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Nature of excited 0^+ states in ^{158}Gd described by the projected shell model

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Excited 0^+ states are studied in the framework of the projected shell model, aiming at understanding the nature of these states in deformed nuclei in general, and the recently observed 13 excited 0^+ states in ^{158}Gd, in particular. The model, which contains projected two- and four-quasiparticle states as building blocks in the basis, is able to reproduce reasonably well the energies for all the observed 0^+ states. The obtained $B(E2)$ values, however, tend to suggest that these 0^+ states might have a mixed nature of quasiparticle excitations coupled to collective vibrations.

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Dynamic perturbations of nuclear shapes around the equilibrium can give rise to physical states at low to moderate excitation energies. Classical examples of such motion are β and γ vibrations [1,2], in which nucleons undergo vibrations in a collective manner. Traditionally, the first excited $K^\pi=0^+$ states and the first excited 2^+ states are interpreted, respectively, as the β and γ vibrational states. While the 2^+ collective excitations are better understood theoretically, the nature of the lowest 0^+ excitation of deformed nuclei still remains under debate [3–7]. The physics of higher 0^+ states is even more complex because, on one side, they can predominantly be multiphonon states based on the single phonons [8], and on the other side, they can be quasiparticle (qp) excitations in nature. The real situation is, perhaps, that the two aspects, collective excitations and qp states, are mixed by residual interactions.

Data on $K^\pi=0^+$ states have been relatively sparse. In a very recent work by Lesher *et al.* [9], a remarkable (p,t) experiment revealed a total of 13 excited 0^+ states in ^{158}Gd, below an excitation energy of $\approx$3.1 MeV. This abundance of 0^+ states in a single nucleus provides significant new information on the poorly understood phenomenon, which has immediately sparked off theoretical interest. For this energy range, one may think about an explanation through collective modes. In fact, Zamfir, Zhang, and Casten [10] suggested that many of the observed 0^+ states may be of two-phonon octupole character. Nevertheless, these authors warned also that, although the mechanism was excluded in their collective models, many of the 0^+ states in this excitation energy range may be predominantly two qp in character.

The purpose of the present paper is to investigate whether one can explain the nature of the observed 0^+ states in terms of qp excitations. In contrast to Ref. [10], our calculation here does not emphasize the aspect of collective excitation. Although, similar to the work of Zamfir, Zhang, and Casten [10], we may provide only a partial image, our results may shed light on the importance of considering the qp aspects in understanding the nature of these 0^+ states.

Our study is based on the projected shell model (PSM) [11]. The PSM is the spherical shell model built on a deformed basis. The PSM calculation usually begins with the deformed Nilsson single-particle states at a deformation ε. Pairing correlations are incorporated into the Nilsson states by BCS calculations. The consequences of the Nilsson-BCS calculations provide us with a set of qp states that define the qp vacuum $|\phi(\varepsilon)\rangle$. One then constructs the shell model bases by building multi-qp states. The broken symmetry in these states is recovered by angular momentum projection [11] (and particle number projection, if necessary) to form a shell model basis in the laboratory frame. Finally, a two-body shell model Hamiltonian is diagonalized in the projected space.

To determine the deformation at which the shell model basis should be built, we first study the bulk properties of ^{158}Gd including the deformation. In a microscopic calculation, one searches for energy minima by varying the deformation parameter. Here, we calculate the angular-momentum-projected energies having the form

$$E^I(\varepsilon)=\frac{\langle\phi(\varepsilon)\hat{H}\hat{P}^I|\phi(\varepsilon)\rangle}{\langle\phi(\varepsilon)\hat{P}^I|\phi(\varepsilon)\rangle}, \tag{1}$$

where P^I is the angular-momentum-projection operator [12] which projects the mean-field vacuum $|\phi(\varepsilon)\rangle$ onto states with good angular momentum. As many previous PSM calculations for the rare earth nuclei, particles in three major shells ($N=4,5,6$ for neutrons and $N=3,4,5$ for protons) are activated in the present calculation for ^{158}Gd. For comparison, unprojected energies

$$E(\varepsilon)=\frac{\langle\phi(\varepsilon)\hat{H}|\phi(\varepsilon)\rangle}{\langle\phi(\varepsilon)|\phi(\varepsilon)\rangle} \tag{2}$$

are also calculated.

As one can see in Fig. 1, the lowest energy for a given angular momentum I is well localized at deformations varying from $\varepsilon\approx0.24$ at $I=0$ to $\varepsilon\approx0.28$ at $I=12$. The ground-state deformation calculated by Möller *et al.* [13] for this nucleus yields $\varepsilon=0.25$. These angular-momentum-projected minima lie at deformations that are slightly larger than the mean-field minimum ($\varepsilon\approx0.23$). Figure 1 indicates that ^{158}Gd is a stably deformed nucleus against rotation, with a pro-

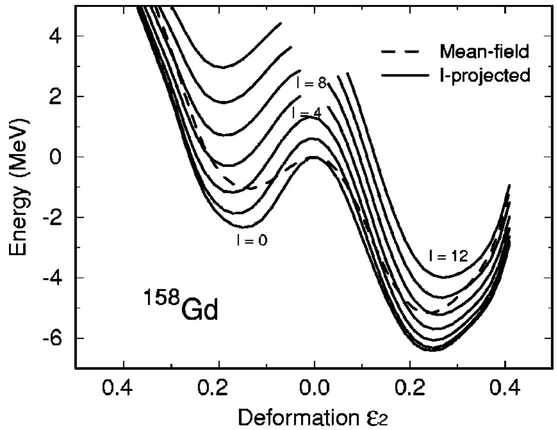

FIG. 1. Angular-momentum-projected energies in ^{158}Gd for the states with I=0 to I=12 as a function of deformation. The unprojected nonrotating energies (denoted as mean field) are also shown for comparison.

nounced energy minimum corresponding to a deformed prolate shape. In the following calculations, we thus construct the shell model basis at the deformation ε=0.26. The vacuum state $|\phi(\varepsilon=0.26)\rangle$ is hereafter written as $|0\rangle$.

Following the spirit of the Tamm-Dancoff method [12], we build the shell model space by including 0-, 2- and 4-qp states:

$$|\Phi_\kappa\rangle = \{|0\rangle, \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}|0\rangle, \alpha^\dagger_{p_k}\alpha^\dagger_{p_l}|0\rangle, \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}\alpha^\dagger_{p_k}\alpha^\dagger_{p_l}|0\rangle\}, \quad (3)$$

where $\alpha^\dagger$ is the creation operator for a qp and the index n (p) denotes neutron (proton) Nilsson quantum numbers which run over the low-lying orbitals. Thus, the projected multi-qp states are the building blocks of our shell model basis:

$$|\Psi^I_M\rangle = \sum_\kappa f^I_\kappa \hat{P}^I_{MK}|\Phi_\kappa\rangle. \quad (4)$$

Here, κ labels the basis states and f^I_κ are determined by configuration mixing.

We then diagonalize the Hamiltonian in the projected multi-qp states given in Eq. (4). In the calculation, we employ a quadrupole plus pairing Hamiltonian, with inclusion of quadrupole-pairing term [11]

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi\sum_\mu \hat{Q}^\dagger_\mu \hat{Q}_\mu - G_M\hat{P}^\dagger\hat{P} - G_Q\sum_\mu \hat{P}^\dagger_\mu \hat{P}_\mu, \quad (5)$$

where $\hat{H}_0$ is the spherical single-particle Hamiltonian which contains a proper spin-orbit force. The quadrupole-quadrupole interaction strength χ is determined by the self-consistent relation with deformation ε. The monopole pairing strength G_M is taken to be $G_M=[20-13(N\mp Z)/A]/A$ with "−" sign for neutrons and "+" sign for protons. Finally, the quadrupole-pairing strength G_Q is assumed to be proportional to G_M, the proportionality constant being fixed to 0.20 in the present work. These interaction strengths are the same as the values used in the previous PSM calculations for the rare earth nuclei [11].

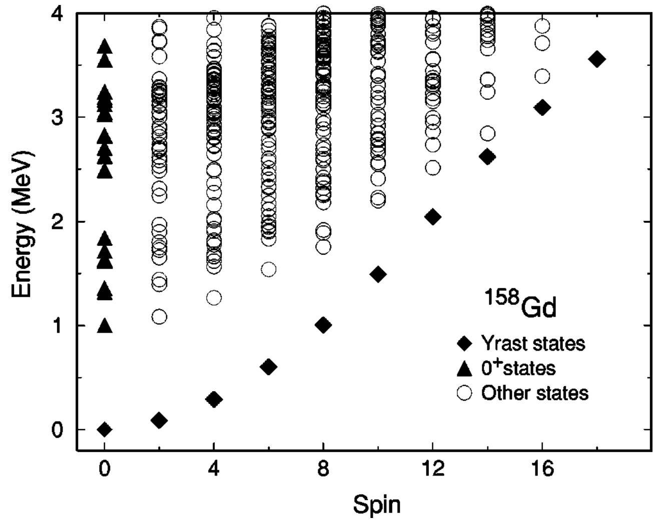

FIG. 2. Theoretical energy spectrum of ^{158}Gd calculated up to E=4 MeV and I=18$\hbar$. Diamonds are the yrast states and filled triangle are excited 0^+ states.

In Fig. 2, we show the partial theoretical spectrum in ^{158}Gd up to 4 MeV in energy and 18$\hbar$ in spin. We emphasize that all these states have been obtained by a single diagonalization, without any adjustment for individual states. Out of these many states, let us concentrate on the lowest one at each spin (the yrast band, denoted by diamonds) and on all the 0^+ states (denoted by filled triangles). Although it is not a focus of our discussion in this paper, we believe that a comparison of the yrast band with known data can provide a strict test of the model and can provide a useful constraint to the calculations of the 0^+ states.

Figure 3 presents the PSM results for the yrast band in ^{158}Gd (the same values shown in Fig. 2 as diamonds), which are compared with the known data [14], in a plot of γ-ray energy versus spin. As can be seen, the data are described very well. The calculations predict a sudden drop in the curve at spin I=16, corresponding to a backbending in the moment of inertia. This sudden change occurs just at the upper part of the band where the current measurement stops.

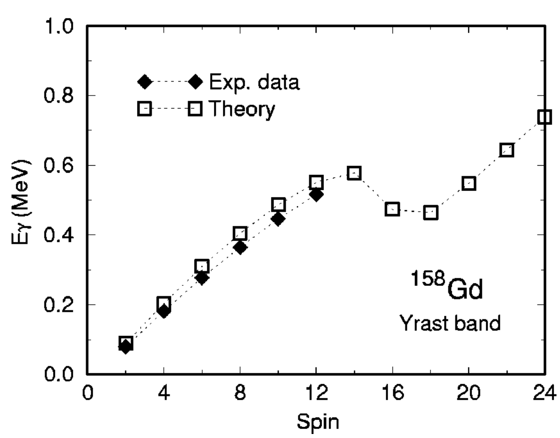

FIG. 3. Comparison of the PSM calculations and data for the yrast states in ^{158}Gd in the form of γ-ray energy $E_\gamma(I)=E(I)-E(I-2)$ vs spin I. (Data are taken from Ref. [14]).

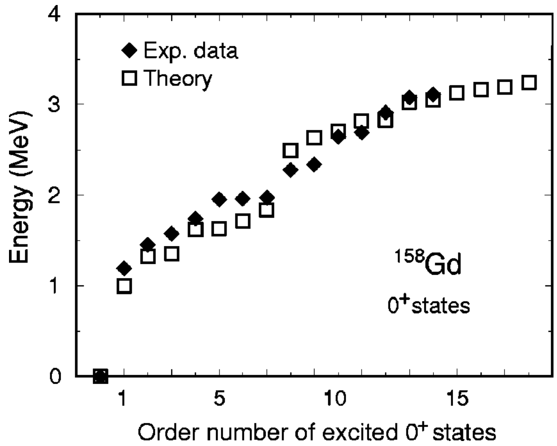

FIG. 4. Comparison of the calculated 0^+ states in ^{158}Gd with data [9].

Extension from the current measurement should see this phenomenon and will provide a strict test of our model prediction. In this regard, we note that in the isotonic chain of nuclei, ^{160}Dy, ^{162}Er, ^{164}Yb, and ^{166}Hf, similar backbending effects have been observed and successfully described elsewhere by the PSM [15,16].

We now turn our discussion to the excited 0^+ states. Lesher *et al.* [9] observed 13 0^+ states in the nucleus ^{158}Gd, within an excitation energy range of 1.2 MeV–3.1 MeV. In Fig. 4, we plot all the theoretical 0^+ states (up to 3.2 MeV in energy; the same values shown in Fig. 2 as filled triangles) in the order of excitation energies. The experimental data [9] are shown in the same plot for comparison. The predicted 0^+ states are found to be in the right energy range, although deviations can be clearly seen between theory and experiment.

What we found impressive is the number of 0^+ states predicted by the calculation. The PSM produces a sufficient number of 0^+ states to be compared with data. As described in Eq. (4), the total wave function $|\Psi^I_M\rangle$ is a linear combination of the (projected) basis states given in Eq. (3). The basis states in Eq. (3) are not arbitrarily selected but are taken from all the neutron and proton Nilsson orbitals that lie close to the Fermi surface. In ^{158}Gd, the relevant orbitals are $\frac{3}{2}[521]\nu$, $\frac{5}{2}[523]\nu$, $\frac{11}{2}[505]\nu$, $\frac{3}{2}[651]\nu$, and $\frac{5}{2}[642]\nu$ for neutrons, and $\frac{1}{2}[411]\pi$, $\frac{3}{2}[411]\pi$, $\frac{5}{2}[413]\pi$, $\frac{5}{2}[532]\pi$, and $\frac{7}{2}[523]\pi$ for protons. In each of these ten near-Fermi orbitals, nucleons having opposite signs for the K quantum number can couple to a 2-qp state with total $K=0$. Combination of a pair of 2-qp states can further give $K=0$ 4-qp states. If one neglects the coupling of these qp states to the collective states, the number of low-lying 0^+ states in deformed nuclei obtained in this way depend solely on the single-particle level density and the level distribution near the Fermi surface. Since similar conditions can also be found in many other rare earth nuclei, we expect such an abundance of 0^+ states as found in ^{158}Gd not to be an isolated case. We predict that such an abundance of 0^+ states should also be observed in many other nuclei.

The large number of 0^+ states is difficult to obtain within collective models. In Ref. [10], one could obtain at most five excited 0^+ states up to 3.2 MeV in calculations with the geometric collective model or the interacting boson model (with only *s* and *d* boson). The reason is that within such collective models, the number of degrees of freedom of collective motion is limited. Only if one considered the odd-parity bosons in an extended boson space, could the authors in Ref. [10] get more excited 0^+ states.

In Table I, we list the 18 calculated 0^+ states below an excitation energy of 3.25 MeV (slightly higher than the highest experimental 0^+ state). Their leading configurations are

TABLE I. Predicted 2-pq and 4-pq 0^+ states (below 3.25 MeV) in ^{158}Gd.

E (MeV)	$B(E2,0_i^+\to 2_g^+)$ (W.u.)	qp states	Configurations
1.004	1.87	2-qp	$-\frac{5}{2}[642]\nu, \frac{5}{2}[642]\nu$
1.321	0.004	2-qp	$-\frac{11}{2}[505]\nu, \frac{11}{2}[505]\nu$
1.360	0.014	2-qp	$-\frac{3}{2}[411]\pi, \frac{3}{2}[411]\pi$
1.621	0.076	2-qp	$-\frac{5}{2}[413]\pi, \frac{5}{2}[413]\pi$
1.636	0.041	2-qp	$-\frac{3}{2}[521]\nu, \frac{3}{2}[521]\nu$
1.716	0.234	2-qp	$-\frac{5}{2}[523]\nu, \frac{5}{2}[523]\nu$
1.841	1.102	2-qp	$-\frac{5}{2}[532]\pi, \frac{5}{2}[532]\pi$
2.492	0.328	2-qp	$-\frac{7}{2}[523]\pi, \frac{7}{2}[523]\pi$
2.631	0.001	4-qp	$-\frac{11}{2}[505]\nu, \frac{11}{2}[505]\nu, -\frac{3}{2}[411]\pi, \frac{3}{2}[411]\pi$
2.707	0.161	2-qp	$-\frac{3}{2}[651]\nu, \frac{3}{2}[651]\nu$
2.818	0	4-qp	$-\frac{11}{2}[505]\nu, \frac{11}{2}[505]\nu, -\frac{5}{2}[413]\pi, \frac{5}{2}[413]\pi$
2.829	0	4-qp	$-\frac{3}{2}[521]\nu, \frac{3}{2}[521]\nu, -\frac{3}{2}[411]\pi, \frac{3}{2}[411]\pi$
3.028	0	4-qp	$-\frac{3}{2}[521]\nu, \frac{3}{2}[521]\nu, -\frac{5}{2}[413]\pi, \frac{5}{2}[413]\pi$
3.048	0	4-qp	$-\frac{5}{2}[523]\nu, \frac{5}{2}[523]\nu, -\frac{3}{2}[411]\pi, \frac{3}{2}[411]\pi$
3.126	0.002	2-qp	$-\frac{1}{2}[411]\pi, \frac{1}{2}[411]\pi$
3.168	0	4-qp	$-\frac{11}{2}[505]\nu, \frac{3}{2}[521]\nu, \frac{5}{2}[413]\pi, \frac{3}{2}[411]\pi$
3.192	0	4-qp	$-\frac{3}{2}[521]\nu, -\frac{5}{2}[523]\nu, \frac{5}{2}[413]\pi, \frac{3}{2}[411]\pi$
3.245	0	4-qp	$-\frac{5}{2}[523]\nu, \frac{5}{2}[523]\nu, -\frac{5}{2}[413]\pi, \frac{5}{2}[413]\pi$

also given. Note that each of the configurations listed here cannot be pure, but is the dominant part in the corresponding wave function. One sees that most 2-qp states coming from the near-Fermi Nilsson levels have lower energies. Due to the varying responses of the residual interactions through configuration mixing, one finds that some 4-qp states are lower in excitation energy than 2-qp states.

Up to now, we have seen that the PSM calculations, which explicitly include 2- and 4-qp states built on the basis but no vibrational degrees of freedom, can reasonably produce a sufficient number of 0^+ states within the right excitation energy range. This fact suggests that these states are qp states in nature contrary to the work of Zamfir, Zhang, and Casten [10]. However, before we do that, we should study the transition properties of these states.

The excited 0^+ states can decay to the 2_g^+ state in the ground-state band through $E2$ transition. One such transition was measured in Ref. [17]. Preliminary results of Lesher *et al.* [18] show 12 such transitions. The PSM calculations of the transition from the ith 0^+ state to the 2_g^+ state, $B(E2, 0_i^+ \rightarrow 2_g^+)$, are listed in Table I. Comparing the theoretical $B(E2)$ values with the experimental data [18], we found that the numbers (in Weisskopf unit) listed in Table I are one or two orders of magnitude too small for many of the transitions.

We note that the $B(E2, 0_i^+ \rightarrow 2_g^+)$ values (a few Weisskopf unit) of Lesher *et al.* [18] are in average of two orders of magnitude smaller than the in-band $B(E2)$ values of the ground-state band (typically, a few hundred Weisskopf unit). These small $B(E2)$ values usually indicate that the observed 0^+ states in Ref. [8] should carry significant quasiparticle components. However, our calculated $B(E2)$ results are much smaller than the values of Lesher *et al.*, which might indicate an insufficient mixing of collectivity to the qp states in our model. This tends to suggest that although most, if not all, of these 0^+ states have significant qp components, they are also mixed with correlations induced by the collective motion [19]. These correlations can be introduced in the PSM framework by inclusion of interactions of higher orders of the multipole type [20], and by addition of the D-pair operators to the vacuum state [21], which takes both quasiparticle and collective degrees of freedom explicitly into account in a shell model basis. Generator coordinate method, which consists of a construction of a linear superposition of different product wave functions, can also be adopted by the PSM.

In summary, the projected shell model was employed to understand the nature of excited 0^+ states in deformed nuclei. The shell model space consists of projected 2- and 4-qp states on top of the deformed BCS vacuum state. Therefore, the calculation emphasized the quasiparticle character of these states, in contrast to previous work [10] using collective models. Our results were compared with the remarkable example of recently observed 13 excited 0^+ states in ^{158}Gd [9]. After performing exact angular momentum projection and configuration mixing calculations (by two-body residual interactions) for all the possible low-lying qp configurations that can give rise to $K^\pi = 0^+$ states, we found that the obtained energy levels as well as the number of the states can reasonably explain the data. Preliminary measurements of $B(E2)$ values suggest that mixing of these qp states with the collective, vibrational motion may not be neglected.

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