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α-decay chains of $^{288}_{173}115$ and $^{287}_{172}115$ in the relativistic mean field theory

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In the recent experiments designed to synthesize the element 115 in the ^{243}Am+^{48}Ca reaction at Dubna in Russia, three similar decay chains consisting of five consecutive α decays and another different decay chain of four consecutive α decays are detected, and the decay properties of these synthesized nuclei are claimed to be consistent with consecutive α decays originating from the parent isotopes of the new element 115, ^{288}115 and ^{287}115, respectively. Here in the present work, the recently developed deformed relativistic mean field+BCS method with a density-independent δ function interaction in the pairing channel is applied to the analysis of these newly synthesized superheavy nuclei. The calculated α-decay energies and half-lives agree well with the experimental values and with those of the macroscopic-microscopic finite-range droplet model with folded-Yukawa single-particle potentials and Yukawa-plus-exponential model with Woods-Saxon single-particle potentials. In the mean field Lagrangian, the TMA parameter set is used. Particular emphasis is laid on the influence to both the ground-state properties and energy surfaces introduced by different treatments of pairing. Two different effective interactions in the particle-particle channel, i.e., the constant pairing and the density-independent δ-function interaction, together with the blocking effect are discussed in detail.

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Since the prediction of the existence of superheavy islands in 1960s [1,2], the synthesis of superheavy elements has been a hot topic in nuclear physics. Following numerous ground breaking technical developments [3] in the last three decades, the process of synthesizing superheavy elements has been sped up dramatically. From 1995–1996, Hofmann *et al.* [3–6] at GSI in Germany successfully produced the elements Z=110, 111, and 112 by using low-energy heavy-ion collisions. In January 1999, the new element Z=114 was reported at Dubna in Russia [7,8]. Two years later, the element Z=116 was also reported at Dubna [9]. In August 2003, in the reaction ^{243}Am+^{48}Ca held at Dubna [10], with a beam dose of 4.3×10^{18} 248 MeV and 253 MeV ^{48}Ca projectiles, nine new odd-Z nuclei originating from the isotopes of the new element 115, ^{288}115 and ^{287}115, were produced. So far, all elements with $110\leqslant Z\leqslant 116$ have been produced successfully in laboratory. All these exciting discoveries have greatly extended our knowledge about superheavy nuclei around the predicted superheavy islands and drawn more and more attention from the theoretical side.

The experimental progress has led to a large-scale investigation of superheavy nuclei by both refined macroscopic-microscopic (MM) models such as the finite-range droplet model with folded-Yukawa single-particle potentials (FRDM +FY) [11] or the Yukawa-plus-exponential model with Woods-Saxon single-particle potentials (YPE+WS) [12], and microscopic models such as the Skyme-Hartree-Fock-Bogoliubov method [13] and the latest relativistic mean field model [14–18]. Exploration for the next so-called "superheavy element island," i.e, the next spherical doubly magic nucleus, is a dream for physicists for the past several decades. There are already many works in this respect (see Refs. [13–15] and references therein). Possible candidates predicted by different theories are $^{298}_{184}$114, $^{292}_{172}$120, or even $^{310}_{184}$126. However, due to the limit of proper projectiles, the superheavy elements synthesized are always neutron deficient and lie in the deformed region. The deformation effects are very important to understand the nuclear structures in superheavy nuclei [13,16,18]. It is known experimentally that the heavy nuclei of the actinum series (Z=93–103) are well deformed and Bohr and Mottelson [19] also pointed out that the deformation can increase the stability of the heavy nuclei. The microscopic and self-consistent relativistic mean field model, due to its natural description of spin-orbit interaction [20–22], which is a purely relativistic effect, has been proved to be a reliable method to describe both exotic and superheavy nuclei [14–18].

In the present work, the recently developed deformed relativistic mean field (RMF)+BCS method with a density-independent δ-function interaction in the pairing channel [23] is adopted to analyze properties of lately synthesized superheavy nuclei ^{288}115, ^{287}115, and their α-decay daughter nuclei. The δ-function interaction has been proved to be very successful to take into account the continuum effect both in relativistic and nonrelativistic self-consistent mean field models [23–28]. In the mean field part, the TMA parameter set [29] is used, which has been proved to be very successful in describing superheavy nuclei [16–18] in the relativistic mean field model.

The RMF calculations have been carried out using the model Lagrangian density with nonlinear terms for both σ and ω mesons as described in detail in Refs. [23,29], which is given by

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TABLE I. The binding energies, B, and α-decay energies, Q_α, of α-decay chains of $^{288}115$ and $^{287}115$. Listed are the RMF+BCS calculations with constant pairing, Const, with the δ-function interaction without blocking, Delta1, and with Blocking, Delta2. FRDM+FY are results from the finite-range droplet model with folded Yukawa single-particle potentials [11]. The last column is the experimental Q_α from Dubna [10]. All energies are in units of MeV.

Nuclei	Const		Delta1		Delta2		FRDM+FY		Experiment
	B	Q_α	B	Q_α	B	Q_α	B	Q_α	Q_α
$^{288}115$	2059.10	9.78	2058.80	9.91	2059.03	10.30	2059.12	10.12	10.61±0.06
$^{284}113$	2040.58	11.22	2040.41	11.04	2041.03	10.74	2040.95	9.15	10.15±0.06
$^{280}111$	2023.50	10.50	2023.15	10.45	2023.47	10.49	2021.81	10.13	9.87±0.06
$^{276}109$	2005.70	9.75	2005.30	9.73	2005.66	9.42	2003.64	9.93	9.85±0.06
$^{272}107$	1987.15	8.16	1986.73	8.27	1986.78	8.60	1985.27	8.88	9.15±0.06
$^{268}105$	1967.01		1966.70		1967.08		1965.86		
$^{287}115$	2051.88	10.96	2051.72	10.82	2053.36	10.90	2052.72	10.25	10.74±0.09
$^{283}113$	2034.54	11.31	2034.24	11.19	2035.96	10.98	2034.68	9.35	10.26±0.09
$^{279}111$	2017.55	10.55	2017.13	10.55	2018.64	10.33	2015.73	10.92	10.52±0.16
$^{275}109$	1999.80	9.67	1999.38	9.67	2000.67	9.52	1998.36	10.06	10.48±0.09
$^{271}107$	1981.17	8.18	1980.75	8.29	1981.89	8.65	1980.13	8.66	
$^{267}105$	1961.05		1960.74		1962.24		1960.49		

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_\sigma\bar{\psi}\sigma\psi - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}g_4(\omega_\mu\omega^\mu)^2 - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu - \frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - g_\rho\bar{\psi}\gamma_\mu\tau^a\psi\rho^{\mu a} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma_\mu\frac{1-\tau_3}{2}A^\mu\psi, \quad (1)$$

where all symbols have their usual meanings. The corresponding Dirac equation for nucleons and Klein-Gordon equations for mesons obtained with the mean field approximation are solved by the expansion method on the widely used axially deformed harmonic-oscillator basis [23,30]. The number of shells used for expansion is chosen as $N_f=N_b=20$. More shells have been tested for convergence considerations. The quadrupole constrained calculations [23,31] have been performed for all the nuclei considered here in order to determine their ground-state deformations and obtain the energy surfaces.

Three kinds of approaches to take into account the pairing correlations have been adopted in the present work. The first is the usual RMF+BCS calculation with a constant pairing interaction. The inputs of pairing gaps are $\Delta_n=\Delta_p=11.2/\sqrt{A}$ and the blocking effect is ignored [18]. The second is the RMF+BCS calculation with a density-independent δ-function interaction, $V=-V_0\delta(\vec{r}_1-\vec{r}_2)$ [23]. Here, the blocking effect is also ignored for comparison. The third is the same as the second one except that the blocking effect is taken into account by the usual blocking method [23,32,33]. The pairing force strengths V_0 are fixed by obtaining similar binding energy for $^{288}115$ as the first approach, i.e., V_0 =280 MeV fm^3 in the second and V_0=330 MeV fm^3 in the third calculations, respectively. The same V_0 has been used for both protons and neutrons. A slight change of the pairing strength, say 10%, only changes the absolute binding energy less than 1.0 MeV and other results are hardly changed. Throughout the paper, the first, second, and third kind of calculations are abbreviated by "Const," "Delta1," and "Delta2."

In Table I, a comparison for binding energies and α-decay energies between the results of our three calculations, Const, Delta1, and Delta2, the results from the macroscopic-microscopic FRDM+FY model [11] and the experimental values for the $^{288}115$ chain and the $^{287}115$ chain is tabulated. More detailed properties obtained from the calculations Delta2 are listed in Table II, where the theoretical half-lives T_α are calculated with the well-known Viola-Seaborg formula [11]. The difference between the predicted Q_α by Const, Delta1, and Delta2, the FRDM+FY model [11], the YPE+WS model [12], and the experimental value, Δ_α(theor)=Q_α(theor)−Q_α(expt.), is plotted in Figs. 1 and 2 for the $^{288}115$ chain and the $^{287}115$ chain, respectively.

For the $^{288}115$ chain, we notice that the quality of agreement between our calculations (particularly Delta2) and the experimental values is similar to those of the MM models (FRDM+FY and YPE+WS). For the last two nuclei in the $^{288}115$ chain, $^{272}107$ and $^{276}109$, results of MM models are closer to experimental values. For $^{280}111$, our calculations are between the FRDM+FY model and the YPE+WS model. For $^{284}113$, the predicted α-decay energy by our calculations, similar to that of the YPE+WS model, is larger than the experimental value while the FRDM+FY model predicts a smaller value. The biggest difference, about 1.0 MeV, between theory and experiment is also found for this nucleus. For $^{288}115$, both our calculations and the FRDM+FY model predict similar values that are smaller than the experimental value, while the result from the YPE+WS model is larger than the experimental value.

For the $^{287}115$ chain, similar things happen. For $^{271}107$, Delta2 and the FRDM+FY model predict similar Q_α, while the YPE+WS model predicts a larger value. Because no ex-

TABLE II. The binding energies, B, neutron and proton quadrupole deformation, β_{2n} and β_{2p}, neutron and proton rms radii, R_n and R_p, the calculated α-decay energies and half-life lives, Q_α and T_α, of superheavy nuclei on the alpha-decay chains of $^{288}115$ and $^{287}115$ obtained from the calculations Delta2. The last two columns are experimental decay energies and lifetimes. All energies are in units of MeV and all radii in units of Fermi.

Nuclei	B	β_{2n}	β_{2p}	R_n	R_p	Q_α	T_α	Q_α(expt.)	T_α(expt.)
$^{288}115$	2059.03	0.48	0.50	6.58	6.41	10.30	6.86 s	10.61±0.06	87^{+105}_{-30} ms
$^{284}113$	2041.03	0.17	0.17	6.37	6.18	10.74	111.96 ms	10.15±0.06	$0.48^{+0.58}_{-0.17}$ s
$^{280}111$	2023.47	0.18	0.19	6.34	6.15	10.49	118.97 ms	9.87±0.06	$3.6^{+4.3}_{-1.3}$ s
$^{276}109$	2005.66	0.20	0.20	6.32	6.12	9.42	25.08 s	9.85±0.06	$0.72^{+0.87}_{-0.25}$ s
$^{272}107$	1986.78	0.20	0.21	6.30	6.09	8.60	1953.31 s	9.15±0.06	$9.8^{+11.7}_{-3.5}$ s
$^{268}105$	1967.08	0.21	0.22	6.28	6.06				16^{+19}_{-6} h
$^{287}115$	2053.36	0.48	0.50	6.56	6.41	10.90	80.57 ms	10.74±0.09	32^{+155}_{-14} ms
$^{283}113$	2035.96	0.18	0.18	6.36	6.18	10.98	12.76 ms	10.26±0.09	100^{+490}_{-45} ms
$^{279}111$	2018.64	0.20	0.20	6.34	6.15	10.33	142.07 ms	10.52±0.16	170^{+810}_{-80} ms
$^{275}109$	2000.67	0.21	0.21	6.32	6.12	9.52	5.77 s	10.48±0.09	$9.7^{+46.}_{-4.4}$ s
$^{271}107$	1981.89	0.21	0.21	6.29	6.09	8.65	604.91 s		
$^{267}105$	1962.24	0.22	0.22	6.27	6.06				73^{+350}_{-33} min

perimental value is observed for this nucleus, prediction of Delta2 is taken as the experimental value for comparison. For $^{275}109$, predictions of all our three calculations deviate from the experimental value more than those of the MM models. While for $^{279}111$ and $^{287}115$, our calculations are closer to experimental values than the MM models. For $^{283}113$, just like the case of $^{284}114$, the difference between theory and experiment is relatively large. Our calculations and the YPE+WS model predict different trends for this nucleus from the FRDM+FY model also.

We note that all our three calculations predict similar α-decay energies for both the $^{288}115$ chain and the $^{287}115$ chain. The calculations Const and Delta1 give essentially the same results for both decay chains while the calculations Delta2 are generally better than the other two calculations. This is more obvious for the odd-odd $^{288}115$ chain than for the odd-even $^{287}115$ chain. Since the main difference between the second and the third calculations is the blocking effect, we could safely conclude that a proper blocking treatment can improve the calculated observables for odd-even or odd-odd nuclei; thus it becomes necessary.

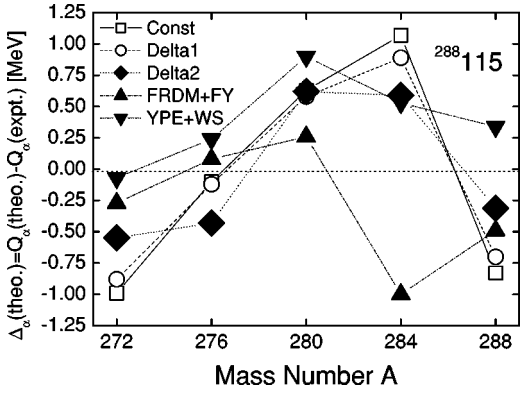

FIG. 1. The difference between calculated Q_α(theor) and experimental Q_α(expt.), Δ_α(theor)=Q_α(theor)−Q_α(expt.), for the $^{288}115$ α-decay chain as a function of mass number A.

Next, we would like to point out the advantage of the RMF method used here over the MM models. As we have seen in the above discussions, predictions of the MM models are closer to the experimental values for the elements 109 and 107, while for the other three elements, our calculations are better than those of the MM models. The reason could be that the MM models depend more on the knowledge of known nuclei, in other words, the parameters of those MM models need to be readjusted in order to account for newly discovered superheavy nuclei. While the RMF model, whose parameters are obtained by fitting properties of a few selected well-known spherical nuclei and remain unchanged thereafter, due to its natural description of spin-orbit interaction, after including deformation, pairing interaction, and blocking effect properly, could be more powerful in predicting the properties of unknown nuclei.

Now let us discuss a bit more the differences between our three different kinds of calculations. We have performed the constrained quadrupole calculations [23,31] for both the $^{288}115$ chain and the $^{287}115$ chain in all the three calculations. The corresponding energy curves are shown in Figs. 3 and 4. We should mention that such calculations are very time consuming. First thing we see is that Const and Delta1

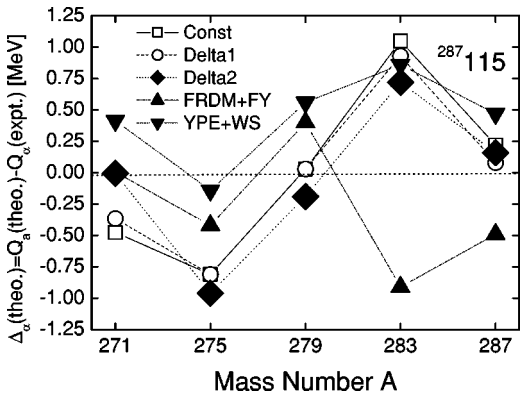

FIG. 2. The same as Fig. 1, but for the $^{287}115$ α-decay chain.

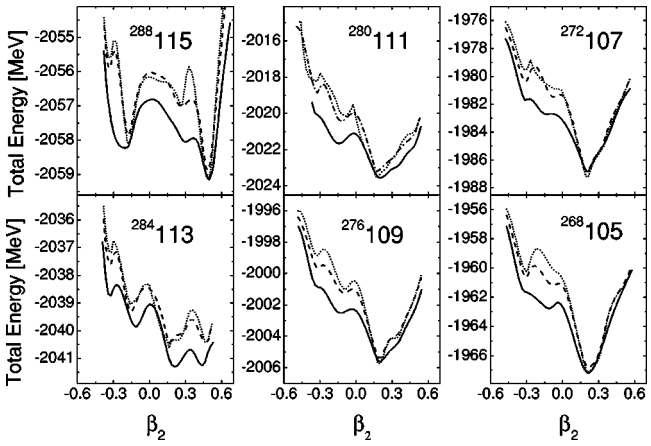

FIG. 3. The energy surfaces for the ^{288}115 α-decay chain as functions of mass quadrupole deformation, β_2, obtained from three calculations: Delta2 (solid line), Delta1 (dashed line), and Const (dotted line).

give quite similar energy curves for both decay chains. In fact they also give almost the same results for all calculated quantities except for the α-decay energies where Delta1 is better (see also Table I and Figs. 1 and 2). Another noticeable difference is that the height of the barrier between different minima can differ a little bit. In most cases, Delta1 gives shallower barriers than Const. Second, we can see that the differences between calculations with and without blocking, Delta2 and Delta1, are relatively large, despite that the ground-state properties are quite similar. This once again shows that proper treatment of blocking effect is necessary for odd-odd or odd-even nuclei. For the ^{287}115 chain, due to the way that we fixed the pairing strength V_0, the absolute binding energies from calculations with and without blocking differ around 1.0 MeV for some nuclei.

Unlike medium or light nuclei where generally only two minima (one oblate minimum and one prolate minimum) or one spherical minimum are observed, the energy curves of superheavy nuclei are more complicated as we can see in Figs. 3 and 4. This is not surprising. As there are more levels in heavy nuclei, level crossing is more frequent to happen and lots of local minima may appear. Except for ^{288}115 and

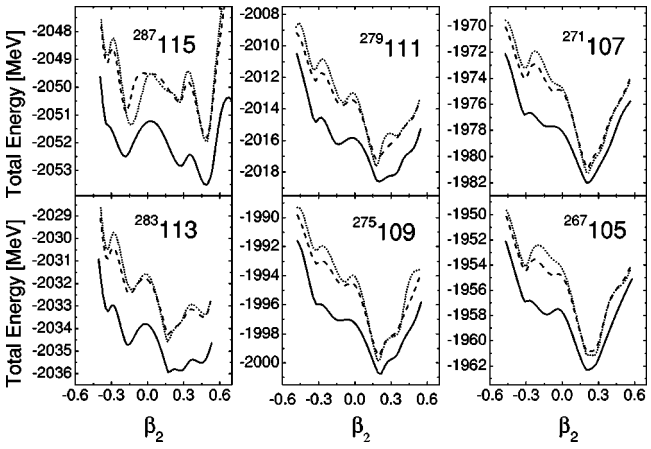

FIG. 4. The same as Fig. 3, but for the ^{287}115 α-decay chain.

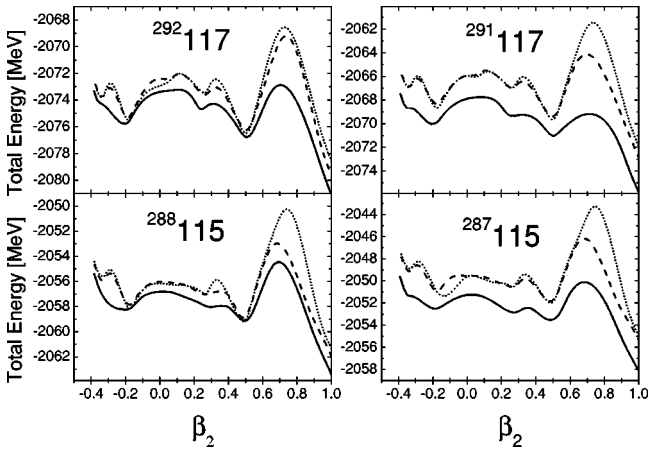

FIG. 5. The same as Fig. 3, but for ^{292}117, ^{288}115, ^{291}117, and ^{287}115.

^{284}113 in the ^{288}115 chain, ^{287}115 and ^{283}113 in the ^{287}115 chain, the ground state of other nuclei can be determined without ambiguity, i.e., around $\beta_2 \sim 0.2$. Similar results have been obtained by the MM YPE+WS model [12], more specifically, β_2=0.200, 0.211, and 0.224 for ^{280}111, ^{276}109, and ^{272}107; β_2=0.202, 0.215, and 0.228 for ^{279}111, ^{275}109, and ^{271}107. The YPE+WS model predicts β_2=0.138 and β_2 =0.149 for ^{284}113 and ^{283}113, which are also close to our calculations β_2=0.17 and β_2=0.18. While for ^{288}115 and ^{287}115, the YPE+WS model predicts β_2=0.072 and β_2 =0.066, respectively, which are quite different from our calculations, $\beta_2 \sim 0.5$ for both these nuclei. This difference can be understood easily because these MM models predict $^{298}_{184}$114 to be the next spherical doubly magic nucleus, while most self-consistent models shift this property to the more proton-rich side [14]. Further experiments are needed to clarify these discrepancies between different theoretical models and different parameter sets in the same model. In our calculations, two other configurations $\beta_2 \sim -0.2$ and β_2 ~ 0.3 are also possible for ^{288}115. That is to say, decay from these two configurations to ^{283}113 are also possible based on the calculated α-decay energy. For ^{283}113, we find that the minima around $\beta_2 \sim 0.2$ and $\beta_2 \sim 0.5$ are close to each other.

Since we see that isotopes of the element 115 are very deformed in our calculations, we would like to have a closer look at this element and the element 117, the mother element of the element 115 in the α-decay chain. The corresponding energy surfaces from all the three calculations are plotted in Fig. 5 for ^{292}117, ^{288}115, ^{291}117, and ^{287}115. It is clearly seen that the configuration around $\beta_2 \sim 0.5$ is still stable against fission even for the element 117, but the barrier is lowered greatly for the calculation Delta2 than the other two calculations. Such an influence to the fission barrier introduced by the blocking effect has been demonstrated by Rutz *et al.* [34] in the RMF model. Here, we notice that the adoption of the density-independent δ-function interaction instead of the constant pairing in the pairing channel further reduces the fission barrier. Further calculations by Delta2 show that α-decay energies of ^{292}117 and ^{291}117 are, respectively, 10.71 MeV (B=2076.62 MeV) and 10.83 MeV (B =2053.36 MeV), with T_α=2.23 s and T_α=0.49 s. It would be

very interesting to synthesize the nuclei $^{292,291}117$ and measure the corresponding α-decay chains because our calculations predict that these nuclei would make α decays.

To summarize, we have studied the newly synthesized superheavy nuclei [10] within the recently developed deformed RMF+BCS model. The calculated α-decay energies, Q_α, are found to agree well with the experimentally observed values and also are close to those of macroscopic-microscopic FRDM+FY model and YPE+WS model. The predicted ground-state deformations agree well with those of macroscopic-microscopic YPE+WS model. The inclusion of blocking effect is found to be able to improve the calculated ground-state properties somewhat. The constrained calculations show that the energy curves are quite complicated for these superheavy nuclei. Further comparisons show that the fission barriers are quite different due to different treatment of pairing, especially from calculations with and without blocking. This suggests that to study superheavy nuclei more reliably one needs to use a more realistic effective interaction in the pairing channel, such as the density-independent δ-function interaction used here, and at the same time include the blocking effect properly.

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