# Proton-neutron pairing in the deformed BCS approach

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(Received 3 April 2003; published 24 November 2003)

We examine isovector and isoscalar proton-neutron-pairing correlations for the ground state of even-even Ge isotopes with mass number $A$=64–76 within the deformed BCS approach. For $N$=$Z$ $^{64}_{32}$Ge the BCS solution with only $T$=0 proton-neutron pairs is found. For other nuclear systems ($N>Z$) a coexistence of $T$=0 and $T$=1 pairs in the BCS wave function is observed. The problem of fixing of strengths of isoscalar and isovector pairing interactions is addressed. The dependence of number of like and unlike pairs in the BCS ground state on the difference between number of neutrons and protons is discussed. We found that for nuclei with $N$ much bigger than $Z$ the effect of proton-neutron pairing is small but not negligible.

DOI: 10.1103/PhysRevC.68.054319 PACS number(s): 21.10.−k, 21.60.Fw, 21.30.Fe, 23.40.Hc

## I. INTRODUCTION

The proton-neutron ($pn$) pairing correlations remain to be a subject of great interest as it is expected that they play an important role in nuclear structure and decay for proton-rich nuclei with $N\approx Z$. In these nuclei proton and neutrons occupy identical orbitals and have maximal spatial overlap. New experimental facilities involving radioactive nuoclear beams offer opportunities to study $N$=$Z$ nuclei up to $^{100}$Sn [1,2]. There is still much to be learned about systems out of the region of stability. New information could be helpful in understanding various phases of stellar evolution including nucleosynthesis and the abundance of elements. Decay properties and nuclear structure are closely related. The influence of the $pn$ pairing on the position and stability of the proton drip line due to the additional $pn$-pairing binding energy are becoming an important issue in nuclear structure [3,4]. The recent progress in sensitivity achieved with the large $\gamma$-ray detector arrays allows one to study the consequences of the $pn$-pair correlations for the rotational spectra [5].

The $pn$-pair correlations have been a major challenge to the nuclear structure models for a long time (for a review of the early work on $pn$-pairing problem see Ref. [6]). In contrast to the proton-proton ($pp$) and the neutron-neutron ($nn$) pairing, the proton-neutron pairing may exist in two different varieties, namely isoscalar ($T$=0) and isovector ($T$=1) pairing. A generalized pairing formalism, which includes $T$=0 and $T$=1 $pn$ correlations, was derived by Chen and Goswami [7]. The interplay of isovector and isoscalar pairing has been studied in various contexts especially for $N$=$Z$ nuclei [8,7,9–16]. In recent publications phenomena such as possible phase transition between different pairing modes, competition of isoscalar and isovector $pn$ pairing, and the ground state properties of both even-even and odd-odd $N$=$Z$ nuclei were studied mostly within schematic models [17–27]. It was also shown that both single- and double-$\beta$ decay transitions are affected by the proton-neutron pairing [28,29].

The aim of this paper is to study the $pn$-pairing effect within the generalized BCS approach with schematic forces by taking into account the deformation degrees of freedom. The main point is to use the advantage of the formalism constructed by Chen and Goswami [7], which is flexible enough to account for both the $T$=1 and $T$=0 pairing correlations between nucleons in time-reversed orbitals, in order to study the interplay and competition of isovector and isoscalar pairing. For this purpose a schematic nuclear Hamiltonian with separated $pp$, $nn$, and $pn$ ($T$=1 and $T$=0) pairing interactions is written. We focus our attention also on the problem whether the $pn$-pairing correlations are restricted only to the vicinity of the $N$=$Z$ line for medium heavy nuclei. Questions related to the fixing of pairing strength parameters will be discussed.

## II. THEORY

The ground state of even-even nuclei is determined by the deformed pairing Hamiltonian, which includes monopole ($K$=0) proton, neutron, and proton-neutron pairing interactions:

$$\begin{aligned} H = &\sum_s (\epsilon^0_{ps} - \lambda_p)\sum_\sigma c^\dagger_{ps\sigma}c_{ps\sigma} + \sum_s (\epsilon^0_{ns} - \lambda_n)\sum_\sigma c^\dagger_{ns\sigma}c_{ns\sigma} \\ &- G^{T=1}_{pp}\sum_{s,s'} S^{T=1\dagger}_{spp}S^{T=1}_{s'pp} - G^{T=1}_{nn}\sum_{s,s'} S^{T=1\dagger}_{snn}S^{T=1}_{s'nn} \\ &- G^{T=1}_{pn}\sum_{s,s'} S^{T=1\dagger}_{spn}S^{T=1}_{s'pn} - G^{T=0}_{pn}\sum_{s,s'} S^{T=0\dagger}_{spn}S^{T=0}_{s'pn}, \end{aligned} \quad (1)$$

where $\epsilon^0_{ps}$ and $\epsilon^0_{ns}$ are the unrenormalized proton and neutron single-particle energies, respectively. $\lambda_p$ ($\lambda_n$) is the proton (neutron) Fermi energy and $S^{T\dagger}_{s\tau\tau'}$ creates isovector ($T$=1) or isoscalar ($T$=0) pairs in time-reversed orbits [27]:

 

$$S_{spp}^{T=1\dagger}=\sum_{\sigma}c_{ps\sigma}^{\dagger}c_{ps\tilde{\sigma}}^{\dagger},\quad S_{snn}^{T=1\dagger}=\sum_{\sigma}c_{ns\sigma}^{\dagger}c_{ns\tilde{\sigma}}^{\dagger},$$

$$S_{spn}^{T=1\dagger}=\sum_{\sigma}\frac{1}{\sqrt{2}}(c_{ps\sigma}^{\dagger}c_{ns\tilde{\sigma}}^{\dagger}+c_{ns\sigma}^{\dagger}c_{ps\tilde{\sigma}}^{\dagger}),$$

$$S_{spn}^{T=0\dagger}=\sum_{\sigma}\frac{1}{\sqrt{2}}(c_{ps\sigma}^{\dagger}c_{ns\tilde{\sigma}}^{\dagger}-c_{ns\sigma}^{\dagger}c_{ps\tilde{\sigma}}^{\dagger}). \tag{2}$$

Here, $c_{\tau s\sigma}^{\dagger}$ and $c_{\tau ps\sigma}$ stand for the creation and annihilation operators of a particle ($\tau=p$ and $\tau=n$ denote proton and neutron, respectively) in the axially symmetric harmonic oscillator potential. These states are completely determined by a principal set of quantum numbers $s=(N,n_z,\Lambda,\Omega)$. $\sigma$ is the sign of the angular momentum projection $\Omega(\sigma=\pm1)$. We note that the intrinsic states are twofold degenerate. The states with $\Omega$ and $-\Omega$ have the same energy as a consequence of the time-reversal invariance. $\sim$ indicates time-reversed states.

The Hamiltonian in Eq. (1) is invariant under Hermitian and time-reversal operations. The four coupling strengths $G_{pp}^{T=1}$, $G_{nn}^{T=1}$, $G_{pn}^{T=1}$, and $G_{pn}^{T=0}$ are real and characterize the associated isovector ($pp$, $nn$, and $pn$) and isoscalar ($pn$) monopole ($K=0$) pairing interactions. The isospin symmetry of the Hamiltonian in Eq. (1) is restored for $\epsilon_{ps}^{0}=\epsilon_{ns}^{0}$ and $G_{pp}^{T=1}=G_{nn}^{T=1}=G_{pn}^{T=1}=G_{pn}^{T=0}$. In the particular case where $G_{pn}^{T=1}=G_{pn}^{T=0}$ we get

$$H=\sum_{s\sigma\tau}(\epsilon_{\tau s}^{0}-\lambda_{\tau})c_{\tau s\sigma}^{\dagger}c_{\tau s\sigma}-\sum_{\tau\tau'}G_{\tau\tau'}\sum_{s\sigma s'\sigma'}c_{\tau s\sigma}^{\dagger}c_{\tau' s\tilde{\sigma}}^{\dagger}c_{\tau' s'\tilde{\sigma}'}c_{\tau s'\sigma'}. \tag{3}$$

It is assumed that $G_{\tau\tau'}=G_{\tau'\tau}$. In this limit one cannot distinguish between $T=0$ and $T=1$ pairing. We note that a similar Hamilton was discussed in Ref. [23], where the representation of the single-particle states with good angular momentum quantum number was considered.

If the proton-proton, neutron-neutron, and proton-neutron pairing correlations are considered for axially symmetric nuclei, the particle ($c_{\tau s\sigma}^{\dagger}$ and $c_{\tau s\sigma}$, $\tau=p,n$) and the quasiparticle ($a_{\rho s\sigma}^{\dagger}$ and $a_{\rho s\sigma}$, $\rho=1,2$) creation and annihilation operators for the deformed shell model states are related each to other by the generalized BCS transformation [10]:

$$\begin{pmatrix} c_{ps\sigma}^{\dagger}\\ c_{ns\sigma}^{\dagger}\\ c_{ps\tilde{\sigma}}\\ c_{ns\tilde{\sigma}}\end{pmatrix}=\begin{pmatrix} u_{s1p} & u_{s2p} & -v_{s1p} & -v_{s2p}\\ u_{s1n} & u_{s2n} & -v_{s1n} & -v_{s2n}\\ v_{s1p} & v_{s2p} & u_{s1p} & u_{s2p}\\ v_{s1n} & v_{s2n} & u_{s1n} & u_{s2n}\end{pmatrix}\begin{pmatrix} a_{1s\sigma}^{\dagger}\\ a_{2s\sigma}^{\dagger}\\ a_{1s\tilde{\sigma}}\\ a_{2s\tilde{\sigma}}\end{pmatrix}, \tag{4}$$

where the occupation amplitudes $u_{s1p}$, $v_{s1p}$, $u_{s2n}$, $v_{s2n}$ are real and $u_{s1n}$, $v_{s1n}$, $u_{s2p}$, $v_{s2p}$ are complex [6]. In the case where only the $T=1$ proton-neutron pairing is considered, all amplitudes are real [6,28]. In the limit in which there is no proton-neutron pairing, $u_{s2p}=v_{s2p}=u_{s1n}=v_{s1n}=0$. Then the isospin generalized BCS transformation in Eq. (4) reduces to two conventional BCS two-dimensional transformations, first for protons ($u_{s1p}=u_{sp}, v_{s1p}=v_{sp}$) and second for neutrons ($u_{s2n}=u_{sn}, v_{s2n}=v_{sn}$).

The diagonalization of Hamiltonian (1) is equivalent to the matrix diagonalization [10]

$$\begin{pmatrix} \epsilon_{ps}-\lambda_p & 0 & \Delta_{pp} & \Delta_{pn}\\ 0 & \epsilon_{ns}-\lambda_n & \Delta_{pn}^{*} & \Delta_{nn}\\ \Delta_{pp} & \Delta_{pn} & -(\epsilon_{ps}-\lambda_p) & 0\\ \Delta_{pn}^{*} & \Delta_{nn} & 0 & -(\epsilon_{ns}-\lambda_n)\end{pmatrix}\begin{pmatrix} u_{s\rho p}\\ u_{s\rho n}\\ v_{s\rho p}\\ v_{s\rho n}\end{pmatrix}=E_{s\rho}\begin{pmatrix} u_{s\rho p}\\ u_{s\rho n}\\ v_{s\rho p}\\ v_{s\rho n}\end{pmatrix} \tag{5}$$

that yields the quasiparticle energies $E_{s\rho}$ and the occupation amplitudes. Here, $\epsilon_{\tau s}$ ($\tau=p,n$) are the renormalized single-particle energies which include terms describing the coupling of the nuclear average field with the characteristics of the pairing interactions [31]. The proton ($\Delta_{pp}$), neutron ($\Delta_{nn}$), and proton-neutron ($\Delta_{pn}$) pairing gaps are given as

$$\Delta_{\tau\tau}=G_{\tau\tau}^{T=1}\sum_{s,\rho}v_{s\rho\tau}u_{s\rho\tau}^{*}=G_{\tau\tau}^{T=1}\sum_{s,\rho}v_{s\rho\tau}^{*}u_{s\rho\tau}(\tau=p,n),$$

$$\Delta_{pn}=\Delta_{pn}^{T=1}+i\Delta_{pn}^{T=0} \tag{6}$$

with

$$\Delta_{pn}^{T=1}=G_{pn}^{T=1}\mathrm{Re}\left\{\sum_{s,\rho}v_{s\rho p}u_{s\rho n}^{*}\right\},$$

$$\Delta_{pn}^{T=0}=G_{pn}^{T=0}\mathrm{Im}\left\{\sum_{s,\rho}v_{s\rho p}u_{s\rho n}^{*}\right\}. \tag{7}$$

The real and imaginary parts of the proton-neutron pairing gap $\Delta_{pn}$ are associated with $T=1$ and $T=0$ pairing modes, respectively. This phenomenon was first pointed out by Goswami [7,10], which made possible almost all subsequent treatments of $pn$ pairing. We note that for $G_{pn}^{T=0}$ equal to zero the occupation amplitudes of the isospin generalized BCS transformations are real. The Langrange multipliers $\lambda_p$ and $\lambda_n$ entering Eq. (5) are adjusted so that the number-conservation relations

$$Z=2\sum_{s\rho}v_{s\rho p}v_{s\rho p}^{*},\quad N=2\sum_{s\rho}v_{s\rho n}v_{s\rho n}^{*} \tag{8}$$

are satisfied.

The ground state energy can be written as

$$H_{\mathrm{g.s.}}=H_0+H_{pair}, \tag{9}$$

where $H_0$ is the BCS expectation value of the single-particle Hamiltonian

$$H_0=2\sum_{\tau s}\epsilon_{\tau s}\sum_{\rho}v_{s\rho\tau}v_{s\rho\tau}^{*} \tag{10}$$

and $H_{pair}$ represents the pairing energy,

$$H_{pair}=-\frac{\Delta_{pp}^2}{G_{pp}^{T=1}}-\frac{\Delta_{nn}^2}{G_{nn}^{T=1}}-\frac{(\Delta_{pn}^{T=1})^2}{G_{pn}^{T=1}}-\frac{(\Delta_{pn}^{T=0})^2}{G_{pn}^{T=0}}. \tag{11}$$

We note that *pp*, *nn*, and *pn* ($T$=0 and $T$=1) pairing modes contribute coherently to the ground state (g.s.) energy $H_{\rm g.s.}$.

In Ref. [22] it has been suggested that the effect of different pairing modes can be quantified by measuring pair numbers in the nuclear wave function [25]. For this purpose we define the operators

$$\mathcal{N}_{pp}=\sum_{s,s'} S^{T=1\dagger}_{spp}S^{T=1}_{s'pp},\quad \mathcal{N}_{nn}=\sum_{s,s'} S^{T=1\dagger}_{snn}S^{T=1}_{s'nn},$$

$$\mathcal{N}^{T=1}_{pn}=\sum_{s,s'} S^{T=1\dagger}_{spn}S^{T=1}_{s'pn},\quad \mathcal{N}^{T=0}_{pn}=\sum_{s,s'} S^{T=0\dagger}_{spn}S^{T=0}_{s'pn},\qquad (12)$$

which are rough measures of the numbers *pp*, *nn*, *pn*, ($T$=1), and *pn* ($T$=0) pairs, respectively. The BCS ground state expectation values of these operators are related with the corresponding pairing gaps. After subtracting the mean field values we find

$$\langle\mathcal{N}_{pp}\rangle\approx\frac{\Delta^2_{pp}}{(G^{T=1}_{pp})^2},\quad \langle\mathcal{N}_{nn}\rangle\approx\frac{\Delta^2_{nn}}{(G^{T=1}_{nn})^2},$$

$$\langle\mathcal{N}^{T=1}_{pn}\rangle\approx\frac{(\Delta^{T=1}_{pn})^2}{(G^{T=1}_{pn})^2},\quad \langle\mathcal{N}^{T=0}_{pn}\rangle\approx\frac{(\Delta^{T=0}_{pn})^2}{(G^{T=0}_{pn})^2}.\qquad (13)$$

We note that the number of these pairs cannot be observed directly.

## III. EMPIRICAL PAIRING GAPS

The magnitude of proton, neutron, and proton-neutron pairing gaps can be determined only indirectly from the experimental data. Usually they are deduced from systematic study of experimental odd-even mass differences:

$$M(Z,N)_{odd\text{-}odd}=\mathcal{M}(Z,N)$$

$$M(Z,N)_{odd\text{-}proton}=\mathcal{M}(Z,N)+\Delta^{emp}_p$$

$$M(Z,N)_{odd\text{-}neutron}=\mathcal{M}(Z,N)+\Delta^{emp}_n$$

$$M(Z,N)_{odd\text{-}odd}=\mathcal{M}(Z,N)+\Delta^{emp}_p+\Delta^{emp}_n-\delta^{emp}_{pn}.\qquad (14)$$

Here, $M(Z,N)$ are the experimental nuclear masses and $\mathcal{M}(Z,N)$ denotes a smooth mass surface formed by a set of even-even nuclei, i.e., for these nuclei the measured mass is identical to the smooth mass. The mass of odd-proton (odd-neutron) nucleus is given by addition of the proton pairing gap $\Delta^{emp}_p$ (neutron pairing gap $\Delta^{emp}_n$) to $\mathcal{M}(Z,N)$. The mass of an odd-odd nucleus is the sum of the smooth mass $\mathcal{M}(Z,N)$ and the sum of the proton and neutron pairing gaps minus the attractive residual proton-neutron interaction energy $\delta^{emp}_{pn}$.

Using the Taylor series expansion of the $\mathcal{M}(Z,N)$, the quantities $\Delta^{emp}_p$, $\Delta^{emp}_n$, and $\delta^{emp}_{pn}$ for even mass nuclei can be expressed as

$$\Delta^{emp}_p=-\tfrac{1}{8}[M(Z+2,N)-4M(Z+1,N)+6M(Z,N)-4M(Z-1,N)+M(Z-2,N)],$$

$$\Delta^{emp}_n=-\tfrac{1}{8}[M(Z,N+2)-4M(Z,N+1)+6M(Z,N)-4M(Z,N-1)+M(Z,N-2)],$$

$$\delta^{emp}_{pn}=\tfrac{1}{4}\{2[M(Z,N+1)+M(Z,N-1)+M(Z-1,N)+M(Z+1,N)]-4M(Z,N)]-[M(Z+1,N+1)+M(Z-1,N+1)+M(Z+1,N-1)+M(Z-1,N-1)].\qquad (15)$$

The first systematic studies of nuclear masses have shown that the average pairing gaps ($\overline{\Delta}_{\tau\tau}$, $\tau=p,n$) decrease slowly with $A^{1/2}$ (traditional model) [32]. Vogel *et al.* found evidence for the dependence of the average pairing gaps on the relative neutron excess $(N-Z)/A$ [33]. The parametrizations of the average pairing gaps and the average proton-neutron residual interaction within these two models are as follows:

$$\overline{\Delta}_\tau=12\ \mathrm{MeV}/A^{1/2},\quad \overline{\delta}_{pn}=20\ \mathrm{MeV}/A\quad \text{(traditional model)}$$

$$\overline{\Delta}_\tau=\left(7.2-44\frac{(N-Z)^2}{A^2}\right)\mathrm{MeV}/A^{1/3},$$

$$\overline{\delta}_{pn}=31\ \mathrm{MeV}/A\quad (\text{Vogel } et\ al).\qquad (16)$$

We note that recently Madland and Nix [34] presented a model for calculation of these average quantities by fixing a larger set of parameters.

In Table I we present the calculated experimental pairing gaps and proton-neutron excitation energies for Ge isotopes with $A$=64–76 and compare them with the averaged ones. We see that a better agreement between empirical and average values is achieved for the model developed by Vogel *et al.* [33]. The differences between empirical and average values are small especially for isotopes close to the valley of $\beta$ stability. It is worthwhile to note that the values of proton-neutron interaction energies are not negligible in comparison with the values of pairing gaps even for isotopes with large neutron excess. This fact is clearly illustrated in Fig. 1. Thus the proton-neutron pairing interaction is expecting to play a significant role in construction of the quasiparticle mean field even for these nuclei. It is supposed that the origin of this phenomenon is associated with the effect of nuclear deformation, which is changing the distribution of proton and neutron single-particle levels inside the nucleus.

For performing a realistic calculation within the deformed BCS approach it is necessary to fix the parameters of the nuclear Hamiltonian in Eq. (1). Following the procedure of Ref. [28] it is done in the following two steps.

The proton (neutron) pairing interaction strength $G^{T=1}_{pp}(G^{T=1}_{nn})$ is adjusted by requiring that the lowest proton (neutron) quasiparticle energy be equal to the empirical proton (neutron) pairing gap $\Delta^{emp}_p(\Delta^{emp}_n)$.

With $G^{T=1}_{pp}$ and $G^{T=1}_{nn}$ already fixed we adjust the proton-neutron pairing interaction strengths $G^{T=1}_{pn}$ and $G^{T=0}_{pn}$ to the empirical proton-neutron interaction energy $\delta^{emp}_{pn}$ using the formula

TABLE I. The empirical [see Eq. (15)] and average [see Eq. (16)] pairing gaps and proton-neutron residual energy for Ge isotopes with $A$=64–76.

| Nucleus | Empirical values | | | Average values | | | |
|---|---|---|---|---|---|---|---|
| | | | | Traditional model | | Vogel *et al.* | |
| | $\Delta_p^{emp}$ (MeV) | $\Delta_n^{emp}$ (MeV) | $\delta_{pn}^{emp}$ (MeV) | $\bar{\Delta}_{p,n}$ (MeV) | $\bar{\delta}_{pn}$ (MeV) | $\bar{\Delta}_{p,n}$ (MeV) | $\bar{\delta}_{pn}$ (MeV) |
| $^{64}$Ge | 1.807 | 2.141 | 1.498 | 1.500 | 0.313 | 1.800 | 0.484 |
| $^{66}$Ge | 1.586 | 1.859 | 0.816 | 1.477 | 0.303 | 1.770 | 0.470 |
| $^{68}$Ge | 1.609 | 1.882 | 0.630 | 1.455 | 0.294 | 1.727 | 0.455 |
| $^{70}$Ge | 1.551 | 1.866 | 0.594 | 1.434 | 0.285 | 1.668 | 0.443 |
| $^{72}$Ge | 1.614 | 1.836 | 0.583 | 1.414 | 0.278 | 1.600 | 0.430 |
| $^{74}$Ge | 1.621 | 1.715 | 0.424 | 1.350 | 0.270 | 1.523 | 0.419 |
| $^{76}$Ge | 1.561 | 1.535 | 0.388 | 1.376 | 0.263 | 1.441 | 0.408 |

$$\delta_{pn}^{theor} = -[(H_{\text{g.s.}}^{(12)} + E_1 + E_2) - (H_{\text{g.s.}}^{(pn)} + E_p + E_n)]. \quad (17)$$

Here, $H_{\text{g.s.}}^{(12)}(H_{\text{g.s.}}^{(pn)})$ is the total deformed BCS ground state energy with (without) proton-neutron pairing and $E_1 + E_2(E_p + E_n)$ is the sum of the lowest two quasiparticles energies with (without) proton-neutron pairing gap $\Delta_{pn}$.

We note that the calculation of ground state energies of odd-odd nuclei within macroscopic pairing models is based on the assumption that there are one unpaired proton and neutron with energies close to the Fermi energies [33,37]. The resulting expectation value of an attractive short-range residual interaction between them, which can be approximated by a $\delta$ force, is considered to be the origin of the proton-neutron interaction energy. Unfortunately, this simplified approach cannot be exploited in microscopic treatment of nuclear properties of open shell nuclei, as the construction of the many-body wave function is required.

In our deformed BCS approach the ground state of the odd-odd nucleus is described as the lowest two quasiparticle excitation of the even-even nucleus. The considered procedure of fixing the pairing strengths has been exploited already in Refs. [28,30]. However, some questions arise about the ambiguity of equating the pairing-gap expressions that

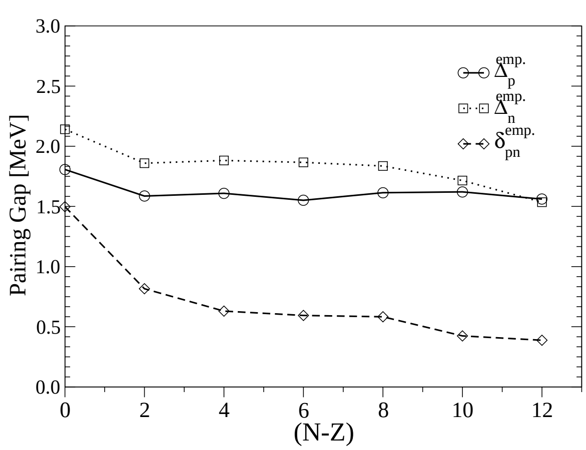


FIG. 1. The experimental proton ($\Delta_p^{emp}$) and neutron ($\Delta_n^{emp}$) pairing gaps and proton-neutron interaction energy ($\delta_{pn}^{emp}$) for even-even Ge isotopes with $A$=64–76 [see Eq. (15)].

are used to determine the strength of pairing matrix elements for microscopic pairing calculations with the macroscopic pairing-gap model that is used to describe average mass differences. Thus, we shall study the importance of the proton-neutron-pairing effect for $N>Z$ nuclei also by assuming a different scenario, namely, commonly used pairing strengths,

$$G_{pp}^{T=1} = G_{nn}^{T=1} = 16/A \text{ MeV}, \quad G_{pn}^{T=0} = 20/A \text{ MeV}, \quad (18)$$

which decrease with increasing neutron excess.

## IV. RESULTS AND DISCUSSION

The starting point of our calculations is the eigenstates of a deformed axially symmetric Woods-Saxon potential with the parametrization of Ref. [35], i.e., spherical symmetry is broken already from the beginning. For description of the ground states of Ge isotopes we use the values of the quadrupole ($\beta_2$) and the hexadecapole ($\beta_4$) nuclear deformation parameters from Ref. [36], which are in good agreement with the predictions of the macroscopic-microscopic model of Möller, Nix, Myers, and Swiatecki [37]. In the BCS calculation the single-particle states are identified with the asymptotic quantum numbers $(N, n_z, \Lambda, \Omega)$. We note that intrinsic states are twofold degenerate. The states with $\Omega$ and $-\Omega$ have the same energy as a consequence of the time-reversal invariance. A truncated model space with $N \leqslant 5$ is considered. As stated in Sec. II only the coupling of nucleon states in time-reversed components of the same orbitals are taken into account.

We performed calculations within the generalized BCS formalism associated with the nuclear Hamiltonian in Eq. (1). The solutions obtained can be classified as follows.

The BCS solution without $pn$ pairing: In this case $\Delta_{pp}$ and $\Delta_{nn}$ are real and $\Delta_{pn}$=0.

The BCS solution with $T$=1 $pn$ pairing: It corresponds to the case the where $\Delta_{pp}$, $\Delta_{nn}$, and $\Delta_{pn}$ are real ($\Delta_{pn}^{T=0}$=0), i.e., all the occupation amplitudes are real.

The BCS solution with $T$=0 $pn$ pairing, which is characterized by real $\Delta_{pp}$ and $\Delta_{nn}$ and purely imaginary $\Delta_{pn}$ ($\Delta_{pn}^{T=1}$=0). In this case the occupation amplitudes associated with $pn$ pairing ($u_{s1n}$, $v_{s1n}$, $u_{s2p}$, and $v_{s2p}$) are imaginary.

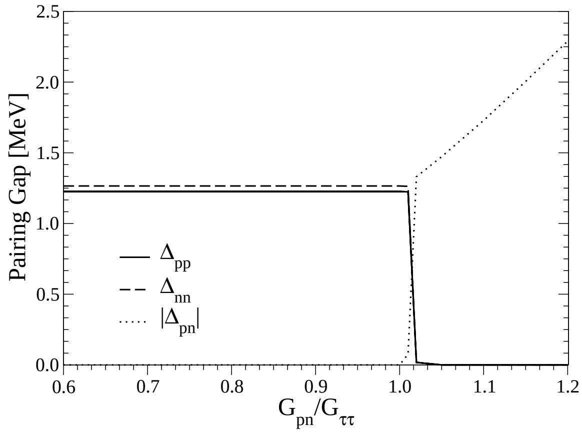


FIG. 2. The proton ($\Delta_{pp}$), neutron ($\Delta_{nn}$), and proton-neutron ($\Delta_{pn}$) pairing gaps as a function of the ratio $G_{pn}/G_{\tau\tau}$ for the $^{64}_{32}$Ge. $G_{\tau\tau}$ represents the proton and neutron-pairing strengths ($G_{\tau\tau}=G_{pp}^{T=1}=G_{nn}^{T=1}$). $G_{pn}$ stands for the larger of $T=0$ ($G_{pn}^{T=0}$) and $T=1$($G_{pn}^{T=1}$) proton-neutron-pairing strengths. $G_{\tau\tau}$ was taken to be 0.250 MeV.

No coexistence of $T=0$ and $T=1$ proton-neutron pairing modes were found. There is a very simple competition between the two kinds of *pn* pairing. For $G_{pn}^{T=1}>G_{pn}^{T=0}$ and $G_{pn}^{T=1}<G_{pn}^{T=0}$, scenarios (ii) and (iii) are realized, respectively. In the particular case $G_{pn}^{T=1}=G_{pn}^{T=0}$ both $T=0$ and $T=1$ pairing modes are indistinguishable as was indicated in Sec. II. We note that the absolute values of the occupation amplitudes associated with solutions (ii) and (iii) are equal to one another if the $T=1$ *pn*-pairing strength used in generating solution (ii) is equal to the $T=0$ *pn*-pairing strength considered in the calculation of solution (iii) (proton and neutron pairing strengths are the same). In the case of $N=Z$ ($^{64}$Ge) for large enough *pn*-pairing strength $G_{pn}^{T=0}$ or $G_{pn}^{T=1}$, a BCS solution without like-particle pairing modes was observed.

In Figs. 2 and 3 we show the BCS gap parameters as a function of the ratio $G_{pn}/G_{\tau\tau}$ for $^{64}$Ge and $^{70}$Ge, respectively. $G_{pn}$ stands for the larger of the $T=1$ ($G_{pn}^{T=1}$) and $T=0$ ($G_{pn}^{T=0}$) proton-neutron-pairing strengths and $G_{\tau\tau}=G_{pp}^{T=1}=G_{nn}^{T=1}$. We stress that there is no coexistence of $T=0$ and $T=1$ proton-neutron-pairing modes and that the absolute value of the *pn*-pairing gap $\Delta_{pn}$ is the same in the case of $T=1$ ($G_{pn}=G_{pn}^{T=1}>G_{pn}^{T=0}$) and $T=0$($G_{pn}=G_{pn}^{T=0}>G_{pn}^{T=1}$) pairing solutions. In the case of $^{64}$Ge($^{70}$Ge), $G_{\tau\tau}$ was assumed to be 0.250 MeV (0.229 MeV). Below some critical value of $G_{pn}/G_{\tau\tau}$ there are only proton and neutron pairing modes. For $^{64}$Ge there is only a narrow region above this critical point in which like-particle and proton-neutron pairs coexist. With additional increase of the ratio $G_{pn}/G_{\tau\tau}$ the system prefers to form only proton-neutron pairs. For nuclei with nonzero neutron excess ($N\neq Z$) such as $^{70}$Ge there is a different situation. In Fig. 3 we notice a less sharp phase transition to the proton-neutron-pairing mode in comparison with that in Fig. 2. In addition, the proton-neutron-pairing mode does exist only in coexistence with the like-particle pairing modes.

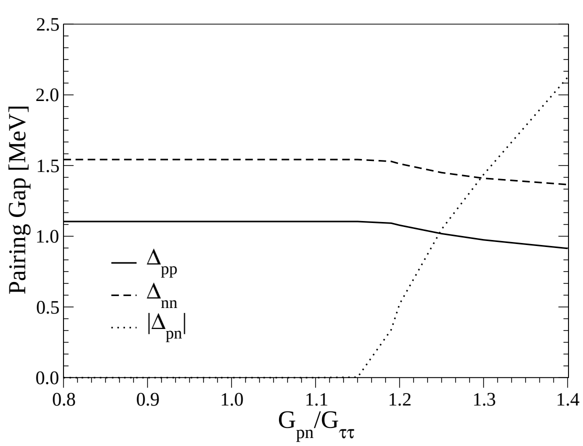


FIG. 3. The proton ($\Delta_p$), neutron ($\Delta_n$), and proton-neutron ($\Delta_{pn}$) pairing gaps as a function of the ratio $G_{pn}/G_{\tau\tau}$ for the $^{70}_{32}$Ge. Conventions are the same as in Fig. 2 and $G_{\tau\tau}$ was equal to 0.229 MeV.

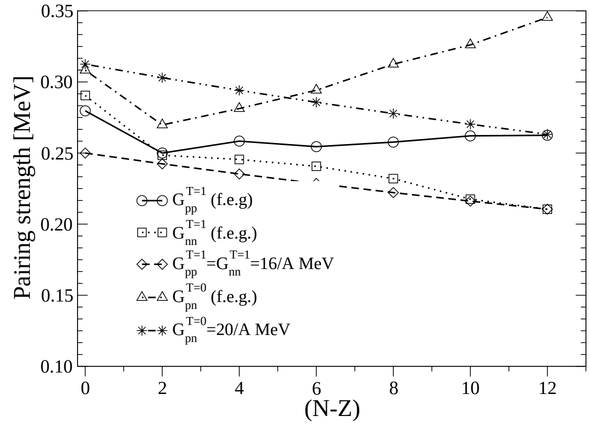


FIG. 4. The proton ($G_{pp}^{T=1}$), neutron ($G_{nn}^{T=1}$), and proton-neutron ($G_{pn}^{T=0}$) pairing strengths as a function of the neutron excess $N-Z$. For the curves f.e.g. (fitted to the experimental gaps) the strength is adjusted to the experimental pairing gap ($\Delta_p^{emp}$ or $\Delta_n^{emp}$) or proton-neutron interaction energy ($\delta_{pn}^{emp}$).

The binding energy gains between a system with no proton-neutron interaction and the system where proton-neutron pairs do exist. The ground state energy decreases monotonically with increasing $G_{pn}^{T=0,1}$. Although the energy gain due to pairing correlations is rather modest, it is expected that *pn* correlations influence many properties of the atomic nuclei. In order to perform corresponding studies the problem of fixing the pairing strength parameters has to be understood.

There is very little known about the $T=0$ and $T=1$ strengths of the *pn* pairing. The $T=0$, $^3S$ pairing force is expected to be stronger in comparison with $T=1$, $^1S$ pairing forces. A strong evidence of this is that the deuteron and many other double even $N=Z$ nuclei prefer this type of coupling due to the strong tensor force contribution. This fact favors solution (iii) in comparison with solution (ii). In Fig. 4 the values of pairing strength adjusted to experimental pairing gaps and proton-neutron interaction energy (see preceding section for details) are presented. By comparing $G_{pp}^{T=1}$ and $G_{nn}^{T=1}$ strengths we see that the isospin invariance is significantly violated especially for isotopes with large neutron excess ($N-Z$). The $T=0$ proton-neutron force $G_{pn}^{T=0}$ is larger in comparison with $T=1$ *pp* and *nn* ($G_{pp}^{T=1}$ and $G_{nn}^{T=1}$) forces for

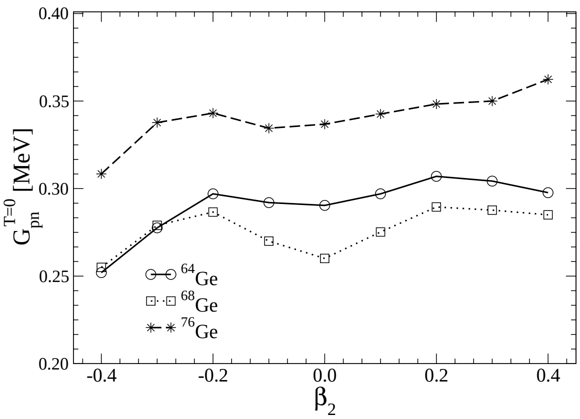


FIG. 5. The $T=0$ proton-neutron-pairing strength $G_{pn}^{T=0}$ as a function of the deformation parameter $\beta_2$ for $^{64}$Ge, $^{68}$Ge, and $^{76}$Ge.

all considered Ge isotopes. The $N=Z$ $^{64}$Ge seems to be a special case. For other Ge isotopes $G_{pp}^{T=1}$ is more or less stable with respect to the $N-Z$ difference and $G_{nn}^{T=1}$ slightly decreases with increasing $N-Z$. The $T=0$ $pn$ force offers a different scenario, namely, $G_{pn}^{T=0}$ slightly growing with increasing neutron excess $N-Z$, which is surprising. It can be due to the fact that only the monopole pair Hamiltonian is considered within the deformed BCS approach or connected with the way of adjusting it. We note that the largest differences among $G_{pp}^{T=1}$, $G_{nn}^{T=1}$, and $G_{pn}^{T=0}$ forces are visible for maximal value of $N-Z=12$. We note that for Ge isotopes with $N-Z<0$ and $N-Z>12$ the pairing strengths cannot be fixed following the procedure presented in the preceding section due to the lack of experimental information about nuclear masses and/or proton and neutron separation energies.

We find it interesting to compare the behavior of the adjusted pairing strengths with the commonly used prescriptions for $G_{pp}^{T=1}$, $G_{nn}^{T=1}$, and $G_{pn}^{T=0}$ [see Eq. (18)]. From Fig. 4 it is evident that the agreement between them, especially, for $G_{pp}^{T=1}$ and $G_{pn}^{T=1}$ forces, is rather poor. The reason can be that the considered strengths are expected to reproduce the general behavior throughout the periodic table as a function of $A$, but not as neutron excess $N-Z$. Other possibilities already announced are the simplicity of the considered nuclear model or the limitations of adjusting the parameters of the microscopic pairing model to those of the macroscopic pairing-gap model.

It is an open issue whether the value of pairing strength $G_{pn}^{T=0}$ depends on the deformation of the considered isotope. In Fig. 5 this point is analyzed for $^{64}$Ge, $^{68}$Ge, and $^{76}$Ge assuming different deformations. $G_{pn}^{T=0}$ is displayed as a function of the deformation parameter $\beta_2$ within the range $-0.4 \leqslant \beta_2 \leqslant 0.4$. We see that $G_{pn}^{T=0}$ is sensitive to the change of the quadrupole parameter $\beta_2$ especially if the shape of the considered nucleus is oblate. From the considered Ge isotopes, $^{68}$Ge exhibits the strongest sensitivity of $G_{pn}^{T=0}$ to $\beta_2$ parameter.

In Fig. 6 the competition among $pp$, $nn$, and $pn$ pairs in the ground state of even-even Ge isotopes is studied, as a function of $N-Z$. The displayed quantities $\langle \mathcal{N}_{pp} \rangle$, $\langle \mathcal{N}_{nn} \rangle$, and $\langle \mathcal{N}_{pn}^{T=0} \rangle$ correspond roughly to the number of $pp$, $nn$, and $T=0$ $pn$ pairs [see Eq. (13)], respectively. These quantities, as it was already stressed above, are closely related to the different contributions to the total pairing energy (11). The number of pairs were measured both for the system with only like-particle pairs (phase i) and for the system where like-particle and proton-neutron pairs coexist (phase iii). In Fig. 6(a) the results obtained with pairing strengths adjusted to experimental pairing gaps ($\Delta_p^{emp}$ and $\Delta_n^{emp}$) and proton-neutron interaction energy ($\delta_{pn}^{emp}$) are presented. We see that in phase i there is a rough constancy of the number of $pp$ pairs for Ge isotopes and that the number of $nn$ pairs is a little bit greater and exhibits some fluctuations. There is a different situation if the system of nucleons prefers the phase iii. $\langle \mathcal{N}_{pp} \rangle$ and $\langle \mathcal{N}_{nn} \rangle$ are equal to zero for $^{64}$Ge and grow up to maximum values about 7.6 and 4.8, respectively, for $^{74}$Ge. We note that the behavior of $\langle \mathcal{N}_{pn}^{T=0} \rangle$ is different. The effect of the proton-neutron-pairing decreases with increasing $N-Z$. For large $N-Z$ the value of $\langle \mathcal{N}_{pn}^{T=0} \rangle$ is significantly

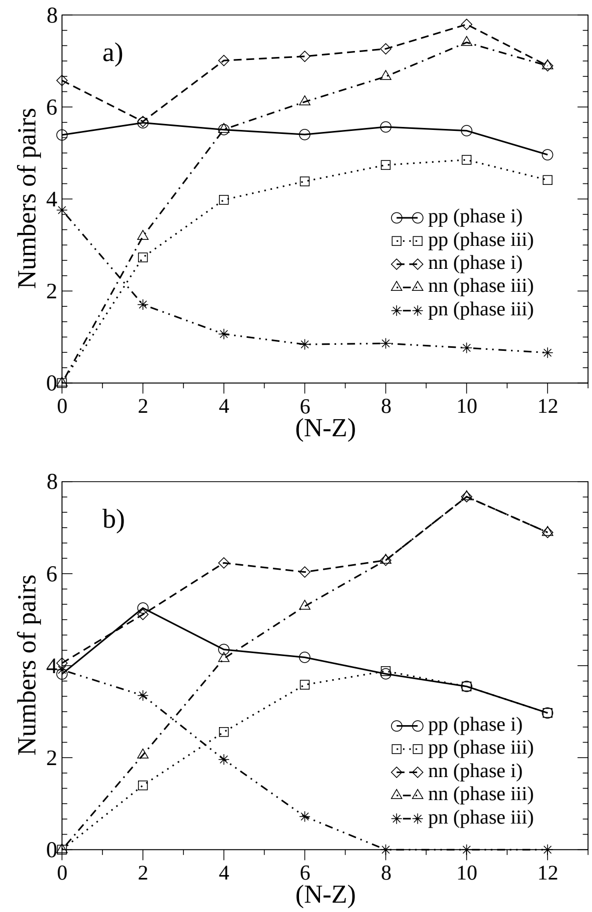


FIG. 6. The quantities $\langle \mathcal{N}_{pp} \rangle$, $\langle \mathcal{N}_{nn} \rangle$, and $\langle \mathcal{N}_{pn}^{T=0} \rangle$ [representing number of $pp$, $nn$, and $pn$ pairs; see Eqs. (13) for definition] for Ge isotopes, as a function of $N-Z$. The results are presented for a pure like-particle pairing phase (phase i) and for a phase where like-particle and $T=0$ proton-neutron pairs coexist (phase iii). The upper panel (a) refers to calculation with pairing strengths adjusted to experimental pairing gaps and proton-neutron interaction energy. The lower panel (b) refers to calculation with pairing strengths given in Eq. (18).

smaller as $\langle \mathcal{N}_{pp} \rangle$ and $\langle \mathcal{N}_{nn} \rangle$, but not negligible. If pairing strengths given in Eq. (18) are used in the BCS calculation, one finds that the effect of proton-neutron pairing disappears at $N-Z \geqslant 8$ in real nuclei as is shown in Fig. 6(b). Then, for these isotopes one fails to explain the nonzero value of the proton-neutron interaction energy $\delta_{pn}^{emp}$ (see Table I). The values of $\delta_{pn}^{emp}$ for all $^{70,72,74,76}$Ge isotopes are of the same order. Thus it is expected that the role of the *pn* pairing for all these isotopes is of comparable importance and not negligible.

From the above discussion it follows that the $T=0$ proton-neutron-pairing correlations should be considered also for medium-heavy nuclei with large neutron excess, i.e., within a procedure proposed in this paper. Usually, correlations between protons and neutrons in medium and heavy nuclei were neglected on the ground that two Fermi levels are apart. Here, it is shown that the proton-neutron-pairing effect is not negligible for such nuclear systems. We strongly suspect that the competition between the different kinds of pairs can affect measurable properties of nuclei, in particular $\beta^+$ strengths. The previous $\beta$- and $\beta\beta$-decay studies [28] performed within the spherical quasiparticle random phase approximation (QRPA) with $T=1$ proton-neutron-pairing support this conclusion as well.

## V. SUMMARY AND CONCLUSIONS

We performed generalized BCS calculation by assuming axial symmetry and the Hamiltonian with schematic $T=1$ and $T=0$ pairing forces in Eq. (1). The system of BCS equations allows three different solutions. There is one solution with only like-particle pairs, and two solutions in which like- and unlike-particle pairs coexist, first with $T=1$ and second with $T=0$ *pn* pairs. We note that none of the observed pairing modes allows simultaneous presence of both $T=0$ and $T=1$ *pn* correlations. The type of the *pn* pairs is determined by the stronger form of $T=0$ and $T=1$ *pn*-pairing interactions of nuclear Hamiltonian. For $N=Z$ $^{64}$Ge pure $T=0$ pairing mode is found and a sharp phase transition from the like-particle pairing mode to the unlike particle-pairing mode is observed, which seems to be a result of a simple monopole pair Hamiltonian. For pair Hamiltonians which are more complex, there is phase coexistence between $T=1$ and $T=0$ pairing in $N=Z$ nuclei, and not a sharp transition from one to the other [19,38]. For other Ge isotopes the phase transition between different pairing modes is much smoother.

A competition between like particles and proton-neutron pairing was studied in even-even Ge isotopes. The pairing strengths were adjusted to reproduce the experimental odd-even mass differences. The diminishing role of the *pn* pairs with increasing $N-Z$ was shown, however, the effect of proton-neutron pairing was found to be important also for isotopes with large neutron excess $N-Z$, in particular for $^{76}$Ge, which undergoes double $\beta$ decay. These results contrast with the general belief that proton-neutron-pairing correlations are restricted only to the vicinity of the $N=Z$ line. The values of the calculated proton-neutron interaction energy $\delta_{pn}^{emp}$ for $N>Z$ isotopes are suggestive and should motivate a greater effort to understand different properties of nuclei in the presence of the $T=0$ proton-neutron-pairing correlations. However, we point out that there is some disagreement between the calculation with pairing strengths adjusted to the experimental pairing gaps and proton-neutron interaction energy and with the commonly used prescription for pairing strengths given in Eq. (18). Within the second scenario the deformed BCS solution with $T=0$ pairing was not found for $N-Z \geqslant 8$.

Of course the effect of *pn* pairing on ground state properties of deformed nuclei can be studied self-consistently by solving the Hartree Fock Bogoliubov (HFB) equations [39]. In this paper we used the advantage of the deformed BCS approach to estimate the effect of *pn* pairing for $N>Z$ nuclei, which can undergo single- or double-$\beta$ decay. At present, a great effort to increase the accuracy and reliability of the calculated single- and double-$\beta$ decay matrix elements is being made. The effects of *pn* pairing and deformation on these matrix elements can be studied within a coupled deformed BCS plus QRPA approach [40,41]. The results of our paper indicate that some of the $\beta$ decay and maybe also the double $\beta$ decay observables might be influenced by the $T=0$ proton-neutron-pairing.

## ACKNOWLEDGMENTS

This work was supported in part by the Deutsche Forschungsgemeinschaft (Grant No. 436 SLK 17/298), by the "Land Baden-Würtemberg" as a "Landesforschungsschwerpunkt" (Grant No. III 1.3-H3-23/74/79/2003): "Low Energy Neutrinos," and by the VEGA grant agency of the Slovac Republic under Contract No. 1/0249/03.

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