(\hat{\mathbf{p}}) = r(r\hat{\mathbf{p}})r_3(r,\hat{\mathbf{p}})$, where

$$r_3(r,\hat{\mathbf{p}}) = r^{-1}(r\hat{\mathbf{p}})rr(\hat{\mathbf{p}}) = \exp[-i\phi(r,\hat{\mathbf{p}})j_3]. \tag{3.7}$$

The fact that $r_3$ is a rotation about the 3-axis follows from the observation that $r_3(r,\hat{\mathbf{p}})\mathbf{e}_3 = \mathbf{e}_3$. It now follows that

$$U(r)|\mathbf{p}\lambda\rangle = |r\mathbf{p},\lambda\rangle\exp[-i\phi(r,\hat{\mathbf{p}})\lambda]. \tag{3.8}$$

We now consider spatial inversion, i.e., $(ct,\mathbf{x})\to(ct,-\mathbf{x})$, which we denote by $a=s$. We denote the unitary operator corresponding to $s$ by $\mathcal{P}=U(s)$. Under spatial inversions the Poincaré generators satisfy [58]

$$\mathcal{P}H\mathcal{P}^{-1} = H,\ \mathcal{P}\mathbf{P}\mathcal{P}^{-1} = -\mathbf{P},\ \mathcal{P}\mathbf{J}\mathcal{P}^{-1} = \mathbf{J},\ \mathcal{P}\mathbf{K}\mathcal{P}^{-1} = -\mathbf{K}. \tag{3.9}$$

Using the inverse relations to Eq. (2.8), which are given by Eqs. (2.7)–(2.9) of Ref. [64], we can show that

$$\mathcal{P}M\mathcal{P}^{-1} = M,\quad \mathcal{P}\mathbf{S}\mathcal{P}^{-1} = \mathbf{S},\quad \mathcal{P}\mathbf{X}\mathcal{P}^{-1} = -\mathbf{X}. \tag{3.10}$$

Since the massive particle rest frame state $|sm\rangle$ is an eigenstate of $\mathbf{S}^2$ and $S_3$ with eigenvalues $s(s+1)$ and $m$, respectively; it follows from Eq. (3.10) that $\mathcal{P}|sm\rangle$ is also such a state; therefore $\mathcal{P}|sm\rangle = |sm\rangle\eta$, where $\eta$ is a phase factor. Application of $S_\pm = S_1 \pm iS_2$ shows that $\eta$ is independent of $m$. Since $\mathcal{P}^2 = U(s^2) = 1$ and therefore $\mathcal{P}^{-1} = \mathcal{P} = \mathcal{P}^\dagger$, we see that $\eta$ is real and $\eta^2 = 1$. It now follows from Eqs. (3.1), (3.2), and (3.9) that

$$\mathcal{P}|\mathbf{p}sm = |-\mathbf{p},sm\rangle\eta,\quad \eta = \pm 1. \tag{3.11}$$

The polar angles for $-\hat{\mathbf{p}}$ are $(\pi-\theta,\phi\pm\pi)$, where the upper sign is used for $0\leqslant\phi<\pi$ and the lower sign for $\pi\leqslant\phi<2\pi$. Since $\mathbf{P}$ is a three-vector operator and $\mathbf{J}\cdot\mathbf{P}$ is a pseudoscalar operator, it follows from Eqs. (3.4) and (3.5) that $|\mathbf{p}\lambda\rangle$ is an eigenstate of $\mathbf{P}$ and $\mathbf{J}\cdot\mathbf{P}$ with eigenvalues $\mathbf{p}$ and $\lambda|\mathbf{p}|$, respectively; while according to Eqs. (3.9) the corresponding eigenvalues for $\mathcal{P}|\mathbf{p}\lambda\rangle$ are $-\mathbf{p}$ and $-\lambda|\mathbf{p}|$. This implies that $\mathcal{P}|\mathbf{p}\lambda\rangle = |-\mathbf{p},-\lambda\rangle\xi(\mathbf{p},\lambda)$ where $\xi(\mathbf{p},\lambda)$ is a phase factor. We can determine this phase factor using results obtained by Tung [58]. Instead of Eq. (3.6a) Tung uses $r(\hat{\mathbf{p}}) = \exp(-i\phi j_3)\exp(-i\theta j_2)$ so our helicity states, Eq. (3.4), are related to his by $|\mathbf{p}\lambda\rangle = |\mathbf{p}\lambda\rangle_{\text{Tung}}\exp(i\lambda\phi)$. It follows from his Eq. (11.3–23) that

$$\mathcal{P}|\mathbf{p}\lambda\rangle = |-\mathbf{p},-\lambda\rangle\eta(-1)^{|\lambda|-\lambda}\exp(2i\lambda\phi). \tag{3.12}$$

For photons $\eta=-1$ and $|\lambda|=1$.

Rather than work with the helicity states, we will work with a linear combination of them, which has simpler transformation properties. We define

$$|\mathbf{p}m) = \sum_{\lambda=\pm s} |\mathbf{p}\lambda\rangle D^{(s)}_{\lambda m}[r^{-1}(\hat{\mathbf{p}})], \quad s = |\lambda| = 1, \tag{3.13}$$

where we have used the curved bracket to distinguish these states from Eq. (3.1). Using Eqs. (3.7) and (3.8), and $D^{(s)}_{\lambda\lambda'}[r_3(r,\hat{\mathbf{p}})] = \delta_{\lambda\lambda'}\exp[-i\phi(r,\hat{\mathbf{p}})\lambda]$, it follows that

$$U(r)|\mathbf{p}m) = \sum_{m'=-s}^{s} |r\mathbf{p}, m')D^{(s)}_{m'm}(r), \tag{3.14}$$

which is just like Eq. (3.3). Using Eq. (3.6) we can derive the identity

$$r^{-1}(\hat{\mathbf{p}}) = \exp(i\pi j_2)\exp(2i\phi j_3)r^{-1}(-\hat{\mathbf{p}}), \tag{3.15}$$

which when combined with Eqs. (3.12) and (3.13) leads to

$$\mathcal{P}|\mathbf{p}m) = |-\mathbf{p}, m)\eta, \tag{3.16}$$

which is just like Eq. (3.11).

## IV. PHOTON-BARYON AND MESON-BARYON STATES

Our photon-baryon states are obtained by boosting photon-baryon states from the photon-baryon center-of-mass (c.m.) frame according to

$$|\mathbf{pq}m_\gamma m_b t_b\rangle = U[l_c(p)]|\mathbf{q}m_\gamma) \otimes |-\mathbf{q}, s_b m_b i_b t_b\rangle, \tag{4.1a}$$

$$\rho = \tanh^{-1}[|\mathbf{p}|/E_{\gamma b}(\mathbf{p},\mathbf{q})], \tag{4.1b}$$

where the boost is the canonical boost defined by Eq. (3.2) but with $\rho$ given by Eq. (4.1b), $\mathbf{p}$ is the total three-momentum, and $\mathbf{q}$ is the photon's c.m. three-momentum. Here $|\mathbf{q}m_\gamma)$ and $|-\mathbf{q}, s_b m_b i_b t_b\rangle$ are defined by Eqs. (3.13) and (3.1), respectively; with the baryon's total isospin and three-component given by $i_b$ and $t_b$, respectively. The total four-momentum is given by

$$p = [E_{\gamma b}(\mathbf{p},\mathbf{q}), \mathbf{p}], \tag{4.2a}$$

$$E_{\gamma b}(\mathbf{p},\mathbf{q}) = [\mathbf{p}^2 + W^2_{\gamma b}(\mathbf{q})], \tag{4.2b}$$

$$W_{\gamma b}(\mathbf{q}) = \omega_\gamma(\mathbf{q}) + \varepsilon_b(\mathbf{q}), \tag{4.2c}$$

with $\omega_\gamma$ and $\varepsilon_b$ the photon and baryon c.m. energy, respectively. It follows from Eqs. (3.3), (3.14), (3.11), (3.16), and (3.9) that under spatial rotations and inversions, the states (4.1) transform according to

$$U(r)|\mathbf{pq}m_\gamma m_b t_b\rangle = \sum_{m'_\gamma m'_b} |r\mathbf{p}, r\mathbf{q}, m'_\gamma m'_b t_b\rangle D^{(s_\gamma)}_{m'_\gamma m_\gamma}(r) D^{(s_b)}_{m'_b m_b}(r), \tag{4.3}$$

$$P|\mathbf{pq}m_\gamma m_b t_b\rangle = |-\mathbf{p}, -\mathbf{q}, m_\gamma m_b t_b\rangle \eta_\gamma \eta_b. \tag{4.4}$$

We refer to the states (4.1) as "plane-wave" states. We define partial-wave states by

$$|\mathbf{p}q(gs_\gamma)ls_b, jmt_b\rangle = \sum_{m_l m_b}\sum_{m_g m_\gamma} \int |\mathbf{pq}m_\gamma m_b t_b\rangle d\Omega_{\mathbf{q}} Y^{m_g}_g(\hat{\mathbf{q}}) \times \langle gs_\gamma m_g m_\gamma | lm_l\rangle\langle ls_b m_l m_b | jm\rangle, \tag{4.5}$$

which transform according to

$$U(r)|\mathbf{p}q(gs_\gamma)ls_b, jmt_b\rangle = \sum_{m'} |r\mathbf{p}, q(gs_\gamma)ls_b, jm't_b\rangle D^{(j)}_{m'm}(r), \tag{4.6}$$

$$\mathcal{P}|\mathbf{p}q(gs_\gamma)ls_b, jmt_b\rangle = |-\mathbf{p}, q(gs_\gamma)ls_b, jmt_b\rangle \eta_\gamma\eta_b(-1)^g. \tag{4.7}$$

Here $Y^{m_g}_g(\hat{\mathbf{q}})$ is a spherical harmonic and the $\langle|\rangle$'s are Clebsch-Gordon coefficients.

For the purpose of dealing with gauge invariance, it is convenient to defined special linear combinations of the states (4.5), which we denote by $|\mathbf{p}qnls_b, jmt_b\rangle$, with $n=0,1,2$; i.e.,

$$|\mathbf{p}q0ls_b, jmt_b\rangle = |\mathbf{p}q(l-1, s_\gamma)ls_b, jmt_b\rangle\sqrt{\frac{l}{2l+1}} - |\mathbf{p}q(l+1, s_\gamma)ls_b, jmt_b\rangle\sqrt{\frac{l+1}{2l+1}}, \tag{4.8a}$$

$$|\mathbf{p}q1ls_b, jmt_b\rangle = |\mathbf{p}q(l, s_\gamma)ls_b, jmt_b\rangle, \tag{4.8b}$$

$$|\mathbf{p}q2ls_b, jmt_b\rangle = |\mathbf{p}q(l-1, s_\gamma)ls_b, jmt_b\rangle\sqrt{\frac{l+1}{2l+1}} + |\mathbf{p}q(l+1, s_\gamma)ls_b, jmt_b\rangle\sqrt{\frac{l}{2l+1}}. \tag{4.8c}$$

These states transform according to

$$U(r)|\mathbf{p}qnls_b, jmt_b\rangle = \sum_{m'} |r\mathbf{p}, qnls_b, jm't_b\rangle D^{(j)}_{m'm}(r), \tag{4.9}$$

$$\mathcal{P}|\mathbf{p}qnls_b, jmt_b\rangle = |-\mathbf{p}, qnls_b, jmt_b\rangle \eta_\gamma\eta_b(-1)^{n+l+1}. \tag{4.10}$$

Meson-baryon states are defined similar to Eq. (4.1), but with $(\gamma, b) \to (\mu, B)$ in Eqs. (4.1b) and (4.2a)–(4.2c). We write

$$|\mathbf{p}qm_\mu m_B t_\mu t_B\rangle = U[l_c(p)]|\mathbf{q}s_\mu m_\mu i_\mu t_\mu\rangle \otimes |-\mathbf{q}, s_B m_B i_B t_B\rangle. \tag{4.11}$$

Partial wave, meson-baryon states $|\mathbf{p}q(gs_\mu)ls_B, jmt_\mu t_B\rangle$ are defined as in Eq. (4.5), and transform similar to Eqs. (4.6) and (4.7).

## V. VERTEX FUNCTIONS, POTENTIALS, AND AMPLITUDES

### A. The transition operator

The transition operator which arises from the mass operator (2.9) is defined by

$$T(z) = U + U\frac{1}{z-M}U, \tag{5.1}$$

where $z$ is a complex parameter which is given by $z=W+i\varepsilon$ for a physical process. The operators **P**, **S**, and **X** commute with $M$ and $U$, and therefore with the transition operator $T(z)$. We assume that the parity operator $\mathcal{P}$ also commutes with $M$, $U$, and $T(z)$. As far as isospin is concerned, we make the standard assumption that $M$, $U$, and $T(z)$ can be decomposed into an isoscalar and the third component of an isovector, e.g.,

$$U = U_s + U_3. \tag{5.2}$$

### B. Photon-baryon↔baryon vertex functions

As a result of the commutation rules (2.10), a partial wave matrix element of the interaction that couples a photon-baryon state to a single-baryon state can be expressed in the form [57,64]

$$\begin{aligned}\langle \mathbf{k}s_Bm_Bi_Bt_B|U|\mathbf{p}qnls_b, jmt_b\rangle &= (2\pi)^3 2[\varepsilon_B(\mathbf{k})E_{\gamma b}(\mathbf{p},q)]^{1/2}\\ &\times \delta^3(\mathbf{k}-\mathbf{p})\delta_{s_Bj}\delta_{m_Bm}\delta_{t_Bt_b}\\ &\times \frac{U_{B,\gamma b}(qnlt_b)}{2[M_BW_{\gamma b}(q)]^{1/2}}, \end{aligned}\tag{5.3a}$$

$$U_{B,\gamma b}(qnlt_b) = 0 \text{ if } \eta_B \neq \eta_\gamma\eta_b(-1)^{n+l+1}. \tag{5.3b}$$

We can justify this form as follows. The commutator $[U,\mathbf{P}]=0$ leads to the $\delta^3(\mathbf{k}-\mathbf{p})$, while the commutator $[U,\mathbf{X}]=0$ implies that $U_{B,\gamma b}(qnlt_b)$ is independent of $\mathbf{k}=\mathbf{p}$. Since $[U,\mathbf{S}]=0$, it follows from Eq. (2.8b) that $[U,\mathbf{J}]=0$ and therefore $U^{-1}(r)UU(r)=U$. Along with Eqs. (3.3) and (4.9), this in turn implies that

$$\begin{aligned}&\langle \mathbf{k}s_Bm_Bi_Bt_B|U|\mathbf{p}qnls_b, jmt_b\rangle\\ &\quad = \sum_{m_B'm'} D^{(s_B)*}_{m_B'm_B}(r)\langle \mathbf{k}s_Bm_B'i_Bt_B|U|\mathbf{p}qnls_b, jm't_b\rangle D^{(j)}_{m'm}(r).\end{aligned}$$

If we integrate over the parameters that label $r$ and use the identity

$$\int d\tau_r D^{(s_B)*}_{m_B'm_B}(r)D^{(j)}_{m'm}(r) = \frac{N}{2j+1}\delta_{s_Bj}\delta_{m_Bm}\delta_{m_B'm'}, \quad N=\int d\tau_r,$$

we have justified the factor $\delta_{s_Bj}\delta_{m_Bm}$ in Eq. (5.3a) along with the fact that $U_{B,\gamma b}(qnlt_b)$ is independent of $m_B=m$.

We now use Eq. (5.3a) to find the structure of the plane-wave matrix elements $\langle \mathbf{k}s_Bm_Bi_Bt_B|U|\mathbf{pq}m_\gamma m_bt_b\rangle$. We introduce spherical three-vectors $\boldsymbol{\varepsilon}_m$ by

$$\boldsymbol{\varepsilon}_\pm = (\mp 1, -i, 0)/\sqrt{2}, \quad \boldsymbol{\varepsilon}_0 = (0,0,1), \tag{5.4}$$

along with $(2s_b+1)$-dimensional baryon spin vectors $\chi^{s_b}_{m_b}$, and use them to define the generalized vector harmonics,

$$\begin{aligned}\mathbf{Y}^m_{(gs_\gamma)ls_bj}(\hat{\mathbf{q}}) &= \sum_{m_lm_b}\sum_{m_gm_\gamma} Y^{m_g}_g(\hat{\mathbf{q}})\boldsymbol{\varepsilon}_{m_\gamma}\chi^{s_b}_{m_b}\langle gs_\gamma m_gm_\gamma|lm_l\rangle\\ &\quad\times\langle ls_bm_lm_b|jm\rangle.\end{aligned}\tag{5.5}$$

In parallel with Eq. (4.8) we also define the vector harmonics

$$\mathbf{Z}^m_{0ls_bj}(\hat{\mathbf{q}}) = \mathbf{Y}^m_{(l-1,s_\gamma)ls_bj}(\hat{\mathbf{q}})\sqrt{\frac{l}{2l+1}} - \mathbf{Y}^m_{(l+1,s_\gamma)ls_bj}(\hat{\mathbf{q}})\sqrt{\frac{l+1}{2l+1}}, \tag{5.6a}$$

$$\mathbf{Z}^m_{1ls_bj}(\hat{\mathbf{q}}) = \mathbf{Y}^m_{(l,s_\gamma)ls_bj}(\hat{\mathbf{q}}), \tag{5.6b}$$

$$\mathbf{Z}^m_{2ls_bj}(\hat{\mathbf{q}}) = \mathbf{Y}^m_{(l-1,s_\gamma)ls_bj}(\hat{\mathbf{q}})\sqrt{\frac{l+1}{2l+1}} + \mathbf{Y}^m_{(l+1,s_\gamma)ls_bj}(\hat{\mathbf{q}})\sqrt{\frac{l}{2l+1}}. \tag{5.6c}$$

By using the orthogonality of the Clebsch-Gordon coefficients and the completeness relation for the spherical harmonics, we can invert Eq. (4.5) to express the plane-wave states $|\mathbf{pq}m_\gamma m_bt_b\rangle$ in terms of the partial-wave states $|\mathbf{p}q(gs_\gamma)ls_b, jmt_b\rangle$, and then with the help of Eqs. (4.8), (5.5), and (5.6), we can show that

$$|\mathbf{pq}m_\gamma m_bt_b\rangle = \sum_{jm}\sum_{nl}|\mathbf{p}qnls_b, jmt_b\rangle \mathbf{Z}^{m\dagger}_{nls_bj}(\hat{\mathbf{q}})\cdot\boldsymbol{\varepsilon}_{m_\gamma}\chi^{s_b}_{m_b}. \tag{5.7}$$

Combining this relation with Eq. (5.3a), we arrive at the plane-wave matrix element

$$\begin{aligned}&\langle \mathbf{k}s_Bm_Bi_Bt_B|U|\mathbf{pq}m_\gamma m_bt_b\rangle\\ &\quad = (2\pi)^3 2[\varepsilon_B(\mathbf{k})E_{\gamma b}(\mathbf{p},q)]^{1/2}\delta^3(\mathbf{k}-\mathbf{p})\delta_{t_Bt_b}\\ &\quad\quad\times\chi^{s_B\dagger}_{m_B}\frac{\mathbf{U}_{B,\gamma b}(\mathbf{q},t_b)}{2[M_BW_{\gamma b}(\mathbf{q})]^{1/2}}\cdot\boldsymbol{\varepsilon}_{m_\gamma}\chi^{s_b}_{m_b},\end{aligned}\tag{5.8}$$

where the vertex function $\mathbf{U}_{B,\gamma b}(\mathbf{q},t_b)$ is given by

$$\mathbf{U}_{B,\gamma b}(\mathbf{q},t_b) = \sum_{m_Bl}\sum_{n=1}^{2}\chi^{s_B}_{m_B}U_{B,\gamma b}(qnlt_b)\mathbf{Z}^{m_B\dagger}_{nls_bs_B}(\hat{\mathbf{q}}). \tag{5.9}$$

We note that we have excluded the $n=0$ term from the sum on $n$. We now show that this is required by gauge invariance.

We define alternative plane-wave states by

$$\begin{aligned}|\mathbf{pq}\lambda_\gamma m_bt_b\rangle &= \sum_{m_\gamma}|\mathbf{pq}m_\gamma m_bt_b\rangle D^{(s_\gamma)}_{m_\gamma\lambda_\gamma}[r(\hat{\mathbf{q}})]\\ &= U[l_c(p)]|\mathbf{q}\lambda_\gamma\rangle\otimes|-\mathbf{q}, s_bm_bi_bt_b\rangle,\end{aligned}\tag{5.10}$$

where the second equality follows from Eqs. (4.1a) and (3.13). Now we have

$$\langle \mathbf{k} s_B m_B i_B t_B | U | \mathbf{p} \mathbf{q} \lambda_\gamma m_b t_b \rangle = (2\pi)^3 2[\varepsilon_B(\mathbf{k}) E_{\gamma b}(\mathbf{p}, q)]^{1/2} \delta^3(\mathbf{k}-\mathbf{p}) \delta_{t_B t_b} \times \chi_{m_B}^{s_B \dagger} \frac{\mathbf{U}_{B,\gamma b}(\mathbf{q}, t_b)}{2[M_B W_{\gamma b}(\mathbf{q})]^{1/2}} \cdot \boldsymbol{\varepsilon}(\mathbf{q}, \lambda_\gamma) \chi_{m_b}^{s_b}, \tag{5.11}$$

where the polarization vector $\boldsymbol{\varepsilon}(\mathbf{q}, \lambda_\gamma)$ is given by

$$\boldsymbol{\varepsilon}(\mathbf{q}, \lambda_\gamma) = \sum_{m_\gamma} \boldsymbol{\varepsilon}_{m_\gamma} D^{(s_\gamma)}_{m_\gamma \lambda_\gamma}[r(\hat{\mathbf{q}})] = r(\hat{\mathbf{q}}) \boldsymbol{\varepsilon}_{\lambda_\gamma}, \quad \lambda_\gamma = \pm 1. \tag{5.12}$$

Gauge invariance requires that Eq. (5.11) should be invariant under the replacement

$$\boldsymbol{\varepsilon}(\mathbf{q}, \lambda_\gamma) \to \boldsymbol{\varepsilon}(\mathbf{q}, \lambda_\gamma) + \text{const}(\mathbf{q}). \tag{5.13}$$

Using the development given in Sec. 25 of Rose [65], we can show that

$$\mathbf{Z}^m_{0 l s_b j}(\hat{\mathbf{q}}) = \hat{\mathbf{q}} Y^m_{l s_b j}(\hat{\mathbf{q}}), \tag{5.14a}$$

$$\mathbf{Z}^m_{1 l s_b j}(\hat{\mathbf{q}}) = \frac{1}{\sqrt{l(l+1)}} (i \nabla_{\mathbf{q}} \times \mathbf{q}) Y^m_{l s_b j}(\hat{\mathbf{q}}), \tag{5.14b}$$

$$\mathbf{Z}^m_{2 l s_b j}(\hat{\mathbf{q}}) = -i \hat{\mathbf{q}} \times \mathbf{Z}^m_{1 l s_b j}(\hat{\mathbf{q}}), \tag{5.14c}$$

with

$$Y^m_{l s_b j}(\hat{\mathbf{q}}) = \sum_{m_l m_b} Y^{m_l}_l(\hat{\mathbf{q}}) \chi^{s_b}_{m_b} \langle l s_b m_l m_b | j m \rangle. \tag{5.15}$$

We see that excluding the $n=0$ term in Eq. (5.9) ensures that Eq. (5.11) is invariant under the replacement Eq. (5.13).

### C. Meson-baryon↔baryon vertex functions

We now consider the meson-baryon↔baryon vertices. We recall that our notation for the meson-baryon, partial-wave states is $|\mathbf{p} q (g s_\mu) l s_B, j m t_\mu t_B\rangle$. Since the strong interaction conserves isospin, it is convenient to define eigenstates of the total isospin by

$$|\mathbf{p} q (g s_\mu) l s_b, j m, i t\rangle = \sum_{t_\mu t_b} |\mathbf{p} q (g s_\mu) l s_b, j m t_\mu t_b\rangle \langle i_\mu i_b t_\mu t_b | i t\rangle. \tag{5.16}$$

By using the same sort of arguments as those that led to Eq. (5.3), we can show that the partial-wave matrix element for the transition $B \to \mu + b$ has the structure

$$\langle \mathbf{p} q (g s_\mu) l s_b, j m, i t | U | \mathbf{k} s_B m_B i_B t_B \rangle = (2\pi)^3 2[E_{\mu b}(\mathbf{p}, q) \varepsilon_B(\mathbf{k})]^{1/2} \delta^3(\mathbf{p}-\mathbf{k}) \delta_{i i_B} \delta_{t t_B} \delta_{j s_B} \delta_{m m_B} \times \frac{U_{\mu b, B}(q g l)}{2[W_{\mu b}(q) M_B]^{1/2}}, \tag{5.17a}$$

$$U_{\mu b, B}(q g l) = 0 \text{ if } \eta_\mu \eta_b (-1)^g \neq \eta_B. \tag{5.17b}$$

The same sort of procedure that led to Eq. (5.7) can be used to show that the meson-baryon, plane-wave states are related to the partial-wave states by

$$|\mathbf{p} \mathbf{q} m_\mu m_b, i t\rangle = \sum_{jm} \sum_{gl} |\mathbf{p} q (g s_\mu) l s_b, j m, i t\rangle Y^{m\dagger}_{(g s_\mu) l s_b j}(\hat{\mathbf{q}}) \zeta^{s_\mu}_{m_\mu} \chi^{s_b}_{m_b}, \tag{5.18}$$

where $Y^m_{(g s_\mu) l s_b j}(\hat{\mathbf{q}})$ is defined similar to Eq. (5.5) but with $\boldsymbol{\varepsilon}_{m_\gamma}$ replaced with the $(2s_\mu+1)$-dimensional meson spin vector $\zeta^{s_\mu}_{m_\mu}$. It follows from Eqs. (5.18) and (5.17) that the plane-wave matrix element for the transition $B \to \mu + b$ is given by

$$\langle \mathbf{p} \mathbf{q} m_\mu m_b, i t | U | \mathbf{k} s_B m_B i_B t_B \rangle = (2\pi)^3 2[E_{\mu b}(\mathbf{p}, q) \varepsilon_B(\mathbf{k})]^{1/2} \delta^3(\mathbf{p}-\mathbf{k}) \delta_{i i_B} \delta_{t t_B} \times \zeta^{s_\mu \dagger}_{m_\mu} \chi^{s_b \dagger}_{m_b} \frac{U_{\mu b, B}(\mathbf{q})}{2[W_{\mu b}(q) M_B]^{1/2}} \chi^{s_B}_{m_B}, \tag{5.19}$$

with

$$U_{\mu b, B}(\mathbf{q}) = \sum_{m_B g l} Y^{m_B}_{(g s_\mu) l s_b s_B}(\hat{\mathbf{q}}) U_{\mu b, B}(q g l) \chi^{s_B \dagger}_{m_B}. \tag{5.20}$$

### D. Photon-baryon↔meson-baryon potentials

For the process $\gamma + b \to \mu + B$ the partial-wave matrix element of the interaction has the form

$$\langle \mathbf{p}' q' (g s_\mu) l' s_B, j' m', i t | U | \mathbf{p} q n l s_b, j m t_b \rangle = (2\pi)^3 2[E_{\mu B}(\mathbf{p}', q') E_{\gamma b}(\mathbf{p}, q)]^{1/2} \times \delta^3(\mathbf{p}'-\mathbf{p}) \delta_{t t_b} \delta_{j' j} \delta_{m' m} \frac{U^j_{\mu B, \gamma b}(q' g l' i, q n l t_b)}{2[W_{\mu B}(q') W_{\gamma b}(q)]^{1/2}}, \tag{5.21a}$$

$$U^j_{\mu B, \gamma b}(q' g l' i, q n l t_b) = 0 \text{ if } \eta_\mu \eta_B (-1)^g \neq \eta_\gamma \eta_b (-1)^{n+l+1}. \tag{5.21b}$$

Using Eqs. (5.18), (5.10), (5.7), and (5.12), we can show that the plane-wave matrix element is given by

$$\langle \mathbf{p}' \mathbf{q}' m_\mu m_B, i t | U | \mathbf{p} \mathbf{q} \lambda_\gamma m_b t_b \rangle = (2\pi)^3 2[E_{\mu B}(\mathbf{p}', \mathbf{q}') E_{\gamma b}(\mathbf{p}, \mathbf{q})]^{1/2} \delta^3(\mathbf{p}'-\mathbf{p}) \delta_{t t_b} \times \zeta^{s_\mu \dagger}_{m_\mu} \chi^{s_B \dagger}_{m_B} \frac{\mathbf{U}_{\mu B, \gamma b}(\mathbf{q}' i, \mathbf{q} t_b)}{2[W_{\mu B}(\mathbf{q}') W_{\gamma b}(\mathbf{q})]^{1/2}} \cdot \boldsymbol{\varepsilon}(\mathbf{q}, \lambda_\gamma) \chi^{s_b}_{m_b}, \tag{5.22}$$

where

$$\mathbf{U}_{\mu B, \gamma b}(\mathbf{q}' i, \mathbf{q} t_b) = \sum_{jm} \sum_{g l' l} \sum_{n=1}^{2} Y^m_{(g s_\mu) l' s_B j}(\hat{\mathbf{q}}') \times U^j_{\mu B, \gamma b}(q' g l' i, q n l t_b) \mathbf{Z}^{m\dagger}_{n l s_b j}(\hat{\mathbf{q}}). \tag{5.23}$$

It should be noted that Eq. (5.23) also gives the most general form for the amplitude for the physical photoproduction process $\gamma+b\rightarrow\mu+B$, and thereby provides a generalization for the CGLN amplitudes for pion-nucleon photoproduction [2].

### E. Photon-nucleon↔pion-nucleon potentials

An obviously important special case of the result (5.23) is for the transition $\gamma+N\rightarrow\pi+N$ for which $b=B=N$, $\mu=\pi$, $s_N=1/2$, $s_\pi=0$, $\eta_\pi=\eta_\gamma=-1$, $\eta_N=1$. Since $s_\pi=0$ we can use $Y^m_{(g0)l',1/2,j}(\hat{\mathbf{q}}')=\delta_{gl'}Y^m_{l,'1/2,j}(\hat{\mathbf{q}}')$ where the right hand side is defined by Eq. (5.15). Looking at Eqs. (5.14) and (5.15) we see that we must have $l',l=j\mp1/2$ which when combined with the parity constraint (5.21b) implies that the nonzero terms in Eq. (5.23) must have $l'=l$ and $n=1$ or $l'=2j-l$ and $n=2$. The result for the transition $\gamma+N\rightarrow\pi+N$ is

$$\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t_N)=\sum_{jml}Y^m_{l,1/2,j}(\hat{\mathbf{q}}')[U^j_{1l}(q'i,qt_N)\mathbf{Z}^{m\dagger}_{1,l,1/2,j}(\hat{\mathbf{q}})+U^j_{2l}(q'i,qt_N)\mathbf{Z}^{m\dagger}_{2,2j-l,1/2,j}(\hat{\mathbf{q}})], \tag{5.24a}$$

$$U^j_{1l}(q'i,qt_N)=U^j_{\pi N,\gamma N}(q',l,l,i;q,1,l,t_N), \tag{5.24b}$$

$$U^j_{2l}(q'i,qt_N)=U^j_{\pi N,\gamma N}(q',l,l,i;q,2,2j-l,t_N). \tag{5.24c}$$

The well-known CGLN representation [2] can be used to obtain formulas for the partial-wave amplitudes $U^j_{1l}$ and $U^j_{2l}$ in terms of the plane-wave amplitudes. We have

$$\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t_N)=\boldsymbol{\sigma}_\perp U_1(\mathbf{q}'i,\mathbf{q}t_N)+i(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}')(\hat{\mathbf{q}}\times\boldsymbol{\sigma})\times U_2(\mathbf{q}'i,\mathbf{q}t_N)+\hat{\mathbf{q}}'_\perp(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})U_3(\mathbf{q}'i,\mathbf{q}t_N)+\hat{\mathbf{q}}'_\perp(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})U_4(\mathbf{q}'i,\mathbf{q}t_N), \tag{5.25a}$$

$$\mathbf{v}_\perp=\mathbf{v}-\hat{\mathbf{q}}(\hat{\mathbf{q}}\cdot\mathbf{v}). \tag{5.25b}$$

The functions $U_n(\mathbf{q}'i,\mathbf{q}t_N)$ are invariant under spatial rotations and inversions. We can show that the various spin factors can be expanded according to

$$\boldsymbol{\sigma}_\perp=-4\pi\sqrt{2}\sum_m Y^m_{0,1/2,1/2}(\hat{\mathbf{q}}')\mathbf{Z}^{m\dagger}_{2,1,1/2,1/2}(\hat{\mathbf{q}}), \tag{5.26a}$$

$$i(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}')(\hat{\mathbf{q}}\times\boldsymbol{\sigma})=4\pi\sqrt{2}\sum_m Y^m_{1,1/2,1/2}(\hat{\mathbf{q}}')\mathbf{Z}^{m\dagger}_{1,1,1/2,1/2}(\hat{\mathbf{q}}), \tag{5.26b}$$

$$\hat{\mathbf{q}}'_\perp(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})=\frac{4\pi}{3\sqrt{2}}\left[2\sum_m Y^m_{1,1/2,1/2}(\hat{\mathbf{q}}')\mathbf{Z}^{m\dagger}_{1,1,1/2,1/2}(\hat{\mathbf{q}})-\sum_m Y^m_{1,1/2,3/2}(\hat{\mathbf{q}}')\mathbf{Z}^{m\dagger}_{1,1,1/2,3/2}(\hat{\mathbf{q}})-\sqrt{3}\sum_m Y^m_{1,1/2,3/2}(\hat{\mathbf{q}}')\mathbf{Z}^{m\dagger}_{2,2,1/2,3/2}(\hat{\mathbf{q}})\right], \tag{5.26c}$$

$$\hat{\mathbf{q}}'_\perp(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}')=-\frac{4\pi\sqrt{2}}{3}\sum_{j,m}Y^m_{2j-1,1/2,j}(\hat{\mathbf{q}}')\mathbf{Z}^{m\dagger}_{2,1,1/2,j}(\hat{\mathbf{q}}). \tag{5.26d}$$

We can expand the functions $U_n(\mathbf{q}'i,\mathbf{q}t_N)$ in the form

$$U_n(\mathbf{q}'i,\mathbf{q}t_N)=\sum_{l,m}Y^m_l(\hat{\mathbf{q}}')U_{nl}(q'i,qt_N)Y^{m*}_l(\hat{\mathbf{q}}), \tag{5.27a}$$

$$U_{nl}(q'i,qt_N)=2\pi\int_{-1}^{1}dxP_l(x)U_n(\mathbf{q}'i,\mathbf{q}t_N),\ x=\hat{\mathbf{q}}'\cdot\hat{\mathbf{q}}. \tag{5.27b}$$

Using the orthonormality of the $Y$'s and $\mathbf{Z}$'s, we can solve Eq. (5.24a) for the partial-wave amplitudes (5.24b) and (5.24c). Combining these results with Eqs. (5.25)–(5.27), we find that the integrals that occur are given by

$$\int d\Omega Y^{m'\dagger}_{l',1/2,j'}(\hat{\mathbf{q}})Y^M_L(\hat{\mathbf{q}})Y^m_{l,1/2,j}(\hat{\mathbf{q}})=\left[\frac{(2l'+1)(2j+1)(2l+1)}{4\pi}\right]^{1/2}(-1)^{j+1/2}\langle l'l00|L0\rangle\times\begin{Bmatrix} j & L & j' \\ l' & 1/2 & l\end{Bmatrix}\langle jLmM|j'm'\rangle, \tag{5.28a}$$

$$\int d\Omega \mathbf{Y}^{m'\dagger}_{(g',1)l',1/2,j'}(\hat{\mathbf{q}})\cdot Y^{M*}_L(\hat{\mathbf{q}})\mathbf{Y}^m_{(g,1)l,1/2,j}(\hat{\mathbf{q}})=\left[\frac{(2j'+1)(2g'+1)(2l'+1)(2l+1)(2g+1)}{4\pi}\right]^{1/2}\times(-1)^{L+l'+l+j'-1/2}\times\langle g'g00|L0\rangle\begin{Bmatrix} l & L & l' \\ g' & 1 & g\end{Bmatrix}\times\begin{Bmatrix} j & L & j' \\ l' & 1/2 & l\end{Bmatrix}\langle j'Lm''M|jm\rangle. \tag{5.28b}$$

Evaluating the Clebsch-Gordon coefficients and the $6j$ symbols, we find that the partial-wave amplitudes are given in terms of the CGLN amplitudes by the relations

$$U^{l+1/2}_{1l}=\frac{\sqrt{l(l+1)}}{l+1}\left[U_{1l}-U_{2,l+1}+\frac{1}{2l+1}(U_{3,l+1}-U_{3,l-1})\right],\quad l=1,2,3,\cdots \tag{5.29a}$$

$$U^{l-1/2}_{1l}=\frac{\sqrt{l(l+1)}}{l}\left[-U_{1l}+U_{2,l-1}+\frac{1}{2l+1}(U_{3,l-1}-U_{3,l+1})\right],\quad l=1,2,3,\cdots \tag{5.29b}$$

$$U_{2l}^{l+1/2}=\frac{\sqrt{(l+1)(l+2)}}{l+1}\left[-U_{1l}+U_{2,l+1}+\frac{l}{2l+1}(U_{3,l+1}-U_{3,l-1})+\frac{l+1}{2l+3}(U_{4,l+2}-U_{4,l})\right],\quad l=0,1,2,\cdots \tag{5.29c}$$

$$U_{2l}^{l-1/2}=\frac{\sqrt{(l-1)l}}{l}\left[U_{1l}-U_{2,l-1}+\frac{l+1}{2l+1}(U_{3,l+1}-U_{3,l-1})+\frac{l}{2l-1}(U_{4l}-U_{4,l-2})\right],\quad l=2,3,4,\cdots. \tag{5.29d}$$

The so-called charge amplitudes, for which both the final meson and baryon have definite three-components of isospin, are related to the amplitudes for which the final state has a definite total isospin by

$$\mathbf{U}_{\mu B,\gamma b}(\mathbf{q}'t_\mu t_B,\mathbf{q}t_b)=\sum_i \langle i_\mu i_B t_\mu t_B|i t_b\rangle \mathbf{U}_{\mu B,\gamma b}(\mathbf{q}'i,\mathbf{q}t_b). \tag{5.30}$$

If we compare this relation for the transition $\gamma+N\rightarrow\pi+N$ with Eq. (3.2) of Arndt *et al.* [66], we find that our isospin amplitudes are related to those commonly used in the literature by

$$\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}',i=1/2;\mathbf{q},t_N)=-\sqrt{3}\;{}_{p,n}H^{1/2},$$
$$\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}',i=3/2;\mathbf{q},t_N)=\sqrt{2/3}\;H^{3/2}. \tag{5.31}$$

## VI. THE *T* MATRIX

Our $T$ matrix is defined by Eqs. (5.1) and (2.9) and according to standard scattering theory [62] it satisfies the Lippmann-Schwinger equations

$$T(z)=U+T(z)\frac{1}{z-M_0}U=U+U\frac{1}{z-M_0}T(z). \tag{6.1}$$

In our model there are one-baryon channels, meson-baryon channels, and photon-baryon channels; accordingly, our interaction has the general structure

$$U=U_{11}+U_{12}+U_{21}+U_{22}, \tag{6.2}$$

where $U_{mn}$ couples an $m$-particle channel to an $n$-particle channel. We have shown previously [57,67] that it is possible to eliminate the one-baryon channels and replace $U$ by an effective interaction $V_{22}(z)$ which only couples two-particle channels. Instead of Eq. (6.1) we can work with the equations

$$T_{22}(z)=V_{22}(z)+T_{22}(z)\frac{1}{z-M_0}V_{22}(z)=V_{22}(z)+V_{22}(z)\frac{1}{z-M_0}T_{22}(z), \tag{6.3}$$

where

$$V_{22}(z)=U_{22}+\sum_{B,m_B,t_B}\int U_{21}|\mathbf{k}s_Bm_Bi_Bt_B\rangle\times\frac{d^3k}{(2\pi)^3 2\varepsilon_B(\mathbf{k})(z-m_B^{(0)})}\langle\mathbf{k}s_Bm_Bi_Bt_B|U_{12}, \tag{6.4}$$

with $m_B^{(0)}$ the bare mass of baryon $B$. For the photoproduction processes we work to first order in $e$, so the relevant $T$ operator is given by

$$T_{\mu B,\gamma b}(z)=V_{\mu B,\gamma b}(z)+\sum_{\mu',B'}T_{\mu B,\mu'B'}(z)\frac{1}{z-M_0}V_{\mu'B',\gamma b}(z)+\cdots. \tag{6.5}$$

When written out in terms of partial-wave amplitudes for a physical process this equation takes on the form

$$\begin{aligned}T^j_{\mu B,\gamma b}(q'gLi,qnlt_b;W+i\varepsilon)&=V^j_{\mu B,\gamma b}(q'gLi,qnlt_b;W)\\&+\sum_{\mu'B'}\sum_{g'L'}\int T^{ij}_{\mu B,\mu'B'}(q'gL,q''g'L';W+i\varepsilon)\\&\times\frac{q''^2dq''V^j_{\mu'B',\gamma b}(q''g'L'i,qnlt_b;W)}{\Delta_{\mu'B'}(q'')2W_{\mu'B'}(q'')[W+i\varepsilon-W_{\mu'B'}(q'')]},\end{aligned} \tag{6.6a}$$

$$W_{\mu B}(q')=W_{\gamma b}(q)=W, \tag{6.6b}$$

$$\Delta_{\mu B}(q)=(2\pi)^3 2\omega_\mu(q)\varepsilon_B(q)/W_{\mu B}(q). \tag{6.6c}$$

Here $T^{ij}_{\mu B,\mu'B'}$ is the $T$-matrix for meson-baryon scattering in the absence of electromagnetic interactions. It follows from Eq. (6.4) that the partial-wave, effective potentials are given by

$$\begin{aligned}V^j_{\mu B,\gamma b}(q'gLi,qnlt_b;W)&=U^j_{\mu B,\gamma b}(q'gLi,qnlt_b)+\sum_{B'}U_{\mu B,B'}(q'gL)\\&\times\frac{\delta_{js_{B'}}\delta_{ii_{b'}}}{2m_B(W-m^{(0)}_{B'})}U_{B',\gamma b}(qnlt_b).\end{aligned} \tag{6.7}$$

By comparing Eq. (5.29) with, for example, Eqs. (B1) and (B2) of Nozawa *et al.* [29], and using the conversion factors in Eq. (5.31), we find that the multipoles given in the literature and in the SAID database [68] are related to our partial-wave, $T$-matrix elements by

$$(L)_{2I,2J}NM=\frac{iC_{2I}}{2(4\pi)^2W\sqrt{L(L+1)}}\times T^J_{\pi N,\gamma N}(q',L,L,I;q,1,L,t_N;W+i\varepsilon), \tag{6.8a}$$

$$(L)_{2I,2J}NE = \frac{2(L-J)iC_{2I}}{2(4\pi)^2 W\sqrt{(2J-L)(2J-L+1)}} \times T^{J}_{\pi N,\gamma N}(q',L,L,I;q,2,2J-L,t_N;W+i\varepsilon), \tag{6.8b}$$

$$W_{\pi N}(q') = W_{\gamma N}(q) = W, \quad C_1 = -1/\sqrt{3}, \quad C_3 = \sqrt{3/2}. \tag{6.8c}$$

Here $L$, $I$, and $J$ are the relative orbital angular momentum, total isospin, and total angular momentum, respectively, of the final pion-nucleon state; $N=n$ or $p$ designates the target nucleon; and $M$ and $E$ refer to magnetic and electric multipoles, respectively.

## VII. INTERACTIONS AND EFFECTIVE LAGRANGIANS

### A. General formalism

We obtain our interactions from effective Lagrangians using a method due to Okubo [56]. The quantum field theory Hamiltonian is divided into a noninteracting part $H_0$ and an interaction $H_1$ according to

$$H_{\rm QFT} = H_0 + H_1, \tag{7.1}$$

where the eigenstates of $H_0$, designated here by $|\zeta\rangle$, are assumed known, and satisfy

$$H_0|\zeta\rangle = E(\zeta)|\zeta\rangle. \tag{7.2}$$

The Fock space of the field theory is divided into a subspace consisting of various single-baryon states, i.e., $|N\rangle, |\Delta\rangle, |R\rangle, \ldots$; and various meson-baryon and photon-nucleon states, i.e., $|\pi N\rangle, |\pi\Delta\rangle, |\gamma N\rangle, \ldots$; and the complement to this subspace. We denote the projection operator onto this subspace by $\Pi$ and the projection operator onto its orthogonal complement by $\Lambda$, so that

$$\Pi + \Lambda = 1. \tag{7.3}$$

The effective Hamiltonian in the $\Pi$ subspace is given to second order in $H_1$ [69] by

$$\langle\zeta|H_\Pi|\zeta'\rangle = \langle\zeta|H_{\rm QFT} + \frac{1}{2}H_1\left[\frac{\Lambda}{E(\zeta)-H_0} + \frac{\Lambda}{E(\zeta')-H_0}\right]H_1|\zeta'\rangle + \cdots. \tag{7.4}$$

The vertex functions for $\gamma+b\rightarrow B$ are obtained from the quantum field theory matrix element

$$\langle \mathbf{p}_B m_B t_B|H_1|\mathbf{p}_\gamma\lambda_\gamma, \mathbf{p}_b m_b t_b\rangle = (2\pi)^3\delta^3(\mathbf{p}_B - \mathbf{p}_\gamma - \mathbf{p}_b)\delta_{t_B t_b} H_{B,\gamma b}(\mathbf{p}_B m_B, \mathbf{p}_\gamma\lambda_\gamma m_b t_b). \tag{7.5}$$

If we put $\mathbf{k}=0$ in Eq. (5.11) and $\mathbf{p}_B=0$ in Eq. (7.5) and compare, we find

$$\chi^{s_B\dagger}_{m_B}\mathbf{U}_{B,\gamma b}(q,t_b)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)\chi^{s_b}_{m_b} = H_{B,\gamma b}(0m_B, \mathbf{q}\lambda_\gamma m_b t_b). \tag{7.6}$$

For the transition $\mu+b\rightarrow B$ the field theory matrix element is

$$\begin{aligned}\langle \mathbf{p}_\mu m_\mu, \mathbf{p}_b m_b, it|H_1|\mathbf{p}_B m_B t_B\rangle &= \sum_{t_\mu,t_b}\langle i_\mu i_b t_\mu t_b it\rangle\langle \mathbf{p}_\mu m_\mu t_\mu, \mathbf{p}_b m_b t_b|H_1|\mathbf{p}_B m_B t_B\rangle \\ &= (2\pi)^3\delta^3(\mathbf{p}_\mu + \mathbf{p}_b - \mathbf{p}_B)\delta_{ii_B}\delta_{tt_B}H_{\mu b,B}(\mathbf{p}_\mu m_\mu m_b, \mathbf{p}_B m_B).\end{aligned} \tag{7.7}$$

Comparing with Eq. (5.19) we find

$$\zeta^{s_\mu\dagger}_{m_\mu}\chi^{s_b\dagger}_{m_b}U_{\mu b,B}(\mathbf{q})\chi^{s_B}_{m_B} = H_{\mu b,B}(\mathbf{q}m_\mu m_b, \mathbf{0}m_B). \tag{7.8}$$

The interactions for $\gamma+b\rightarrow\mu+B$ are obtained from the matrix elements

$$\begin{aligned}\langle \mathbf{p}_\mu m_\mu, \mathbf{p}_B m_B, it|H_\Pi|\mathbf{p}_\gamma\lambda_\gamma, \mathbf{p}_b m_b t_b\rangle &= \sum_{t_\mu,t_B}\langle i_\mu i_B t_\mu t_B|it\rangle\langle \mathbf{p}_\mu m_\mu t_\mu, \mathbf{p}_B m_B t_B|H_\Pi|\mathbf{p}_\gamma\lambda_\gamma, \mathbf{p}_b m_b t_b\rangle \\ &= (2\pi)^3\delta^3(\mathbf{p}_\mu + \mathbf{p}_B - \mathbf{p}_\gamma - \mathbf{p}_b)\delta_{tt_b} \\ &\quad\times H_{\mu B,\gamma b}(\mathbf{p}_\mu m_\mu, \mathbf{p}_B m_B, i;\mathbf{p}_\gamma\lambda_\gamma, \mathbf{p}_b m_b t_b).\end{aligned} \tag{7.9}$$

We set $\mathbf{p}_\gamma+\mathbf{p}_b=\mathbf{p}=\mathbf{0}$ and compare to Eq. (5.22) with $\mathbf{p}=\mathbf{0}$ to obtain

$$\zeta^{s_\mu\dagger}_{m_\mu}\chi^{s_B\dagger}_{m_B}\mathbf{U}_{\mu B,\gamma b}(\mathbf{q}'i,\mathbf{q}t_b)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)\chi^{s_b}_{m_b} = H_{\mu B,\gamma b}(\mathbf{q}'m_\mu, -\mathbf{q}', m_B, i;\mathbf{q}\lambda_\gamma, -\mathbf{q}, m_{b,}t_b). \tag{7.10}$$

In order to deduce the forms $\mathbf{U}_{B,\gamma b}(q,t_b)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)$ and $\mathbf{U}_{\mu B,\gamma b}(\mathbf{q}'i,\mathbf{q}t_b)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)$ from the quantum field theory matrix elements we exploit the fact that these matrix elements are of the form $J\cdot\varepsilon$ with $k\cdot\varepsilon=0$, and in principle, $J\cdot k=0$. Assuming this is so, we can write $\varepsilon^0=-\mathbf{q}\cdot\boldsymbol{\varepsilon}/\omega_\gamma(\mathbf{q})=-\hat{\mathbf{q}}\cdot\boldsymbol{\varepsilon}$ and $J^0=-\hat{\mathbf{q}}\cdot\mathbf{J}$, which in turn implies that

$$J\cdot\varepsilon(k,\lambda) = -[\mathbf{J}-(\mathbf{J}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}]\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda) = -\mathbf{J}_\perp\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda). \tag{7.11}$$

### B. The Lagrangians

Here we give the effective Lagrangians that are used to construct our mass operator interactions. It is important to do so as there is some variation in the literature having to do with the coupling constant normalizations and the signs of some of the terms. We begin with the Lagrangians that lead to the so-called Born term for pion photoproduction.

$$\mathcal{L}_{\gamma NN}(x) = -e\bar{N}(x)\left[\frac{1+\tau_3}{2}A\!\!\!/(x) + \left(\frac{1+\tau_3}{2}\kappa_p + \frac{1-\tau_3}{2}\kappa_n\right)\frac{\sigma_{\mu\nu}}{2m_N}\partial^\mu A^\nu(x)\right]N(x), \tag{7.12}$$

$$\mathcal{L}_{\pi NN}(x) = -\frac{g_{\pi NN}}{2m_N}\bar{N}(x)\gamma_5\gamma_\mu \boldsymbol{\tau} N(x)\cdot\partial^\mu \boldsymbol{\pi}(x), \quad (7.13)$$

$$\mathcal{L}_{\pi N\gamma N}(x) = -\frac{g_{\pi NN}}{2m_N}e\bar{N}(x)\gamma_5\gamma_\mu[\boldsymbol{\tau}\times\boldsymbol{\pi}(x)]_3 N(x)A^\mu(x), \quad (7.14)$$

$$\mathcal{L}_{\gamma\pi\pi}(x) = e\{[\partial_\mu \boldsymbol{\pi}(x)]^\dagger \times \boldsymbol{\pi}(x)\}_3 A^\mu(x), \quad (7.15)$$

$$\mathcal{L}_{\rho NN}(x) = f_{\rho NN}\bar{N}(x)\left[\left(\gamma_\mu + \frac{\kappa_\rho}{2m_N}\sigma_{\mu\nu}\partial^\nu\right)\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu(x)\right]N(x), \quad (7.16)$$

$$\mathcal{L}_{\rho\pi\gamma}(x) = \frac{g_{\rho\pi\gamma}}{m_\pi}\varepsilon_{\alpha\beta\gamma\delta}[\partial^\alpha A^\beta(x)]\boldsymbol{\pi}(x)\cdot\partial^\gamma\boldsymbol{\rho}^\delta(x), \quad (7.17)$$

$$\mathcal{L}_{\omega NN}(x) = f_{\omega NN}\bar{N}(x)\left[\left(\gamma_\mu + \frac{\kappa_\omega}{2m_N}\sigma_{\mu\nu}\partial^\nu\right)\omega^\mu(x)\right]N(x), \quad (7.18)$$

$$\mathcal{L}_{\omega\pi\gamma}(x) = \frac{g_{\omega\pi\gamma}}{m_\pi}\varepsilon_{\alpha\beta\gamma\delta}[\partial^\alpha A^\beta(x)]\pi_0(x)\cdot\partial^\gamma\omega^\delta(x). \quad (7.19)$$

For the Lagrangians that describe the couplings to the $\Delta = P_{33}(1232)$ resonance we [10] take

$$\mathcal{L}_{\gamma N\Delta}(x) = \mathcal{L}^{(1)}_{\gamma N\Delta}(x) + \mathcal{L}^{(2)}_{\gamma N\Delta}(x), \quad (7.20a)$$

$$\mathcal{L}^{(1)}_{\gamma N\Delta}(x) = \frac{ieg_{1\gamma N\Delta}}{2m_N}\bar{\Delta}^\mu(x)\Theta_{\mu\lambda}(X)\gamma_\nu\gamma_5 T^\dagger_{N\Delta,3}N(x)F^{\nu\lambda}(x) + (\dagger), \quad (7.20b)$$

$$\mathcal{L}^{(2)}_{\gamma N\Delta}(x) = -\frac{eg_{2\gamma N\Delta}}{4m_N^2}\bar{\Delta}^\mu(x)\Theta_{\mu\nu}(Y)\gamma_5 T^\dagger_{N\Delta,3}[\partial_\lambda N(x)]F^{\nu\lambda}(x) + (\dagger), \quad (7.20c)$$

$$\Theta_{\mu\nu}(X) = g_{\mu\nu} + \left[\frac{1}{2}(1+4X)A + X\right]\gamma_\mu\gamma_\nu, \quad (7.20d)$$

$$F^{\mu\nu}(x) = \partial^\nu A^\mu(x) - \partial^\mu A^\nu(x), \quad (7.20e)$$

$$\mathcal{L}_{\pi N\Delta}(x) = -\frac{g_{\pi N\Delta}}{m_\pi}\bar{\Delta}^\mu(x)\mathbf{T}^\dagger_{N\Delta}N(x)\cdot\partial_\mu\boldsymbol{\pi}(x) + (\dagger). \quad (7.21)$$

Here the vector matrix $\mathbf{T}_{N\Delta}$ is an isospin transition operator [57].

For the Lagrangians that describe the coupling to the Roper resonance $R = P_{11}(1440)$, we take

$$\mathcal{L}_{\gamma NR}(x) = -e\bar{R}(x)\left(\frac{1+\tau_3}{2}\kappa_{Rp} + \frac{1-\tau_3}{2}\kappa_{Rn}\right) \times \frac{\sigma_{\mu\nu}}{m_N + m_R}\partial^\mu A^\nu(x)N(x) + (\dagger), \quad (7.22)$$

$$\mathcal{L}_{\pi NR}(x) = -\frac{g_{\pi NR}}{m_N + m_R}\bar{N}(x)\gamma_5\gamma_\mu\boldsymbol{\tau}R(x)\cdot\partial^\mu\boldsymbol{\pi}(x) + (\dagger), \quad (7.23)$$

$$\mathcal{L}_{\pi R\Delta}(x) = -\frac{g_{\pi R\Delta}}{m_\pi}\bar{\Delta}^\mu(x)\mathbf{T}^\dagger_{R\Delta}R(x)\cdot\partial_\mu\boldsymbol{\pi}(x) + (\dagger). \quad (7.24)$$

For the couplings to the $D = D_{13}(1520)$ resonance we use [13,14]

$$\mathcal{L}^{(1)}_{\gamma ND}(x) = -\frac{ie}{2m_N}\bar{D}^\mu(x)\Theta_{\mu\lambda}\gamma_\nu\left(\kappa_{Dp1}\frac{1+\tau_3}{2} + \kappa_{Dn1}\frac{1-\tau_3}{2}\right)N(x)F^{\nu\lambda}(x) + (\dagger), \quad (7.25a)$$

$$\mathcal{L}^{(2)}_{\gamma ND}(x) = \frac{e}{4m_N^2}\bar{D}^\mu(x)\Theta_{\mu\nu}\left(\kappa_{Dp2}\frac{1+\tau_3}{2} + \kappa_{Dn2}\frac{1-\tau_3}{2}\right) \times [\partial_\lambda N(x)]F^{\nu\lambda}(x) + (\dagger), \quad (7.25b)$$

$$\mathcal{L}_{\pi ND}(x) = \frac{g_{\pi ND}}{m_\pi}\bar{D}^\mu(x)\Theta_{\mu\nu}\boldsymbol{\tau}\gamma_5 N(x)\cdot\partial^\nu\boldsymbol{\pi}(x) + (\dagger), \quad (7.26)$$

$$\mathcal{L}_{\pi\Delta D}(x) = \frac{g_{\pi\Delta D}}{m_\pi}\bar{D}^\mu(x)[\mathbf{T}_{D\Delta}\cdot\boldsymbol{\pi}(x)]\Delta_\mu(x) + (\dagger). \quad (7.27)$$

We note that both the $R$ and the $D$ couple to not only the $\gamma N$ and $\pi N$ channels, but also to the $\pi\Delta$ channel.

For the $S = S_{11}(1535)$ couplings we take [13,14]

$$\mathcal{L}_{\gamma NS}(x) = -e\bar{S}(x)\left(\frac{1+\tau_3}{2}\kappa_{Sp} + \frac{1-\tau_3}{2}\kappa_{Sn}\right)\frac{\gamma_5\sigma_{\mu\nu}}{m_S - m_N}\partial^\mu A^\nu(x)N(x) + (\dagger), \quad (7.28)$$

$$\mathcal{L}_{\pi NS}(x) = \frac{g_{\pi NS}}{m_S - m_N}\bar{S}(x)\gamma_\mu\boldsymbol{\tau}N(x)\cdot\partial^\mu\boldsymbol{\pi}(x) + (\dagger), \quad (7.29)$$

$$\mathcal{L}_{\eta NS}(x) = \frac{g_{\eta NS}}{m_S - m_N}\bar{S}(x)\gamma_\mu N(x)\cdot\partial^\mu\eta(x) + (\dagger). \quad (7.30)$$

We see that the $S$ couples to the $\gamma N$, $\pi N$, and $\eta N$ channels.

For each interaction Lagrangian $\mathcal{L}_I(x)$, we take for the corresponding interaction Hamiltonian

$$H_I = -\int d^3x \mathcal{L}_I(x)\big|_{t=0}. \tag{7.31}$$

### C. The $\gamma+N\leftrightarrow N$ and $\pi+N\leftrightarrow N$ vertex functions

As our first application of the above relations, we construct vertex functions for $\gamma+N\leftrightarrow N$ and $\pi+N\leftrightarrow N$ from the effective Lagrangian densities (7.12) and (7.13).

It follows from Eq. (7.5) that

$$H_{N,\gamma N}(\mathbf{0}m', q\lambda mt) = e\bar{u}(p', m')\left[\delta_{tp}\gamma_\nu - i(\delta_{tp}\kappa_p + \delta_{tn}\kappa_n)\frac{\sigma_{\mu\nu}k^\mu}{2m_N}\right]\varepsilon^\nu(k,\lambda)u(p,m), \tag{7.32a}$$

$$p' = (m_N, \mathbf{0}),\ k = [\boldsymbol{\omega}_\gamma(\mathbf{q}), \mathbf{q}],\ p = [\varepsilon_N(\mathbf{q}), -\mathbf{q}]. \tag{7.32b}$$

Now using Eqs. (7.6) and (7.11) we find

$$\chi_{m'}^{1/2\dagger}\mathbf{U}_{N,\gamma N}(\mathbf{q},t)\chi_m^{1/2} = e\bar{u}(p',m')\boldsymbol{\gamma}_\perp\left[-\delta_{tp} + (\delta_{tp}\kappa_p + \delta_{tn}\kappa_n)\frac{W_{\gamma N}(\mathbf{q})\gamma^0 - m_N}{2m_N}\right]u(p,m), \tag{7.33}$$

and putting in the explicit forms for the Dirac spinors, we obtain the vertex function

$$\mathbf{U}_{N,\gamma N}(\mathbf{q},t) = ie\left[\frac{2m_N}{\varepsilon_N(\mathbf{q}) + m_N}\right]^{1/2}\left[\delta_{tp} + (\delta_{tp}\kappa_p + \delta_{tn}\kappa_n)\frac{W_{\gamma N}(\mathbf{q}) + m_N}{2m_N}\right](\mathbf{q}\times\boldsymbol{\sigma}). \tag{7.34}$$

Using Eq. (5.14) and the identity

$$\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}Y^m_{l,1/2,j}(\hat{\mathbf{q}}) = -Y^m_{2j-l,1/2,j}(\hat{\mathbf{q}}), \tag{7.35}$$

we can write

$$\mathbf{Z}^m_{1,1,1/2,1/2}(\hat{\mathbf{q}}) = -\frac{1}{\sqrt{2}}(i\nabla_{\mathbf{q}}\times\mathbf{q})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})Y^m_{0,1/2,1/2}(\hat{\mathbf{q}}) = \frac{i}{\sqrt{8\pi}}(\mathbf{q}\times\boldsymbol{\sigma})\chi_m^{1/2}, \tag{7.36}$$

which allows us to expand Eq. (7.34) in the form (5.9) and deduce that the partial-wave, vertex function is given by

$$U_{N,\gamma N}(qnlt) = -\delta_{n1}\delta_{l1}\sqrt{8\pi}eq\left[\frac{2m_N}{\varepsilon_N(q) + m_N}\right]^{1/2}\left[\delta_{tp} + (\delta_{tp}\kappa_p + \delta_{tn}\kappa_n)\frac{W_{\gamma N}(q) + m_N}{2m_N}\right]. \tag{7.37}$$

Using Eqs. (7.13), (7.7), and (7.8), we find

$$\chi_m^{1/2\dagger}U_{\pi N,N}(\mathbf{q})\chi_{m'}^{1/2} = -\sqrt{3}\ i\frac{g_{\pi NN}}{2m_N}\bar{u}(p_N,m)\gamma_5 \not{p}_\pi u(p'_N,m'), \tag{7.38a}$$

$$p_\pi = [\omega_\pi(\mathbf{q}), \mathbf{q}],\ p_N = [\varepsilon_N(\mathbf{q}), -\mathbf{q}],\ p'_N = (m_N, \mathbf{0}). \tag{7.38b}$$

Putting in the explicit forms for the Dirac spinors we find that the $\pi+N\leftrightarrow N$ vertex function can be expanded in the form (5.20) with

$$U_{\pi N,N}(qgl) = \delta_{gl}\delta_{l1}\sqrt{12\pi}\ ig_{\pi NN}\left[\frac{2m_N}{\varepsilon_N(q) + m_N}\right]^{1/2}\times\frac{W_{\pi N}(q) + m_N}{2m_N}q. \tag{7.39}$$

### D. The $\gamma+N\rightarrow\pi+N$ interactions

In constructing the interactions for the process $\gamma+N\rightarrow\pi+N$ that follow from Eqs. (7.9) and (7.10) it is convenient to introduce the notation

$$\langle k'u', p'm't'|H_{\Pi}|k\lambda, pmt\rangle = (2\pi)^3\delta^3(\mathbf{k}' + \mathbf{p}' - \mathbf{k} - \mathbf{p})\times V(k'u', p'm't'; k\lambda, pmt), \tag{7.40a}$$

where

$$k' = p'_\pi,\quad u' = t'_\pi,\quad p' = p'_N,\quad m' = m'_N,\quad t' = t'_N,$$

$$k = p_\gamma,\quad \lambda = \lambda_\gamma,\quad p = p_N,\quad m = m_N,\quad t = t_N. \tag{7.40b}$$

With this notation Eq. (7.10) becomes

$$\chi_{m'}^{1/2\dagger}\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}'i, \mathbf{q}t)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)\chi_m^{1/2} = \sum_{u',t'}\langle 1, 1/2, u't'|it\rangle V(k'u', p'm't'; k\lambda, pmt), \tag{7.41a}$$

$$k' = (\omega', \mathbf{q}'),\quad p' = (\varepsilon', -\mathbf{q}'),\quad k = (\omega, \mathbf{q}),\quad p = (\varepsilon, -\mathbf{q}), \tag{7.41b}$$

$$\omega' = \omega_\pi(\mathbf{q}'),\quad \varepsilon' = \varepsilon_N(\mathbf{q}'),\quad \omega = \omega_\gamma(\mathbf{q}),\quad \varepsilon = \varepsilon_N(\mathbf{q}). \tag{7.41c}$$

A $\gamma+N\rightarrow\pi+N$ interaction that comes from the $H_1$ term in Eq. (7.4) arises from the Lagrangian density (7.14) which leads to the so-called contact interaction

$$V^{\text{contact}}_{\pi N,\gamma N}(k'u', p'm't'; k\lambda, pmt) = i\frac{g_{\pi NN}}{2m_N}e\chi_{t'}^{1/2\dagger}(-u'\tau^\dagger_{u'})\chi_t^{1/2}\times\bar{u}(p',m')\gamma_5\gamma_\mu u(p,m)\varepsilon^\mu(k,\lambda). \tag{7.42}$$

We now consider interactions that come from the second-order term in Eq. (7.4). Here we use a slight variation of the Okubo method and replace the second-order term by a sum of terms of the form

$$\langle k'u',p'm't'|H_{\mu NN}\frac{\Lambda}{\omega+\varepsilon-H_0}H_{em} + H_{em}\frac{\Lambda}{\omega'+\varepsilon'-H_0}H_{\mu NN}|k\lambda,pmt\rangle. \tag{7.43}$$

For the direct and crossed nucleon interactions $\mu=\pi$ and $H_{em}=H_{\gamma NN}$. For the $\pi$ exchange interaction $\mu=\pi$ and $H_{em}=H_{\gamma\pi\pi}$. For the $\rho$ or $\omega$ exchange interactions $\mu=\rho$ or $\omega$ and $H_{em}=H_{\rho\pi\gamma}$ or $H_{\omega\pi\gamma}$, respectively. This procedure, which was introduced in Ref. [50], leads to a photoproduction Born term that looks like a Feynman diagram result with the four-momentum conserved at the electromagnetic vertex but not necessarily at the strong interaction vertex. Moreover, the complete Born term is gauge invariant. The various contributions are given by

$$V^{\text{direct}}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) = i\frac{g_{\pi NN}}{2m_N}e\chi^{1/2\dagger}_{t'}\tau^\dagger_{u'}\chi^{1/2}_t\bar{u}(p',m')\gamma_5\not{k}'\frac{\not{p}+\not{k}+m_N}{(p+k)^2-m_N^2}\times\left[\delta_{tp}\gamma_\mu+(\delta_{tp}\kappa_p+\delta_{tn}\kappa_n)i\sigma_{\mu\nu}\frac{k^\nu}{2m_N}\right]u(p,m)\varepsilon^\mu(k,\lambda), \tag{7.44}$$

$$V^{\text{cross}}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) = i\frac{g_{\pi NN}}{2m_N}e\chi^{1/2\dagger}_{t'}\tau^\dagger_{u'}\chi^{1/2}_t\bar{u}(p',m')\times\left[\delta_{t'p}\gamma_\mu+(\delta_{t'p}\kappa_p+\delta_{t'n}\kappa_n)i\sigma_{\mu\nu}\frac{k^\nu}{2m_N}\right]\times\frac{\not{p}'-\not{k}'+m_N}{(p'-k)^2-m_N^2}\gamma_5\not{k}'u(p,m)\varepsilon^\mu(k,\lambda), \tag{7.45}$$

$$V^{\pi}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) = i\frac{g_{\pi NN}}{2m_N}e\chi^{1/2\dagger}_{t'}(-u'\tau^\dagger_{u'})\chi^{1/2}_t\times\bar{u}(p',m')\gamma_5\frac{(\not{k}'-\not{k})(k-2k')_\mu}{(k'-k)^2-m_\pi^2}u(p,m)\varepsilon^\mu(k,\lambda), \tag{7.46}$$

$$V^{\rho}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) = ig_{\rho\pi\gamma}\frac{f_{\rho NN}}{m_\pi}\chi^{1/2\dagger}_{t'}\tau^\dagger_{u'}\chi^{1/2}_t Q^\rho_{\pi N,\gamma N}(k',p'm';k\lambda,pm), \tag{7.47}$$

$$V^{\omega}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) = ig_{\omega\pi\gamma}\frac{f_{\omega NN}}{m_\pi}\chi^{1/2\dagger}_{t'}\delta_{u'0}\chi^{1/2}_t Q^\omega_{\pi N,\gamma N}(k',p'm';k\lambda,pm), \tag{7.48}$$

with

$$Q^{\mu}_{\pi N,\gamma N}(k',p'm';k\lambda,pm) = \bar{u}(p',m')\left[\gamma_\delta+\frac{\kappa_\mu}{2m_N}i\sigma_{\delta\nu}(k'-k)^\nu\right]\times i\varepsilon_{\xi\alpha\beta\zeta}(k'-k)^\alpha k^\beta D^{\zeta\delta}(k'-k,m_\mu)u(p,m)\varepsilon^\xi(k,\lambda),\quad \mu=\rho,\omega, \tag{7.49a}$$

$$D^{\zeta\delta}(q,m)=\frac{-g^{\zeta\delta}+q^\zeta q^\delta/m^2}{q^2-m^2+i\varepsilon}. \tag{7.49b}$$

We collect the interactions that involve the coupling $g_{\pi NN}e$ in defining

$$\chi^{1/2\dagger}_{m'}\mathbf{U}^{\pi\pi N}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)\chi^{1/2}_m = \sum_{u',t'}\langle 1,1/2,u't'|it\rangle[V^{\text{direct}}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) + V^{\text{cross}}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) + V^{\text{contact}}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt) + V^{\pi}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt)], \tag{7.50}$$

and write for the vector meson interactions

$$\chi^{1/2\dagger}_{m'}\mathbf{U}^{\mu}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t)\cdot\boldsymbol{\varepsilon}(\mathbf{q},\lambda_\gamma)\chi^{1/2}_m = \sum_{u',t'}\langle 1,1/2,u't'|it\rangle V^{\mu}_{\pi N,\gamma N}(k'u',p'm't';k\lambda,pmt). \tag{7.51}$$

Upon inserting Eqs. (7.44)–(7.46) in Eq. (7.50) and using Eq. (7.41b), we find

$$\mathbf{U}^{\pi\pi N}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t) = i\frac{g_{\pi NN}}{2m_N}e(\varepsilon'+m_N)^{1/2}(\varepsilon+m_N)^{1/2}(-\sqrt{3})\times\sum_{u',t'}\langle 1,1/2,u',t'|i,t\rangle\langle 1,1/2,u',t'|1/2,t\rangle\times\{\delta_{tp}\mathbf{A}(\mathbf{q}',\mathbf{q})+(\delta_{tp}\kappa_p+\delta_{tn}\kappa_n)\mathbf{B}(\mathbf{q}',\mathbf{q}) + \delta_{t'p}\mathbf{C}(\mathbf{q}',\mathbf{q})+(\delta_{t'p}\kappa_p+\delta_{t'n}\kappa_n)\mathbf{D}(\mathbf{q}',\mathbf{q}) - u'[\mathbf{E}^c(\mathbf{q}',\mathbf{q})+\mathbf{E}(\mathbf{q}',\mathbf{q})]\}, \tag{7.52}$$

where we have also used

$$\chi^{1/2\dagger}_{t'}\tau^\dagger_{u'}\chi^{1/2}_t = -\sqrt{3}\langle 1,1/2,u',t'|1/2t\rangle. \tag{7.53}$$

The functions that appear are given by

$$\mathbf{A}(\mathbf{q}',\mathbf{q})=\frac{W'-m_N}{W+m_N}\boldsymbol{\sigma}_\perp+\frac{W'+m_N}{W-m_N}(\boldsymbol{\sigma}.\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma}), \tag{7.54a}$$

$$\mathbf{B}(\mathbf{q}',\mathbf{q})=-\frac{(W'-m_N)(W-m_N)}{2m_N(W+m_N)}\boldsymbol{\sigma}_\perp+\frac{(W'+m_N)(W+m_N)}{2m_N(W-m_N)}(\boldsymbol{\sigma}.\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma}), \tag{7.54b}$$

$$\mathbf{C}(\mathbf{q}',\mathbf{q})=\frac{1}{2(\varepsilon'\omega+\mathbf{q}'\cdot\mathbf{q})}\Big\{-2\mathbf{q}'_\perp(\boldsymbol{\sigma}\cdot\mathbf{x})(W'-W-2m_N)-2\mathbf{q}'_\perp(\boldsymbol{\sigma}.\mathbf{x}')(W'-W+2m_N)+[\boldsymbol{\sigma}_\perp(W-m_N)-(\boldsymbol{\sigma}.\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma})(W+m_N)](W'+W)\frac{m_N}{W}-2(\varepsilon'\omega+\mathbf{q}'\cdot\mathbf{q})[\boldsymbol{\sigma}_\perp+(\boldsymbol{\sigma}.\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma})]\Big\}, \tag{7.54c}$$

$$\mathbf{D}(\mathbf{q}',\mathbf{q})=\frac{1}{2(\varepsilon'\omega+\mathbf{q}'\cdot\mathbf{q})}\Big\{[\mathbf{q}'_\perp(\boldsymbol{\sigma}\cdot\mathbf{x})(W+m_N)+\mathbf{q}'_\perp(\boldsymbol{\sigma}.\mathbf{x}')\times(W-m_N)+\boldsymbol{\sigma}_\perp m_N(W-m_N)-(\boldsymbol{\sigma}.\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma})m_N(W+m_N)]\frac{(W'+W)}{W}-2(\varepsilon'\omega+\mathbf{q}'\cdot\mathbf{q})\times\Big[\boldsymbol{\sigma}_\perp\frac{(W'+m_N)}{2m_N}-(\boldsymbol{\sigma}.\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma})\frac{(W'-m_N)}{2m_N}\Big]\Big\}, \tag{7.54d}$$

$$\mathbf{E}^c(\mathbf{q}',\mathbf{q})=\boldsymbol{\sigma}_\perp+(\boldsymbol{\sigma}\cdot\mathbf{x}')i(\mathbf{x}\times\boldsymbol{\sigma}), \tag{7.54e}$$

$$\mathbf{E}(\mathbf{q}',\mathbf{q})=\frac{1}{\omega'\omega-\mathbf{q}'\cdot\mathbf{q}}[-\mathbf{q}'_\perp(\boldsymbol{\sigma}\cdot\mathbf{x})(W'-W-2m_N)-\mathbf{q}'_\perp(\boldsymbol{\sigma}\cdot\mathbf{x}')(W'-W+2m_N)]. \tag{7.54f}$$

Here

$$W=\omega+\varepsilon,\ W'=\omega'+\varepsilon', \tag{7.55}$$

$$\mathbf{x}=\frac{\mathbf{q}}{\varepsilon+m_N},\ \mathbf{x}'=\frac{\mathbf{q}'}{\varepsilon'+m_N}. \tag{7.56}$$

It should be noted that some care must be taken in combining the interaction (7.52) with the contributions coming from the second terms on the right hand sides of Eqs. (6.4) and (6.7) since these terms already contain poles when $z=W=m_N^{(0)}$, where $m_N^{(0)}$ is the bare nucleon mass. The bare pole term gets dressed by the interactions and leads to a pole at $z=W=m_N$ [67], therefore in order not to include the dressed pole twice we must drop the pole terms in Eqs. (7.54a) and (7.54b). This we do by making the replacements

$$\mathbf{A}(\mathbf{q}',\mathbf{q})\rightarrow\frac{W'-m_N}{W+m_N}\boldsymbol{\sigma}_\perp, \tag{7.57a}$$

$$\mathbf{B}(\mathbf{q}',\mathbf{q})\rightarrow-\frac{(W'-m_N)(W-m_N)}{2m_N(W+m_N)}\boldsymbol{\sigma}_\perp. \tag{7.57b}$$

After a great deal of algebra we find that the vector-meson potentials are given by

$$\mathbf{U}^\mu_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t)=\boldsymbol{\sigma}_\perp U_1^\mu(\mathbf{q}'i,\mathbf{q}t)+i(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}')(\hat{\mathbf{q}}\times\boldsymbol{\sigma})\times U_2^\mu(\mathbf{q}'i,\mathbf{q}t)+\hat{\mathbf{q}}'_\perp(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})U_3^\mu(\mathbf{q}'i,\mathbf{q}t)+\hat{\mathbf{q}}'_\perp(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}')U_4^\mu(\mathbf{q}'i,\mathbf{q}t), \tag{7.58}$$

where

$$U_n^\rho(\mathbf{q}'i,\mathbf{q}t)=ig_{\rho\pi\gamma}\frac{f_{\rho NN}}{m_\pi}\delta_{i,1/2}(-\sqrt{3})\times(\varepsilon'+m_N)^{1/2}(\varepsilon+m_N)^{1/2}F_n^\rho(\mathbf{q}',\mathbf{q}), \tag{7.59a}$$

$$U_n^\omega(\mathbf{q}'i,\mathbf{q}t)=ig_{\omega\pi\gamma}\frac{f_{\omega NN}}{m_\pi}\left[\delta_{i,1/2}\left(-\frac{2t}{\sqrt{3}}\right)+\delta_{i,3/2}\sqrt{\frac{2}{3}}\right]\times(\varepsilon'+m_N)^{1/2}(\varepsilon+m_N)^{1/2}F_n^\omega(\mathbf{q}',\mathbf{q}), \tag{7.59b}$$

with

$$F_1^\mu(\mathbf{q}',\mathbf{q})=\left\{-\left[(W'-m_N)+\frac{\kappa_\mu m_\pi^2}{2m_N}\right](W-m_N)+\left[1+\frac{\kappa_\mu}{2m_N}(W'+W-2m_N)\right](k'\cdot k)\right\}\times\frac{1}{(k'-k)^2-m_\mu^2}, \tag{7.60a}$$

$$F_2^\mu(\mathbf{q}',\mathbf{q})=\left\{-\left[(W'+m_N)-\frac{\kappa_\mu m_\pi^2}{2m_N}\right](W+m_N)+\left[1-\frac{\kappa_\mu}{2m_N}(W'+W+2m_N)\right](k'\cdot k)\right\}\times\frac{|\mathbf{x}'||\mathbf{x}|}{(k'-k)^2-m_\mu^2}, \tag{7.60b}$$

$$F_3^\mu(\mathbf{q}',\mathbf{q})=\left[1+\frac{\kappa_\mu}{2m_N}(W'-m_N)\right](W+m_N)\frac{q'|\mathbf{x}|}{(k'-k)^2-m_\mu^2}, \tag{7.60c}$$

$$F_4^\mu(\mathbf{q}',\mathbf{q})=\left[1-\frac{\kappa_\mu}{2m_N}(W'+m_N)\right](W-m_N)\frac{q'|\mathbf{x}'|}{(k'-k)^2-m_\mu^2}. \tag{7.60d}$$

### E. The Δ resonance

With the help of Eqs. (7.20), (7.31), (7.5), and (7.6), and the analysis of Ref. [57], we find that the vertex function for $\gamma+N\rightarrow\Delta$ is given by

$$\mathbf{U}_{\Delta,\gamma N}(\mathbf{q},t)=e\langle 1,1/2,0,t|3/2t\rangle\sqrt{2m_\Delta}[\varepsilon_N(\mathbf{q})+m_N]^{1/2}\times\Bigg\{\frac{g_{1\gamma N\Delta}}{2m_N}[-\hat{\mathbf{q}}\times(\hat{\mathbf{q}}\times\mathbf{S}^\dagger_{N\Delta})q(\boldsymbol{\sigma}\cdot\mathbf{x})+\mathbf{S}^\dagger_{N\Delta}(\boldsymbol{\sigma}\cdot\mathbf{q})-(\mathbf{S}^\dagger_{N\Delta}\cdot\mathbf{q})\boldsymbol{\sigma}]+\frac{g_{2\gamma N\Delta}}{4m_N^2}[qW_{\gamma N}(\mathbf{q})\hat{\mathbf{q}}\times(\hat{\mathbf{q}}\times\mathbf{S}^\dagger_{N\Delta})(\boldsymbol{\sigma}\cdot\mathbf{x})]\Bigg\}. \tag{7.61}$$

Here the spin transition vector matrix is defined by

$$\mathbf{X}_{bb'}=\sum_{m,n,n'}\boldsymbol{\varepsilon}_m\chi^b_n\langle 1,b,m,n|b',n'\rangle\chi^{b'\dagger}_{n'}=(-1)^{b-b'}\sqrt{\frac{2b'+1}{2b+1}}\mathbf{X}^\dagger_{b'b},\ \mathbf{X}=\mathbf{S},\mathbf{T}. \tag{7.62}$$

According to Eqs. (5.3a) and (4.8) the only nonzero, partial-wave vertex functions $U_{\Delta,\gamma N}(qnlt)$ are those for which $l=1,2$, which in combination with Eq. (5.3b) implies that for $n=1$ we have $l=1$ while for $n=2$ we have $l=2$. It follows from Eqs. (5.14) and (7.35), and the identity

$$(\mathbf{S}_{N\Delta}\cdot\hat{\mathbf{q}})\chi^{3/2}_m=\sqrt{4\pi/3}Y^m_{1,1/2,3/2}(\hat{\mathbf{q}}) \tag{7.63}$$

that

$$\mathbf{Z}^m_{1,1,1/2,3/2}(\hat{\mathbf{q}})=-i\sqrt{\frac{3}{8\pi}}(\hat{\mathbf{q}}\times\mathbf{S}_{N\Delta})\chi^{3/2}_m, \tag{7.64}$$

$$\mathbf{Z}^m_{2,2,1/2,3/2}(\hat{\mathbf{q}})=\frac{1}{\sqrt{8\pi}}[\hat{\mathbf{q}}\times(\hat{\mathbf{q}}\times\boldsymbol{\sigma})(\mathbf{S}_{N\Delta}\cdot\hat{\mathbf{q}})+(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}\times(\hat{\mathbf{q}}\times\mathbf{S}^\dagger_{N\Delta})]\chi^{3/2}_m. \tag{7.65}$$

Using the fact that $\boldsymbol{\sigma}=-\sqrt{3}\mathbf{S}_{NN}$ [57] and the definition (7.62), we can show that $\boldsymbol{\sigma}\cdot\mathbf{S}_{N\Delta}=0$. By writing out $0=(\boldsymbol{\sigma}\cdot\mathbf{V})(\boldsymbol{\sigma}\cdot\mathbf{S}_{N\Delta})$ we can show in turn that $\boldsymbol{\sigma}\times\mathbf{S}_{N\Delta}=-i\mathbf{S}_{N\Delta}$. Now by manipulating $-i\hat{\mathbf{q}}\times\mathbf{S}_{N\Delta}$ we arrive at the alternative expressions

$$\mathbf{Z}^m_{1,1,1/2,3/2}(\hat{\mathbf{q}})=\sqrt{\frac{3}{8\pi}}[\boldsymbol{\sigma}(\mathbf{S}_{N\Delta}\cdot\hat{\mathbf{q}})-(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\mathbf{S}_{N\Delta}]\chi^{3/2}_m,=\sqrt{\frac{3}{8\pi}}[(\boldsymbol{\sigma}\,.\,\hat{\mathbf{q}})\hat{\mathbf{q}}\times(\hat{\mathbf{q}}\times\mathbf{S}_{N\Delta})-\hat{\mathbf{q}}\times(\hat{\mathbf{q}}\times\boldsymbol{\sigma})\times(\mathbf{S}_{N\Delta}\cdot\hat{\mathbf{q}})]\chi^{3/2}_m. \tag{7.66}$$

Combining Eqs. (7.61), (7.64), and (7.65), we obtain the partial-wave vertex functions

$$U_{\Delta,\gamma,N}(q,n=1,l=1,t)=-\sqrt{\frac{4\pi}{3}}e\frac{\sqrt{3}q}{m_N}\sqrt{m_\Delta W_{\gamma N}(q)}G_M[W_{\gamma N}(q)], \tag{7.67a}$$

$$U_{\Delta,\gamma,N}(q,n=2,l=2,t)=-\sqrt{\frac{4\pi}{3}}e\frac{3q}{m_N}\sqrt{m_\Delta W_{\gamma N}(q)}G_E[W_{\gamma N}(q)], \tag{7.67b}$$

with

$$G_M(W)=\frac{m_N}{3}\left[(3W+m_N)\frac{g_{1\gamma N\Delta}}{2m_NW}-(W-m_N)\frac{g_{2\gamma N\Delta}}{4m_N^2}\right], \tag{7.68a}$$

$$G_E(W)=\frac{m_N}{3}(W-m_N)\left(\frac{g_{1\gamma N\Delta}}{2m_NW}-\frac{g_{2\gamma N\Delta}}{4m_N^2}\right). \tag{7.68b}$$

These results for $G_M(W)$ and $G_E(W)$ agree with Eq. (B7) of Nozawa *et al.* [29] when $W=m_\Delta$. It should be noted that there is ambiguity in these functions. For real photons, the Jones and Scadron [70] analysis of the $\gamma+N\rightarrow\Delta$ vertex, which is based on a consideration of covariants, leads to the following alternatives:

$$\tilde{G}_M(W)=\frac{m_N}{3}\left[(3W+m_N)\frac{g_{1\gamma N\Delta}}{2m_NW}-(W-m_N)\frac{W+m_\Delta}{2W}\frac{g_{2\gamma N\Delta}}{4m_N^2}\right], \tag{7.69a}$$

$$\tilde{G}_E(W)=\frac{m_N}{3}(W-m_N)\left(\frac{g_{1\gamma N\Delta}}{2m_NW}-\frac{W+m_\Delta}{2W}\frac{g_{2\gamma N\Delta}}{4m_N^2}\right). \tag{7.69b}$$

The two sets agree when $W=m_\Delta$.

Using Eqs. (7.7), (7.8), and (7.21), we find that the $\pi+N\rightarrow\Delta$ vertex function is given by

$$U_{\pi N,\Delta}(\mathbf{q})=-i\frac{g_{\pi N\Delta}}{m_\pi}[\varepsilon_N(\mathbf{q})+m_N]^{1/2}(2m_\Delta)^{1/2}(\mathbf{q}\cdot\mathbf{S}_{N\Delta}), \tag{7.70}$$

which with the help of Eq. (7.63), is easily shown to lead to the partial-wave vertex function

$$U_{\pi N,\Delta}(qgl)=-\delta_{gl}\delta_{l1}i\sqrt{\frac{4\pi}{3}}\frac{g_{\pi N\Delta}}{m_\pi}[\varepsilon_N(\mathbf{q})+m_N]^{1/2}(2m_\Delta)^{1/2}q. \tag{7.71}$$

The Lagrangian density (7.21) also leads to a vertex function for the virtual process $\pi+\Delta\rightarrow N$. According to Eq. (5.17), the only non zero partial-wave, vertex function $U_{\pi\Delta,N}(qgl)$ has $g=l=1$. We find

$$U_{\pi\Delta,N}(qgl) = -\delta_{gl}\delta_{l1} i4\sqrt{\frac{\pi}{3}}\frac{g_{\pi N\Delta}}{m_\pi} q[\varepsilon_\Delta(q)+m_\Delta]^{1/2} \times (2m_N)^{1/2}\frac{W_{\pi\Delta}(q)}{m_\Delta}. \tag{7.72}$$

Including the virtual process $N\to\pi+\Delta$ leads to the photoproduction of a $\pi$ and a $\Delta$ through the process $\gamma+N\to N\to\pi+\Delta$.

### F. The Roper resonance

The derivations of the Roper resonance, $R=P_{11}(1440)$, vertex functions are very similar to those for the nucleon since $R$ and $N$ have the same spin and isospin. For the processes $\gamma+N\to R$, $R\to\pi+N$, $R\to\pi+\Delta$ we find, respectively,

$$U_{R,\gamma N}(qnlt) = -\delta_{n1}\delta_{l1}\sqrt{8\pi} eq\left[\frac{2m_R}{\varepsilon_N(q)+m_N}\right]^{1/2} \times(\delta_{tp}\kappa_{Rp}+\delta_{tn}\kappa_{Rn})\frac{W_{\gamma N}(q)+m_N}{m_R+m_N}, \tag{7.73}$$

$$U_{\pi N,R}(qgl) = \delta_{gl}\delta_{l1}\sqrt{12\pi} i g_{\pi NR}\left[\frac{2m_R}{\varepsilon_N(q)+m_N}\right]^{1/2} \times\frac{W_{\pi N}(q)+m_N}{m_R+m_N}q, \tag{7.74}$$

$$U_{\pi\Delta,R}(qgl) = -\delta_{gl}\delta_{l1} i4\sqrt{\frac{\pi}{3}}\frac{g_{\pi R\Delta}}{m_\pi} \times q[\varepsilon_\Delta(q)+m_\Delta]^{1/2}(2m_R)^{1/2}\frac{W_{\pi\Delta}(q)}{m_\Delta}. \tag{7.75}$$

In our model the Roper resonance is involved in the processes $\gamma+N\to R\to\pi+N$ and $\gamma+N\to R\to\pi+\Delta$.

### G. The $D_{13}(1520)$ resonance

With the help of Eqs. (7.25), (7.31), (7.5), (7.6), (7.62), and (5.25b), we find that the vertex function for $\gamma+N\to D$ is given by

$$\mathbf{U}_{D,\gamma N}(\mathbf{q},t) = e(2m_D)^{1/2}[\varepsilon_N(\mathbf{q})+m_N]^{1/2} \times\Bigg\{\frac{(\kappa_{Dp1}\delta_{tp}+\kappa_{Dn1}\delta_{tn})}{2m_N}[(\mathbf{S}^\dagger_{ND})_\perp[W_{\gamma N}(\mathbf{q})-m_N] - (\mathbf{S}^\dagger_{ND}.\hat{\mathbf{q}})i(\hat{\mathbf{q}}\times\boldsymbol{\sigma})[\varepsilon_N(\mathbf{q})-m_N]] - \frac{(\kappa_{Dp2}\delta_{tp}+\kappa_{Dn2}\delta_{tn})}{4m_N^2}[(\mathbf{S}^\dagger_{ND})_\perp W_{\gamma N}(\mathbf{q})q]\Bigg\}. \tag{7.76}$$

We now make a partial-wave expansion of the form (5.9) for this vertex function. According to Eqs. (5.3) and (4.8) the only terms that contribute are those for which $n=1$, $l=2$ and $n=2$, $l=1$. Using Eqs. (5.14) and (7.63), we find

$$\mathbf{Z}^m_{1,2,1/2,3/2}(\hat{\mathbf{q}}) = \frac{i}{\sqrt{8\pi}}[(\hat{\mathbf{q}}\times\boldsymbol{\sigma})(\hat{\mathbf{q}}\cdot\mathbf{S}_{ND})+(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})(\hat{\mathbf{q}} \times\mathbf{S}_{ND})]\chi^{3/2}_m, \tag{7.77a}$$

$$\mathbf{Z}^m_{2,1,1/2,3/2}(\hat{\mathbf{q}}) = \sqrt{\frac{3}{8\pi}}[\mathbf{S}_{ND}-\hat{\mathbf{q}}(\hat{\mathbf{q}}\cdot\mathbf{S}_{ND})]\chi^{3/2}_m. \tag{7.77b}$$

Comparing Eqs. (7.64) and (7.65), and using a property of the Pauli matrices, we can derive the identity

$$i(\boldsymbol{\sigma}.\hat{\mathbf{q}})(\hat{\mathbf{q}}\times\mathbf{S}_{ND}) = \mathbf{S}_{ND}-\hat{\mathbf{q}}(\hat{\mathbf{q}}.\mathbf{S}_{ND})+i(\hat{\mathbf{q}}\times\boldsymbol{\sigma})(\hat{\mathbf{q}}.\mathbf{S}_{ND}), \tag{7.78}$$

which allows us to rewrite Eq. (7.84) as

$$\mathbf{Z}^m_{1,2,1/2,3/2}(\hat{\mathbf{q}}) = \frac{i}{\sqrt{8\pi}}[\mathbf{S}_{ND}-\hat{\mathbf{q}}(\hat{\mathbf{q}}\cdot\mathbf{S}_{ND})+2i(\hat{\mathbf{q}}\times\boldsymbol{\sigma}) \times(\hat{\mathbf{q}}.\mathbf{S}_{ND})]\chi^{3/2}_m. \tag{7.79}$$

It is now straightforward to show that the nonzero partial-wave, vertex functions are given by

$$U_{D,\gamma N}(q,n=1,l=2,t) = \frac{e}{2m_N}\sqrt{2\pi}(2m_D)^{1/2}(\kappa_{Dp1}\delta_{tp}+\kappa_{Dn1}\delta_{tn})\frac{q[W_{\gamma N}(q)-m_N]}{[2W_{\gamma N}(q)]^{1/2}}, \tag{7.80a}$$

$$U_{D,\gamma N}(q,n=2,l=1,t) = \frac{e}{2m_N}\sqrt{\frac{2\pi}{3}}(2m_D)^{1/2}q[2W_{\gamma N}(q)]^{1/2} \times\left[(\kappa_{Dp1}\delta_{tp}+\kappa_{Dn1}\delta_{tn})\frac{3W_{\gamma N}(q)+m_N}{2W_{\gamma N}(q)} - (\kappa_{Dp2}\delta_{tp}+\kappa_{Dn2}\delta_{tn})\frac{W_{\gamma N}(q)+m_N}{2m_N}\right]. \tag{7.80b}$$

Using results from Ref. [57], we can show that the $D\to\pi+N$ and $D\to\pi+\Delta$ partial-wave vertex functions are given, respectively, by

$$U_{\pi N,D}(q,g=2,l=2) = -\sqrt{4\pi} i\frac{g_{\pi ND}}{m_\pi}\left[\frac{2m_D}{\varepsilon_N(\mathbf{q})+m_N}\right]^{1/2} q^2, \tag{7.81}$$

TABLE I. Pole term contributions.

| Baryon | Vertex functions | Equations |
|---|---|---|
| $N$ | $U_{N,\gamma N}$ | (7.37) |
| | $U_{\pi N,N}$ | (7.39) |
| | $U_{\pi\Delta,N}$ | (7.72) |
| $\Delta=P_{33}(1232)$ | $U_{\Delta,\gamma N}$ | (7.67) and (7.69) |
| | $U_{\pi N,\Delta}$ | (7.71) |
| $R=P_{11}(1440)$ | $U_{R,\gamma N}$ | (7.73) |
| | $U_{\pi N,R}$ | (7.74) |
| | $U_{\pi\Delta,R}$ | (7.75) |
| $D=D_{13}(1520)$ | $U_{D,\gamma N}$ | (7.80) |
| | $U_{\pi N,D}$ | (7.81) |
| | $U_{\pi\Delta,D}$ | (7.82) |
| $S=S_{11}(1535)$ | $U_{S,\gamma N}$ | (7.83) |
| | $U_{\mu N,S}$ | (7.84) |

$$U_{\pi\Delta,D}(q,l,l) = -\sqrt{8\pi}\, i\frac{g_{\pi\Delta D}}{m_\pi}[\varepsilon_\Delta(q)+m_\Delta]^{1/2}(2m_D)^{1/2} \times[W_{\pi\Delta}(q)-m_\Delta]\left\{\delta_{l0}\left[1+\frac{\varepsilon_\Delta(q)-m_\Delta}{3m_\Delta}\right] - \delta_{l2}\frac{\varepsilon_\Delta(q)-m_\Delta}{3m_\Delta}\right\}. \tag{7.82}$$

### H. $S_{11}(1535)$ The resonance

The derivations of the vertex functions for the $S=S_{11}(1535)$ resonance from the Lagrangians (7.28)–(7.30) are very similar to those for the nucleon. For the processes $\gamma+N\to S$, $S\to\pi+N$, $S\to\eta+N$ we find, respectively,

$$U_{S,\gamma N}(qnlt) = \delta_{n2}\delta_{l1}\sqrt{8\pi}e(2m_S)^{1/2}[\varepsilon_N(q)+m_N]^{1/2}(\delta_{tp}\kappa_{Sp} + \delta_{tn}\kappa_{Sn})\frac{W_{\gamma N}(q)-m_N}{m_S-m_N}, \tag{7.83}$$

$$U_{\mu N,S}(qgl) = \delta_{gl}\delta_{l0}\begin{pmatrix}\sqrt{3}\\ -1\end{pmatrix}\sqrt{4\pi} i g_{\mu NS}[\varepsilon_N(q) + m_N]^{1/2}(2m_S)^{1/2}\frac{W_{\gamma N}(q)-m_N}{m_S-m_N}, \quad \mu=\pi,\ \eta. \tag{7.84}$$

In Eq. (7.84) the upper and lower factors go with $\pi$ and $\eta$ respectively. In our model the $S_{11}(1535)$ resonance is involved in the processes $\gamma+N\to S\to\pi+N$ and $\gamma+N\to S\to\eta+S$.

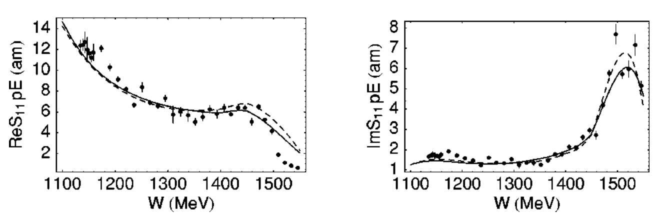


FIG. 1. $S_{11}pE$ multipoles. The solid lines are theory. The dashed lines and points with error bars are the SM95 [68,71] energy-dependent and single-energy values, respectively. The am unit (attometer)=mfm (milli Fermi).

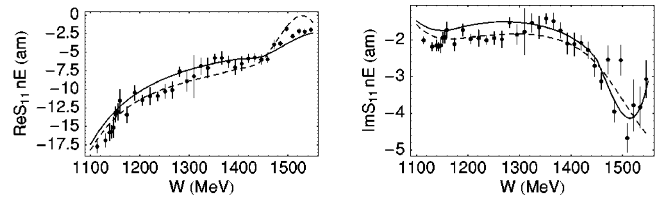


FIG. 2. $S_{11}nE$ multipoles. Curves and points as in Fig. 1.

## VIII. RESULTS

Here we present the results of our multipole calculations for the process $\gamma+N\to\pi+N$. The multipoles we calculate are designated by $(L)_{2I,2J}N(M \text{ or } E)$ where $(L)_{2I,2J}$ specifies the final pion-nucleon state with $L$ the relative orbital angular momentum, $I$ the total isospin, and $J$ the total angular momentum. The target is specified by $N=n$ or $p$, and $M$ and $E$ indicate magnetic and electric multipoles, respectively. These multipoles are related to the $T$-matrix elements by Eq. (6.8). The $T$-matrix elements are calculated from Eq. (6.6) with the electromagnetic potentials determined by Eq. (6.7). In Eq. (6.6) $b=N$, $\mu B=\pi N$, and $\mu' B'=\pi N$, $\pi\Delta$, $\eta N$. We recall that the quantum number $g$ that appears in Eq. (6.6) is the relative orbital angular momentum of $\mu$ and $B$, and $L$ is obtained by coupling $s_\mu$, the spin of $\mu$, to $g$. Since here $s_\mu=s_\pi=0$, we have $g=L$. In the rescattering term in Eq. (6.6a), $\mu'=\pi$ or $\eta$, so $g'=L'$.

In Eq. (6.7)

$$U^j_{\mu B,\gamma b}(q'LLi, qnlt_b) = \delta_{\mu\pi}\delta_{BN}\delta_{bN}U^j_{\pi N,\gamma N}(q'LLi, qnlt_N), \tag{8.1}$$

where the partial-wave matrix elements are obtained from the plane-wave matrix elements $\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}'i, \mathbf{q}t_N)$ by using Eqs. (5.24), (5.25), (5.27), and (5.29). These plane-wave matrix elements are given by

$$\mathbf{U}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t_N) = \mathbf{U}^{\pi\pi N}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t_N) + \sum_{\mu=\rho,\omega}\mathbf{U}^{\mu}_{\pi N,\gamma N}(\mathbf{q}'i,\mathbf{q}t_N) \tag{8.2}$$

with $\mathbf{U}^{\pi\pi N}_{\pi N,\gamma N}$ given by Eqs. (7.52), (7.54c)–(7.54f), and (7.57); and with $\mathbf{U}^{\mu}_{\pi N,\gamma N}$ given by Eqs. (7.58)–(7.60). The pole terms in Eq. (6.7) arise from the baryons listed in Table I, where the associated vertex functions and the equations that define them are also indicated. The strong interaction $T$ matrix $T^{ij}_{\mu B,\mu' B'}$ that appears in the rescatter-

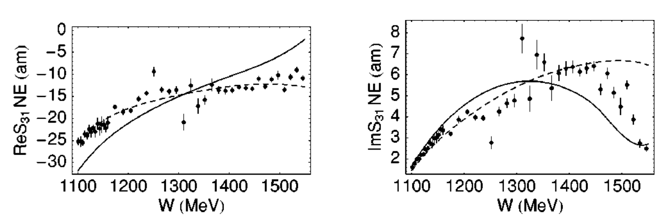


FIG. 3. $S_{31}NE$ multipoles. Curves and points as in Fig. 1.

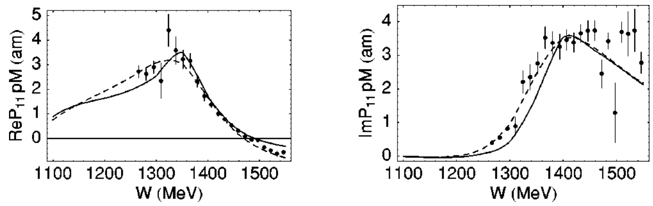


FIG. 4. $P_{11}pM$ multipoles. Curves and points as in Fig. 1.

ing term in Eq. (6.6a) is taken from the Elmessiri-Fuda model of pion-nucleon scattering [57] except for modifications in the parameters associated with the $P_{33}$ partial wave. In this partial wave the $\sigma$-exchange parameters were changed to improve the fit to the $P_{33}$ phase shifts and inelasticites. The new parameters are $g_{\sigma\pi\pi}g_{\sigma NN}/4\pi = 489.71$, $\tilde{g}_{\sigma\pi\pi}g_{\sigma NN}/4\pi = 541.06$, $m_\sigma = 1617.0$ MeV, $\Lambda_{\sigma\pi\pi} = 3347.5$ MeV, $\Lambda_{\sigma NN} = 3846.1$ MeV, and $m_\Delta^{\text{threshold}} = 1200.1$ MeV.

The strong interaction vertex functions given by the indicated equations in Table I are modified in practice in two ways. First of all, the coupling constants are replaced by bare coupling constants since the vertices are dressed by the interactions [67,57]. To be consistent with the notation of Ref. [57] we let $g_{\pi NN} \to g^{(0)}_{\pi NN}$, $g_{\pi N\Delta} \to g^{(0)}_{\pi N\Delta}$, etc. Second we modify the strong vertex functions in Eq. (6.7) by multiplying them by cutoff functions according to Ref. [57],

$$U_{\mu B,B'}(q'gL) \to f_{\mu B,B'}(q')U_{\mu B,B'}(q'gL), \tag{8.3}$$

$$f_{\mu B,B'}(q') = \left[\frac{\Lambda^{(0)2}_{\mu BB'} + q'^2_{\text{pole}}}{\Lambda^{(0)2}_{\mu BB'} + q'^2}\right]^n, \tag{8.4a}$$

$$q'^2_{\text{pole}} = [(m_\mu^2 + m_B^2 - m_{B'}^2)^2 - 4m_\mu^2 m_B^2]/(2m_{B'})^2. \tag{8.4b}$$

Here $\Lambda^{(0)}_{\mu BB'}$ is a cutoff mass and $q'_{\text{pole}}$ is determined by the condition

$$W_{\mu B}(q'_{\text{pole}}) = \omega_\mu(q'_{\text{pole}}) + \varepsilon_B(q'_{\text{pole}}) = m_{B'}, \tag{8.5}$$

which normalizes the cutoff function so that $f_{\mu B,B'}(q'_{\text{pole}}) = 1$. The strong interaction coupling constants $g^{(0)}_{\mu BB'}$, the cutoff masses $\Lambda^{(0)}_{\mu BB'}$, and the bare masses $m_B^{(0)}$, which appear in Eq. (6.7) are given in Table II of Ref. [57].

We also modify the electromagnetic vertex for the process $\gamma + N \leftrightarrow \Delta$ by making the replacement

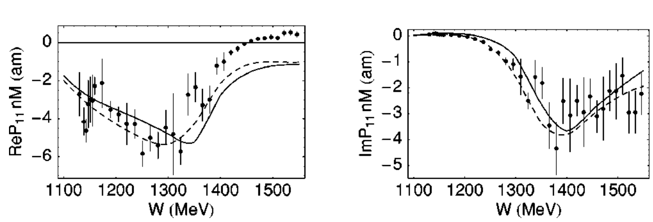


FIG. 5. $P_{11}nM$ multipoles. Curves and points as in Fig. 1.

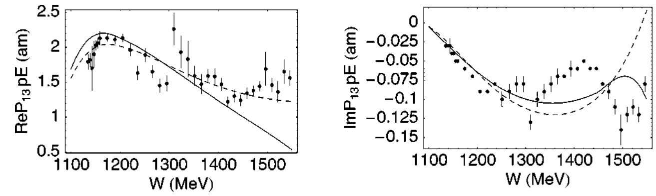


FIG. 6. $P_{13}pE$ multipoles. Curves and points as in Fig. 1.

$$g_{j\gamma N\Delta} \to g^{(0)}_{j\gamma N\Delta} f_{j\gamma N,\Delta}(q), \quad j = 1, 2 \tag{8.6}$$

in Eq. (7.69). This greatly improves our ability to fit the $P_{33}NE$ multipoles.

We put in a strong interaction cutoff function in the antinucleon contributions **A** and **B**, that appear in Eq. (7.52). Specifically we modify the **A** and **B** given by Eq. (7.57) according to

$$\mathbf{A}(\mathbf{q}', \mathbf{q}) \to f_{\pi N,\bar{N}}(|\mathbf{q}'|)\mathbf{A}(\mathbf{q}', \mathbf{q}),$$

$$\mathbf{B}(\mathbf{q}', \mathbf{q}) \to f_{\pi N,\bar{N}}(|\mathbf{q}'|)\mathbf{B}(\mathbf{q}', \mathbf{q}), \tag{8.7}$$

where $f_{\pi N,\bar{N}}$ is given by Eq. (8.4). The contact interaction term $\mathbf{E}^c$ that appears in Eq. (7.52) also requires a cutoff function, which we introduce according to

$$\mathbf{E}^c(\mathbf{q}', \mathbf{q}) \to \left(\frac{\Lambda_c^2}{\Lambda_c^2 + \mathbf{q}'^2}\right)^2 \mathbf{E}^c(\mathbf{q}', \mathbf{q}). \tag{8.8}$$

For the crossed contributions **C** and **D** and the exchange contribution **E** in Eq. (7.52); as well as the vector-meson exchange interactions $\mathbf{U}^\mu_{\pi N,\gamma N}$ given by Eqs. (7.58)–(7.60); we use cutoff functions defined by

$$F(Q, m_x, \Lambda_x) \equiv \left[\frac{\Lambda_x^4}{\Lambda_x^4 + (Q^2 - m_x^2)^2}\right]^2. \tag{8.9}$$

Here $m_x$ is the mass of the intermediate exchanged particle, $\Lambda_x$ is a cutoff mass, and $Q$ is a four-momentum associated with the exchanged particle. Since these cutoffs are associated with the strong interaction vertex, we determine $Q$ by conservation of four-momentum at the strong interaction vertex. The replacements are given by

$$\mathbf{C}(\mathbf{q}', \mathbf{q}) \to F(p - k', m_N, \Lambda_N)\mathbf{C}(\mathbf{q}', \mathbf{q}), \tag{8.10}$$

$$\mathbf{D}(\mathbf{q}', \mathbf{q}) \to F(p - k', m_N, \Lambda_N)\mathbf{D}(\mathbf{q}', \mathbf{q}), \tag{8.11}$$

$$\mathbf{E}(\mathbf{q}', \mathbf{q}) \to F(p' - p, m_\pi, \Lambda_\pi)\mathbf{E}(\mathbf{q}', \mathbf{q}), \tag{8.12}$$

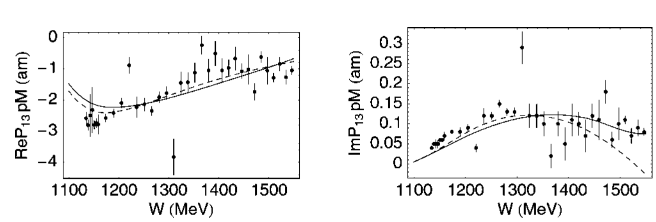


FIG. 7. $P_{13}pM$ multipoles. Curves and points as in Fig. 1.

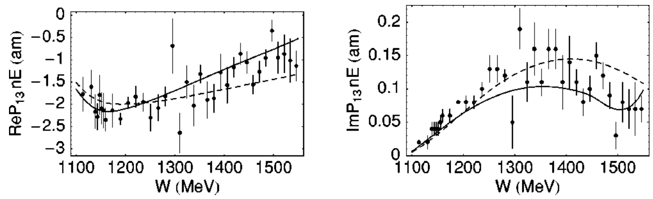


FIG. 8. $P_{13}nE$ multipoles. Curves and points as in Fig. 1.

$$\mathbf{U}^{\mu}_{\pi N,\gamma N}(\mathbf{q}',\mathbf{q}) \to F(p'-p,m_\mu,\Lambda_\mu)\mathbf{U}^{\mu}_{\pi N,\gamma N}(\mathbf{q}',\mathbf{q}), \quad \mu=\rho,\omega. \tag{8.13}$$

The four-momenta that appear in Eqs. (8.10)–(8.13) are defined in Eqs. (7.41b) and (7.41c). On-shell, i.e., when $k+p=k'+p'$, these cutoff functions become one at the pole of the accompanying Feynman propagator.

In our model there are a number of fixed parameters. The physical masses of the various particles are given in Ref. [57] except for the mass of the $\omega$ meson which we take to be $m_\omega$=782.6 MeV. The coupling constants that are fixed are given [24] by

$$e=\sqrt{4\pi/137}, \quad \kappa_p=1.79, \quad \kappa_n=-1.97,$$

$$g_{\pi NN}=\sqrt{4\pi(3.7815)}, \quad g_{\rho\pi\gamma}=0.103e, \quad g_{\omega\pi\gamma}=0.314e. \tag{8.14}$$

The parameters that were varied in fitting the multipoles are the strong interaction coupling constants for the vector mesons, the electromagnetic coupling constants for the $\Delta=P_{33}(1232)$, $R=P_{11}(1440)$, $D=P_{13}(1520)$, and $S=S_{11}(1535)$ resonances, and the cutoff parameters $\Lambda$ and $n$. We fit to the SM95 analysis [68,71] of the photoproduction data. We chose this analysis because the phases of the multipoles for energies below the two-pion production threshold are the pion-nucleon phase shifts to which we fit our model for pion-nucleon scattering [57]. Our fits are shown in Figs. 1–14 and the resulting parameters are given in Table II. We have also calculated the $E2/M1$ ratio from our fits. We find $E2/M1=P_{33}NE/P_{33}NM=-2.09\%$, which is consistent with the range $(-2.5\pm0.5)\%$ given by the Particle Data Group [72].

We note that the majority of the parameters in Table II are associated with the form factors, which are purely phenomenological. In commenting on the role of form factors in their photoproduction model, Surya and Gross [25] state "Unfortunately, our results are sensitive to the form factors, which are purely phenomenological." This has also been our experience. In Sec. IX we discuss how we propose to improve on the treatment of the form factors. Drechsel *et al.*

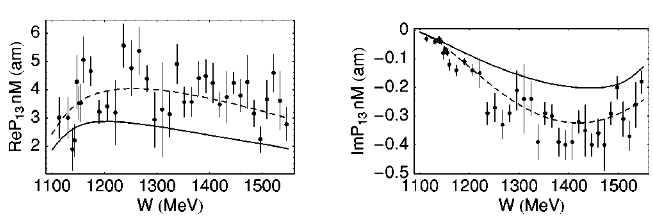


FIG. 9. $P_{13}nM$ multipoles. Curves and points as in Fig. 1.

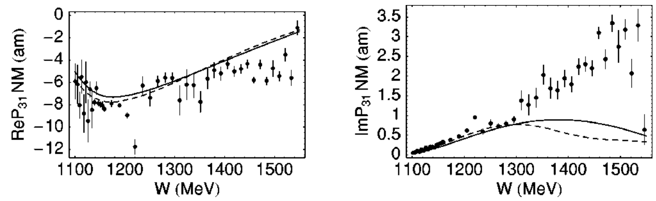


FIG. 10. $P_{31}NM$ multipoles. Curves and points as in Fig. 1.

[24] point out that the vector-meson strong coupling constants are not well determined. In surveying earlier results they found $8\leqslant f_{\omega NN}\leqslant 20$, $-1\leqslant\kappa_\omega\leqslant 0$, $1.8\leqslant f_{\rho NN}\leqslant 3.2$, $4.3\leqslant\kappa_\rho\leqslant 6.6$. For their fit to the multipoles they found $f_{\omega NN}=21.0$, $\kappa_\omega=-0.57$, $f_{\rho NN}=2.0$, $\kappa_\rho=6.5$. Our $f_{\omega NN}$, $\kappa_\omega$, and $f_{\rho NN}$ are in reasonable agreement with the ranges they have indicated, however, our $\kappa_\rho$ is quite different. In contemplating the bare electromagnetic coupling constants that characterize the strength of the various resonances, it should be kept in mind that these parameters are not observables. Wilhelm *et al.* [73] have shown that it is possible to introduce into a model such as ours a unitary transformation that alters the relative mix of the background and resonance contribution to a multipole without changing the total multipole amplitude. As a result of this the electromagnetic coupling constants given in Table II only have significance within the context of the present model.

In Figs. 1–14 the dots with the bars through them are the SM95 [68,71] single-energy values, the dashed lines are the SM95 energy-dependent fits, and the solid lines are our fits to the SM95 energy-dependent fits. We have labeled the isospin triplet plots with $N$ rather than $n$ or $p$ since these multipoles are independent of the target nucleon. There is generally good agreement between theory and the SM95 energy-dependent fits for the following multipoles: $S_{11}(p,n)E$, $P_{11}(p,n)M$, Re$[P_{13}pM]$, Re$[P_{31}NM]$, Im$[P_{33}NE]$, $P_{33}NM$, and Im$[D_{13}(p,n)E]$. There is mediocre agreement for $S_{31}NE$, however, it is interesting to note that the theoretical Im$[S_{31}NE]$ agrees better at high energies with the SM95 single-energy values then the SM95 energy-dependent fit does. The agreement for the various $P_{13}$ multipoles is mediocre except for Re$[P_{13}pM]$. It should be noted, however, that these multipoles are quite small and therefore get less attention in the least-squares fitting procedure. For Im$[P_{31}NM]$ both theory and the SM95 energy-dependent fit do not follow the single-energy values at high energies. At both the low- and high-energy ends the theory for Re$[P_{33}NE]$ lies above the SM95 energy-dependent fits.

It is satisfying that in the imaginary parts of the $P_{33}$, $P_{11}$, $D_{13}$, and $S_{11}$ multipoles the resonances show up very clearly at their physical masses. The nucleon and these resonances

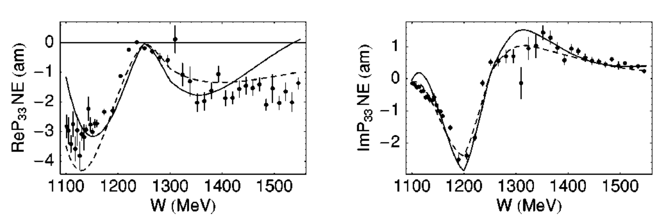


FIG. 11. $P_{33}NE$ multipoles. Curves and points as in Fig. 1.

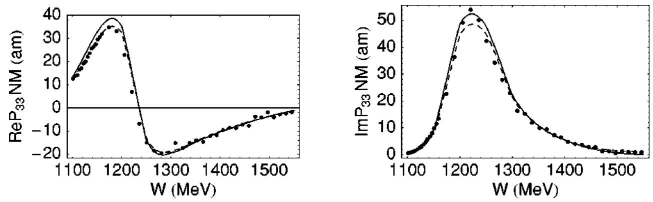


FIG. 12. $P_{33}NM$ multipoles. Curves and points as in Fig. 1.

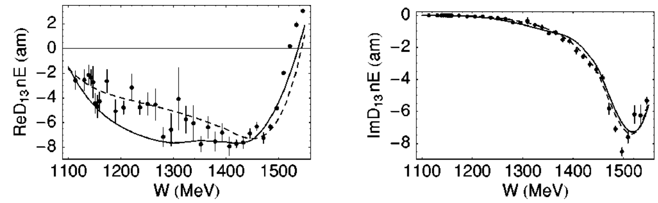


FIG. 14. $D_{13}nE$ multipoles. Curves and points as in Fig. 1.

contribute to the electromagnetic interaction through the second term on the right hand side of Eq. (6.7). The poles in these terms are determined by the bare masses of the particles, which are quite different from the physical masses (see Table II of Ref. [57]). The physical masses arise from the dynamics of the model and are not put in by hand, so it is quite clear that our way of treating resonances is satisfactory.

Recently, the multipole analysis of the photoproduction multipoles has been updated [74] to SM02. It would be inconsistent for us to fit to the SM02 analysis since the pion-nucleon phase shifts to which we fit our model [57] for $\pi N$ elastic scattering differ somewhat from the phases of the SM02 multipoles for energies below the $2\pi$ production threshold. It is interesting to note that our theoretical result for $\mathrm{Im}[S_{31}NE]$ agrees very well with the SM02 analysis. The $\mathrm{Re}[P_{33}NE]$ energy-dependent fit has changed quite a bit in going from SM95 to SM02. In fact, our low-energy $\mathrm{Re}[P_{33}NE]$ agrees quite well with SM02.

We do not wish to imply that our disagreements with SM95 are due simply to problems with the analysis of the experimental data. There are exchange processes that can be added to our model and these may very well improve agreement with the energy-dependent fits to experiment [68,71,74]. The present model only includes crossed processes with $\gamma\pi N$ and $NN\bar{N}$ intermediate states. Clearly, there are crossed processes with $\gamma\pi B$ and $NN\bar{B}$ intermediate states where $B$ is any of the resonances in the energy range. Also in the present model electromagnetic coupling to the inelastic channels is limited to the processes $\gamma N \to (N,R,D) \to \pi\Delta$ and $\gamma N \to S \to \eta N$. We should also consider crossed processes such as $\gamma N \to (\gamma\pi N, \Delta N\bar{N}) \to \pi\Delta$. We plan to improve our photoproduction model accordingly and to refit our $\pi N$ elastic scattering model [57] and the improved photoproduction model to the more recent analyses of the experimental data [74]. This will take some time.

It is of interest to compare the results of our photoproduction model with other dynamical models for photoproduction. The models that are closest to ours are the Hamiltonian models [27–33], but unfortunately there is not a lot to compare. The oldest models [27–29] employ separable potentials to describe $\pi N$ elastic scattering and so are not complete exchange models. The more recent models [30–33] are complete exchange models, but the authors do not give a complete set of multipoles. References [30] and [31] only give threshold results, while the Sato-Lee [32,33] papers only give $P_{33}NM$ and $P_{33}NE$ multipoles. The spaces of these models are limited to $\Delta \oplus \pi N \oplus \gamma N$ or $N \oplus \Delta \oplus \pi N \oplus \gamma N$, and do not include $\pi\Delta$ and/or $\eta N$ channels.

It appears that the only dynamical model that is comparable to ours is the Surya-Gross model [25]. This model goes up to a maximum photon lab energy of 770 MeV, while ours goes up to 810 MeV. They include the following direct poles: $N$, $P_{33}(1232)=\Delta$, $P_{11}(1440)=R$, and $D_{13}(1520)=D$. We include these, but also $S_{11}(1535)=S$. They include crossed $N$ and $R$ interactions, whereas we include the crossed $N$ but not the crossed $R$. We both include a contact or Kroll-Ruderman interaction [75], as well as $\pi$, $\rho$, and $\omega$ exchanges. They use a mixture of pseudoscalar and pseudovector $\pi\pi N$ coupling, while we use pure pseudovector coupling. In their model electromagnetic coupling to an inelastic channel is provided by the processes $\gamma + N \to (R,D) \to \sigma^* + N$ where $\sigma^*$ is an artificial scalar meson with the mass of two pions. Our inelastic electromagnetic coupling is due to the processes $\gamma + N \to (N,R,D) \to \pi + \Delta$ and $\gamma + N \to S \to \eta + N$. The dynamical equations in the two models are quite different. Gross and co-workers use a three-dimensional reduction [25,45,76] of the Bethe-Salpeter [43] equation in which the intermediate pion is put on-mass-shell, except for one of the pion pole $\gamma N$ driving terms. We on the other hand use standard Lippmann-Schwinger equations as described in Sec. VI. We feel that it

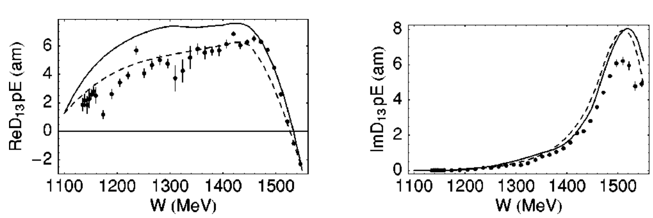


FIG. 13. $D_{13}pE$ multipoles. Curves and points as in Fig. 1.

TABLE II. Adjusted parameters.

| Interaction | parameters Cutoff masses (MeV) |
|---|---|
| $U^{\pi\pi N}_{\pi N,\gamma N}$ | $\Lambda^{(0)}_{\pi N\bar{N}}=813.4$, $n=10$, |
| | $\Lambda_N=815.7$, $n=5$, |
| | $\Lambda_c=1727.0$, $n=10$, |
| | $\Lambda_\pi=752.5$, $n=1$ |
| $U^{\rho}_{\pi N,\gamma N}$ | $f_{\rho NN}=3.206$, $\kappa_\rho=0.8935$, |
| | $\Lambda_\rho=1933.0$, $n=4$ |
| $U^{\omega}_{\pi N,\gamma N}$ | $f_{\omega NN}=7.119$, $\kappa_\omega=-1.092$, |
| | $\Lambda_\omega=2748.0$, $n=5$ |
| $U_{\Delta,\gamma N}$ | $g_{1\gamma N\Delta}=3.105$, $\Lambda^{(0)}_{1\gamma N\Delta}=1290.0$, $n=9$, |
| | $g_{2\gamma N\Delta}=-30.74$, $\Lambda^{(0)}_{2\gamma N\Delta}=442.6$, $n=9$ |
| $U_{R,\gamma N}$ | $\kappa_{Rp}=-0.2105$, $\kappa_{Rn}=0.9063$ |
| $U_{D,\gamma n}$ | $\kappa_{Dp1}=-67.21$, $\kappa_{Dp2}=-88.87$, |
| | $\kappa_{Dn1}=66.45$, $\kappa_{Dn2}=87.12$ |
| $U_{S,\gamma N}$ | $\kappa_{Sp}=0.2833$, $\kappa_{Sn}=-0.2026$ |

is fair to say that the Surya-Gross dynamical scheme requires a much more complicated treatment of gauge invariance than does our approach. In particular, the Surya-Gross model requires extra driving terms to satisfy gauge invariance, whereas in our approach gauge invariance is satisfied as long as the electromagnetic vertex interactions $\mathbf{U}_{B,\gamma b}$ and potentials $\mathbf{U}_{\mu B,\gamma b}$ have the structures (5.9) and (5.23), respectively.

Surya and Gross present results for the following multipoles: $S_{11}pE$, $S_{31}NE$, $P_{11}pM$, $P_{31}NM$, $P33NM$, $P33NE$, and $D_{13}pE$. Our $S_{11}pE$ results are quite similar to theirs, however, their $\mathrm{Im}[S_{11}pE]$ is off at the high energy end since they do not include the $S_{11}(1535)$ resonance. It must be admitted that their $\mathrm{Re}[S_{31}NE]$ is superior to ours, however, their $\mathrm{Im}[S_{31}NE]$ differs from SM02 [74] at high energies. Our $P_{11}pM$ results are quite similar to each other although our $\mathrm{Im}[P_{11}pM]$ agrees somewhat better with the SM95 energy-dependent analysis. The two fits for the $P_{31}NM$ and $P33NM$ multipoles are of comparable quality. The two fits for $P33NE$ are quite similar, however, the Surya-Gross result for $\mathrm{Re}[P33NE]$ is somewhat better than ours at the high energy end. The two fits for $D_{13}pE$ are of similar quality. Surya and Gross do not give results for the $P_{13}$ multipoles. In summary, we think it is fair to say that the two models give the same overall level of agreement with the multipoles extracted from the data. It is of some comfort that two such very different dynamical schemes yield such similar results.

## IX. DISCUSSION

The work of Nozawa and Lee [77] makes it quite clear that the formalism developed here can be extended to electroproduction. As is well known, electroproduction can be viewed as photoproduction by a space-like virtual photon. The mass operator developed here describes transitions between physical particle states, so in particular, the photon is lightlike. This would appear to present a problem, however, this is not the case. According to (6.6) and (6.7), the photon's four-momentum occurs only in the electromagnetic potentials $U_{\mu B,\gamma b}$ and vertex interactions $U_{B,\gamma b}$. These interactions can easily be extended to spacelike photons. In the approach pursued here, which is three dimensional in character, relative three-momenta play an essential role. The relative three-momentum of a particle such as the photon or pion is defined as the three-momentum of the particle in a c.m. frame. The relative three-momentum of the virtual photon can still be defined since the total four-momentum of the virtual photon –initial baryon system is timelike. The unphysical nature of the photon in the electroproduction process is due not only to its spacelike four-momentum, but also to the fact that it can have polarizations that are not transverse; they can also be scalar or longitudinal. Again, because of the fact that the photon's properties only appear in the electromagnetic potentials $U_{\mu B,\gamma b}$ and vertex interactions $U_{B,\gamma b}$, these unphysical polarizations do not cause a problem. Among other things, we will have to include the longitudinal vector functions $\mathbf{Z}^{m}_{0ls_bj}(\hat{\mathbf{q}})$ given by Eq. (5.14a) in the general expansions of $\mathbf{U}_{\mu B,\gamma b}$ and $\mathbf{U}_{B,\gamma b}$.

In the present work we have derived our mass operator interactions from effective Lagrangians using a variation of Okubo's method [50,56]. The resulting interactions have been modified by the introduction of purely phenomenological form factors or cutoff functions so as to take into account the extension of the hadrons. At the present time we are pursuing a more microscopic approach that starts with the constituent quark model. The virtue of such an approach is that the form factors emerge as a consequence of the quark wave functions. Other authors have already achieved some success in deriving meson-baryon and photon-baryon interactions from constituent quark models. At the present time it appears that the most tractable approach is based on what have been called *elementary meson emission* models (see, e.g., Refs. [22,78] ). In this class of models the mesons $\mu$ are treated as elementary particles that couple directly to the quarks $q$ through a vertex that describes the process $q \leftrightarrow q' + \mu$. This approach can be thought of as originating from an effective theory of hadrons discussed by Manohar and Georgi [79] in which a Lagrangian is constructed that describes the coupling of constituent quarks, gluons, and Goldstone bosons. Starting from such a Lagrangian along with one that describes the electromagnetic coupling of quarks and photons, it is possible to derive analytic expressions for meson-baryon and photon-baryon interactions that take into account the extended nature of the hadrons. These interactions can then be used to calculate meson-baryon reactions as well as the photoproduction and electroproduction of mesons from baryons. What we can bring to such an approach is a general framework that ensures that the probabilities for the various processes are Poincaré and gauge invariant.


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