# *R*-matrix formulas for three-body decay widths

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(Received 26 June 2003; published 5 November 2003)

For three-body decay of a nuclear level, it is suggested that instead of the total width being the sum of the partial widths calculated from *R*-matrix formulas the contributions from different decay channels should be added coherently. Two-proton and two-neutron decay widths are considered, and also widths of levels of $^{9}$Be and $^{9}$B.

DOI: 10.1103/PhysRevC.68.054602 PACS number(s): 23.50.+z, 27.20.+n

## I. INTRODUCTION

The $0^+$ ground state of $^{12}$O undergoes three-body decay to the $^{10}$C ground state plus two protons. *R*-matrix formulas based on the one-level approximation of Lane and Thomas [1] have been used to calculate upper limits on the contributions to the $^{12}$O width due to sequential decay through the $1/2^+$ ground state of $^{11}$N [2] and due to diproton decay [3], which is another form of sequential decay. The same formulas have been used to calculate the two-proton decay width of a $1^-$ excited state of $^{18}$Ne [4], of the $3/2^+$ ground state of $^{45}$Fe and the $0^+$ ground state of $^{48}$Ni [5], and of the $0^+$ ground state and $2^+$ first excited state of $^{6}$Be [6]. Some contributions to the width of the $0^+$ ground state of $^{8}$C, which decays to $^{4}$He plus four protons, have also been calculated [6]. Revised values of the $^{6}$Be and $^{8}$C widths are given in Ref. [7].

In each of these cases except $^{48}$Ni, experimental width values are available, and the sum of the calculated sequential and diproton decay widths is less than the experimental width. In the $^{6}$Be($2^+$) case, the calculated diproton width [7] is wrong, as the decay angular momentum was taken as 0 instead of 2; correction of this leads to a still smaller calculated width (see Table I).

Two recent experiments [8,9] have investigated the $1/2^+$ ground state of $^{5}$H, and given values of its energy and width that are rather inconsistent. The *R*-matrix formula for sequential decay [2], and that for diproton decay [3] modified to apply to dineutron decay, are used below to calculate the width of the $^{5}$H ground state. The sum of the calculated sequential and dineutron decay widths is less than the experimental value in one case and greater in the other.

The *R*-matrix formula for sequential decay [2] may also be used to calculate widths of $^{9}$Be and $^{9}$B levels, as described below. All levels except the $^{9}$Be ground state are particle unstable, and undergo three-body decay to two $\alpha$ particles plus a neutron or a proton. For most of these levels, the widths calculated from the sum of contributions due to decay through the $0^+$ ground state or $2^+$ first excited state of $^{8}$Be, or through the $3/2^-$ ground state of $^{5}$He or $^{5}$Li, are less than the experimental values as given in the latest compilation [10].

These discrepancies between calculated and experimental widths could be attributed to direct, nonsequential, three-body decay. Here we suggest, however, that the above procedure of taking the total width as the sum of the partial widths in the various decay channels may not be justified for three-body decay, and that there could be coherence between the various contributions. This could affect the amount (if any) of nonsequential contributions needed.

In conventional *R*-matrix theory [1], the total width of a level is the sum of the partial widths. This depends essentially on two of the four broad assumptions that Lane and Thomas [1] give as the basis of *R*-matrix theory; these are "absence or unimportance of all processes in which more than two product nuclei are formed" and "the existence, for any pair of nuclei $c$, of some finite radial distance of separation $a_c$, beyond which neither nucleus experiences any polarizing potential from the other." Lane and Thomas showed that a relaxation of the first of these assumptions is possible in an approximate treatment of three-body decays, assumed to proceed as a succession of two-body decays. The second assumption seems reasonable for decays to two stable nuclei; for example, if eight nucleons are assembled to form $^{7}$Li(g.s.)+$p$, they are unlikely to rearrange to two $\alpha$ particles without the intermediate step of going through a $^{8}$Be compound nucleus. For three-body decay, however, it is plausible that the 12 nucleons in $^{12}$O(g.s.). could rearrange from $^{11}$N(g.s.)+$p$ to $^{10}$C(g.s.)+$^{2}$He even when the proton is separated from the $^{11}$N(g.s.). by more than the usual value of the interaction radius $a_c$, because the $^{11}$N and $^{2}$He themselves decay. In other words, the decays of $^{12}$O into $^{11}$N+$p$ and into $^{10}$C+$^{2}$He are not really independent. As far as the width of $^{12}$O(g.s.) is concerned, this probably means that the contributions coming from decay through $^{11}$N+$p$ and through $^{10}$C+$^{2}$He should not be treated as incoherent with the partial widths being added, giving $\Gamma^0_{\rm inc}$ say, but that the contributions are coherent to some extent at least. If there is coherence, the total width could be less than or greater than $\Gamma^0_{\rm inc}$, but an upper limit is obtained by assuming maximum coherence, giving $\Gamma^0_{\rm coh}$. Thus we take [6]

$$\Gamma^0_{\rm inc} = \sum_c \Gamma^0_c \tag{1}$$

with

$$\Gamma^0_c = \frac{\Gamma_c}{1 + \sum_{c'} \gamma^2_{1c'} \bar{S}'_{c'}}, \quad \Gamma_c = 2\gamma^2_{1c} \bar{P}_c, \tag{2}$$

and

 

TABLE I. Widths for two-proton decay. In this table, $0.102\times10^{-2}$ is abbreviated to 0.102E-2, etc.

| Case | Ref. | $c$[a] | $\mathcal{S}_c$ | $\gamma^2_{1c}$ (MeV) | $\overline{P}_c$ | $\overline{S'_c}$ (MeV$^{-1}$) | $\Gamma_c$ (keV) | $\Gamma^0_c$ (keV) | $\Gamma^0_{\rm inc}$ (keV) | $\Gamma^0_{\rm coh}$ (keV) | $\Gamma_{\rm expt}$ (keV) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $^6$Be($0^+$) | [6,7] | $s$ | 2.25 | 3.25 | 0.102E-2 | 0.213 | 6.6 | 2.8 | | | |
| | | $d$ | 0.981 | 2.03 | 0.372E-1 | 0.324 | 151.0 | 64.3 | 67.1 | 94 | 92±6 |
| $^6$Be($2^+$) | [6,7] | $s$ | 2.24 | 4.46 | 0.977E-1 | 0.243 | 871 | 384 | | | |
| | | $d$ | 0.780 | 0.74 | 0.416E-1[b] | 0.246[b] | 62 | 27 | 411 | 615 | 1160±60 |
| $^8$C($0^+$) | [6,7] | $s$ | 3.55 | 3.91 | 0.370E-2 | 0.255 | 28.9 | 13.9 | | | |
| | | $d$ | 0.173 | 0.242 | 0.478E-1 | 0.345 | 23.1 | 11.1 | 25.0 | 50 | 230±50 |
| $^{12}$O($0^+$) | [2] | $s$ | 0.4[c] | 0.67 | 0.161E-1 | 0.411 | 21.4 | 14.3 | | | |
| | [3] | $d$ | 1.0[c] | 0.63 | 0.465E-2 | 0.352 | 5.9 | 3.9 | 18.2 | 33 | 400±250 |
| | | | | | | | | | | | 578±205 |
| $^{18}$Ne($1^-$) | [4] | $s$ | 0.026[d] | 0.038 | 0.143E-3 | 0.422 | 0.0108 | 0.0106 | | | |
| | | $d$ | 0.043[d] | 0.022 | 0.135E-3 | 0.326 | 0.0059 | 0.0058 | 0.0164 | 0.032 | 0.021−0.057 |
| $^{45}$Fe($3/2^+$) | [5] | $s$ | 1.0[c] | 0.271 | 0.0 | 0.324 | 0.0 | 0.0 | | | |
| | | $d$ | 0.195 | 0.0086 | 0.638E-18 | 0.286 | 0.110E-16 | 0.101E-16 | 0.101E-16 | 0.10E-16 | $(1.2^{+0.3}_{-0.4})$E-16 |
| $^{48}$Ni($0^+$) | [5] | $s$ | 1.0[c] | 0.0692 | 0.0 | 0.174 | 0.0 | 0.0 | | | |
| | | $d$ | 0.14 | 0.0051 | 0.567E-17 | 0.282 | 0.58E-16 | 0.57E-16 | 0.57E-16 | 0.57E-16 | |

[a] $s$—sequential decay; $d$—diproton decay.
[b] Corrected value.
[c] Assumed value.
[d] MK interaction.

$$\Gamma^0_{\rm coh}=\left[\sum_c (\Gamma^0_c)^{1/2}\right]^2. \tag{3}$$

Calculated values of $\Gamma^0_{\rm inc}$ and $\Gamma^0_{\rm coh}$ are compared with experimental width values $\Gamma_{\rm expt}$ for the cases of two-proton decay in Table I, for the two-neutron decay of $^5$H in Table II, and for levels of $^9$Be and $^9$B with $E_x<7$ MeV in Table III. In Table I, parameter values are taken from the previous publications [2–7]. In all cases, we have used conventional values of the interaction or channel radius $a_c=1.45\ (A_1^{1/3}+A_2^{1/3})$fm, although it was found in some cases of two-proton decay that larger values of $a_c$ led to values of $\Gamma^0_{\rm inc}$ in better agreement with experiment [7]. We have also used "best" experimental values of level energies, although again improved agreement was found in some cases for energies varied within experimental uncertainties [5,7].

## II. TWO-PROTON DECAY WIDTHS

The values of $\Gamma^0_{\rm coh}$ in Table I are to be regarded as upper limits of the calculated width for the parameter values used; they are still, however, less than or at most about equal to the experimental values. For the $^6$Be cases, values of $\Gamma^0_{\rm coh}$ greater than $\Gamma_{\rm expt}$ can be obtained by decreasing the values of $Q_{1ps}$ and/or by increasing the channel radii by reasonable amounts.

For $^8$C, $\Gamma^0_{\rm coh}$ is much less than the experimental width. Contributions from only two of the possible decay channels have been included. It would be surprising if the contributions from other sequential decay channels were sufficiently large, and the degree of coherence sufficiently high, for $\Gamma^0_{\rm coh}$ to attain the value of $\Gamma_{\rm expt}$.

The calculated value of $\Gamma^0_{\rm coh}$ for $^{12}$O is an order of magnitude less than the experimental width values. Similarly Grigorenko *et al.* [11] calculated a width of about 60 keV from their three-body models. It is suggested in Ref. [2] that the experimental values may be in error.

For $^{18}$Ne, consistency with experiment is possible if the roughly comparable sequential and diproton contributions are coherent. The values in Table I are for the MK interaction (see Ref. [4]); for the alternative WBP and WBT interactions, $\Gamma^0_{\rm coh}$=0.038 keV and 0.039 keV, respectively.

For the $^{45}$Fe and $^{48}$Ni cases, the penetration factor for sequential decay is taken to be zero, so that the values of $\Gamma^0_{\rm inc}$

TABLE II. Widths for two-neutron decay of $^5$H $1/2^+$ ground state.

| Ref. | $E_m$ (MeV) | $E_r$ (MeV) | $c$[a] | $\mathcal{S}_c$ | $\gamma^2_{1c}$ (MeV) | $\overline{P}_c$ | $\overline{S'_c}$ (MeV$^{-1}$) | $\Gamma_c$ (MeV) | $\Gamma^0_c$ (MeV) | $\Gamma^0_{\rm inc}$ (MeV) | $\Gamma^0_{\rm coh}$ (MeV) | $\Gamma_{\rm expt}$ (MeV) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [8] | 3.0 | 3.30 | $s$ | 2.5[b] | 6.63 | 0.0881 | 0.165 | 1.17 | 0.41 | | | |
| | | | $d$ | 1.39[b] | 4.14 | 0.617 | 0.177 | 5.11 | 1.81 | 2.22 | 3.95 | ≈3 |
| [9] | 1.8 | 1.99 | $s$ | 2.5[b] | 5.49 | 0.0115 | 0.102 | 0.126 | 0.050 | | | |
| | | | $d$ | 1.39[b] | 4.21 | 0.347 | 0.229 | 2.92 | 1.16 | 1.21 | 1.66 | ⩽0.5 |

[a] $s$—sequential decay; $d$—dineutron decay.
[b] Assumed value.

TABLE III. Widths of $^9$Be and $^9$B levels.

| Nucl | $J^\pi$ | $E_x$ (MeV) | $c$[a] | $l_c$ | $\mathcal{S}_c$ | $\theta^2_{\rm sp}$ | $\gamma^2_{1c}$ (MeV) | $\overline{P}_c$ | $\overline{S'_c}$ (MeV$^{-1}$) | $\Gamma_c$ (keV) | $\Gamma^0_c$ (keV) | $\Gamma^0_{\rm inc}$ (keV) | $\Gamma^0_{\rm coh}$ (keV) | $\Gamma_{\rm expt}$ (keV) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $^9$Be | $1/2^+$ | 1.684 | 0 | 0 | 0.694 | 1.202 | 2.06 | 0.1241 | 0.000 | 511 | 476 | | | |
| | | | 2 | 2 | 0.266 | 0.254 | 0.167 | $\approx 10^{-16}$ | 0.155 | 0 | 0 | | | |
| | | | $\alpha$ | 1 | 0.722 | 0.557 | 0.111 | $\approx 10^{-11}$ | 0.430 | 0 | 0 | 476 | 476 | $217\pm10$ |
| | $5/2^-$ | 2.429 | 0 | 3 | 0.0054[b] | 0.262 | 0.0035 | $0.733\times10^{-3}$ | 0.179 | 0.0051 | 0.0039 | | | |
| | | | 2 | 1 | 1.122 | 0.331 | 0.916 | $0.475\times10^{-5}$ | 0.214 | 0.009 | 0.007 | | | |
| | | | $\alpha$ | 2 | 0.997 | 0.333 | 0.273 | $0.107\times10^{-3}$ | 0.423 | 0.059 | 0.045 | 0.056 | 0.13 | $0.77\pm0.15$ |
| | $1/2^-$ | 2.78 | 0 | 1 | 0.750 | 0.715 | 1.323 | 0.452 | 0.224 | 1195 | 817 | | | |
| | | | 2 | 1 | 0.417 | 0.362 | 0.372 | $0.556\times10^{-4}$ | 0.227 | 0.04 | 0.03 | | | |
| | | | $\alpha$ | 2 | 0.560 | 0.372 | 0.171 | $0.187\times10^{-2}$ | 0.473 | 0.64 | 0.44 | 817 | 855 | $1080\pm110$ |
| | $5/2^+$ | 3.049 | 0 | 2 | 0.501 | 0.457 | 0.564 | $0.979\times10^{-1}$ | 0.311 | 110.5 | 63.2 | | | |
| | | | 2 | 0 | 0.276 | 0.758 | 0.516 | $0.116\times10^{-2}$ | 0.301 | 1.2 | 0.7 | | | |
| | | | | 2 | 0.191 | 0.296 | 0.132 | $0.105\times10^{-4}$ | 0.175 | 0.0 | 0.0 | | | |
| | | | $\alpha$ | 1 | 0.774 | 0.878 | 0.559 | $0.515\times10^{-1}$ | 0.693 | 57.6 | 32.9 | | | |
| | | | | 3 | 0.119 | 0.229 | 0.0225 | $0.682\times10^{-3}$ | 0.370 | 0.0 | 0.0 | 97 | 211 | $282\pm11$ |
| | $3/2^+$ | 4.704 | 0 | 2 | 0.266 | 0.572 | 0.359 | 0.424 | 0.215 | 305 | 172 | | | |
| | | | 2 | 0 | 0.457 | 1.198 | 1.350 | 0.171 | 0.392 | 460 | 259 | | | |
| | | | | 2 | 0.207 | 0.364 | 0.186 | $0.499\times10^{-2}$ | 0.227 | 2 | 1 | | | |
| | | | $\alpha$ | 1 | 0.201 | 1.344 | 0.214 | 0.735 | 0.346 | 315 | 178 | | | |
| | | | | 3 | 0.375 | 0.358 | 0.110 | $0.940\times10^{-1}$ | 0.471 | 21 | 12 | 622 | 2210 | $743\pm55$ |
| | $3/2^-$ | 5.59 | 0 | 1 | 0.053 | 0.960 | 0.125 | 1.358 | 0.046 | 338 | 204 | | | |
| | | | 2 | 1 | 1.131 | 0.688 | 1.918 | 0.214 | 0.289 | 820 | 494 | | | |
| | | | $\alpha$ | 0 | 0.064 | 1.536 | 0.081 | 1.388 | 0.191 | 225 | 135 | | | |
| | | | | 2 | 0.308 | 0.949 | 0.241 | 0.698 | 0.355 | 336 | 202 | 1035 | 3886 | $1330\pm360$ |
| | $7/2^-$ | 6.38 | 2 | 1 | 0.183 | 0.773 | 0.349 | 0.404 | 0.219 | 282 | 244 | | | |
| | | | $\alpha$ | 2 | 0.219 | 1.106 | 0.199 | 1.011 | 0.280 | 402 | 347 | | | |
| | | | | 4 | 0.605 | 0.136 | 0.068 | 0.130 | 0.402 | 18 | 15 | 606 | 1453 | $1210\pm230$ |
| | $9/2^+$ | 6.76 | 2 | 2 | 0.891 | 0.508 | 1.117 | 0.1361 | 0.246 | 304 | 205 | | | |
| | | | $\alpha$ | 3 | 0.939 | 0.751 | 0.580 | 0.596 | 0.364 | 691 | 465 | 670 | 1287 | $1330\pm90$ |
| $^9$B | $3/2^-$ | 0.0 | 0 | 1 | 0.564 | 0.440 | 0.613 | $0.568\times10^{-3}$ | 0.430 | 0.696 | 0.483 | | | |
| | | | 2 | 1 | 0.751 | 0.268 | 0.498 | $0.229\times10^{-12}$ | 0.180 | 0.000 | 0.000 | | | |
| | | | $\alpha$ | 0 | 0.562 | 0.376 | 0.174 | $0.271\times10^{-10}$ | 0.355 | 0.000 | 0.000 | | | |
| | | | | 2 | 0.554 | 0.187 | 0.085 | $0.363\times10^{-12}$ | 0.298 | 0.000 | 0.000 | 0.483 | 0.48 | $0.54\pm0.21$ |
| | $5/2^-$ | 2.361 | 0 | 3 | 0.0031[c] | 0.278 | 0.0021 | $0.156\times10^{-1}$ | 0.196 | 0.066 | 0.047 | | | |
| | | | 2 | 1 | 1.122 | 0.384 | 1.062 | $0.288\times10^{-2}$ | 0.260 | 6.1 | 4.4 | | | |
| | | | $\alpha$ | 2 | 0.997 | 0.350 | 0.287 | $0.124\times10^{-1}$ | 0.441 | 7.1 | 5.1 | 9.5 | 21 | $81\pm5$ |
| | $1/2^-$ | 2.78[d] | 0 | 1 | 0.750 | 0.715 | 1.324 | 0.815 | 0.128 | 2160 | 1553 | | | |
| | | | 2 | 1 | 0.417 | 0.396 | 0.408 | $0.103\times10^{-1}$ | 0.292 | 8 | 6 | | | |
| | | | $\alpha$ | 2 | 0.560 | 0.405 | 0.186 | 0.214 | 0.550 | 80 | 58 | 1617 | 2448 | |
| | $5/2^+$ | 2.788 | 0 | 2 | 0.501 | 0.488 | 0.603 | 0.238 | 0.256 | 288 | 168 | | | |
| | | | 2 | 0 | 0.276 | 0.844 | 0.575 | $0.377\times10^{-1}$ | 0.389 | 43 | 25 | | | |
| | | | | 2 | 0.191 | 0.333 | 0.165 | $0.966\times10^{-3}$ | 0.193 | 0 | 0 | | | |
| | | | $\alpha$ | 1 | 0.774 | 0.876 | 0.558 | 0.124 | 0.548 | 139 | 81 | 274 | 727 | $550\pm40$ |
| | $7/2^-$ | 6.97 | 2 | 1 | 0.183 | 0.888 | 0.402 | 0.760 | 0.143 | 611 | 535 | | | |
| | | | $\alpha$ | 2 | 0.219 | 1.222 | 0.220 | 1.200 | 0.242 | 528 | 462 | | | |
| | | | | 4 | 0.605 | 0.174 | 0.087 | 0.254 | 0.368 | 44 | 38 | 1035 | 2579 | $2000\pm200$ |

[a] $0-{}^8$Be$(0^+)+N$; $2-{}^8$Be$(2^+)+N$; $\alpha-{}^5$He$(3/2^-)$ or ${}^5$Li$(3/2^-)+\alpha$.
[b] Chosen to make branching ratio to ${}^8$Be$(0^+)+n=7.0\%$ [10].
[c] Chosen to make branching ratio to ${}^8$Be$(0^+)+p=0.5\%$ [10].
[d] Assumed value.

and $\Gamma^0_{\rm coh}$ are equal. As discussed in Ref. [5], better agreement with experiment is obtained for $^{45}$Fe if $Q$ is increased from the best value of 1.14 MeV to the top of the experimental range at 1.19 MeV.

### III. WIDTH OF $^5$H GROUND STATE

Two recent investigations of the $1/2^+$ ground state of $^5$H have found rather different values for its width. Meister *et al.* [8] measured the energy spectrum of the $t+n+n$ system following one-proton knockout from $^6$He, finding a broad structure peaked at 3 MeV above the $t+n+n$ threshold with a full width at half maximum (FWHM) of about 6 MeV. They compare their measured spectrum with values calculated by Shul'gina *et al.* [12] in a three-body model. This calculation predicts the peak energy due to the ground state of $^5$H at 2.5–3.0 MeV, and the FWHM as 3–4 MeV (estimated for cross sections after subtraction of the contribution connected with the plane waves). The full $1/2^+$ contribution is considerably wider, and indeed appears to be wider than the structure observed by Meister *et al.* (see Fig. 3 of Ref. [8]). Thus the measurements seem to indicate a FWHM due to the $1/2^+$ ground state of about 3 MeV. This is reasonably consistent with the calculation by Descouvemont and Kharbach [13], who used a microscopic generator-coordinate model.

Golovkov *et al.* [9] used the two-neutron transfer reaction $t(t,p)\,^5$H. They find a resonance at 1.8±0.1 MeV above the $t+n+n$ threshold, with a width of $\leqslant$0.5 MeV.

In our calculation of the width of the $1/2^+$ state of $^5$H using $R$-matrix formulas, we use the parameter values of Meister *et al.* [8] for the ground state of $^4$H (or more probably for the $2^-$ ground state plus the $1^-$ first excited state at $E_x$=0.3 MeV [14]); otherwise we use conventional values of the channel radii. As reasonable upper limits on the spectroscopic factors, for $^5{\rm H}(1/2^+)\rightarrow{}^4{\rm H}(2^-+1^-)+n$ we take $\mathcal{S}_{41}=2.0\times5/4=2.5$ (where 5/4 is the c.m. correction factor [15]), and for $^5{\rm H}(1/2^+)\rightarrow{}^3{\rm H}(1/2^+)+2n(0^+)$ we take $\mathcal{S}_{32}=1.0\times25/18=1.39$. For the density-of-states function of the dineutron $(2n)$, we use hard-core effective-range parameter values that fit the $n-p$ $^1S_0$ phase shift given in Table VI of Ref. [16] for $E_{\rm lab}\leqslant100$ MeV. In the notation of Ref. [3], we find $c$=0.109 fm, $A=-0.0423$ fm$^{-1}$, and $B$=1.104 fm.

Resultant values are given in Table II. In each case, $\Gamma^0_{\rm coh}$ exceeds $\Gamma_{\rm expt}$.

### IV. WIDTHS OF $^9$Be AND $^9$B LEVELS

In Table III, the widths of $^9$Be and $^9$B levels are calculated using the $R$-matrix formula for sequential decay [2]. The spectroscopic factors for positive-parity levels and $s$- or $d$-wave nucleon channels are taken from Glickman *et al.* [17], and for negative-parity levels and $p$-wave nucleon channels from Kumar [18]; for all levels and $\alpha$-decay channels we use values given by Millener [19]. The energy $Q_{1ps}$ and width $\Gamma^0_2(Q_{1ps})$ of the $^5$He and $^5$Li ground states are taken from Ref. [20]. Otherwise conventional values are used for channel radii and Woods-Saxon potential radius and diffuseness parameters. The values of $E_x$ and $\Gamma_{\rm expt}$ are taken from the latest compilation [10].

In most of the cases in Table III, $\Gamma^0_{\rm inc}$ is less than $\Gamma_{\rm expt}$, whereas $\Gamma^0_{\rm coh}$ is greater than or at least not much less than $\Gamma_{\rm expt}$, and in the latter cases consistency could probably be obtained by reasonable changes in the values of the channel radii, spectroscopic factors, and/or energy and width of the $^5$He or $^5$Li ground state. There are some cases, however, where there seems to be a significant discrepancy.

For the $1/2^+$ first excited state of $^9$Be, the $^8$Be$(2^+)$ and $^5$He$(3/2^-)$ channels contribute to the width mainly by a renormalization of the wave function, giving the observed width $\Gamma^0_c$ slightly less than the formal width $\Gamma_c$. Then $\Gamma^0_{\rm coh}$ is the calculated width (not an upper limit). It may be noted that, in this case, $P_c$ for the $^8$Be$(0^+)$ channel is calculated for a definite energy, and is not averaged over a density-of-states function. The value $\Gamma^0_{\rm coh}=\Gamma^0_{\rm inc}=476$ keV is much greater than the experimental value $\Gamma_{\rm expt}=217$ keV [10]. The values of $E_x$ and $\Gamma_{\rm expt}$ in Table III for the $1/2^+$ state originate solely from Kuechler *et al.* [21], and have been criticized in Refs. [22,23]. Most of the experimental values of the width given in the compilations of Ajzenberg-Selove and co-workers [24,20] are also about 200 keV. A recent fit [23] to $^9$Be$(\gamma,n)^8$Be data has, however, given widths of about 135 keV for the best fits. In spite of the apparent discrepancies, it seems possible to reconcile these various values.

The experimental widths of about 200 keV come from reactions, such as $^{11}$B$(d,\alpha)^9$Be, in which $^9$Be is formed as an unstable product nucleus, and the widths should be taken (approximately) as the FWHM $\Gamma_\rho$ of the density-of-states function [25]

$$\rho(E)=c_1\frac{E^{1/2}}{(E_r-E)^2+\epsilon E},\qquad(4)$$

where $E_r$ and $E$ are measured from the $^8$Be+$n$ threshold (at $E_x$=1.665 MeV), and $\epsilon=2\mu a^2\gamma^4/\hbar^2$ ($\mu$= reduced mass, $a$= channel radius, $\gamma^2$= reduced width).

The widths from Ref. [23] are the FWHM of the $^9$Be$(\gamma,n)^8$Be cross section $\sigma$ which in standard $R$-matrix theory is given by [25]

$$\sigma(E)=c_2\,E_\gamma\,\rho(E),\qquad(5)$$

however, when channel contributions are included, as in Ref. [23], the form of $\sigma$ is more complicated. The values of $E_r$ and $\gamma^2$ given in Table 1 of Ref. [23] can be used to calculate $\Gamma_\rho$. For example, for the Kuechler fit A, which uses the standard formula (5), one finds $\Gamma_\rho$=123 keV, which is less than the FWHM of $\sigma$ (135 keV) as expected, and is far from the experimental values of about 200 keV. For Kuechler fit D (nonstandard), one gets $\Gamma_\rho$=216 keV, which is compatible with the experimental values. $\Gamma^0_{\rm coh}$ in Table III is a value of the $R$-matrix observed width (which in the one-channel approximation is identical with the formal width) evaluated at the energy $E_m$ at which $\rho(E)$ is a maximum. This width is

$$\Gamma_R(E_m)=2(\epsilon E_m)^{1/2}.\qquad(6)$$

The value 511 keV is obtained for the parameter values $E_m$=0.019 MeV and $\gamma^2$=2.06 MeV. To get $E_m$=0.019 MeV from Eq. (4), one needs $E_r$=0.239 MeV, and then one finds $\Gamma_\rho = 229$ keV, again consistent with the experimental values.

The other obvious disagreements in Table III are for the $5/2^-$ second excited states of $^9$Be and $^9$B. In each case, the calculated value of $\Gamma^0_{\rm coh}$ is roughly 20% of the experimental width. Even with favorable but reasonable choices of channel parameters (lower values of the energies of the $^5$He and $^5$Li ground states and $^8$Be first excited state, and larger values of channel radii), and with maximum coherence between the channel contributions, our values of $\Gamma^0_{\rm coh}$ are still only 60–70 % of $\Gamma_{\rm expt}$. Such a discrepancy between theory and experiment for the $5/2^-$ level of $^9$Be had already been pointed out in Ref. [26]. The discrepancy has also appeared recently in the description of $^9$B levels populated in the $\beta$ decay of $^8$C [27]. Buchmann *et al.* [27] used an *R*-matrix formula for sequential decay similar to that used here, and found that the calculated width of the $5/2^-$ level was too small to fit the data for any choice of reduced width amplitudes; their solution of the problem, to reduce the energy of the $^5$Li ground state to 0.3 MeV (compared with 1.97 MeV in Ref. [20]), is hardly acceptable.

A microscopic cluster-model calculation [28] gave widths of the $5/2^-$ levels of $^9$Be and $^9$B that “reasonably reproduced” the experimental values, with the calculated width about twice the experimental value for $^9$Be and about half for $^9$B. A more recent calculation [29], however, gives calculated widths that are much less than the experimental values; for $^9$Be, different methods give widths of $7\times10^{-8}$ keV and $<0.1$ keV ($\Gamma_{\rm expt}=0.77$ keV), while for $^9$B, the calculated values are 0.07 keV and 6 keV ($\Gamma_{\rm expt}=81$ keV). These discrepancies may be attributed in part but probably not entirely to the calculated energies of the states being too low by about 0.4 MeV.

## V. CONCLUSION

In summary, it seems that agreement with the experimental widths may be possible for the various cases of two-proton and two-neutron decay, and for the $^9$Be and $^9$B levels apart from the $5/2^-$ levels, provided that there is coherence between the *R*-matrix contributions from the various decay channels. If such coherence exists, however, the predictive power of *R*-matrix calculations for three-body decay widths is lessened, unless the contribution from one decay channel is dominant.

## ACKNOWLEDGMENT

I am grateful to D. J. Millener for helpful correspondence, and for renewing my interest in the widths of $^9$Be and $^9$B levels.

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