Relativistic calculation of hard bremsstrahlung $pp\to pp\gamma$ to discriminate among different kinds of NN interactions

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Previous nonrelativistic calculations have demonstrated a high sensitivity of hard bremsstrahlung $pp\to pp\gamma$ at the beam energies of 350–500 MeV to the kind of nucleon-nucleon potential (meson exchange potentials versus the Moscow one). Here, bremsstrahlung calculations are generalized by means of relativistic considerations (point form dynamics). The necessary formal technique is presented. Resulting cross sections become smaller in comparison with nonrelativistic theory and their angular dependence changes. However, the high sensitivity to the kind of potential continues to exist and is characteristic of the differential cross section even at a relatively low beam energy of E_0=280 MeV where corresponding experimental data do exist. Our calculations give some preliminary indication that one of the versions of the Moscow potential may be valid here.

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I. INTRODUCTION

There are two principally different kinds of models of the nucleon-nucleon interaction. The first traditional model represented by meson exchange potential (MEP) goes conceptually back to the famous idea by Yukawa. Such phenomenological potentials contain a repulsive core representing the exchange of vector mesons. We will use two popular potentials of this kind, namely, the Paris one [1] and the Nijmegen one [2]. Some quark model potentials obtained both by resonating group method (RGM) [3] and by superposition of different shell-model $6q$ configurations [3,4] are pragmatically rather close to the above meson potentials.

The second kind of potential is represented by the strongly attractive Moscow potential (MP) with forbidden states [5–8]. Here, a hard core does not exist; instead, the wave function nodes appear for S and P partial waves, characterizing their short-range oscillations. Such a short-range potential is based on the application of $6q$ symmetries in the quark models (see below). This contrasts significantly with the concept of an "effective field theory" [9], where it is assumed that at the energies below 1 GeV the quark degrees of freedom can be integrated away so that the nucleon-nucleon interaction should be analyzed entirely in terms of meson exchange between nucleons.

The Moscow potential (with the inclusion of an imaginary part rising with energy) is able to describe the NN-scattering data (differential cross sections and vector polarizations) in the laboratory energy range up to 5–6 GeV [6], while both S and P phase shifts remain positive throughout this energy range (say, ${}^3S_1$ phase shift equals 2π at the zero energy in accordance with the generalized Levinson theorem). This property of the MP seems now urgent in connection with increasing interest in the intermediate energy NN interaction [10,11].

Microscopically, the MP corresponds to an excited quark configuration such as $s^4p^2[42]_x[42]_{CS}$ or $s^4p^2[42]_x[42]_{ST}$ in the nucleon-nucleon overlap region (see Refs. [12,13] where the notations of Young tableaus in different spaces are also explained). Namely, it was stressed that $6q$ configuration $s^4p^2[42]_x[2^3]_C[42]_{CS}$ offers a way to an enhanced virtual decay $N(2S)N\to D_C(0S)D_{\overline{C}}$ accompanied by a very strong attraction between the colored dipoles D_C and $D_{\overline{C}}$, $|D_C\rangle = |s^2p[21]_xL=1,[21]_CC=1,[3]_{ST}\rangle$ (the terms $(0S)$ and $(2S)$ symbolize the mutual motion wave functions) [14,15]. The first preliminary RGM treatment of this kind has been made recently [16].

Furthermore, $s^4p^2[42]_x[42]_{ST}$ configuration may be predominant if a strong nonperturbative instanton-induced interaction between quarks does exist [13].

The excited quark configurations should be seen directly in a series of high-energy nuclear reactions involving the investigation of the baryon-baryon (BB) composition of the deuteron, viz., a quasielastic knockout such as ${}^2\mathrm{H}(e,e'p)\mathrm{B}$ [17] with energies of final protons around 2 GeV, the polarization transfer in $d+p$ exclusive and inclusive high-energy backward scattering [18], etc. Namely, the $s^4p^2[42]_x$ configuration of deuteron produces the BB components $N^*(1/2^-,3/2^-)(1P)N$, $N^*(1/2^-,3/2^-)(0S)N^*(1/2^-,3/2^-)$, $N^{**}(1/2^+)(0S)N$, etc. with probabilities of the order of 1% in comparison with that of the predominant $n(2S)p$ channel [17]. In particular, the $N^*(1/2^-,3/2^-)(1P)N$ component appears to be just suitable to explain the cited polarization data [18].

As regards the short-range $\Delta\Delta$ component in the deuteron, which is also investigated in high- and intermediate-energy regions [19,20], it can have both a meson-exchange [19,20] and six-quark [17] origin ($s^6[6]_x[2^3]_{ST}$ configuration con-

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nected to $\Delta\Delta$ component is also responsible for a repulsive core in the *NN* interaction within the framework of the quark approach [4]). It is interesting that in the most reliable high-energy experiment [19] this component is not seen.

In the previous papers [15,21–23] we have shown that there is a rather efficient independent way to determine the kind of the *NN* potential (the MEP versus the MP) by means of the hard bremsstrahlung (HBS) $pp \to pp\gamma$ at rather moderate energies of 350–500 MeV. Such an opportunity is based on the fact that the MEP and MP are not phase-shift equivalent potentials and this nonequivalence is seen in theoretical papers very clearly if the maximally broad energy range up to E_{lab}=5–6 GeV values is considered [6]. However, at moderate energies the *S* and *P* phase shifts for the MP are simply displaced upward at 180° in comparison with those for the MEP, and it is only HBS that discriminates between these potentials well in contrast to the nucleon-nucleon elastic scattering data. Namely, HBS is sensitive to the shape of the short-range part of a relative motion wave function (the nodal radial wave function with a well-developed loop for *S* and *P* partial waves versus the short-range suppression of wave functions).

So, we discuss here a new opportunity, which was not analyzed in the previous papers on bremsstrahlung [24–28]: some kinds of *NN* potentials, which describe equally well the elastic *NN*-scattering data within rather broad energy range of, say, 800 MeV, may not be phase-shift equivalent (πn differences for the lowest phase shifts), but this bright nonequivalence may be efficiently revealed by investigation of the hard bremsstrahlung $pp \to pp\gamma$. It is well known [24–28] that various versions of meson-exchange potentials, which are phase-shift equivalent, practically cannot be distinguished by means of bremsstrahlung $pp \to pp\gamma$: their off-shell difference does not appear to be not essential here.

In the present paper we continue our investigation of HBS $pp \to pp\gamma$ passing from the nonrelativistic treatment to a relativistic one. In this way we eliminate the nonuniqueness of the nonrelativistic results, depending on the choice of the center-of-mass (c.m.) system of two protons (initial c.m. system or final one). This nonuniqueness (the difference of the corresponding cross sections) was almost invisible at the beam energy E_0=280 MeV, rather noticeable at E_0=450 MeV and large at E_0=500 MeV [23], which demonstrates the urgency of the relativistic treatment.

Our present analysis shows that relativistic effects are quite significant even at the lowest inspected energy E_0=208 MeV and are rather large at higher energies. However, the most important result is that our previous conclusion about a high sensitivity of HBS to the kind of *NN* interaction remains valid albeit the values of cross sections and their angular dependencies are changed. It is important to note that both initial and final state *pp* interactions have a profound influence here.

II. FORMALISM

Our consideration is based on the relativistic quasipotential equation [29] (c.m. system):

$$\left(\frac{\hat{\vec{p}}^2}{m} + \hat{V}\right)\chi(\vec{r}) = \frac{M^2 - 4m^2}{4m}\chi(\vec{r}) = \frac{1}{2}E_{lab}\chi(\vec{r}). \tag{1}$$

Here, $\hat{V}$ is the nucleon-nucleon potential, $M \equiv M(\vec{q}) = 2w(\vec{q}) = 2\sqrt{m^2+\vec{q}^2}$ is the mass of *NN* system; $\vec{q}$ relative momentum, $E_{lab} = \sqrt{m^2+\vec{p}_1^2} - m$ the kinetic energy of a bombarding proton, and $\vec{p}_1$ is its momentum.

The quasicoordinate representation [30] corresponds to the realization $\hat{\vec{p}} = -i\partial/\partial\vec{r}, \hat{V} = \hat{V}(\vec{r})$ and offers an opportunity to use partly our previous formalism of the nonrelativistic coordinate representation [15,23].

The principal relativistic effects are contained in the operator of electromagnetic current, which will be discussed below.

The bremsstrahlung amplitude A_{if} is determined by the familiar expression [15]

$$\delta(E_i - E_f - k_0)\delta^3(\vec{P}_i - \vec{P}_f - \vec{k})A_{if} = \sqrt{\frac{2\pi}{k_0}} \int d^4x \langle P_f\chi_f|\varepsilon^*_\mu \hat{J}^\mu(x)|P_i\chi_i\rangle e^{-ikx}, \tag{2}$$

where $k = (k_0, \vec{k})$ is the 4-momentum of a photon and $\varepsilon = (\varepsilon_0, \vec{\varepsilon})$ is its polarization 4-vector; the total energy is $E = \sqrt{M^2+\vec{P}^2}$ [31]; $\vec{P}$ the total 3-momentum of the system [in the laboratory system of coordinates (lsc), $\vec{P} = \vec{p}_1$; and $E = \sqrt{m^2+\vec{p}_2^1} + m$]. We rely upon the point form of relativistic dynamics [32–34], where interaction is only present in the components of the total 4-momentum *P* of a two-nucleon system and is not involved in the generators of boosts and rotations of this system [34].

Equation (1) can be written in terms of the point form dynamics as

$$\hat{M}^2\chi = M^2\chi.$$

It deals with the space of internal variables of *pp* system, and the squared mass operator $\hat{M}^2$ is defined as

$$\hat{M}^2 = 4(\hat{\vec{p}}^2 2 + mV + m^2). \tag{3}$$

Such structure of $\hat{M}^2$ causes the dependence of the current operator $\hat{J}(x)$ on interaction (see below).

The formulas for the matrix elements of the current appear especially simple in the coordinate frame, where

$$\vec{G}_i + \vec{G}_f = 0, \tag{4}$$

$\vec{G} = \vec{P}/M$. This frame is just suitable for operations with the solutions of the quasipotential, Eq. (1), as far as boosts of the wave function from both the initial and final center-of-mass frame to the frame (4) preserve the equal time framework $t_1 = t_2$ (it can be easily seen by the Lorentz transformation of coordinates). Such boosts are necessary to calculate the matrix element (2) [33,34]. Here appears an opportunity to separate the c.m. and internal coordinates of the system and to write [34]

$$\langle P_f\chi_f|\hat{J}_\mu(x)|P_i\chi_i\rangle = 2\sqrt{M_iM_f}e^{i(P_f-P_i)x}\langle\chi_f|\hat{j}_\mu(x)|\chi_i\rangle. \quad (5)$$

As a result, Eq. (2) can be rewritten as

$$A_{if} = \sqrt{\frac{2\pi}{k_0}}16\pi^3\sqrt{M_iM_f}\langle\chi_f|\varepsilon^*_\mu\hat{j}^\mu(\vec{k})|\chi_i\rangle, \quad (6)$$

with the integration performed only over the internal variables of the pp system.

Using the transverse gauge

$$\varepsilon = (0,\vec{\varepsilon}), \quad (\vec{\varepsilon}\vec{k}) = 0, \quad (7)$$

we exclude the $j_0(\vec{k})$ component of the current.

It is shown in the Appendix that the expression for remaining 3-current $\vec{j}(\vec{k})$ can be written as [we mean the coordinate frame (4)]

$$\begin{aligned}\hat{\vec{j}}(\vec{h}) &= 2iF_me\left(\frac{m}{w}[\vec{S}\times\vec{h}] + \frac{1}{w(w+m)}[\vec{q}\times\vec{h}](\vec{q}\cdot\vec{S})\right)\\ &+ 2ie\left(\frac{F_m}{mw} + \frac{F_e}{w(w+m)}\right)(\vec{h}\cdot[\vec{q}\times\vec{S}])\vec{q} + \frac{F_ee}{w}\vec{q}[I_1(\vec{h})\\ &- I_2(\vec{h})] - \frac{2F_ee(\vec{h}\cdot\vec{q})}{wm}\vec{q} + \frac{M_i-M_f}{M_i+M_f}2ie\left(\frac{F_m}{m} + \frac{F_e}{w+m}\right)\\ &\times[\vec{q}\times\vec{T}]. \end{aligned} \quad (8)$$

Here, $\vec{S}=\vec{s}_1+\vec{s}_2$, $\vec{T}=\vec{s}_1-\vec{s}_2$ ($\vec{s}_1$ and $\vec{s}_2$ being the spins of proton), $\vec{h}=2(M_fM_i)^{1/2}(M_f+M_i)^{-2}\vec{k}$ [$|\vec{h}|\ll 1$ for $E_\gamma < 500$ MeV and Eq. (8) corresponds to the first order on $|\vec{h}|$]. Further, $F_e(\vec{k})$ and $F_m(\vec{k})$ are the electric and magnetic form factors of the proton, respectively. Finally, the operators $I_j(\vec{h})$ represent the shifts of momentum,

$$I_j(\vec{h})\chi(\vec{q}) = \begin{cases}\chi(\vec{d}_1) = \chi\left[\vec{q} - \dfrac{2\vec{h}}{1-\vec{h}^2}[w(\vec{q}) - \vec{h}\cdot\vec{q}]\right], & j=1\\[2ex] \chi(\vec{d}_2) = \chi\left[\vec{q} + \dfrac{2\vec{h}}{1-\vec{h}^2}[w(\vec{q}) - \vec{h}\cdot\vec{q}]\right], & j=2.\end{cases} \quad (9)$$

The components of the vector terms in Eq. (8), orthogonal to $\vec{k}$, are only important since the scalar product $\vec{\varepsilon}\hat{\vec{j}}$ with the condition (7) enters the theory.

In the derivation of Eq. (8), the current conservation equation is used, and the generators of the Poincaré group $\hat{P}^\mu$ enter into consideration (see the Appendix). They include an interaction between particles, which is concentrated, within the point form dynamics, in the mass operator $\hat{M}$ [see Eq. (3)], $\vec{P}=\vec{G}M$ [33,34]. $(M_i-M_f)/(M_i+M_f)$ factor in Eq. (8) just represents the result of the operator $\hat{M}$ action. So, the current operator (8) is not merely the sum of the operators of two independent particles, it also includes the effect of the NN interaction (which, however, appears to be rather modest; see the Appendix).

Concerning the manifestation of relativistic effects in our general formalism, the central role here plays the operation of boosting from the initial c.m. reference frame and from the final reference frame to the single reference frame (4). Indeed, we have seen in Ref. [5] how big can be the difference of results calculated in two reference frames mentioned above. Concerning the current operator (8) itself, the main contribution here gives the terms reflecting the convection current $\{\sim F_e\vec{q}[I_1(\vec{h})-I_2(\vec{h})]\}$, which appeared to be very enhanced in case of MP due to the short-range oscillations of radial wave functions for S and P waves (and their large derivatives here) and spin magnetic current $(\sim F_m[\vec{S}\times\vec{h}])$. The relativistic effects in the cross section connected to the structure of the current (8) originate mainly from the relativistic features of these operators and from the interference products of their matrix elements and of relativistic components of other terms in Eq. (8).

In our previous papers [15,21–23], the nonrelativistic limit ($|\vec{k}|\ll m$ and $|\vec{q}|\ll m$) of Eq. (8) was used.

In the coordinate representation, the action of the operators $I_j(\vec{h})$ can be expressed as

$$I_j(\vec{h})\chi(\vec{r}) = e^{\mp 2i(\vec{\rho}\vec{h})\sqrt{m^2+\hat{\vec{p}}^2}}\chi(\vec{r})\big|_{\vec{\rho}=\vec{r}}, \quad (10)$$

with $\hat{\vec{p}}=-i\nabla$, which can be verified by the example of a plane wave. In fact, the series expansion

$$I_j(\vec{h}) = e^{\mp 2i(\vec{\rho}\vec{h})m}\left[1 + i(\vec{\rho}\vec{h})\frac{\hat{\vec{p}}^2}{m} + \cdots\right]_{\vec{\rho}=\vec{r}} \quad (11)$$

is used here.

The matrix element entering Eq. (6) can be transformed, for convenience, as follows:

$$\begin{aligned}\langle\chi_f|\varepsilon^*_\mu\hat{j}^\mu(\vec{h})|\chi_i\rangle &= \langle\chi_f|\vec{\varepsilon}^*\hat{\vec{j}}(\vec{h})|\chi_i\rangle\\ &= \langle\chi_f-\phi_f|\vec{\varepsilon}^*\hat{\vec{j}}(\vec{h})|\chi_i-\phi_i\rangle + \langle\phi_f|\vec{\varepsilon}^*\hat{\vec{j}}(\vec{h})|\chi_i\rangle\\ &+ \langle\chi_f|\vec{\varepsilon}^*\hat{\vec{j}}(\vec{h})|\phi_i\rangle.\end{aligned} \quad (12)$$

Here $|\phi_i\rangle$ and $|\phi_f\rangle$ are plane waves [they are also eigenfunctions of the operator $\hat{\vec{j}}(\vec{h})$] representing mutual motion with the momenta $\vec{q}_i$ and $\vec{q}_f$, respectively. Calculation of the matrix elements such as $\langle\phi_f|\vec{\varepsilon}^*\hat{\vec{j}}(\vec{h})|\chi_i\rangle$ is comparatively simple, while the use of the $\chi(\vec{r})-\phi(\vec{r})$ combinations makes it possible to accelerate convergence of the partial wave expansion.

The action of, e.g., the operator $\hat{\vec{p}}I_i(\vec{h})$ [see Eq. (8)] can be clarified here as

$$\begin{aligned}\hat{\vec{p}}I_i(\vec{h})|\chi_i-\phi_i\rangle = -i\vec{\nabla}\big[&e^{\mp 2i(\vec{r}\vec{h})\sqrt{m^2+\hat{\vec{q}}_i^2-m\hat{V}_{\tilde{E}}(\vec{r})}}\chi_i\\ &- e^{\mp 2i(\vec{r}\vec{h})\sqrt{m^2+\hat{\vec{q}}_i^2}}\phi_i\big],\end{aligned} \quad (13)$$

where Eq. (1) is used. The corresponding radial integrals are divided into two parts, $\int_0^R$ and $\int_R^\infty$, while R is chosen to be "minimally large" for the inequality $|mV_{\tilde{E}}(R)|\ll m^2+\vec{q}^2$ to hold (e.g., $R=3$ fm for the MP). For calculating the first integral $\int_0^R$, the exponential in the first term of Eq. (13) is expanded into a series on the powers $[\vec{q}_i^{\,2} - mV_E^{\sim}(R)]^n$, similar to Eq. (11). A good convergence is observed, and it is sufficient to take $n \leqslant 3$. A calculation technique for the other components of current operator is demonstrated in the Appendix.

The given procedure of numerical realization seems to be somewhat cumbersome, but it is not more complicated than that for momentum representation, where, e.g., an unwieldy technical detail is also present in considering the integration over the solid angles of both the vector $\vec{q}$ and the vector $\vec{q} \pm 2\vec{h}w(\vec{q})$, which is performed to obtain finally one-dimensional radial integrals.

In the literature, there are no results based on consideration of some *NN* potentials within a complete relativistic formalism [33,34] used in the present paper. However, some partial relativistic effects were considered [24,28]. In the paper [24], analysis of relativistic spin corrections resulting from introduction of the Dirac bispinors for free protons (i.e., within the impulse approximation for the electromagnetic current) is given. In our formalism, the above effect corresponds to an anomalous magnetic moment of proton, which is a long wave limit of its magnetic form factor [see Eq. (8)]. Even for the lowest considered proton beam energy $E_0 = 280$ MeV such correction (25% decrease of the cross section [24]) is a few times smaller than our figure of the total relativistic effect (see below).

Concerning the origin of this discrepancy, we can mention, first, that within formalism of Ref. [24] the *T* matrices for the initial state and for the final state are calculated in different reference frames, corresponding to the initial and to the final center-of-mass *pp* systems, respectively. Such procedure is not quite consistent; a single coordinate frame should be used here with the principal role of boosting procedure (see the discussion of Ref. [8]). Second, the relativistic content of other terms in Eq. (8) besides the spin-magnetic term is important (see the above discussion).

In Ref. [28], the *T*-matrix formalism based on the single-coordinate frame was used with the boost of the *T*-matrix from, say, the initial c.m. reference frame to the final reference frame when calculating the matrix element of $pp \to pp\gamma$ transition. But the equal time framework $t_1 = t_2$, connected to the use of quasipotential equations in Ref. [28], is lost after such boost, $t_1' \neq t_2'$, and the situation is outside the quasipotential approximation.

III. RESULTS AND DISCUSSION

The differential cross sections for the bremsstrahlung reaction $pp \to pp\gamma$ for six chosen values of the beam energy E_0 are presented in Figs. 1–6. Everywhere, the geometry of experiment does correspond to the hardest photons (it is just realized in the experiment of Ref. [35]).

The general features of the results presented in Figs. 1–6 are as follows.

First, relativistic effects are essential even at the beam energy of 280 MeV, while for higher energies they change even the general picture, and the cross section becomes a few times smaller compared with the nonrelativistic treatment.

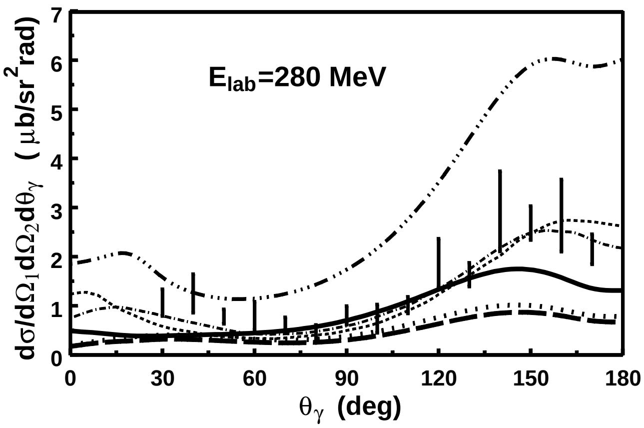

FIG. 1. Differential cross section for a $pp \to pp\gamma$ reaction as a function of the laboratory photon emission angle ν_γ at the proton beam energy $E_0 = 280$ MeV and at laboratory proton emission angles fixed at $\theta_1 = 12.4°$ and $\theta_2 = 12°$. A photon is emitted towards the side of θ_2 angle (coplanar geometry). Nonrelativistic calculation (here, the results in the initial and final c.m. frames are very close to each other): the short-dashed thin line, the MP-92; the dot-dashed thin line, Paris potential. Relativistic calculation: the solid thick line, the MP-92; the dash-three points thick line, the MP-97; the long dashed thick line, the Paris potential; the dotted thick line, the Nijmegen potential.

Second, as in the nonrelativistic case [15,21–23], the short-range loop of the radial *P*-wave function (characteristics of the Moscow potential) has a bright manifestation. Namely, due to the above-mentioned loop the values of the cross section for the MP are a few times larger than those for the MEPs. But in the nonrelativistic case this property of a *P* wave also gave impressive forward and backward maxima of their angular difference [15,21–23] for both the MP-92 [7] and MP-97 [8]. Now this feature is significantly damped and, in reality, still remains for the MP-97 only since the amplitude of the above-mentioned *P*-wave oscillation is especially large here (see below).

It is interesting to note also that recently proposed potential of the *NN* interaction (generalization of the cloudy-bag

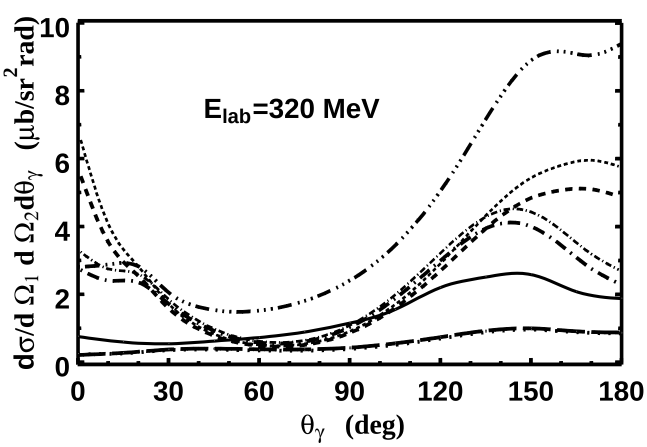

FIG. 2. Same as in Fig. 1, but for $E_0 = 320$ MeV. Nonrelativistic results: the upper curve of each of the two kinds corresponds to the final c.m. frame; and the lower curve to the initial c.m. frame.

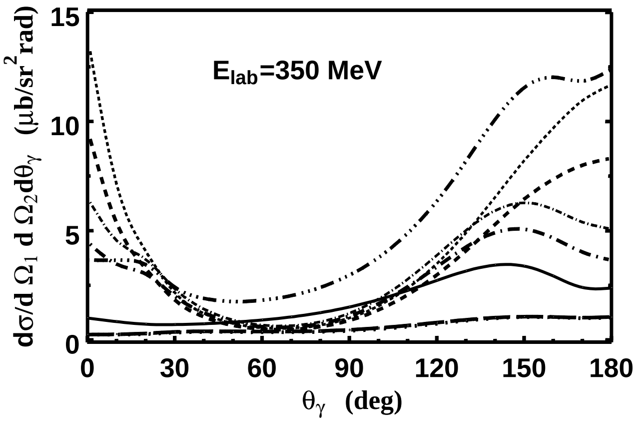

FIG. 3. Same as in Fig. 2, but for E_0=350 MeV.

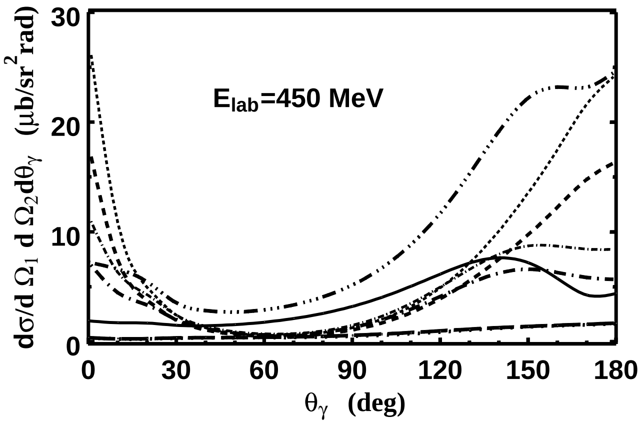

FIG. 5. Same as in Fig. 2, but for E_0=450 MeV.

model of the nucleon to a two-nucleon system) [36], conversely, is characterized by a very shallow loop in the P wave (keeping, however, the well-pronounced loop in S wave, which is typical for the Moscow potential), and the cross section for the $pp\to pp\gamma$ reaction for this potential is practically indistinguishable from that for the MEP (our previous nonrelativistic results [23] should be corrected at this point [37]).

Third, the problem of difference between the cross sections calculated in the initial and in the final c.m. systems, naturally, disappears.

The theoretical results for E_0=280 MeV presented in Fig. 1 seem now to be especially urgent since the existing experimental data [35] in the relativistic treatment become discriminative with respect to the kind of the NN potential. Namely, the nonrelativistic theoretical curves for the MP-92 and the MEP [15,21–23] are very close to each other here but the relativistic cross section in the backward hemisphere for the MP-92, which is much closer to the experiment, is twice larger than that for the MEP. The relativistic results for the MP-97, which are also shown in Fig. 1, are important from the methodological point of view—they do not agree with the experiment but they demonstrate what a tremendous effect can be produced by an increase of the P-wave loop amplitude (an increase in the potential depth) still compatible with the phase shift analysis data. So, it may be concluded that a very moderate rearrangement of the P-wave part of the MP-92 (slight increase of the P-wave loop and amplitude) will result in a good agreement of the corresponding theoretical curve with the experiment. By comparison, the mesonic potentials do not possess this feature.

All the aforementioned reasons show that a careful experimental examination of the theoretical concepts at various energies should be very urgent—HBS $pp\to pp\gamma$ at moderate beam energies of 300–500 MeV is indeed an efficient tool for discriminating between the two kinds of the nucleon-nucleon potentials. The hard bremsstrahlung $pp\to pp\gamma$ is practically insensitive to the node of the radial S-wave function at R=0.5–0.6 Fm, which is the original property of the MP along with the node in the P wave. The important question concerning the node in the S wave should be clarified when the nucleon momentum distribution in the deuteron is extracted from the experiment [38], where the quasielastic knockout reaction ${}^2\mathrm{H}(e,e'p)n$ was investigated at a beam energy of a few GeV within a broad range of recoil momentum values up to 1 GeV. Theoretical analysis of this experiment should imply a strong final state pn interaction characteristic of the MP. We are planning to accomplish this in the future.

Another essential experiment ${}^2\mathrm{H}+\gamma\to n+p$ at moderately high energies of $E_\gamma\geqslant 2$ GeV [39] should also be analyzed

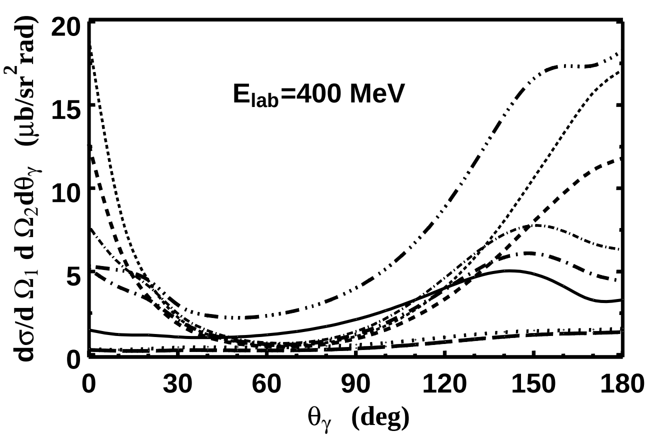

FIG. 4. Same as in Fig. 2, but for E_0=400 MeV.

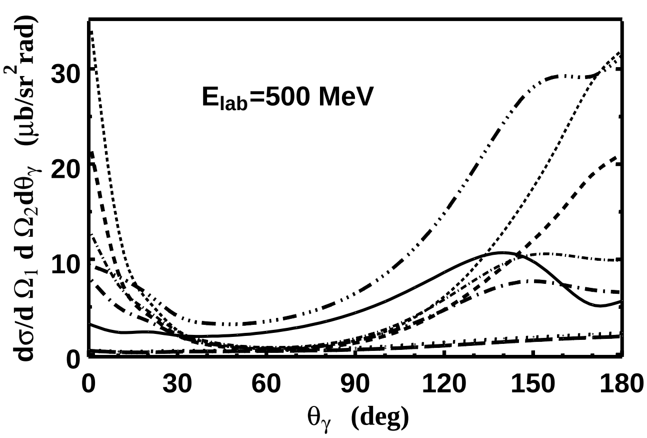

FIG. 6. Same as in Fig. 2, but for E_0=500 MeV.

because unlike lower energies the meson-exchange currents are strongly suppressed here (meson electroproduction data testify that, e.g., the pion cutoff parameter Λ_π is 0.6 GeV [40,41]). Hence, here the influence of the discussed radial short-range oscillations in S and P partial waves is expected to be seen too (the pn final state interaction is important, as before).

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APPENDIX

Here, the derivation of Eq. (8) is based on the results of the review Ref. [34] to supplement them. Having in mind the nucleon with the 4-momentum p, let us define the Lorentz transformation associated with a boost g as $p\rightarrow L[\alpha(g)]p$, where [42]

$$\alpha(g)=\frac{g^0+1+\vec{\sigma}\vec{g}}{\sqrt{2(g^0+1)}}, \tag{A1}$$

g giving 4-velocity, and $\sigma=(\sigma_x,\sigma_y,\sigma_z)$, the Pauli matrices. Introducing, next, the $\check{p}$ matrix as $\check{p}=\sigma^\mu p_\mu$ with the interrelations

$$p_0=\frac{1}{2}(\check{p}_{11}+\check{p}_{22}),\quad p_1=\frac{1}{2}(\check{p}_{12}+\check{p}_{21}),$$

$$p_2=\frac{1}{2i}(-\check{p}_{12}+\check{p}_{21}),\quad p_3=\frac{1}{2}(\check{p}_{11}-\check{p}_{22}), \tag{A2}$$

we carry out the matrix transformation

$$\check{p}\rightarrow\alpha(g)\check{p}\alpha(g)^+, \tag{A3}$$

which in combination with the formulas (A2), just describes the boost operation $p\rightarrow L[\alpha(g)]p$.

The Poincare group transformation $\mathrm{U}(a,l)$ is characterized [34] by the 4-shift a and 4-rotation l:

$$\mathrm{U}(\alpha,l)\varphi(g)=\exp(img'a)D[\vec{s};\alpha(g)^{-1}l\alpha(g')]\varphi(g'). \tag{A4}$$

Here, $\varphi(g)$ is a normalized spinor function, $\vec{s}$ the spin operator, and $g'=L(l)g$. In our case of spin $s=1/2$ particles, we deal with the fundamental representation [42], i.e.,

$$D[\vec{s};\alpha(g)^{-1}l\alpha(g')]=\alpha(g)^{-1}l\alpha(g'). \tag{A5}$$

The operator l will be specified for our two-particle system. Here, $p_i=m_ig_i$ $(i=1,2)$ and the center-of-mass velocity is G. In the center-of-mass coordinate frame the momenta of particles are expressed [34] as

$$q_i=L[\alpha(G)]^{-1}m_ig_i,\quad \vec{q}_1=\vec{q}=-\vec{q}_2. \tag{A6}$$

Bearing in mind our nuclear reaction $pp\rightarrow pp\gamma$ and choosing the coordinate frame (3), we can specify the general expression for the current operator in the system of the non interacting particles, which corresponds to the point form dynamics [34]

$$\begin{aligned}j^\nu(\vec{h})=&\sum_{i=1,2}L\left(L[\alpha(f)]\frac{q_i}{m_i},L[\alpha(f')]\frac{d_i}{m_i}\right)^\mu_\nu D[\vec{s}_k;\alpha(q_k/m_k)^{-1}\alpha(f)^{-1}\alpha(f')\alpha(d_{ki}/m_k)]\\&\times D[\vec{s}_i;\alpha(q_i/m_i)^{-1}\alpha(f)^{-1}\alpha(L[\alpha(f)]q_i/m_i,L[\alpha(f')]d_i/m_i)\alpha(f_i)]j_i^\nu(\vec{h})\\&\times D[\vec{s}_i;\alpha(f_i')^{-1}\alpha(L[\alpha(f)]q_i/m_i,L[\alpha(f')]d_i/m_i)\alpha(f')\alpha(d_i/m_i)]\frac{m_iw_i(\vec{q}_i)}{w_i(\vec{d}_i)}\left(\frac{M(\vec{d}_i)}{M(\vec{q})}\right)^{3/2}I_i(\vec{h}).\end{aligned} \tag{A7}$$

Here, $k=2$ if $i=1$, and, conversely, $k=1$ if $i=2$. Next,

$$f=L(G,G')^{-1}G,\quad f'=L(G,G')^{-1}G' \tag{A8}$$

represent the 4-velocities of the two-nucleon c.m. in the initial and final states, respectively, meaning the coordinate frame (3). The following formal aspects should be mentioned here: $f^2=f'^2=1$, $\vec{f}+\vec{f}'=0$, $f_0=f_0'=(1+\vec{f}^2)^{1/2}$; $L(G,G')=L[\alpha(G,G')]$, $\alpha(G,G')=\alpha[(G+G')/|G+G'|]$; $d_1=[w_1(\vec{d}_1),\vec{d}_1]$, $d_2=[w_2(\vec{d}_2),\vec{d}_2]$, $d_{12}=L[\alpha(f')^{-1}\alpha(f)]q_2=[w_2(\vec{d}_1),-\vec{d}_1]$, $d_{21}=L[\alpha(f')^{-1}\alpha(f)]q_1=[w_1(\vec{d}_2),\vec{d}_2]$; and the 3-vectors $\vec{d}_1$ and $\vec{d}_2$ are given by Eq. (8). Finally, $j_i^\nu(\vec{h})$ is 4-current of the particle i,

$$j_i^0(\vec{h}) = eF_e\left(-\frac{4m^2\vec{h}_i^2}{\sqrt{1-\vec{h}_i^2}}\right),$$

$$\vec{j}_i(\vec{h}) = -\frac{ie}{\sqrt{1-\vec{h}_i^2}}F_m\left(-\frac{4m^2\vec{h}_i^2}{\sqrt{1-\vec{h}_i^2}}\right)(\vec{h}_i\times\vec{s}_i), \quad \text{(A9)}$$

where the vectors $\vec{h}_i$ are defined below.

The photon energies $E_\gamma < 500$ MeV considered in our paper correspond to the inequality $|\vec{h}| \ll 1$. In such first-order approximation $F_e(\vec{h}) \approx F_e(0) = 1$, $F_m(\vec{h}) \approx F_m(0) = 2.793$, and, further ($\vec{h}$ is directed along the z axis): $d_1 = (w - 2hq_z, q_x, q_y, q_z - 2hw)$, $d_2 = (w + 2hq_z, -q_x, -q_y, -q_z - 2hw)$, and $d_{12} = (w - 2hq_z, -q_x, -q_y, -q_z + 2hw)$, $d_{21} = (w + 2hq_z, q_x, q_y, q_z + 2hw)$,

$$\frac{1}{w(\vec{d}_1)}\left(\frac{M(\vec{d}_1)}{M(\vec{q})}\right)^{3/2} = \frac{w-hq_z}{w^2},$$

$$\frac{1}{w(\vec{d}_2)}\left(\frac{M(\vec{d}_2)}{M(\vec{q})}\right)^{3/2} = \frac{w+hq_z}{w^2},$$

$$h_1 = h_2 = f_1 = f_2 = \left(1, -\frac{2hq_zq_x}{m(w+m)}, -\frac{2hq_zq_y}{m(w+m)}, -\frac{2h[m(w+m)+q_x^2+q_y^2]}{m(w+m)}\right),$$

$$f_1' = f_2' = \left(1, \frac{2hq_zq_x}{m(w+m)}, \frac{2hq_zq_y}{m(w+m)}, -\frac{2h[m(w+m)+q_x^2+q_y^2]}{m(w+m)}\right),$$

$$L[\alpha(f')]\frac{q_1}{m} = m^{-1}(w - 3hq_z, q_x, q_y, q_z - 3hw), \quad \text{(A10)}$$

$$L[\alpha(f)]\frac{q_1}{m} = m^{-1}(w + hq_z, q_x, q_y, q_z + hw).$$

Introducing 4-vectors

$$z_1 = \frac{L[\alpha(f)]\frac{q_1}{m} + L[\alpha(f')]\frac{d_1}{m}}{\left|L[\alpha(f)]\frac{q_1}{m} + L[\alpha(f')]\frac{d_1}{m}\right|} = m^{-1}(w - hq_z, q_x, q_y, q_z - hw) \quad \text{(A11)}$$

and z_2, where $\vec{q}$ is replaced by $-\vec{q}$, we can write the 4-vector

$$L\left(L[\alpha(f)]\frac{q_i}{m_1}, L[\alpha(f')]\frac{d_i}{m_1}\right)j_i(h_i) \equiv L(z_i)j_i(h_i) \simeq \left[\frac{F_e e}{m}[w \mp (\vec{h}\cdot\vec{q})] \pm \frac{4iF_m e}{m^2}w(\vec{h}\cdot[\vec{q}\times\vec{s}_i]), \pm\frac{F_e e}{m}(\vec{q} \mp w\vec{h}) + 4iF_m e\left([\vec{s}_i\times\vec{h}] + \frac{(\vec{h}\cdot[\vec{q}\times\vec{s}_i])q}{m^2} + \frac{(\vec{q}\cdot\vec{s}_i)[\vec{q}\times\vec{h}]}{m(w+m)}\right)\right], \quad \text{(A12)}$$

where the 4-vectors d_i were presented as $d_i \simeq [w \mp 2(\vec{h}\vec{q}), \pm\vec{q} - 2w\vec{h}]$.

Finally, following the realization (A5), we obtain

$$D[\vec{s}_k; \alpha(q_k/m_k)^{-1}\alpha(f)^{-1}\alpha(f')\alpha(d_{ki}/m_k)] \simeq 1 \pm \frac{i}{w+m}(\vec{h}\cdot[\vec{\sigma}_k\times\vec{q}]), \quad \text{(A13)}$$

$$D[\vec{s}_i; \alpha(q_i/m_i)^{-1}\alpha(f)^{-1}\alpha\{L[\alpha(f)]q_i/m_i, L[\alpha(f')]d_i/m_i\}\alpha(f_i)] \simeq D[\vec{s}_i; \alpha(f_i')^{-1}\alpha\{L[\alpha(f)]q_i/m_i, L[\alpha(f')]d_i/m_i\} \times \alpha(f')\alpha(d_i/m_i)] \simeq 1 \pm \frac{i(m+2w)}{2m(w+m)}(\vec{h}\cdot[\vec{\sigma}_i\times\vec{q}]). \quad \text{(A14)}$$

Substituting all these intermediate results into Eq. (A7), we arrive at the expression (not interacting particles, first order approximation with respect to $|\vec{h}|$)

$$j^0(\vec{h}) = 2i\left(\frac{F_m}{m} - \frac{F_e}{w+m}\right)e(\vec{h}\cdot[\vec{q}\times\vec{T}]) + 2F_e e, \quad \text{(A15)}$$

$$\vec{j}(\vec{h}) = 2iF_m e\left(\frac{m}{w}[\vec{S}\times\vec{h}] + \frac{1}{w(w+m)}[\vec{q}\times\vec{h}](\vec{q}\cdot\vec{S})\right) + 2ie\left(\frac{F_m}{mw} + \frac{F_e}{w(w+m)}\right)(\vec{h}\cdot[\vec{q}\times\vec{S}])\vec{q} + \frac{F_e e}{w}\vec{q}[I_1(\vec{h}) - I_2(\vec{h})] - \frac{2F_e e(\vec{h}\cdot\vec{q})}{wm}\vec{q} - 2F_e e\vec{h}.$$

Now, we should take into account the current conservation equation

$$\frac{\hat{J}^\mu(x)}{\partial x^\mu} = 0. \quad \text{(A16)}$$

Using also the 4-shift

$$\hat{J}^\mu(x) = \exp(i\hat{P}x)\hat{J}^\mu(0)\exp(-i\hat{P}x), \quad \text{(A17)}$$

we obtain an important relation

$$[\hat{P}_\mu, \hat{J}^\mu(0)] = 0. \quad \text{(A18)}$$

In terms of the internal variables of pp system, Eq. (A18) can be reduced to the matrix element

$$\langle\chi_f|[\hat{M}, G^0\hat{j}^0(\vec{h}) - \vec{G}\hat{\vec{j}}(\vec{h})]|\chi_i\rangle = 0, \tag{A19}$$

which can be rewritten in the form

$$\langle\chi_f|(\vec{h}\cdot\hat{\vec{j}}(\vec{h}))|\chi_i\rangle = \frac{M_i - M_f}{M_i + M_f}\langle\chi_f|\hat{j}^0(\vec{h})|\chi_i\rangle, \tag{A20}$$

as far as $\vec{G}_i=\vec{h}$, $\vec{G}_f=-\vec{h}$, $\vec{P}_i=M_i\vec{h}$, $\vec{P}_f=-M_f\vec{h}$, $\hat{M}|\chi_i\rangle=M_i|\chi_i\rangle$, and $\hat{M}|\chi_f\rangle=M_f|\chi_f\rangle$. The current operator (A15) does not satisfy Eq. (A20) and needs some modification. To discuss the problem we decompose the current operator for a system of two noninteracting particles into three different components [33,34]:

$$\vec{j}(\vec{h}) = \vec{j}(0) + \frac{\vec{h}}{|\vec{h}|}j_{\parallel}(\vec{h}) + \vec{j}_{\perp}(\vec{h}),$$

where $\vec{h}\vec{j}_{\perp}(\vec{h})=0$.

When the interaction between particles is included, the current operator changes $\vec{j}(\vec{h})\to\hat{\vec{j}}(\vec{h})$, $\vec{j}(0)\to\hat{\vec{j}}(0)$, $j_{\parallel}(\vec{h})\to\hat{j}_{\parallel}(\vec{h})$, and $\vec{j}_{\perp}(\vec{h})\to\hat{\vec{j}}_{\perp}(\vec{h})$.

The structure of Eq. (A20) shows that the $\hat{\vec{j}}_{\perp}(\vec{h})$ component is not influenced by the current conservation, so it is natural to take it from Eq. (A15) [33,34]. To get out of this simplest method here it would be necessary to use explicitly the microscopic picture of interaction [43] and the diagrams describing the interaction current (see, e.g., Ref. [44] for such description of meson-exchange currents). Our phenomenological quark-induced quasipotential model offers no such microscopic picture of interaction albeit some systematical work is in progress here [13,16,45]. So, there is still no basis for building up of the transverse component of the interaction current [by the way, the contribution of the interaction current to the $\hat{\vec{j}}(0)$ component can be reconstructed by means of the current conservation Eq. (A20) and its role appears to be rather modest—see below].

Nevertheless, our model has a very interesting property, reflecting the effect of the deep attractive Moscow NN potential with forbidden states—it shows the very increased role of $\vec{\nabla}$ operator (i.e., $\vec{q}$ in momentum representation) in Eq. (A20) in comparison to MEPs due to the short range oscillations of radial wave functions for S and P waves of the pp scattering. This fact is just responsible for the few times increase of cross section of hard bremsstrahlung process $pp\to pp\gamma$ for the case of MP in comparison to that of MEPs (see the main text). The radiation of soft photons ($k\ll q^2/m$) is not suitable for discriminating the above potentials as far as the reaction amplitude is described here [46] in terms of partial amplitudes T_l of elastic pp scattering and their E derivatives which at proton beam energies of a few hundreds MeV are practically indistinguishable if the potentials of the above two kinds are compared—see the Introduction. But at higher energies of 2–3 GeV some sensitivity of the soft bremsstrahlung to the kind of NN potential can appear.

Further, it is seen from Eqs. (6) and (7) that the $\hat{j}_{\parallel}(\vec{h})$ component of the current plays no role in our electromagnetic process [this is because the term $-2F_e e\vec{h}$ in Eqs. (A15) and (A21) is omitted in Eq. (8)]. But the $\hat{\vec{j}}(0)$ part of the current both contributes to the amplitude of the reaction (8) and is influenced by the current conservation condition (A20). It appears as the necessary two-body modification of $\vec{j}(0)$ component of the single-particle current (A15) to satisfy Eq. (A20):

$$\hat{\vec{j}}(0) = \vec{j}(0) + \frac{M_i - M_f}{M_i + M_f}2ie\left(\frac{F_m}{m} - \frac{F_e}{m+w}\right)[\vec{q}\times\vec{T}],$$

while $\hat{j}^0(\vec{h})=j^0(\vec{h})$. The real contribution of the above two-body term to the cross section of hard bremsstrahlung $pp\to pp\gamma$ reaction within the kinematics of Figs. 1–6 is rather modest, about 5–10 %.

So, the modified expression (A15) for the current, which contains the terms of the zero- and first-order magnitude with respect to $|\vec{h}|$ including the (partial) contribution of two-body current, can be written as

$$\hat{j}^0(\vec{h}) = j^0(\vec{h}), \tag{A21}$$

$$\hat{\vec{j}}(\vec{h}) = \vec{j}(\vec{h}) + \frac{M_i - M_f}{M_i + M_f}2ie\left(\frac{F_m}{m} - \frac{F_e}{m+w}\right)[\vec{q}\times\vec{T}].$$

So, we come to Eq. (8) of the main text.

The nonrelativistic limit ($|\vec{q}|/m\ll 1$) of Eq. (A21) looks like

$$\hat{\vec{j}}_{nr}(\vec{h}) = 2ieF_m[\vec{S}\times\vec{h}] + \frac{eF_e}{m}\vec{q}[I_1(\vec{h}) - I_2(\vec{h})] + \frac{ie}{m}|\vec{h}|(2F_m - F_e)[\vec{q}\times\vec{T}], \tag{A22}$$

while $I(\vec{h})\simeq\exp(\pm i\vec{k}\vec{r}/2)$ here [coordinate representation, see Eq. (10)].

In particular, the contribution of two-body current corresponds to the third term in r.h.s. of Eq. (A22).

Comparing Eq. (A22) to that used in our nonrelativistic treatment [Eqs. (3)–(6) and (A2) of Ref. [15], coordinate representation] we see that the amplitude (6) of Ref. [15] is really of second order in magnitude with respect to $|\vec{k}|/m$ (it gives a small contribution) and the term of such content in Eq. (A22) is absent. Further, the term corresponding to Eq. (5) of Ref. [15] is evidently the first term in the rhs of Eq. (A22). As a next step, we will show now that the term equivalent to Eq. (4) of Ref. [15] is the second term in the rhs of Eq. (A22). Indeed, the matrix element corresponding to this term in Eq. (A22) should be written within a constant factor as

$$\int d^3r\bar{\varphi}_f\vec{\nabla}[\exp(-i\vec{k}\vec{r}/2) - \exp(i\vec{k}\vec{r}/2)]\varphi_i. \tag{A23}$$

Using the important property that for the wave packets, we have [47]

$$\int d^3r\overline{\varphi}_f(\vec{\nabla}\varphi_i) = \int d^3r(\vec{\nabla}\overline{\varphi}_f)\varphi_i,$$

and that the vector amplitudes parallel to $\vec{k}$ can be omitted, we just obtain from Eq. (4) of Ref. [15] our above expression (A23)).

Finally, it should be noted that the real contribution of the two-body term in Eq. (A22) is small, it accounts for approximately 5% of the cross section. This figure is just comparable with the measure of accuracy of all our formal procedure. So nonrelativistic results of Figs. 1–6 are practically the same as these of Ref. [15], where this term was not taken into account.

The current operator (A21) is meant to calculate the matrix elements $\langle\chi_f|\hat{\vec{j}}(\vec{h})|\chi_i\rangle$.

Now, by a few examples, we illustrate the calculation technique for the matrix elements of various components of the relativistic current operator

$$(\vec{\nabla}\cdot\vec{S})\nabla_\mu = -\frac{1}{\sqrt{3}}[[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(0)}\times\vec{S}^{(1)}]^{(1)}_\mu - \frac{\sqrt{5}}{\sqrt{3}}[[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(2)}\times\vec{S}^{(1)}]^{(1)}_\mu, \quad (A24)$$

$$(\vec{h}\cdot[\vec{\nabla}\times\vec{S}])\nabla_\mu = -i\frac{\sqrt{6}}{3}\Bigg(\frac{\sqrt{15}}{2}[[[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(2)}\times\vec{S}^{(1)}]^{(2)} \times\vec{h}^{(1)}]^{(1)}_\mu + [[[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(0)}\times\vec{S}^{(1)}]^{(1)} \times\vec{h}^{(1)}]^{(1)}_\mu - \frac{\sqrt{5}}{2} + [[[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(2)} \times\vec{S}^{(1)}]^{(1)}\times\vec{h}^{(1)}]^{(1)}_\mu\Bigg), \quad (A25)$$

$$\langle L_f,S_f=1;J_fM_f|[[[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(k)}\times\vec{S}^{(1)}]^{(1)}]^{(n)}_\mu f(r)|L_i,S_i = 1;J_iM_i\rangle = C^{J_fM_f}_{J_iM_in\mu}\begin{Bmatrix} L_f & 1 & J_f\\ L_i & 1 & J_i\\ k & 1 & n\end{Bmatrix}\sqrt{6(2J_i+1)(2n+1)} \times\langle L_f||[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(k)}f(r)||L_i\rangle, \quad (A26)$$

$$\begin{aligned}\langle L_f||[\vec{\nabla}^{(1)}\times\vec{\nabla}^{(1)}]^{(2)}\frac{f(r)}{r}||L_i\rangle &= \frac{\sqrt{2L_f+1}}{\sqrt{6}C^{L_f0}_{L_i020}}\frac{1}{r}\Bigg[\delta_{L_iL_f}\Bigg(-1+\frac{3(2L_i^2+2L_i-1)\sqrt{2(2L_i+1)}}{(2L_i-1)(2L_i+1)(2L_i+3)}\Bigg)\\ &\times\left(\frac{d^2}{dr^2}-\frac{L_i(L_i+1)}{r^2}\right)f(r)\\ &+\delta_{L_iL_f-2}\frac{3(L_i+1)(L_i+2)\sqrt{2(2L_i+1)}}{(2L_i+1)(2L_i+3)(2L_i+5)}\\ &\times\left(\frac{d^2}{dr^2}-\frac{(2L_i+3)}{r}\frac{d}{dr}+\frac{(L_i+3)(L_i+1)}{r^2}\right)f(r)\\ &+\delta_{L_iL_f+2}\frac{3L_i(L_i-1)\sqrt{2(2L_i+1)}}{(2L_i+1)(2L_i-3)(2L_i-1)}\\ &\times\left(\frac{d^2}{dr^2}-\frac{(2L_i-1)}{r}\frac{d}{dr}+\frac{L_i(L_i-2)}{r^2}\right)f(r)\Bigg].\end{aligned} \quad (A27)$$

In these expressions, the upper index in brackets, e.g., Eq. (2), means the tensor rank of the operator.

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