PHYSICAL REVIEW C **68**, 061601(R) (2003)

# $\eta$-$\pi$ mixing close to the $\eta$-helium threshold

A. M. Green*
*Department of Physical Sciences and Helsinki Institute of Physics, P.O. Box 64, FIN–00014 University of Helsinki, Finland*

S. Wycech†
*Soltan Institute for Nuclear Studies, Warsaw, Poland*
(Received 25 August 2003; published 22 December 2003)

A $K$-matrix formalism is used to relate the amplitudes for the three reactions $pd \to {}^3\text{He}\,\eta$, $\pi^+\,{}^3\text{H} \to {}^3\text{He}\,\eta$, and $pd \to {}^3\text{H}\,\pi^+$. Free parameters are fitted to the available experimental data and an extrapolation below the $\eta^3$He threshold is made to see the origin of the $\eta^3$He threshold enhancement. The existence of a virtual—and not a quasibound—state finds support in the data. The $K$ matrix permits a discussion of $\eta$-$\pi$ mixing. A mixing parameter of 0.010(5), i.e., a mixing angle $\theta$=0.6(3)°, is extracted from a best fit to the very recent $pd \to {}^3\text{He}\,\pi^0$ reaction data.

DOI: 10.1103/PhysRevC.68.061601 PACS number(s): 13.75.−n, 25.80.−e, 25.40.Ve

In this paper we concentrate on the few-body interactions of $\eta$ mesons in three-nucleon systems. These complement our knowledge on the $\eta$-nucleon interaction and properties of the $\eta$ meson itself. Considerable experimental, phenomenological, and theoretical work has been devoted to understand the $\eta^3$He system. Early SATURNE experiments found a large cross section for the $pd \to \eta^3$He reaction close to the $\eta$ threshold [1]. One interesting feature of this process is an enhancement of the meson formation amplitude in the few million electron volt energy region close to the threshold. A similar effect was also noted in the studies of the $pn \to d\eta$ reactions made at CELSIUS [2]. Both of these reactions indicate the possibility for virtual or quasibound states to be formed in these systems. It is expected that in the deuteron the state of interest is a virtual state, whereas in helium it may also be virtual but there a bound state is not ruled out [3–5]. While the slopes of the formation amplitudes indicate the existence of such states, more detailed properties may be found only with an extrapolation of these amplitudes below the $\eta^3$He threshold. This is made possible with the pion production experiments $pd \to {}^3\text{H}\pi^+$ and $pd \to {}^3\text{He}\pi^0$ undertaken by COSY [6,7]. These results obtained at backward angles supplement the older SATURNE cross sections measured some distance away from the threshold [8].

In this work we present a multiple-channel $K$-matrix analysis of $\eta^3$He formation. In addition to the reactions listed above, we also include the data from the $\pi^+\,{}^3\text{H} \to {}^3\text{He}\,\eta$ process studied at Brookhaven [9]. Altogether the data comprise 17 measurements. The number of important $K$-matrix parameters, obtained with some minor theoretical input, amounts to four. Unfortunately the system is not strongly constrained, and so new data are welcome.

A parallel study is devoted to the effect of $\eta$-$\pi$ mixing in these systems. It has been suggested in Ref. [6] that such a mixing is enhanced by the existence of a $\eta^3$He bound state. This enhancement has been found in a subsequent experiment [7]. These two related questions are discussed in this paper. A full description, in the isospin symmetric limit, within a zero range approximation requires at least five real $K$-matrix elements. Couplings to the open three-body and four-body channels induce phases. Since the present data are not sufficient for a full determination of all the relevant $K$-matrix elements, a model of $S$(1535) dominance is used to remove some ambiguities.

Let the isospin invariant states of $\eta$ and $\pi^0$ mesons be described by $|\overline{\eta})$ and $|\overline{\pi})$. Due to some isospin mixing interaction $H_m$ these states mix into the physical states $|\eta)$ and $|\pi)$. The relation of these two equivalent sets of states is

$$|\eta) = N[|\overline{\eta}) - \theta|\overline{\pi})] \quad \text{and} \quad |\pi) = N[|\overline{\pi}) + \theta|\overline{\eta})], \tag{1}$$

where $\theta$ is a mixing parameter and $N = 1/\sqrt{1+\theta^2}$ is the normalization factor. These two sets of states form a complete orthonormal basis with the relations

$$(\eta|\pi) = 0 = (\overline{\eta}|\overline{\pi}) \quad \text{and} \quad (\overline{\eta}|\pi) = N\theta = -(\overline{\pi}|\eta). \tag{2}$$

The mixing parameter follows from interactions at the quark level due to differences in the light quark masses and to electromagnetic effects [10].

The transitions in the few-body systems may be analyzed in terms of the scattering matrices $T$ that lead to the physical $\eta$ and $\pi$ in the final states

$$T(\eta) = (pd|T|\eta^3\text{He}) \quad \text{and} \quad T(\pi) = (pd|T|\pi^3\text{He}). \tag{3}$$

On the other hand, when discussing the formation processes and final state $\eta$ interactions, that are supposed to be isospin conserving, it is more convenient to use the isospin basis

$$T(\overline{\eta}) = (pd|T|\overline{\eta}^3\text{He}) \quad \text{and} \quad T(\overline{\pi}) = (pd|T|\overline{\pi}^3\text{He}). \tag{4}$$

The simple relationship between these amplitudes is

$$T(\eta) = T(\overline{\eta})(\overline{\eta}|\eta) + T(\overline{\pi})(\overline{\pi}|\eta) \approx T(\overline{\eta}) - \theta T(\overline{\pi}), \tag{5}$$

*Email address: anthony.green@helsinki.fi
†Email address: wycech@fuw.edu.pl

 

$$T(\pi) = T(\overline{\eta})(\overline{\eta}|\pi) + T(\overline{\pi})(\overline{\pi}|\pi) \approx T(\overline{\pi}) + \theta T(\overline{\eta}). \qquad (6)$$

The physical $T$-matrix elements are used to describe the experimental data. However, to parametrize these data we are going to use the isospin basis amplitudes for the scattering matrix $T$ and for the reaction matrix $K$.

We now introduce the $S$-wave scattering matrix $T$ for the *idealized* four-channel two-body system $|pd)$, $|^3\mathrm{He}\overline{\eta})$, $|^3\mathrm{He}\overline{\pi}^0)$, $|^3\mathrm{H}\pi^+)$. These channels are denoted by the suffices $p, \eta, \pi, \pi^+$, respectively. The important couplings to open few-body channels are at first forgotten. The scattering matrix and the basic interactions are described and parametrized in terms of a real $K$ matrix. Next, the Heitler equation for the $T$ matrix is solved. This becomes a simple matrix equation

$$T_{j,k} = K_{j,k} + i\Sigma_l K_{j,l} q_l T_{l,k}, \qquad (7)$$

where $j,k,l$ are the channel indices and $q_l$ is a diagonal matrix of the center-of-mass momenta in each channel. Here the region of interest spans from about 10 MeV below the $\eta^3$He threshold to some 10 MeV above it. For these low energies the experimental data exists. Also, in this region the $K_{j,k}$ elements are believed to be constant. The model used here supposes the interactions to conserve isospin. In this way the $K$ matrix is calculated in between the $|\overline{\eta})$, $|\overline{\pi})$, and $|\overline{\pi}^+)$ states. Since the isospin symmetry for the $\pi^+$ is supposedly not violated one has $|\overline{\pi}^+)=|\pi^+)$ and, in addition, some simple symmetries relate the matrix elements in the $|^3\mathrm{He}\overline{\pi}^0)$ and $|^3\mathrm{H}\pi^+)$ channels. In this way the parameters needed to fix the $4\times 4$ $K$ matrix are reduced from ten to six. These are $K_{p,p}, K_{p,\eta}, K_{p,\pi}, K_{\pi,\eta}, K_{\eta,\eta}, K_{\pi,\pi}$. In practice, only five of these matrix elements are needed, since the coupling to the entrance $pd$ channel is very small and so one finds $K_{p,p}$ to be irrelevant. Other elements are related by the isospin symmetry: $K_{p,\pi^+}=\sqrt{2}K_{p,\pi}$, $K_{\pi^+,\eta}=\sqrt{2}K_{\pi,\eta}$, $K_{\pi^+,\pi^+}=2K_{\pi,\pi}$, $K_{\pi^+,\pi}=\sqrt{2}K_{\pi,\pi}$.

The solution of the four-dimensional Eq. (7) may be brought to a typical form

$$T(p,\overline{\eta}) = \frac{A_{p,\eta}}{1-iq_\eta A_{\eta,\eta}}, \quad A_{\eta,\eta} = K_{\eta,\eta} + \frac{i3q_\pi K_{\pi,\eta}^2}{1-i3q_\pi K_{\pi,\pi}},$$
$$A_{p,\eta} = K_{p,\eta} + \frac{i3q_\pi K_{p,\pi}K_{\pi,\eta}}{1-i3q_\pi K_{\pi,\pi}}, \qquad (8)$$

where $A_{\eta,\eta}$ is the $\eta^3$He scattering length and $A_{p,\eta}$ the $pd \to {}^3\mathrm{He}\eta$ transition length. For the pion production amplitudes one obtains

$$T(p,\overline{\pi}) = \frac{A_{p,\pi}}{1-iq_\eta A_{\eta,\eta}} = -\frac{1}{\sqrt{2}}T(p,\overline{\pi}^+), \quad \text{where}$$
$$A_{p,\pi} = \frac{K_{p,\pi}[1-iq_\eta K_{\eta,\eta}] + iq_\eta K_{p,\eta}K_{\eta,\pi}}{1-i3q_\pi K_{\pi,\pi}} \qquad (9)$$

and the isospin relationship between the $\overline{\pi}^+$ and $\overline{\pi}$ is satisfied. Equations (8) and (9) contain expressions which change rapidly in the small $q_\eta$ region. One such term involves the scattering length $A_{\eta,\eta}$. A quasibound state, if it exists, is given by the condition

$$1 - iq_\eta A_{\eta,\eta} = 0, \qquad (10)$$

which is to be satisfied by a complex momentum $q_\eta^B$. For a large $A_{\eta,\eta}$ this momentum is close to the threshold and the factor $1/(1-iq_\eta A_{\eta,\eta})$ induces rapid energy dependence of the formation amplitudes in this region. This has been found in the $\eta$ production experiment [1,3], and one can expect a similar behavior in other channels. However, in the pion production amplitudes another factor arises which tends to suppress such an effect. It is due to the $A_{p,\pi}$ of Eq. (9). There the existence of a quasibound state is expressed as the dominance of $K_{\eta,\eta}$. Therefore, if $K_{\eta,\eta}$ were to differ from $A_{\eta,\eta}$ in a significant way, then one would expect a sizable energy dependence in the pion formation amplitude. However, as will be seen later, the indications are that $K_{\eta,\eta}\approx A_{\eta,\eta}$. This question is discussed below.

The real physical situation involves the coupling of the two-body channels to the continuum spectrum of three-body $NNN$, $Nd\pi$ and four-body $NNN\pi$ channels. The coupling to these systems may be strong and it is not easy to calculate. On phenomenological grounds it requires additional terms in the $K$ matrix, so that

$$K_{j,k} \rightarrow K_{j,k} + \Sigma_c K_{j,c}\frac{iq_c}{1-iK_{c,c}q_c}K_{c,k}, \qquad (11)$$

where the summation (an integration) over the continuum few-body channels $c$ is to be performed. This equation induces complex contributions to the real two-channel matrix elements. Here we estimate the magnitude of these contributions. The phases that arise are calculated in terms of a model or left to a best fit determination. The fine tuning of the parameters is done later. The large matrix elements are those related to the low-energy $\eta^3$He channel. These enter essentially in the form of final-state interaction factors in the $\eta^3$He system. One can visualize the relationship between the $K$-matrix parametrization and a model description of the meson formation via an expression

$$T(i,\overline{\eta}) = \int d\mathbf{r}d\mathbf{r}'\,\psi_i(\mathbf{r}')U_{i,\eta}(\mathbf{r}',\mathbf{r})\left[j_0(q_\eta r) + \exp(iq_\eta r)\frac{T_{\eta,\eta}}{r}\right]. \qquad (12)$$

Here, $\psi$ denotes a wave function in the initial channel $i$ and $U_{i,\eta}$ is an operator responsible for the meson formation. The term in square brackets is the final-state wave function expressed in terms of the $\eta$He scattering matrix $T_{\eta,\eta}=A_{\eta,\eta}/(1-iq_\eta A_{\eta,\eta})$. Up to terms linear in the final momentum $q_\eta$, one obtains from Eqs. (8) and (12)

$$A_{i,\eta} = \overline{U}_{i,\eta}\left[1 + \frac{A_{\eta,\eta}}{R}\right]. \qquad (13)$$

The $\overline{U}_{i,\eta}$ and $\overline{U}_{i,\eta}/R$ are results of the integrations in Eq. (12). The radius $R$ reflects the range of final-state interactions and is expected to be close to the $^3$He radius. Formula (13) contains effects from all the channels characterized by high intermediate momenta, i.e., all $K$-matrix elements other than $K_{\eta,\eta}$ or $A_{\eta,\eta}$, which have already been specified explicitly. Now, we show that the other matrix elements are small.

The SATURNE cross section [1] for the $pd \to {}^3\mathrm{He}\ \eta$ reaction is given by $T(p,\bar{\eta})$ of Eq. (8). These data fix $|A_{p,\eta}|$ rather precisely to the value of 0.013 fm. On the other hand, provided $|A_{\eta,\eta}| \approx 5$ fm, there is a whole region of Re $A_{\eta,\eta}$ and Im $A_{\eta,\eta}$ values that are equally likely. This result is consistent with the findings of Ref. [11]. For example, the modulus of Eq. (8) is invariant with respect to the sign change Re $A_{\eta,\eta} \leftrightarrow -$Re $A_{\eta,\eta}$. These two possibilities describe different physics. Large positive values of Re $A_{\eta,\eta}$ correspond to a virtual state, an analog of the $NN$ spin 0 state at low energies and the singularity of the scattering matrix given by Eq. (10) is located in the third quadrant of the complex $q_\eta$ plane. The other option, a negative length, signifies a quasibound state, which is analogous to the deuteron. One sees that the formation cross section cannot distinguish between these two possibilities.

The Brookhaven data, for the $\pi^+\ {}^3\mathrm{H} \to {}^3\mathrm{He}\,\eta$ process [9], permit one to extract two small $K$-matrix elements. These data are described by the amplitude

$$-\frac{1}{\sqrt{2}}T(\pi,\eta) = \frac{K_{\pi,\eta}}{(1-iq_\eta K_{\eta,\eta})(1-iq_\pi K_{\pi,\pi}) - q_\eta q_\pi K^2_{\pi,\eta}}, \tag{14}$$

giving $|K_{\pi,\eta}| \approx 0.07$ fm, a value that depends only slightly on the choice of $K_{\eta,\eta}$. The coupling of ${}^3\mathrm{He}\,\eta$ to ${}^3\mathrm{He}({}^3\mathrm{H})\pi$ is rather weak, and an inspection of Eq. (8) tells us that its contribution to Im $A_{\eta,\eta}$ is quite small. Therefore, the Brookhaven experiment implies that the $\eta{}^3\mathrm{He}$ state decays mainly into three- or four-body systems. Now, both $\eta$ formation experiments permit a simple description of the two-body channels. Since it is well established that low-energy $\eta N$ physics is dominated by the $S(1535)$ resonance we extend this dominance to few-body systems. Thus the channel coupling is given by

$$U_{i,j} = F(q_i)\sqrt{\gamma_i}\left\langle \frac{1}{E_S - E}\right\rangle \sqrt{\gamma_j}F(q_j), \tag{15}$$

where the $\gamma_i$ couple the resonance to the meson-nucleon channels, $\langle\cdots\rangle$ denotes a suitable average of the resonance propagator over the binding and recoil energies, and the $F(q_j)$ are the form factors for the meson-${}^3$He (${}^3$H) systems. The latter are expected to be about unity in the $\eta$ channel and small in the $\pi$ channels due to the high momenta involved. The ratio $\gamma_\pi/\gamma_\eta$ may be extracted from $S(1535)$ decay. Next, Eq. (15)—when combined with the $(\pi,\eta)$ data—yields $|K_{\pi,\pi}| \approx 0.001$ fm, a negligible value. There is another consequence of Eq. (15), the phase of $U_{\pi,\eta}$ is given by the phase of the $S(1535)$ propagator. For low-energy $\eta{}^3$He scattering, the relevant energies in the meson-nucleon system fall well below the resonance. Therefore, the dominant mode of decay is closed and $U_{\pi,\eta}$ is almost real. The uncertainty in the relative sign in the coupling constants $\sqrt{\gamma_i}$ and $\sqrt{\gamma_j}$ may be removed by the $S(1535)$ state wave function, where the SU(3) coupling mechanism of Ref. [12] gives a positive sign for $K_{\pi,\eta}$.

We are now ready to study the $pd \to {}^3\mathrm{H}\pi^+$ reaction [7]. Given a small value of $K_{\pi,\eta}$, along with a negligible $K_{\pi,\pi}$, then Eqs. (9) and the experimental data from Ref. [7] yield a crude estimate of $|A_{p,\pi}| \approx 0.000\,21$ fm.

The data used consist of eight measurements of the ${}^3\mathrm{He}\,\eta$ cross section in the threshold region [1], four measurements of the ${}^3\mathrm{H}\pi^+$ cross section [7], four measurements of the ${}^3\mathrm{He}\pi^0$ cross section [7], and one result for the $\pi^+\ {}^3\mathrm{H} \to {}^3\mathrm{He}\ \eta$ reaction [9]. We limit the available data to the low-energy $\eta$ region to stay with the approximation of a constant $K$ matrix. The first two reactions yield absolute values of the $K_{p,\eta}$ and $K_{p,\pi}$ matrix elements. However, the relative phase of these has to be left to an experimental determination. We set

$$K_{p,\pi} = \frac{K_{p,\eta}}{\omega}\exp(i\psi), \tag{16}$$

where $\omega = |K_{p,\eta}/K_{p,\pi}| = 5.54(50)$ is well determined from the $\eta$ and $\pi^+$ formation experiments. The phase $\psi$ and the $\eta$-$\pi$ mixing angle $\theta$ are free parameters. To elucidate the interference pattern in the equations of the preceding section, the $\pi$ formation amplitudes are now presented in a simplified form. Forgetting an irrelevant overall phase up to terms linear in small $K_{\pi,\eta}$ one has

$$-\frac{1}{\sqrt{2}}T(p,\bar{\pi}^+) = K_{p,\pi}\left[\exp(i\psi) + \frac{iq_\eta K_{\pi,\eta}}{1-iq_\eta A_{\eta,\eta}}\omega\right],$$

$$T(p,\bar{\pi}^0) = K_{p,\pi}\left[\exp(i\psi) + \frac{iq_\eta K_{\pi,\eta}+\theta}{1-iq_\eta A_{\eta,\eta}}\omega\right]. \tag{17}$$

In these equations the two $K$-matrix parameters are real. A positive sign for $K_{\pi,\eta}$ is preferred by the $S(1535)$ dominance and also by the best fit to the experimental data.

Using the MINUIT minimization package, an overall best fit search to the data yields $K_{p,\eta}$=0.0115(9)fm, $K_{p,\pi}$=0.00207(3)fm, $K_{\pi,\eta}$=0.067(11)fm, $K_{\pi,\pi}$=0, $K_{\eta,\eta}$=4.24(29)+$i$0.69(81)fm, $A_{\eta,\eta}$=4.24(29)+$i$0.72(81)fm, $\psi$=4.14(27), and $\theta$=0.010(0.005)=0.6(3)°. The small difference between Im $K_{\eta,\eta}$ and Im $A_{\eta,\eta}$ is due to the explicit inclusion of $K_{\pi,\eta}$. As discussed in the preceding section, the large error in Im $A_{\eta,\eta}$ arises since the $\eta$ formation cross section is not restrictive on the values of $A_{\eta,\eta}$. The real part of the ${}^3\mathrm{He}\,\eta$ scattering length is seen to be large and positive—signalling the existence of a virtual state in this system.

The idea behind the detection of $\eta$-$\pi$ mixing at COSY was to exploit the ratio of the charged and neutral pion cross sections $R_{mix}$, [6]. According to isospin invariance, $R_{mix}$ should equal two. However, the mixing induces corrections such that

$$R_{mix} \equiv \frac{|T(\pi^+)|^2}{|T(\pi^0)|^2} = \frac{2|T(\bar{\pi})|^2}{|T(\bar{\eta})(\bar{\eta}|\pi) + T(\bar{\pi})(\bar{\pi}|\pi)|^2}$$

$$= \frac{2}{N^2|1+\theta T(\bar{\eta})/T(\bar{\pi})|^2}. \tag{18}$$

There is an additional reason for studying the ratio $R_{mix}$, rather than the separate cross sections, since in this way

TABLE I. The experimental and calculated amplitudes $|T(\pi^+)|^2[10^{-7}\,\text{fm}^2]$ for the $\pi^+$ production. Other columns give ratios $R=|T(\pi^+)|^2/|T(\pi^0)|^2$ calculated with a mixing angle of $\theta$=0.010. The experimental results are from COSY [7]. The first column gives the proton laboratory momentum in giga-electron-volt.

| $p_{lab}$ | $\lvert T(\pi^+)\rvert^2_{expt}$ | $\lvert T(\pi^+)\rvert^2_{calc}$ | $R_{expt}$ | $R_{calc}$ |
|---|---|---|---|---|
| 1.560 | 47.4(3.9) | 49.7 | 2.05(0.17) | 2.04 |
| 1.570 | 45.8(6.6) | 48.8 | 1.84(0.27) | 2.08 |
| 1.571 | 47.5(2.2) | 46.0 | 2.24(0.11) | 2.17 |
| 1.590 | 62.6(8.6) | 47.6 | 2.57(0.27) | 2.11 |

most of the systematic errors are removed. Because of that we limit our discussion to the data obtained in one laboratory. The amplitudes obtained in the preceding section and $\theta$=0.010(5)[$\theta$=0.6(3)°] reproduce the trends in the measured values to a fair degree as shown in Table I. The data and overall fit from the model display a maximum just above the $\eta$ threshold. However, in the data a minimum below the threshold is also indicated, but it is not reproduced here. For comparison, in Fig. 1, the $pd \to {}^3\text{He}\pi^0$ cross section calculated with a mixing angle of $\theta$=0.010 is plotted against the experimental results [7].

The last step in this analysis has been a more detailed extraction of the $\eta$-$\pi$ mixing parameter $\theta$. Note that Eqs. (17), which are essential for that procedure, have a very simple structure at the threshold $q_\eta$=0. This point determines the unknown phase $\psi$. We find two basic solutions that differ by the sign of $\theta$. The negative sign is ruled out by all present day theories for $\theta$ [10,14–17]. The actual value of $\theta$ is then extracted by the energy dependence of the ratio $R_{mix}$ and the energy independence in $|T(p,\bar{\pi}^+)|$ below the threshold indicated in Table I. However, the best fit parameters are not well determined and their errors are large. The need for more precise and more numerous data is evident.

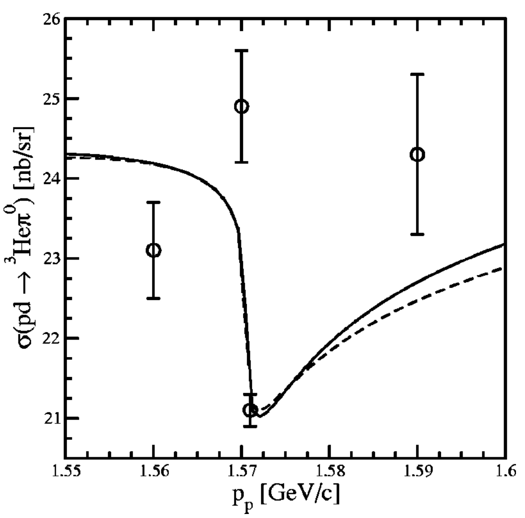


FIG. 1. The $pd \to {}^3\text{He}\pi^0$ cross section calculated with the mixing angle $\theta$=0.010 and λ=1.04. The experimental results are from COSY [7]. The dashed curve is the $K$-matrix fit, where the error bars of the $pd \to {}^3\text{H}\pi^+$ and $pd \to {}^3\text{He}\pi^0$ data have been doubled in an attempt to simulate systematic errors in that data.

The value of the mixing parameter obtained here is smaller than the $\theta$=1.5(4)° extracted from the $\pi d \to \eta NN$ reaction [13]. Theoretical calculations yield values in the region of (0.75−0.85°) [14–16], although angles twice as large have also been suggested [17].

So far we have assumed that all the isospin violation is due to the $\eta$-$\pi$ mixing. However, there are other sources of such a violation. These can enter in two ways.

(1) The two reactions $pd \to {}^3\text{H}\pi^+$ and $pd \to {}^3\text{He}\pi^0$ need not be exactly in the ratio 2 to 1 as is assumed in Eq. (18), since they contain explicit isospin violation effects such as the difference between the ${}^3$He and ${}^3$H nuclear wave functions, Coulomb interactions and different meson momenta. This effect is analyzed in terms of an additional free parameter λ, by assuming $|T(\bar{\pi}^+)|^2/|T(\bar{\pi}^0)|^2$ to be 2λ. The best fit value of λ=1.04(5) is found to improve our $R_{mix}$ ratio in Fig. 1. In this way, the mixing parameter is reduced to $\theta$ =0.007(5). However, this procedure uses a very limited data base. An extension of the data could possibly lead to a different value of λ. On the other hand, qualitative theoretical arguments in Ref. [18] do seem to suggest a value of λ that is greater than unity (≈1.10). One conclusion from this type of overall renormalization is that the error in $\theta$ could well be larger.

(2) An additional isospin violation effect in the $pd \to {}^3\text{He}\eta$ reaction could also arise from $\rho$-$\omega$ mixing. However, since our approach is based on phenomenological $K$-matrix parameters such an effect would not change directly the present determination of $\theta$. Presumably this would contribute to an overall normalization correction and so is taken into account by the above λ correction.

The conclusion is that, even though these two effects could be at a 10% level, they do not lead to any dramatic effect at the $\eta$ threshold and so are incorporated in the multiplicative factor λ.

In addition to the above corrections, it should be remembered that Eq. (18) is written down for $S$ waves under the assumption that these dominate in the backward scattering, whereas a more correct expression would involve the effect of higher partial waves. That possibility was incorporated by simply adding an additional "background" contribution as a complex constant $c$ to $T(p,\bar{\pi})$. However, the best fit procedure indicated that $c$ was very small and so ruled out any significant contribution of this kind.

The $K$-matrix formalism developed here is able to account for the structure seen at the $\eta$ threshold in the experimental ratio $|T(\bar{\pi}^+)|^2/|T(\bar{\pi}^0)|^2$ as a manifestation of $\eta$-$\pi$ mixing. It also enables an estimate of 0.010(5) to be made of the mixing angle. Unfortunately, at present this estimate has a large uncertainty, which could be significantly reduced by the removal of several uncertain systematic effects in the available experimental data. This clearly exposes the need for more precise data over the energy range covering the $\eta$ threshold. Such data should be detailed within the range $p_{lab}$=1.55 −1.59 GeV. In addition, some data points further from the threshold would be very valuable in order to study the nonthreshold value of $|T(\bar{\pi}^+)|^2/|T(\bar{\pi}^0)|^2$ and so tie down more precisely model parameters such as λ.

Hospitality of the Helsinki University Department of Physical Sciences and the Helsinki Institute of Physics is acknowledged by S.W. We wish to thank Andrzej Magiera for his collaboration and advice and Dan-Olof Riska and Christoph Hanhard for discussions. This project was financed by the Academy of Finland Contract No. 54038, the KBN Grant No. 5P03B04521, and the European Community Human Potential Program HPRN-CT-2002-00311 EURIDICE.

[1] B. Mayer *et al.*, Phys. Rev. C **53**, 2068 (1996).
[2] H. Calén *et al.*, Phys. Rev. Lett. **80**, 2069 (1989).
[3] C. Wilkin, Phys. Rev. C **47**, R938 (1993).
[4] S. Wycech, A. M. Green, and J. A. Niskanen, Phys. Rev. C **52**, 544 (1995).
[5] V. B. Belyaev, S. A. Rakityanski, S. A. Sofianos, M. Braun, and W. Sandhas, Few-Body Syst., Suppl. **8**, 309 (1995).
[6] A. Magiera and H. Machner, Nucl. Phys. **A674**, 515 (2000).
[7] M. Abdel-Bary *et al.*, Phys. Rev. C **68**, 021603(R) (2003).
[8] J. Banaigs *et al.*, Phys. Lett. **45B**, 394 (1973); P. Berthet *et al.*, Nucl. Phys. **443A**, 589 (1985); C. Kerboul *et al.*, Phys. Lett. B **181**, 28 (1986).
[9] J. C. Peng *et al.*, Phys. Rev. Lett. **63**, 2353 (1989).
[10] G. A. Miller, B. W. Nefkens, and L. Slaus, Phys. Rep. **194**, 1 (1990).
[11] V. Baru, J. Haidenbauer, C. Hanhart, and J. A. Niskanen, Phys. Rev. C **68**, 035203 (2003).
[12] D. O. Riska and G. E. Brown, Nucl. Phys. **A679**, 577 (2001).
[13] W. B. Tippens *et al.*, Phys. Rev. D **63**, 052001 (2001).
[14] J. Piekarewicz, Phys. Rev. C **48**, 1555 (1993).
[15] Chuang-Tsung Chan, E. M. Henley, and T. Meissner, Phys. Lett. B **343**, 7 (1995).
[16] H. Leutwyler, Phys. Lett. B **374**, 181 (1996).
[17] B. Bagchi, Phys. Rev. D **41**, 2871 (1990).
[18] H. S. Kohler, Phys. Rev. **118**, 1345 (1960).