Nonresonant capture cross sections of ^{11}B(n,γ) and ^{12}C(n,γ) at stellar energies

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The nonresonant direct radiative capture (DRC) cross sections of ^{11}B(n,γ) and ^{12}C(n,γ) at stellar energies are deduced by means of the asymptotic normalization coefficient (ANC) method. The ANC factors were extracted from the peripheral transfer reactions of ^{11}B(d,p) and ^{12}C(d,p). The deduced DRC cross sections of ^{12}C(n,γ) are in good agreement with experimental results of direct measurement. The DRC cross sections of ^{11}B(n,γ) at low energies are obtained for the first time.

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The ^{11}B(n,γ) and ^{12}C(n,γ) reactions of direct radiative capture (DRC) processes are two important reactions in stellar nucleosynthesis [1–3]. Unfortunately, it is quite difficult to perform the direct measurement at stellar energy (usually less than 500 keV) due to their small cross section (about a few microbarns). Up to now, only a few experimental data have been obtained for the ^{12}C(n,γ) reaction [4,5]. Stimulated by these experimental results, many efforts were devoted to predict or to reproduce them [6–9] in theoretical calculations. For the ^{11}B(n,γ) reaction, the experimental cross sections are still nonexistent, and less attention was paid to the theory. Very strong $E1$ γ transitions associating with the incoming p-wave neutrons capture into the $2s_{1/2}$ orbits of ^{13}C were observed [4,5] in the ^{12}C(n,γ) reactions. It indicates that halo structures occur in these s orbits. By using the wave function with halo feature, the experimental capture cross sections were perfectly reproduced by Otsuka *et al.* [7]. These authors further pointed out that “The present low-energy direct capture is of indispensable significance as a new and presently unique tool for investigating the halo structure particularly in excited states.” In the same year, Xu *et al.* [10] proposed an indirect method, i.e., asymptotic normalization coefficient (ANC) method, to obtain the DRC cross section through the peripheral transfer reaction. Since then, the ANC method has become useful in evaluating the astrophysical S factor or the DRC cross sections of (p,γ) and (n,γ) processes at low energies [11–15]. Very recently, the ANC method was employed to study the halo structure [16–18]. In the previous papers [17,18], we reported that the halo structures exist in the second and third excited states of ^{12}B and the first excited state of ^{13}C by means of the ANC method through the transfer reactions of ^{11}B(d,p) and ^{12}C(d,p). Almost at the same time, Imai *et al.* [19] measured the angular distribution of ^{12}C$(d,p)^{13}$C$(1/2^+)$ at the same incident energy of 11.8 MeV, extracted the ANC factor of ^{13}C$(1/2^+)$, and evaluated the DRC cross sections of ^{12}C(n,γ) at low energies. Their work confirmed again the availability of ANC method. In view of the above work, in the present work we use the ANC factors reported in the previous papers [17,18] to calculate the DRC cross sections of astrophysical interest.

The experimental data of ^{12}C(n,γ) clearly show that the DRC processes are essentially the nonresonant reactions. Some theoretical work [6–9] have pointed out that the dominant component of the DRC process is the capture of the incoming p-wave neutron into the s orbit through $E1$ transition. The contributions of other transitions are very small at low energies. It is reasonable to think that similar nonresonant processes occur in the ^{11}B(n,γ) reactions because the bound states of ^{12}B have similar structures to that of ^{13}C. Therefore, only the nonresonant processes of $p\to s$ transitions are considered in the present work.

The DRC process can be treated as a transition from continuum to bound state in the frame of the classical transition theory [20]. The transition cross section is given by

$$\sigma_{lm}=\frac{8\pi(l+1)}{l[(2l+1)!!]^2}\left(\frac{\omega}{c}\right)^{2l+1}\frac{1}{\hbar v}|Q_{lm}^2|, \qquad (1)$$

where $\hbar\omega$ is the energy of the emitted photon, v is the relative velocity, Q_{lm} is the transition matrix element, l and m are the angular momentum and its z projection of the photon, respectively. In the present case, $l=1$. Taking ^{12}C(n,γ) as an example, Q_{lm} can be written as

$$Q_{1m}=\langle^{13}\mathrm{C}(1/2^+)|T^{(E1)}|^{12}\mathrm{C}(0^+)\mathbf{n}\rangle, \qquad (2)$$

where $T^{(E1)}$ is the operator of $E1$ transition and $|\mathbf{n}\rangle$ denotes the wave function of incoming neutron. If the interaction between the incoming neutron and the ^{12}C core can be neglected, which is true for neutron captures at very low energies, then $|\mathbf{n}\rangle$ can be replaced by the plane wave (PW) $|\mathbf{k}\rangle$. However, the incoming plane wave is strongly distorted by the nuclear potential when the incident energy increases, and results in an obvious phase shift. In this case, the exact calculation of distorted wave (DW) is necessary. Neglecting the center-of-mass motion, the $E1$ transition operator is expressed as

$$T^{(E1)}=-e\sqrt{3/4\pi}(Z/A)\mathbf{r}, \qquad (3)$$

where Z and A are the charge number and mass number of the nucleus after capture, and $\mathbf{r}$ implies the coordinates of the relative motion. The wave function of the first excited state $(2s_{1/2^+})$ of ^{13}C can be decomposed into

$$|^{13}\mathrm{C}(1/2^+)\rangle=\zeta_1|^{12}\mathrm{C}(0^+)n(2s_{1/2^+})\rangle+\cdots, \qquad (4)$$

where ζ_1 is an amplitude, $|^{12}\mathrm{C}(0^+)\rangle$ and $|n(2s_{1/2^+})\rangle$ denote the 0^+ ground state of the ^{12}C core and the $2s_{1/2^+}$ orbit of

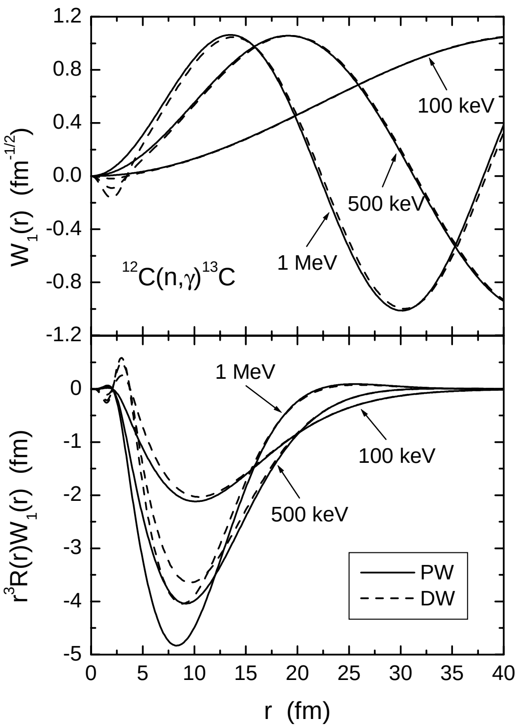

FIG. 1. The p-wave components (top panel) and their contributions to the $E1$ transitions (bottom panel) of incoming PW (solid lines) and DW (dashed lines) for the $^{12}\mathrm{C}(n,\gamma)^{13}\mathrm{C}$ reactions at the incident energies of 100 keV, 500 keV, and 1 MeV.

the last neutron, respectively. For $E1$ transition, the core has no contribution to the matrix element. Then Eq. (2) becomes

$$Q_{1m} = -\delta_{m,0}\zeta_1\sqrt{3}ie(Z/A)\int_0^{\infty} r^3R(r)W_1(kr)dr, \qquad (5)$$

where $R(r)$ is the radial wave function of the last neutron in the $2s_{1/2^+}$ orbit and $W_1(kr)$ represents the p-wave component of the incoming neutron wave. It is known that the $2s_{1/2^+}$ state of ^{13}C is a halo state [7,17,18,21], i.e., its radial wave function has a long tail. In this case, Q_{10} becomes the predominant component at low energy. Detailed discussions can be found in Ref. [7]. If we neglect the distortion of incoming wave, i.e., the PW approximation, $W_1(kr)$ can be simply written as the spherical Bessel function $J_1(kr)$. In order to test the availability of PW, the incoming DW was calculated by DWUCK4 code [22] and the comparison between them was made, as illustrated in the top panel of Fig. 1 at the incident energies of 100 keV, 500 keV, and 1 MeV. The parameters of optical-model potential are extracted from the Wilmore and Hodgson systematics [23] in the DW calculations. The wave functions are barely distinguishable between PW and DW when the distance is larger than the interaction radius. However, their contributions to the transition, i.e., the integrand in Eq. (5), are obviously different as shown in the bottom panel of Fig. 1. It shows that the capture cross sections are somewhat overestimated by the PW approximation. For this reason, we prefer using DW in the calculations.

According to the ANC definition [10,13,14], the single-particle overlap function between $|^{12}\mathrm{C}(0^+)\rangle$ and $|n(2s_{1/2^+})\rangle$ for the first excited state of ^{13}C has the following asymptotic behavior:

$$\psi_{An} \rightarrow r > R_N C_{s_{1/2}}^{(sp)} W(2\kappa_B r)/r, \qquad (6)$$

where the subscript A and n are the core nucleus and the outside valence neutron, respectively, R_N is the interaction radius, $C_{s_{1/2}}^{(sp)}$ is the ANC factor of single-particle state, $W(2\kappa_B r)$ the Whittaker function, and κ_B is the wave number of bound state. In the neutron case, the Whittaker function is reduced to the Hankel function $H(\kappa_B r)$. Therefore, the radial wave function $R(r)$ in Eq. (5) can be replaced by the Hankel function. $C_{s_{1/2}}^{(sp)}$ connects with the ANC of nuclear state by $C_{s_{1/2}} = C_{s_{1/2}}^{(sp)}\sqrt{S_{s_{1/2}}^{(sp)}}$, where $S_{s_{1/2}}^{(sp)}$ is the single-particle spectroscopic factor. Note that $\sqrt{S_{s_{1/2}}^{(sp)}}$ equals to ζ_1 in Eq. (4). Finally, the transition cross section is expressed as

$$\sigma_{n,\gamma}^{(E1)} = \frac{16\pi}{3}e^2\left(\frac{Z}{A}\right)^2\left(\frac{\hbar\omega}{\hbar c}\right)^3\frac{1}{\hbar v} \times C_{s_{1/2}}^2\left[\int_{R_N}^{\infty} r^2 H(\kappa_B r)W_1(kr)dr\right]^2. \qquad (7)$$

Apparently this equation is nearly model independent, only slightly dependent on the optical-model potential of the entrance channel. It expresses the main idea of so-called indirect ANC method: the DRC cross section can be determined from the peripheral transfer reaction through the ANC factor. The above equation is deduced by using the $^{12}\mathrm{C}(n,\gamma)$ reaction as an example. It is also appropriate for the $^{11}\mathrm{B}(n,\gamma)$ reaction.

Before applying Eq. (7) to calculate the DRC cross sections, it is necessary to evaluate the impact of the integral truncated by the interaction radius R_N. One can define

$$D(R_N) = \frac{\int_0^{R_N} r^3R(r)W_1(kr)dr}{\int_0^{\infty} r^3R(r)W_1(kr)dr} \qquad (8)$$

to estimate the contribution of inner integral. The radial wave function $R(r)$ was calculated by solving the bound-state Schrödinger equation in the frame of the single-particle potential model [24]. The Woods-Saxon potential was employed with radius $R = r_0A^{1/3}$, $r_0 = 1.25$ fm, diffuseness $a = 0.65$ fm, and Thomas form factor of spin-orbit potential $\lambda = 25$. The potential well depth V_0 was automatically adjusted to reproduce the binding energy. For the first excited state of ^{13}C, $V_0 = 57.62$ MeV. The interaction radius is taken as $R_N = 1.25(A_c^{1/3}+1)$ fm with A_c being the mass number of core nucleus, which is consistent with our previous definition [17,18,24]. For $^{12}\mathrm{C}+n$ and $^{11}\mathrm{B}+n$, R_N is equal to 4.11 fm and 4.03 fm, respectively. The inner contributions varying with energies of the incident neutrons for the $^{12}\mathrm{C}(n,\gamma)$ processes are illustrated in Fig. 2.

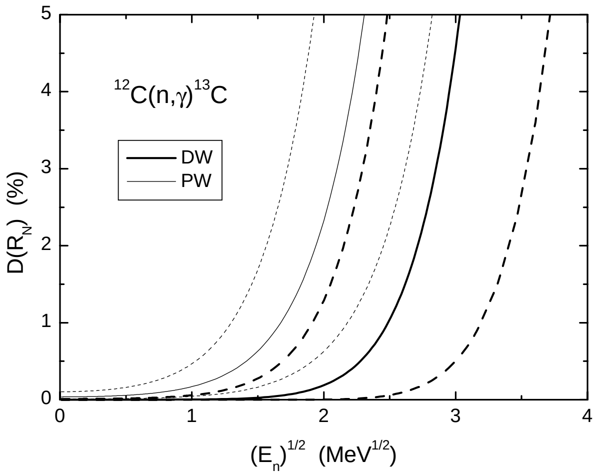

FIG. 2. The inner contributions of the final bound states to the $E1$ transitions vary with the root of the incident energies. The solid and dashed lines correspond to the cases of R_N=4.11 fm and R_N =4.11±0.5 fm, respectively, and the thick and thin lines are the results calculated by DW and PW, respectively.

The solid line represents the case of R_N=4.11 fm. In order to see the influence induced by varying the cutoff radius, R_N are arbitrarily set to 4.11±0.5 fm, as the dashed lines shown in Fig. 2. It is clear that the contributions of inner parts are rather small at low energies. At 1 MeV, $D(R_N)$ are typically less than 0.05% and 0.5% for DW (thick lines) and PW (thin lines), respectively. But when energies exceed 1 MeV, the inner contributions rise rapidly and cannot be neglected anymore. In the present work, we only consider the case of energies less than 1 MeV, in the region of which Eq. (7) works well.

In our previous work [17,18], the angular distributions of the transfer reactions of ^{11}B(d,p) and ^{12}C(d,p) at the incident energy of 11.8 MeV were measured with high precision by a Q3D magnetic spectrograph. The ANC factors for some bound states of ^{12}B and ^{13}C were extracted from the cross sections at the forward angles close to 0°. For each state, the root-mean-square radius was obtained and the contribution of the external part was estimated as a criterion of halo formation. As a result, the first exited state of ^{13}C was confirmed to be a halo state and two new halo states were found in the second and third excited states of ^{12}B. The last neutrons of these halo states are nonexceptionally in the $2s_{1/2}$ orbits, therefore one can use Eq. (7) to calculate the DRC cross sections of ^{11}B(n,γ) and ^{12}C(n,γ) at low energies. Results of the ^{12}C$(n,\gamma)^{13}$C$(2s_{1/2^+})$ processes are shown as the solid line in Fig. 3. In the figure, the error ranges are denoted by two dashed lines, which come from the error of ANC factor. For the ^{13}C$(2s_{1/2^+})$ state, the value of ANC factor, $C_{s_{1/2}}$=1.84±0.16 fm$^{-1/2}$, consistent with that of Imai *et al.* [19] within the experimental error. The results displayed in Fig. 3 clearly show that the deduced results of such an indirect measurement are in good agreement with the experimental results of the direct measurement [4,5] within the experimental errors. In addition, the results calculated by PW are shown as the dotted line in the figure for comparison. The results of PW lie within the error range of the results of DW

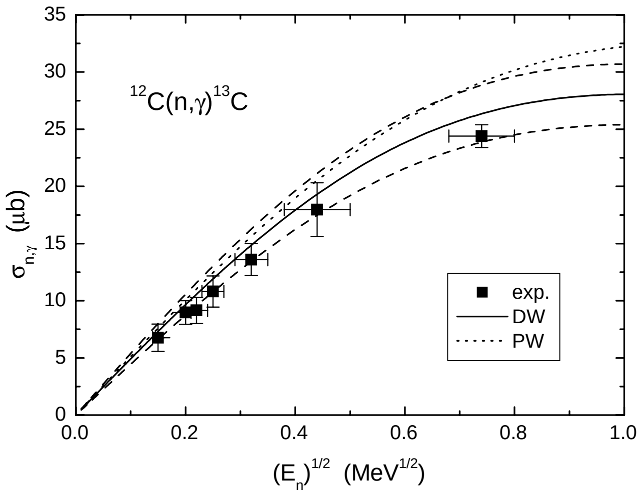

FIG. 3. The cross sections of direct neutron capture to the first excited states of ^{13}C at energies below 1 MeV. The results of DW are represented by the solid line and their error ranges are denoted by the dashed lines. The results of PW are shown by the dotted line for comparison. The experimental data (solid squares) are obtained from Refs. [4,5].

at lower energies, for example, less than 500 keV, but exceed them at higher energies. So only at very low energies one can conclude that the capture process is essentially determined by the structure of the final neutron state. The extracted ANC values for the two halo states of ^{12}B, i.e., the second and third excited states, are 1.34±0.12 fm$^{-1/2}$ and 0.94±0.08 fm$^{-1/2}$, respectively. In the same way, the DRC cross sections corresponding to these two states are deduced and illustrated in Fig. 4 with the labels of σ_{Ex2} and σ_{Ex3}, respectively. The total cross sections σ_{tot} are the sums of these two. The present results of the DRC cross sections for the ^{11}B(n,γ) reaction at low energies are obtained for the first time, and we are waiting for comparison with the results of direct measurement.

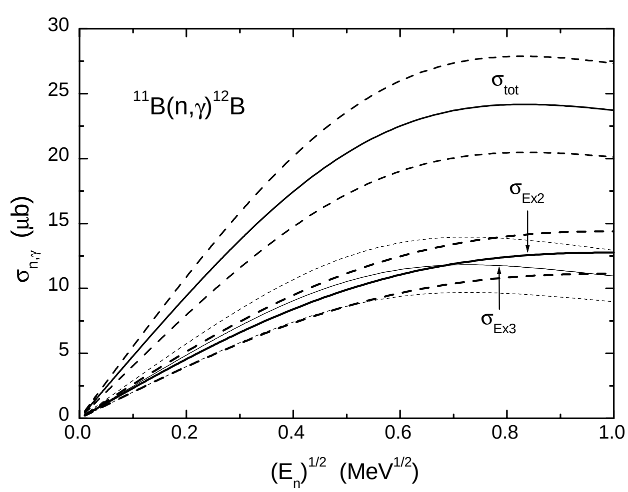

FIG. 4. Same as Fig. 2 but for captures to the second (σ_{Ex2}, thick lines) and third (σ_{Ex3}, thin lines) excited states of ^{12}B. σ_{tot} is the sum of σ_{Ex2} and σ_{Ex3}.

In summary, the cross sections of $p \rightarrow s$ transitions in ^{11}B(n,γ) and ^{12}C(n,γ) reactions at low energies are deduced by means of ANC method, where the ANC factors were extracted from the peripheral transfer reactions of ^{11}B(d,p) and ^{12}C(d,p). These cross sections can be considered as the nonresonant DRC cross sections at stellar energies. The deduced cross sections of ^{12}C(n,γ) are in good agreement with the directly measured results within experimental errors. The DRC cross sections of ^{11}B(n,γ) are obtained for the first time. The present results would be useful to calculate the reaction rates and to understand the procedure of stellar nucleosynthesis.

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