Asymmetric nuclear matter in the relativistic mean-field approach with vector cross interaction

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Asymmetric nuclear matter is studied in the frame of relativistic mean-field theory, using scalar-isoscalar σ, vector-isoscalar ω meson together with their self-interactions, vector-isovector ρ meson with its cross interaction with ω meson too, and scalar-isovector δ meson as degrees of freedom. The model is used to parametrize the nuclear matter property results calculated by more fundamental Dirac-Brueckner-Hartree-Fock theory, and thus to provide an effective Dirac-Brueckner-Hartree-Fock model applicable also to finite nuclei. Vector ω-ρ cross interaction seems to be an useful degree of freedom for describing of the asymmetric nuclear matter, mostly due to its impact on density dependence of the symmetry energy.

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I. INTRODUCTION

The study of nuclear matter—hypothetical uniform infinite system of nucleons interacting via strong forces—has already been an important part of the nuclear physics development for several decades. It is a good starting point for both nuclear physics of finite systems (e.g., structure and properties of finite nuclei, dynamics of heavy-ion collisions) and astrophysics (e.g., structure and evolution of stars). The early attempts were based on the nonrelativistic Brueckner-Hartree-Fock (BHF) theory (see, e.g., Refs. [1] for review and references therein), and related the bare nucleon-nucleon (*NN*) interaction to nuclear ground-state properties in a parameter-free way with a limited success.

The breakthrough was achieved when the relativistic extension of the BHF theory [so called Dirac-Brueckner-Hartree-Fock (DBHF) approach] was developed [2–4] and successfully applied to nuclear matter problems. An essential feature of the DBHF is the incorporation of the relativistic dynamics, governed by the Dirac equation with strong scalar and vector fields. The explicit treatment of the lower components of Dirac spinors gives rise to strongly density-dependent relativistic effects. They shift the nuclear matter saturation points ("Coester band") towards empirical values [5]. Subsequently, a great effort has been devoted to the successful DBHF description of both symmetric and (to lesser extent) isospin-asymmetric nuclear matter.

Thus, the DBHF approach is currently considered to be a microscopic parameter-free nuclear model based on realistic *NN* interaction. However, due to its complexity, this sophisticated approach is successfully manageable for nuclear matter properties only; finite nuclei are at present beyond the scope of this model. To overcome this restriction, several approaches were developed which relate the DBHF output for nuclear matter to the parameters of the relativistic mean-field (RMF) theory [6,7]. The RMF approach is widely used and powerful phenomenological tool for various aspects of nuclear many-body problems which provides an effective framework for calculation of both nuclear matter and (contrary to DBHF) finite nuclei. The nonlinear RMF approach [8] has already been proven to be a reliable tool for the calculation of normal nuclei close to the valley of stability [9], exotic and superheavy nuclei [10].

Now new experimental facilities are available to study properties of exotic nuclei with high isospin asymmetry. Additionally, increasingly more precise observations and measurements of properties of neutron stars and supernovas have been carried out. This naturally brings a need for better description of isospin degree of freedom, which can be done by enhancing the isovector meson sector. The isovector scalar δ meson [11] and vector cross interactions [12] were included into RMF for this purpose.

The goal of this paper is obtaining effective parametrization applied to asymmetric nuclear matter, using different degrees of freedom in order to study influence of δ meson and vector meson cross interaction on quality of the reproduction of DBHF results, as well as their influence on calculated nuclear matter properties, especially on density dependence of nuclear symmetry energy.

II. THEORETICAL FRAMEWORK

The starting point of the model is Lagrangian density that introduces nucleon field ψ, isoscalar-scalar meson field σ, isoscalar-vector meson field $\boldsymbol{\omega}$, isovector-vector meson field $\boldsymbol{\rho}$ and isovector-scalar meson field δ (pion field does not contribute, because it is pseudoscalar, and nuclear matter is parity invariant), and takes a form

$$\begin{aligned}\mathcal{L}(\psi,\sigma,\boldsymbol{\omega},\boldsymbol{\rho},\delta) &= \bar{\psi}[\gamma_\mu(i\partial^\mu - g_\omega\boldsymbol{\omega}^\mu - (M-g_\sigma\sigma)]\psi + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)\\ &- \tfrac{1}{4}\boldsymbol{\omega}_{\mu\nu}\boldsymbol{\omega}^{\mu\nu} + \tfrac{1}{2}m_\omega^2\boldsymbol{\omega}_\mu\boldsymbol{\omega}^\mu - \tfrac{1}{3}b_\sigma M(g_\sigma\sigma)^3\\ &- \tfrac{1}{4}c_\sigma(g_\sigma\sigma)^4 + \tfrac{1}{4}c_\omega(g_\omega^2\boldsymbol{\omega}_\mu\boldsymbol{\omega}^\mu)^2 + \tfrac{1}{2}(\partial_\mu\delta\partial^\mu\delta - m_\delta^2\delta^2)\\ &+ \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \tfrac{1}{2}\Lambda_V(g_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu)\\ &\times(g_\omega^2\boldsymbol{\omega}_\mu\boldsymbol{\omega}^\mu) - g_\rho\boldsymbol{\rho}_\mu\bar{\psi}\gamma^\mu\boldsymbol{\tau}\psi + g_\delta\delta\bar{\psi}\boldsymbol{\tau}\psi,\end{aligned} \tag{1}$$

where antisymmetric field tensors are given by

$$\boldsymbol{\omega}_{\mu\nu} \equiv \partial_\nu\boldsymbol{\omega}_\mu - \partial_\mu\boldsymbol{\omega}_\nu,$$

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$$\boldsymbol{\rho}_{\mu\nu} \equiv \partial_\nu \boldsymbol{\rho}_\mu - \partial_\mu \boldsymbol{\rho}_\nu,$$

and the symbols used have their usual meaning. The parameters entering the Lagrangian are M, denoting the nucleon free mass, whereas m_σ, m_ω, m_ϱ, and m_δ are masses assigned to the meson fields. The first term together with the last two ones describe interaction of isoscalar and isovector mesons with nucleons where the strength of these interactions is determined by dimensionless coupling constants g_σ, g_ω, g_ϱ, and g_δ. Three terms in the third line represent cubic and quartic scalar self-interactions [8] and quartic vector self-couplings [13,14], the strength of which is also given by dimensionless self-interaction coupling constants b_σ, c_σ, and c_ω. The second and fourth lines represent free (noninteracting) Lagrangian for all mesons, and the fifth line realizes cross interaction between ω and ρ mesons characterized by cross-coupling constant Λ_V [12].

The constraint of stationarity of the action leads to the well-known Euler-Lagrange field equations and equations of motion follow after their application to Lagrangian (1). This produces the Dirac equation for nucleon field,

$$[\gamma_\mu(i\partial^\mu - g_\omega \boldsymbol{\omega}^\mu - g_\rho \boldsymbol{\rho}_\mu \,.\, \boldsymbol{\tau}) - (M - g_\sigma \sigma - g_\delta \boldsymbol{\delta} \,.\, \boldsymbol{\tau})]\psi = 0. \tag{2}$$

Isoscalar meson field σ, ω are then described by Klein-Gordon and Proca equations, respectively,

$$(\partial_\mu \partial^\mu + m_\sigma^2)\sigma = g_\sigma[\bar{\psi}\psi - b_\sigma M(g_\sigma \sigma)^2 - c_\sigma (g_\sigma \sigma)^3], \tag{3}$$

$$\begin{aligned}\partial_\mu \boldsymbol{\omega}^{\mu\nu} + m_\omega^2 \boldsymbol{\omega}^\nu = g_\omega[&\bar{\psi}\gamma^\nu\psi - c_\omega g_\omega^3(\boldsymbol{\omega}_\mu \boldsymbol{\omega}^\mu \boldsymbol{\omega}^\nu) \\ &- \Lambda_V g_\rho^2 \boldsymbol{\rho}_\mu \,.\, \boldsymbol{\rho}^\mu g_{\boldsymbol{\omega}} \omega_\mu].\end{aligned} \tag{4}$$

Analogically, isovector ρ and δ meson fields read,

$$\partial_\mu \boldsymbol{\rho}^{\mu\nu} + m_\rho^2 \boldsymbol{\rho}^\nu = g_\rho[\bar{\psi}\gamma^\mu \boldsymbol{\tau}\psi - \Lambda_V g_\rho \boldsymbol{\rho}_\mu g_\omega^2 \boldsymbol{\omega}_\mu \boldsymbol{\omega}^\mu], \tag{5}$$

$$(\partial_\mu \partial^\mu + m_\delta^2)\delta = g_\delta \bar{\psi}\boldsymbol{\tau}\psi. \tag{6}$$

Due to the fact that these equations are nonlinear, nowadays no suitable method is known to solve them. The way to avoid this is to replace the operators of meson fields by their expectation values, the so called mean-field approximation. The fields are thus treated as classical c numbers. Its reasonability increases with increasing baryon density. The second approximation introduced is the nonsea approximation which does not take account of the Dirac sea of negative energy states.

In this model we are dealing with static, homogenous, infinite nuclear matter that allows us to consider some other simplifications due to translational invariance and rotational symmetry of nuclear matter. This causes the expectation values of spacelike components of vector fields vanish and only zero components—ρ_0 and ω_0—survive [9]. In addition, rotational invariance around third axis of isospin space results in taking into account only the third component of isovector fields—$\rho^{(3)}$ and $\delta^{(3)}$ [7,11]. The above mentioned can formally be written as

$$\sigma \to \langle\sigma\rangle \equiv \bar{\sigma},$$

$$\boldsymbol{\omega}_\mu \to \langle \boldsymbol{\omega}_\mu \rangle \equiv \delta_{\mu 0} \bar{\boldsymbol{\omega}}_\mu = \bar{\omega}_0,$$

$$\boldsymbol{\rho}_\mu \to \langle \boldsymbol{\rho}_\mu \rangle \equiv \bar{\rho}_0^{(3)},$$

$$\delta \to \langle \delta \rangle \equiv \bar{\delta}^{(3)}.$$

Having inserted the above simplifications the field equations are reduced and we can easily obtain potentials of both isoscalar meson fields,

$$U_\sigma \equiv -g_\sigma \bar{\sigma} = -\frac{g_\sigma^2}{m_\sigma^2}[\rho_S - b_\sigma M(g_\sigma \bar{\sigma})^2 - c_\sigma (g_\sigma \bar{\sigma})^3], \tag{7}$$

$$U_\omega \equiv g_\omega \bar{\omega}_0 = \frac{g_\omega^2}{m_\omega^2}[\rho_B - c_\omega (g_\omega \bar{\omega}_0)^3 - U_\rho^2 \Lambda_V (g_\omega \bar{\omega}_0)], \tag{8}$$

and isovector meson fields,

$$\begin{aligned}U_\rho \equiv g_\rho \bar{\rho}_0^{(3)} &= \frac{g_\rho^2}{m_\rho^2}[\bar{\psi}\gamma^0 \tau_3 \psi - g_\rho \bar{\rho}_0^{(3)} \Lambda_V (g_\omega \bar{\omega}_0)^2] \\ &= \frac{g_\rho^2}{m_\rho^2}\left[\left(2\frac{Z}{A} - 1\right)\rho_B - g_\rho \bar{\rho}_0^{(3)} \Lambda_V U_\omega^2\right],\end{aligned} \tag{9}$$

$$U_\delta \equiv -g_\delta \bar{\delta}^{(3)} = -\frac{g_\delta^2}{m_\delta^2}\bar{\psi}\tau_3\psi = \frac{g_\delta^2}{m_\delta^2}(\rho_n^S - \rho_p^S), \tag{10}$$

where scalar density ρ_S is expressed as the sum of proton (p) and neutron (n) part

$$\rho_S = \langle \bar{\psi}\psi \rangle = \rho_p^S + \rho_n^S, \tag{11}$$

which are given by

$$\rho_i^S = \frac{2}{(2\pi)^3}\int_0^{\boldsymbol{k}_i} d^3k \frac{M_i^*}{(\boldsymbol{k}^2 + M_i^{*2})^{1/2}}, \quad i = p, n. \tag{12}$$

In Eq. (12) k_i is nucleons' Fermi momentum and M_p^*, M_n^* denote proton and neutron effective masses, respectively, which can be written as

$$M_p^* = M - g_\sigma \bar{\sigma} - g_\delta \bar{\delta}^{(3)}, \tag{13}$$

$$M_n^* = M - g_\sigma \bar{\sigma} + g_\delta \bar{\delta}^{(3)}. \tag{14}$$

One can see that condensed scalar σ meson field generates a shift of nucleon mass, in consequence of which nuclear matter is described as a system of pseudonucleons with masses M^* moving in classical fields $\bar{\sigma}$, $\bar{\omega}_0$, and $\rho_0{}^{(3)}$, where additionally δ meson field is responsible for splitting of proton and neutron effective masses, which is an important feature of δ meson influence on the nuclear matter saturation mechanism and its properties. The δ meson seemed to be an useful degree of freedom in describing of asymmetric nuclear matter, indicated by its influence on, e.g., stiffness of equation of state, slope, and curvature of symmetry energy and properties of warm asymmetric nuclear matter [11,15].

The solution requires to be performed self-consistently, which can be clearly seen from Eqs. (7)–(10), where σ potential (7) must be solved using iterations.

The baryon density is given by

$$\rho_B = \langle \bar{\psi}\gamma^0\psi\rangle = \frac{4}{(2\pi)^3}\int_0^{k_F} d^3k = \frac{2}{3\pi^2}k_F^3, \tag{15}$$

with k_F being an average Fermi momentum. It can be seen that scalar density (11) is less than baryon density due to the term $M_i^*/(\boldsymbol{k}^2+M_i^{*2})^{1/2}$ that causes reduction of the contribution of rapidly moving nucleons to scalar source term. This mechanism is responsible for nuclear matter saturation in the mean-field theory and essentially distinguishes relativistic models from nonrelativistic ones.

Cross coupling of the ω and ρ mesons requires also self-consistent calculation of Eqs. (8) and (9), with iterative procedure for ω potential.

By reason that δ field splits nucleon effective masses, the proton and neutron Fermi momenta will be also split, while they have to fulfill

$$\rho_B = \rho_p + \rho_n = \frac{2}{(2\pi)^3}\int_0^{k_p} d^3k + \frac{2}{(2\pi)^3}\int_0^{k_n} d^3k, \tag{16}$$

where k_F is average Fermi moment of the matter, and k_p, k_n are Fermi momenta of protons and neutrons, respectively. The different values of Fermi momenta have consequences for transport properties of asymmetric nuclear matter.

To obtain formula for energy density of nuclear matter it is essential to have cognizance of the energy tensor, in continuum mechanics defined [16] as

$$T_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \frac{\partial \Phi_i}{\partial \mathbf{x}^\nu}\frac{\partial \mathcal{L}}{\partial(\partial \Phi_i/\partial \mathbf{x}_\mu)}, \tag{17}$$

where Φ_i generally denotes physical fields. The energy density of such a system is the zero component of the energy tensor $\varepsilon = \langle T_{00}\rangle$, and finally the binding energy per nucleon is related to energy density by

$$E_b = \frac{\varepsilon}{\rho_B} - M. \tag{18}$$

The energy density per nucleon is a starting quantity for further properties of nuclear matter. Incompressibility is given as its second derivative with respect to baryon density, at the saturation point

$$K = 9\left[\rho^2 \frac{\partial^2}{\partial \rho^2}\left(\frac{\varepsilon}{\rho_B}\right)\right]_{\rho=\rho_0}. \tag{19}$$

Symmetry energy of nuclear matter is defined as a second derivative of binding energy per nucleon with respect to the asymmetry parameter $\alpha = (\rho_p - \rho_n)/(\rho_p + \rho_n)$:

$$\varepsilon(\rho, \alpha) = \varepsilon(\rho, 0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4, \tag{20}$$

where parameters S_2 and S_4 are defined as

$$S_2 = \frac{1}{2}\left[\frac{\partial^2 \varepsilon(\rho, \alpha)}{\partial \alpha^2}\right]_{\alpha=0}, \tag{21}$$

$$S_4 = \frac{1}{24}\left[\frac{\partial^4 \varepsilon(\rho, \alpha)}{\partial \alpha^4}\right]_{\alpha=0}. \tag{22}$$

The parameter S_2 is often used as symmetry energy itself, argumented by negligible contribution of higher order S_4 parameter, especially for densities relevant for common nuclei.

III. RESULTS AND DISCUSSION

The mean-field parametrizations were obtained by calculation using three different DBHF results for nuclear matter as initial data—results of Li, Machleidt, and Brockmann [17], results of Lee, Kuo, Li, and Brown [18,19], and finally calculations of Huber, Weber, and Weigel [20,21].

Based on the realistic and relativistic *NN* interaction of the Bonn group, in Ref. [17] DBHF calculations are shown, which yield an effective *NN* interaction, and subsequently the single-particle potentials, equations of state, nucleon effective masses, and speed of sound for both symmetric and neutron matter were studied. The Bonn-A potential reproduced quantitatively the empirical saturation properties of nuclear matter as well as nucleon effective mass.

Reference [18] deals with asymmetric matter also using the DBHF approach with Bonn A one-boson-exchange *NN* interaction. Not only saturation properties, but in addition even the empirical value of the symmetry energy at the saturation density were reproduced satisfactorily. Isoscalar meson potentials for symmetric matter are calculated in Ref. [19].

Finally, energy per nucleon for several asymmetries using DBHF approach is calculated in Ref. [20], together with proton and neutron scalar and vector potentials from Ref. [21] making possible to fit also the mean-field parameter for coupling of δ meson to nucleons. Furthermore, in these two works momentum dependence of self-energies is also tested; however, due to momentum independence of mean-field theory coupling constants, the momentum independent results were chosen to fit.

As it is easily seen from equations for meson potentials and for energy per nucleon, the squares of coupling constants appear exclusively in ratios with meson masses and thus one can fix the meson masses to experimental values without any physical restriction of the RMF. The meson masses considered in this work are m_σ=550 MeV, m_ω=783 MeV, m_ρ =770 MeV, and m_δ=980 MeV.

The first fit was performed using σ, ω, ρ mesons and scalar-isoscalar cubic and quartic self-interactions, vector-isoscalar quartic self-interactions as well as cross interactions between vector mesons, fitting energies per nucleon for symmetric and neutron matter, and symmetric isoscalar potentials. The corresponding parameter set obtained either with vector meson cross interaction (MA) or without it (MB) is listed in Table I. This parametrization reproduces the DBHF results satisfactorily in the whole fitting range of densities relevant for common nuclei, which can be said both for en-

TABLE I. Parameter sets resulting from the RMF fit to DBHF results of Machleidt and co-workers [17].

	MA	MB
g_σ^2	106.85	112.27
g_ω^2	180.61	204.36
g_ρ^2	18.445	9.4932
b_σ	−0.0025823	−0.0029820
c_σ	0.011529	0.013345
c_ω	0.015849	0.020449
Λ_V	0.25857	
χ^2/N	2.76	9.95

TABLE II. Parameter sets resulting from the fit of Lee *et al.* DBHF calculations [18,19].

	LA	LB
g_σ^2	103.91	102.11
g_ω^2	147.84	146.73
g_ρ^2	17.432	9.6697
b_σ	0.00097186	0.00083559
c_σ	0.0012694	0.0012411
c_ω	0.0054204	0.0051878
Λ_V	0.18790	
χ^2/N	1.69	2.62

ergy per nucleon and also for isoscalar σ and ω meson potentials. Growth of the ρ potential is decreasing with baryon density, being result of the isovector cross-interactions. This has impact on the energy per nucleon especially for extreme asymmetries and consequently on curvature of symmetry energy, which is plotted in the Fig 1, upper panel. The cross interactions significantly affect density dependence of the symmetry energy—they increase rise of symmetry energy below 0.24 fm^{-3} and decrease it above this density. Symmetry energy at the saturation density is 33.3 MeV, which is in accordance with experimental value of about 34 MeV. The incompressibility of symmetric matter at the saturation density is 347 MeV.

For better description of asymmetry behavior of matter it is of course profitable to use not only symmetric and neutron case, but also other nonsymmetric cases with partial fraction

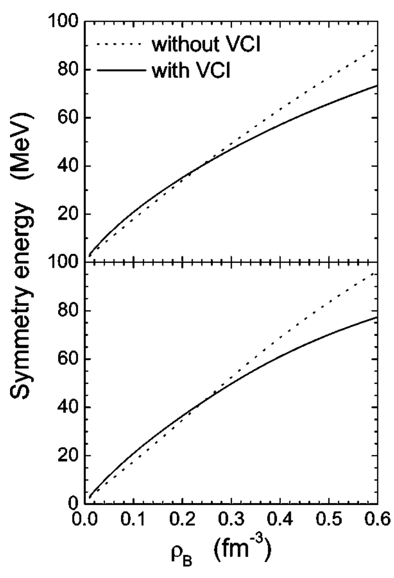

FIG. 1. Density dependence of the symmetry energy for two different parametrization sets. Upper panel shows results for the fit of Machleidt and co-workers DBHF results [17], with mean-field parametrizations listed in Table I, where σ, ω mesons with their self-interactions and ρ mesons were used as degrees of freedom, and both with and without inclusion of vector-meson cross interactions (VCI), plotted with solid and dotted lines, respectively. The lower panel displays analogical calculation results for the fit of Lee *et al.* DBHF results [18,19], with parametrizations in Table II.

of protons, as was calculated with DBHF theory by Lee *et al.* [18,19]. Results of the fit with the same degrees of freedom as were in the previous case, performed for binding energy per nucleon for several asymmetries as well as for symmetric matter scalar and vector potentials, are listed in Table II (parametrization LA), also with parameter set obtained using model without vector-meson cross interactions (LB). All of the relevant physical quantities for all of the asymmetries are reproduced closely. Also in these parametrizations there is strong influence of cross interaction, which is displayed in Fig. 1, lower panel. There is increasing growth of symmetry energy below approximately 0.25 fm^{-3} and deceleration in higher density region. This fact will be commented more closely in the last dataset.

In the theoretical framework we were dealing also with isovector-scalar sector of nucleon-nucleon effective interaction. In Ref. [20] binding energy per nucleon was calculated for several asymmetries in the DBHF approach, using Bonn potential B with density independent self-energies. Additionally, in Ref. [21] authors performed calculation also for proton and neutron scalar and vector potentials, enabling us to fit not only ρ potential but also δ potential value. The fit was thus using σ, ω mesons, their self-interactions, ρ meson with its cross interaction to ω meson, and finally even δ meson as degree of freedom. Results of the fit for binding energy per nucleon for several considered asymmetries are drawn in Fig. 2. Fit values (represented by lines) follow closely the DBHF results (scatter symbols) for all asymmetries. Corresponding parameter set is listed in Table III (HA), together with fit parameters obtained without vector cross interactions (HB).

Negative values of the quartic self-interaction constants are consequence of the fact that dependence of symmetric scalar and vector potentials ensued from DBHF calculations on the baryon density is almost linear, which brings some difficulties into the determination of the self-interaction force of isoscalar mesons, and thus results in negative and positive second derivative of vector and scalar potential, respectively. One of the explanations of this verity could be the fact that the Van Hove theorem [22] is, unlike for the DBHF theory, fully consistent with the mean-field model only. The fit of Lee *et al.* DBHF calculations is not the case (see Table II—all self-interaction constants have positive sign), due to more distinct curvature of density dependence of the potentials

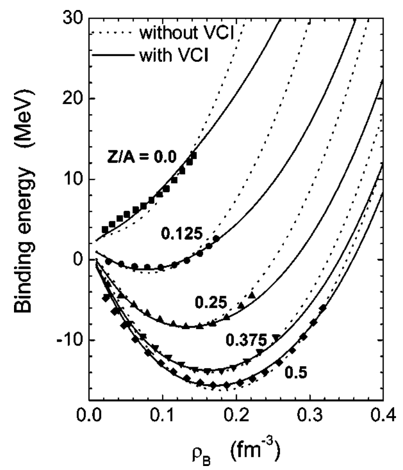

FIG. 2. Density dependence of the binding energy per nucleon for five different asymmetries from pure neutron matter to symmetric nuclear matter, resulting from mean-field theory fit (represented by lines) of Huber *et al.* DBHF results [20,21] (scatter symbols), with corresponding parametrizations listed in Table III. Isoscalar σ and ω mesons with their self-interaction, isovector ρ mesons with (solid lines, parametrization HA) and without (dotted lines, parametrization HB) cross interaction with ω mesons, and δ mesons was used as degrees of freedom.

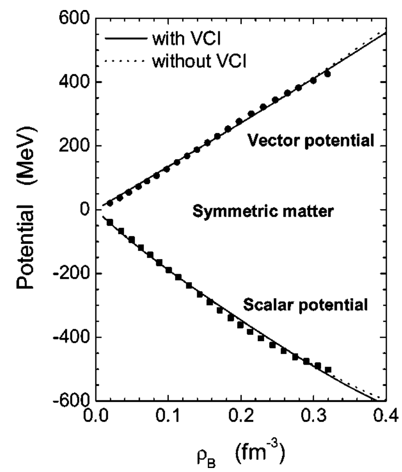

FIG. 3. Isoscalar potentials of symmetric nuclear matter, resulting from the fit of Huber *et al.* DBHF results [20,21], with the same parametrization and notation as in Fig 2.

(not shown in this paper). The incompressibility of nuclear matter is 235 MeV.

Simultaneously with binding energies fit to the symmetric scalar and vector potentials (Fig. 3) and also to above-mentioned proton and neutron scalar and vector potentials for proton fraction 0.125 (Fig. 4) was performed. As can be seen, all of these potentials are reproduced satisfactorily within several MeV.

Evaluation of ρ-ω cross-interaction influence on symmetry energy and its comparison with δ meson influence is possible from Fig. 5. There it is drawn density dependence of the symmetry energy, where each of the lines is correspondent to different degrees of freedom used (see Table III): HA contains both cross interactions and δ meson contribution; then inclusion of the δ meson without vector cross interaction is accomplished by HB; HC considers the vector cross interaction but without δ meson, and finally basic model with σ and ω mesons with their self-interactions and ρ mesons only (HD). Vector cross interaction has significant impact on symmetry energy, where its influence has analogical nature as in the previous two datasets, thus supporting those results also in the case of δ meson inclusion. It increases growth of the symmetry energy in this case below approximately 0.13 fm^{-3}, which is important for description of properties of exotic nuclei near the dripline and is similar as consequences of density dependence of the isovector couplings due to Fock contributions, which was calculated in Ref. [23]. Above 0.13 fm^{-3} it decreases symmetry energy rise, thus implicating an impact on higher density behavior of matter (high energy beam collisions, supernova explosions, and neutron stars properties), and it is also in agreement with recent Hartree calculations [24], where similar high-density influence of cross interactions was concluded. In comparison, δ meson contributes relatively slightly to the behavior of symmetry energy—for lower and intermediate densities its contribution is almost inconspicuous, thus not significantly affecting properties of atomic nuclei. For higher densities in absence

TABLE III. Parameter sets resulting from the fit of Huber *et al.* DBHF results [20,21].

	HA	HB	HC	HD
g_σ^2	90.532	86.432	91.110	87.591
g_ω^2	108.95	106.89	109.26	107.61
g_ρ^2	36.681	28.795	20.804	15.335
g_δ^2	28.739	25.170		
b_σ	0.0043852	0.0033779	0.0044388	0.0035745
c_σ	−0.0052045	−0.0037762	−0.0052076	−0.0039753
c_ω	−0.0001421	−0.0010509	−0.0000385	−0.0007753
Λ_V	0.10647		0.34805	
χ^2/N	2.05	3.80	5.85	6.89

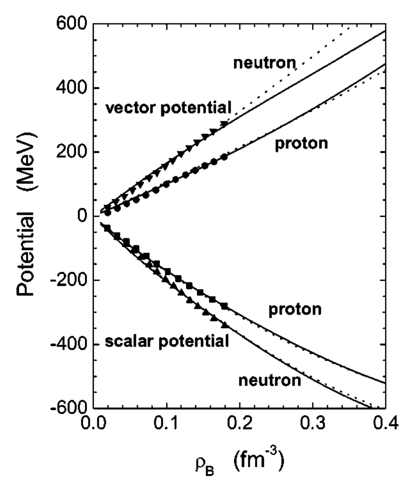

FIG. 4. Scalar and vector potentials of protons and neutrons of the asymmetric matter with proton fraction Z/A=0.125, for the parametrization and notation identical with previous figures.

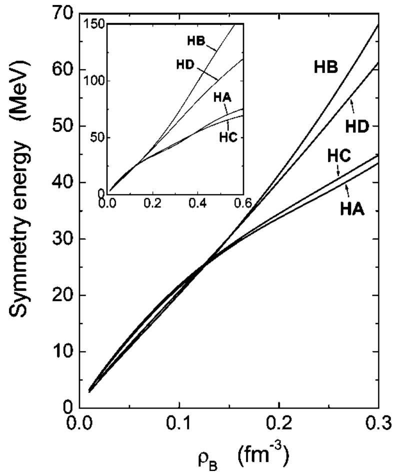

FIG. 5. Density dependence of the symmetry energy of nuclear matter, as resulted from the fit of Huber *et al.* DBHF results [20,21] for several considered degrees of freedom (all models include also isoscalar scalar and vector self-interactions, see Table III): HA: σ, ω, ρ, and δ mesons and ρ-ω cross interaction; HB: σ, ω, ρ, and δ mesons; HC: σ, ω, ρ mesons, and ρ-ω cross-interaction; HD: σ, ω, and ρ mesons. The inset picture shows the symmetry energy behavior in a wider density region.

of cross interaction it slightly increases symmetry energy. Also this result is in concordance with recent calculations of other authors [25]. However, the presence of cross-interaction causes an opposite influence of δ meson—for higher densities it softly fortifies decrease of symmetry energy growth. This leads to the conclusion that ρ-ω cross-interactions seems to be an important degree of freedom which should be used in further calculations.

IV. SUMMARY

In this work the relativistic mean-field theory was used to obtain an effective parametrization of the properties of asymmetric nuclear matter calculated by more fundamental Dirac-Brueckner-Hartree-Fock theory. The energy per nucleon together with the symmetric isoscalar potentials were fitted, and simultaneously also proton and neutron scalar and vector potentials. Isoscalar σ, ω mesons with their selfinteractions, and isovector ρ, δ mesons with ρ-ω cross interaction were used as degrees of freedom and parameters of the fit. Generally a good reproduction of both the energy and the potentials was reached, and thus the parameter sets are representing an effective DBHF description of asymmetric nuclear matter at normal baryon densities applicable as well for calculation of finite nuclei properties. The cross interaction between ρ and ω mesons turns out to improve reproduction of properties of asymmetric nuclear matter. Additionally, it increases symmetry energy in common nuclei density region, e.g., approximately below saturation density, and decreases this energy above saturation point. This has consequences for properties of finite nuclei, especially with large isospin asymmetry, and also for description of nuclear matter at higher densities, relevant in high energy nuclear collisions and several astrophysical processes and phenomena (e.g., neutron star properties and supernova explosions). Isovector δ meson also improves quality of the mean-field model, but without such a strong impact on density dependence of symmetry energy. These results imply that ρ-ω cross interaction is very useful for better description of nuclear matter and that it could be important for calculations of properties of finite nuclei with high isospin asymmetry.

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