Deuteron anapole moment with heavy mesons

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Parity-nonconserving two-body currents due to vector meson exchange are considered with the aim of determining the related contributions to the anapole moment. Particular attention is given to the requirement of current conservation which is essential for a reliable estimate of this quantity. An application is made to the deuteron case.

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I. INTRODUCTION

The anapole moment (AM) of a quantum system, first introduced by Zel'dovich [1], involves both the electromagnetic interaction and parity nonconservation (PNC). It was not before studies by Flambaum and Khriplovich [2] that the concept acquired a practical interest. In their work, these authors especially emphasized that the AM would grow with the size of the nucleus, making heavy nuclei natural candidates for observation. An anapole measurement is not easy because it involves the hyperfine structure of an atom; the first clear evidence came only a few years ago in the ^{133}Cs nucleus [3].

The deuteron AM has also recently received some attention [4–8]. Though its interest is largely academic (an experiment is not feasible in a near future), it offers the advantage of a laboratory where methods and ingredients can be studied in detail. These studies have been concerned with the pion-exchange component of the PNC nucleon-nucleon (*NN*) force. Calculations were based on assuming an effective-field-theory description [4,5], zero-range *NN* strong forces [7], or more realistic *NN* strong forces [8]. The use of an alternative field-theory description was also proposed [6]. The next step concerns the extension of these results to include the component of the PNC force due to vector-meson exchange that could contribute as much as, if not more than, the pion exchange.

Determining the AM requires the calculation of the effective current that couples to the photon. As it involves parity nonconservation, the current necessarily has an axial character, making the requirement of current conservation nontrivial. Individual contributions are proportional to the weak coupling G_F, while fulfilling the above property implies that the effective current contains the factor $G_F q^2$, which vanishes in the limit of a zero momentum transfer. Getting a reasonable anapole result demands particular care about ensuring gauge invariance. When dealing with vector mesons, this task becomes less straightforward. In particular, it has to be done consistently with the PNC interaction model that is employed in calculations. Some contributions had been given in Ref. [9]. In the present work, we intend to complete this study with the double aim of satisfying gauge invariance and consistency with the PNC interaction model. The DDH potential, given by Desplanques, Donoghue, and Holstein, will be our choice [10]. Some estimates of the vector-meson-exchange contributions to the deuteron anapole moment will be presented.

The plan for this paper is as follows. In Sec. II, we first present the various ingredients pertinent to the interaction: parity-conserving (PC), parity-nonconserving, and electromagnetic (EM) ones. We subsequently provide the expressions for the PNC two-body currents at the lowest $1/m_N$ order and show how the current conservation is fulfilled. Section III is devoted to the calculation of the deuteron anapole moment. This includes the deuteron description, especially the determination of the PNC components; the anapole matrix elements from both the one- and two-body currents; and a numerical estimate in terms of the PNC meson-nucleon coupling constants. A discussion of the results is given in Sec. IV. This is completed by the Appendix that contains expressions of the two-body currents in configuration space.

II. PNC *NN* INTERACTION, CURRENTS, AND CURRENT CONSERVATION

The anapole moment is a special electromagnetic property of a system in which parity conservation is violated; therefore, the first step in the anapole calculation is to determine the EM current operators. Throughout this work, which concerns low-energy nuclear systems, we assume the validity of the nonrelativistic (NR) limit and keep only terms of leading order.

The one-body current ($\rho^{(1)}$, $\boldsymbol{j}^{(1)}\equiv\boldsymbol{j}_{spin}+\boldsymbol{j}_{conv}$), which is the sum of the contributions from each nucleon (for the deuteron, $A=2$), takes the form in momentum space as

$$\rho^{(1)} = e\sum_{i=1}^{A}\frac{1+\tau_i^z}{2}(2\pi)^3\delta^{(3)}(\boldsymbol{k}+\boldsymbol{p}_i'-\boldsymbol{p}_i), \tag{1}$$

$$\mathbf{j}_{spin} = e\sum_{i=1}^{A}\frac{\mu_i}{2m_N}i\boldsymbol{\sigma}_i\times(\boldsymbol{p}_i'-\boldsymbol{p}_i)(2\pi)^3\delta^{(3)}(\boldsymbol{k}+\boldsymbol{p}_i'-\boldsymbol{p}_i), \tag{2}$$

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$$\mathbf{j}_{conv} = e\sum_{i=1}^{A} \frac{1+\tau_i^z}{4m_N}(\boldsymbol{p}_i' + \boldsymbol{p}_i)(2\pi)^3\delta^{(3)}(\boldsymbol{k}+\boldsymbol{p}_i'-\boldsymbol{p}_i), \quad (3)$$

where μ_i is defined as $(\mu_S+\tau_i^z\mu_V)/2$ with $\mu_S=0.88$ and $\mu_V=4.71$; and the vectors $\boldsymbol{k}$, $\boldsymbol{p}_i'$, and $\boldsymbol{p}_i$ denote the 3-momentum of the outgoing photon, outgoing ith nucleon, and incoming ith nucleon, respectively. Besides the one-body current, the canonical meson-exchange picture of the NN interaction suggests additional nuclear EM currents due to exchange effects, which are two-body in character. Thus, in order to reduce theoretical uncertainties and to reach consistency with the chosen NN potential model, it is important to construct these two-body currents which should be constrained by current conservation and phenomenology.

For the PC NN interaction, we choose the Argonne v_{18} potential (Av_{18}) [11]. This potential gives good fits to the scattering data and deuteron properties, but it is not straightforward to construct the corresponding exchange currents (ECs) because the connection with the meson exchange picture is not clear for some parts of this potential. One traditional way to construct the ECs for such cases is implementing the NR minimal coupling (MC) to the potential, i.e.,

$$\boldsymbol{p} \to \boldsymbol{p} - \frac{e}{2}(1+\tau^z)\boldsymbol{A}, \quad H \to H + \frac{e}{2}(1+\tau^z)A^0,$$

then identifying the EM currents from the interaction Hamiltonian density, $ej_\mu A^\mu$. However, this procedure only constrains the longitudinal components, while giving no information about the transverse components which are conserved by themselves. Some uncertainty about these transverse terms comes from potentials involving quadratic velocity-dependent components as discussed in Ref. [12]. Moreover, the derivation of exchange currents for a model like Av_{18}, employed in Ref. [13] for instance, requires further elaboration: the potential contains a Gaussian type component while the above MC prescription is usually applied to Yukawa potentials. Therefore, we leave the fully conserved PC EC as an open question for future work and follow the much-simplified treatment of Ref. [8] to examine: (1) to what degree current conservation is broken by the omission of PC ECs, and (2) how much the inclusion of PC ECs due to the one-pion exchange, which gives the long-range part in Av_{18}, could restore the conservation. By making this exercise, one can get a qualitative handle on this problem. The detail will be discussed in Sec. IV.

For the PNC NN interaction, our choice is the potential based on a one-boson exchange scheme involving π, ρ, and ω mesons, suggested by Desplanques, Donoghue, and Holstein (DDH) [10]. Because this potential has a close tie with the exchange picture, a more field-theoretical formalism, the so-called S-matrix approach [14–16,9], is used to construct all the corresponding ECs. As will be shown later, some transverse components arise naturally in this derivation.

For clarity, we divide the following discussion into three parts: first, the model Lagrangian, consistent with the DDH scheme, is constructed; second, the PNC ECs are derived; and finally, we show how these exchange currents fulfill the current conservation condition with the DDH potential.

A. Model Lagrangian

The total Lagrangian density is divided as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{PC}} + \mathcal{L}_{\mathrm{PNC}} + \mathcal{L}_{\mathrm{EM}} + \delta\mathcal{L}, \quad (4)$$

where $\delta\mathcal{L}$ contains all the terms not relevant for this discussion. The free Lagrangian density of the nucleon (N), pions ($\boldsymbol{\pi}$), rho mesons ($\boldsymbol{\rho}$), and omega meson (ω), is

$$\begin{aligned}\mathcal{L}_0 = {} & \bar{N}'(i\partial\!\!\!/ - m_N)N + \frac{1}{2}(\partial_\mu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 \\ & - \frac{1}{4}\boldsymbol{F}^{(\rho)}_{\mu\nu}\cdot\boldsymbol{F}^{(\rho)\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{2\xi}(\partial_\mu\boldsymbol{\rho}^\mu)\cdot(\partial_\nu\boldsymbol{\rho}^\nu) \\ & - \frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{2\xi}(\partial_\mu\omega^\mu)(\partial_\nu\omega^\nu), \quad (5)\end{aligned}$$

where $\boldsymbol{F}^{(\rho)}_{\mu\nu}$ and $F^{(\omega)}_{\mu\nu}$ are the field tensors of ρ and ω mesons, and the R_ξ gauge-fixing terms for vector mesons are kept explicit. The PC and PNC meson-nucleon interaction Lagrangian densities are

$$\begin{aligned}\mathcal{L}_{PC} = {} & ig_{\pi NN}\bar{N}'\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N - g_{\rho NN}\bar{N}'\left(\gamma_\mu - i\frac{\chi_V}{2m_N}\sigma_{\mu\nu}q^\nu\right)\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu N \\ & - g_{\omega NN}\bar{N}'\left(\gamma_\mu - i\frac{\chi_S}{2m_N}\sigma_{\mu\nu}q^\nu\right)\omega^\mu N, \quad (6)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{PNC} = {} & -\frac{h_\pi^1}{\sqrt{2}}\bar{N}'(\boldsymbol{\tau}\times\boldsymbol{\pi})^z N + \bar{N}'\Bigg[h_\rho^0\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu + h_\rho^1\rho^{z\mu} \\ & + \frac{h_\rho^2}{2\sqrt{6}}(3\tau^z\rho^{z\mu} - \boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\Bigg]\gamma_\mu\gamma_5 N + \bar{N}'(h_\omega^0\omega^\mu \\ & + h_\omega^1\tau^z\omega^\mu)\gamma_\mu\gamma_5 N, \quad (7)\end{aligned}$$

where q^μ is the 4-momentum carried by the outgoing boson; the strong couplings g_{XNN} as well as the weak couplings $h_X^{(i)}$ are defined as in DDH's work (except that their PNC pion coupling, f_π, is renamed as h_π^1 here); and the anomalous strong isoscalar and isovector magnetic moments of the nucleon, χ_S and χ_V, are assumed to be the same as the EM values, −0.12 and 3.70, by vector meson dominance.

The EM interactions are obtained by applying the covariant MC,

$$p_\mu \to p_\mu - \frac{e}{2}(1+\tau^z)A_\mu,$$

and only terms of first-order in e are included in $\mathcal{L}_{\mathrm{EM}}$. From $\mathcal{L}_0$, one gets

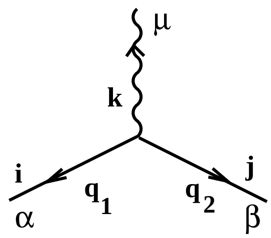

FIG. 1. The vertex factor for vector-meson-photon coupling, where α, β, and γ are the Lorentz indexes; i and j are isospin indexes.

$$\mathcal{L}_{\rm EM}^{(NN\gamma)} = -e\overline{N'}\left[\left(F_1^{(S)}(Q^2)\frac{1}{2} + F_1^{(V)}(Q^2)\frac{\tau^z}{2}\right)\gamma_\mu - i\frac{1}{2m_N}\left(F_2^{(S)}(Q^2)\frac{1}{2} + F_2^{V}(Q^2)\frac{\tau^z}{2}\right)\sigma_{\mu\nu}q^\nu\right]NA^\mu, \quad (8)$$

$$\mathcal{L}_{\rm EM}^{(\pi\pi\gamma)} = -e(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^z A^\mu, \quad (9)$$

$$\mathcal{L}_{\rm EM}^{(\rho\rho\gamma)} = -e(\boldsymbol{\rho}^\nu\times\boldsymbol{F}_{\nu\mu}^{(\rho)})^z A^\mu - \frac{1}{\xi}e(\boldsymbol{\rho}_\mu\times\partial_\nu\boldsymbol{\rho}^\nu)^z A^\mu. \quad (10)$$

Note that in order to account for the nucleon structure, the nucleon EM form factors, $F_{1,2}^{(S,V)}$ (superscript "S" for isoscalar and "V" for isovector; subscript "1" for Dirac and "2" for Pauli) are added. At $Q^2=-q^2=0$, $F_1^{(S)}(0)=F_1^{(V)}(0)=1$, $F_2^{(S)}(0)=-0.12$, and $F_2^{(V)}(0)=3.70$. In principle, one should also take into account the meson structure; however, they are still poorly constrained so we simply assume the mesons are elementary.

Due to the momentum-dependent coupling of a ρ meson to the nucleon anomalous magnetic moment, a Kroll-Ruderman type contact interaction [17],

$$\mathcal{L}_{\rm EM}^{(NN\rho\gamma)} = -e\frac{g_{\rho NN}\chi_V}{2m_N}\overline{N'}\sigma_{\mu\nu}(\boldsymbol{\tau}\times\boldsymbol{\rho}^\nu)^z NA^\mu, \quad (11)$$

also arises. This leads to a seagull current which is important for current conservation, but was ignored in Ref. [9].

It is worthwhile to point out that the EM interactions obtained above depend on the model Lagrangian one starts with. For example, compared with the result of QHD II [18,19] one observes a larger $\rho\rho\gamma$ interaction in QHD II by the amount

$$\Delta\mathcal{L}_{\rm EM}^{(\rho\rho\gamma)} = \tfrac{1}{2}e(\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu)^z F^{(\gamma)\mu\nu},$$

which modifies the $\rho\rho\gamma$ vertex in an interesting way as we are going to explain.

By fixing the gauge parameter $\xi=1$, i.e., 't Hooft-Feynman gauge, we obtained the same $\mathcal{L}_{EM}^{(\rho\rho\gamma)}$ as in Ref. [20], and this gives a vertex factor (see Fig. 1)

$$\epsilon_{3ij}[(q_1-q_2)^\mu g^{\alpha\beta} + k^\alpha g^{\beta\mu} - k^\beta g^{\mu\alpha}], \quad (12)$$

with $k+q_1+q_2=0$. The last two terms together give an amplitude conserved by itself (actually, they correspond to magnetic dipole couplings which explain the transversality). Adding $\Delta\mathcal{L}_{\rm EM}^{(\rho\rho\gamma)}$ simply doubles the self-conserved terms so that we have

$$\epsilon_{3ij}[(q_1-q_2)^\mu g^{\alpha\beta} + 2k^\alpha g^{\beta\mu} - 2k^\beta g^{\mu\alpha}], \quad (13)$$

for the vertex. As the ρ meson is a spin-one particle, it has charge (c), magnetic dipole (μ), and charge quadrupole (Q) couplings to the EM field. When assuming that it is an elementary particle: $c=e$, $\mu=e/m_\rho$, and $Q=-e/m_\rho^2$, the vertex factor appears to be Eq. (13) [21,22], which implies that the MC result underpredicts the ρ meson magnetic moment by 2. This factor of 2 difference in self-conserved terms between MC and chiral Lagrangian approaches has been pointed out in Refs. [23,16] and was attributed to the model dependency. However, in order to have a closer contact with phenomenology, we use Eq. (13) instead as the $\rho\rho\gamma$ vertex.

The modification mentioned above is just an example of model dependency in constructing ECs. Since these purely transverse terms, often called non-Born (NB) terms, could not be constrained by current conservation, it is not easy to set up criteria *a priori* to judge which ones should be included, unless comparisons are made with experiments [24]. For this derivation, we include the $\rho\pi\gamma$ and $\omega\pi\gamma$ interactions

$$\mathcal{L}_{\rm EM}^{(\rho\pi\gamma)} = e\frac{g_{\rho\pi\gamma}}{2\,m_\rho}\epsilon_{\alpha\beta\gamma\delta}F^{(\gamma)\alpha\beta}(\boldsymbol{\rho}^\gamma\cdot\partial^\delta\boldsymbol{\pi}), \quad (14)$$

$$\mathcal{L}_{\rm EM}^{(\omega\pi\gamma)} = e\frac{g_{\omega\pi\gamma}}{2\,m_\omega}\epsilon_{\alpha\beta\gamma\delta}F^{(\gamma)\alpha\beta}(\omega^\gamma\partial^\delta\pi^z), \quad (15)$$

where the total antisymmetric tensor is defined as $\epsilon_{0123}=-1$ [25,15], because the corresponding meson-nucleon coupling constants, $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$, can be determined from the decay data (except for signs). We ignore all the nucleon isobaric excitations because they are not the main theoretical emphasis of this work. The present work could easily be extended if necessary.

To sum up, the total EM Lagrangian density we consider is

$$\mathcal{L}_{\rm EM} = L_{\rm EM}^{(NN\gamma)} + L_{\rm EM}^{(\pi\pi\gamma)} + \left\{L_{\rm EM}^{(\rho\rho\gamma)} + \tfrac{1}{2}e(\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu)^z F^{(\gamma)\mu\nu}\right\} + \mathcal{L}_{\rm EM}^{(\rho\pi\gamma)} + \mathcal{L}_{\rm EM}^{(\omega\pi\gamma)}. \quad (16)$$

B. PNC meson exchange currents

Diagrammatically, the ECs could be classified according to Fig. 2 as (a) norm-recoil, (b) pair, (c) mesonic, (d) seagull, (e) isobaric, and (f) NB mesonic types. The division into norm-recoil and pair terms simply comes from the separation of positive- and negative-energy components in the covariant nucleon propagator. Confusion sometimes arises when it comes to pair and seagull diagrams. If the PC πNN coupling is formulated as pseudovector, the seagull term is $O(1/m_N)$, while the pair term is higher order in $1/m_N$. On the other hand, if the pseudoscalar coupling is adopted as we do here, there is no seagull term; however, at the leading order, the pair term looks exactly as the seagull term in the pseudo-

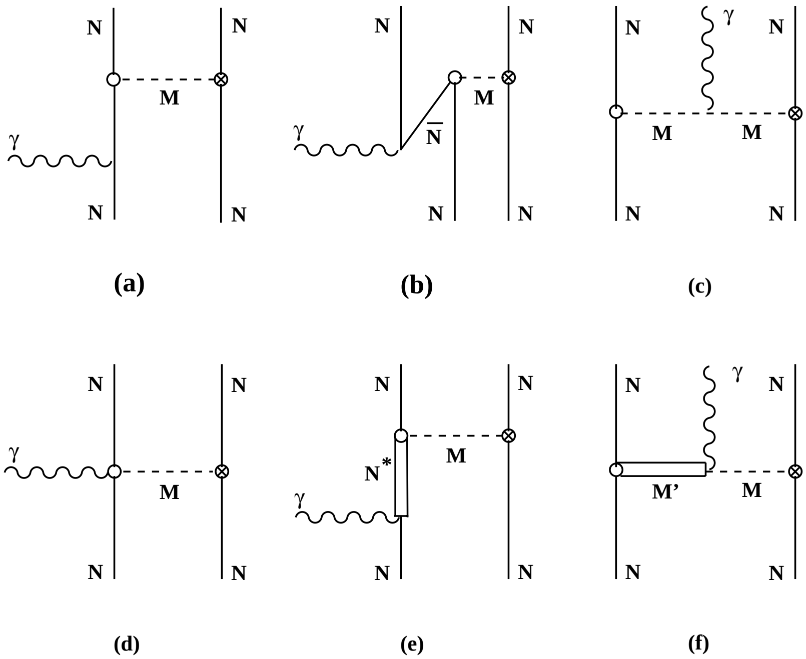

FIG. 2. Classification of meson-exchange currents: (a) norm-recoil, (b) pair, (c) mesonic, (d) seagull, (e) isobaric, and (f) non-Born mesonic, where N and N^* denote nucleon and nucleon excited state; M and M' denote mesons.

vector scheme. Therefore, as far as the NR approximation is valid, these two formalisms are equivalent [26,12]. For the case of ρ mesons, both pair and seagull diagrams have $O(1/m_N)$ contributions. As for the NB contributions from (e) and (f), we only consider the latter as explained above.

Before one applies Feynman rules to evaluate these diagrams and extract the corresponding ECs, the gauge parameter has to be fixed. Though physical results should be gauge independent, a proper choice may greatly simplify the calculation. Here, we adopt the 't Hooft-Feynman gauge, ξ=1, for the following reasons. First, the propagator is simpler,

$$\langle A_\mu A_\nu \rangle = \left(g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2 - \xi m^2 + i\epsilon}\right)\frac{-i}{q^2 - m^2 + i\epsilon} = \frac{-ig_{\mu\nu}}{q^2 - m^2 + i\epsilon} \quad (\text{for } \xi = 1).$$

Second, the PNC potential, constructed from a NR reduction of the one-boson exchange diagrams, corresponds to the form given by DDH. The last and most important, the contribution from the norm-recoil diagram represents how the one-body EM matrix element is modified by the presence of the NR potential [16]. This term should not be double counted if one has already taken care of it by using the perturbed wave function—the route we will follow.

In momentum space and to the order of $1/m_N$, the pair and ρ-seagull (KR) 3-currents are

$$\boldsymbol{j}^{\pi}_{pair} = \frac{-e g_{\pi NN} h^1_\pi}{2\sqrt{2} m_N}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \tau_1^z \tau_2^z)[\boldsymbol{\sigma}_1]\frac{(2\pi)^3 \delta^{(3)}(\cdots)}{\boldsymbol{q}_2^2 + m_\pi^2} + (1 \leftrightarrow 2), \tag{17}$$

$$\boldsymbol{j}^{\rho}_{pair+KR} = \frac{-e g_{\rho NN}}{2m_N}\Bigg\{\Bigg(h^0_\rho(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \tau_2^z) + \frac{h^2_\rho}{2\sqrt{6}}(3\tau_1^z\tau_2^z - \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + 2\tau_2^z)\Bigg)[\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2] + h^1_\rho(1+\tau_1^z)[\tau_2^z\boldsymbol{\sigma}_1 - \tau_1^z\boldsymbol{\sigma}_2] + (1 + \chi_V)\Bigg(h^0_\rho - \frac{h^2_\rho}{2\sqrt{6}}\Bigg)(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^z[\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2]\Bigg\} \times \frac{(2\pi)^3\delta^{(3)}(\cdots)}{q_2^2 + m_\rho^2} + (1 \leftrightarrow 2), \tag{18}$$

$$\boldsymbol{j}^{\omega}_{pair} = \frac{-e g_{\omega NN}}{2M_N}(1+\tau_1^z)(h^0_\omega[\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2] + h^1_\omega[\tau_1^z\boldsymbol{\sigma}_1 - \tau_2^z\boldsymbol{\sigma}_2])\frac{(2\pi)^3\delta^{(3)}(\cdots)}{\boldsymbol{q}_2^2 + m_\omega^2} + (1 \leftrightarrow 2), \tag{19}$$

where $\boldsymbol{q}_{1,2}$=$\boldsymbol{p}'_{1,2}$−$\boldsymbol{p}_{1,2}$ and the δ function imposes the total 3-momentum conservation, $\boldsymbol{k}$+$\boldsymbol{q}_1$+$\boldsymbol{q}_2$=0. The ρ-seagull 3-current corresponds to the term involving χ_V in Eq. (18). All the pair charges are of higher order in $1/m_N$ compared with the nucleon charge, which is $O(1)$, so they are neglected. The mesonic 3-currents are

$$\boldsymbol{j}^{\pi}_{mesonic} = \frac{-e g_{\pi NN} h^1_\pi}{2\sqrt{2} m_N}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \tau_1^z\tau_2^z)[\boldsymbol{q}_2 - \boldsymbol{q}_1]\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1\frac{(2\pi)^3\delta^{(3)}(\cdots)}{(\boldsymbol{q}_1^2 + m_\pi^2)(\boldsymbol{q}_2^2 + m_\pi^2)} + (1 \leftrightarrow 2), \tag{20}$$

$$\mathbf{j}^{\rho}_{mesonic} = \frac{-eg_{\rho NN}}{2m_N}\left(h^0_\rho - \frac{h^2_\rho}{2\sqrt{6}}\right) i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z([\boldsymbol{q}_1-\boldsymbol{q}_2]\boldsymbol{\sigma}_2\cdot[(\boldsymbol{p}'_2 + \boldsymbol{p}_2)-(\boldsymbol{p}'_1+\boldsymbol{p}_1) - i(1+\chi_V)\boldsymbol{\sigma}_1\times\boldsymbol{q}_1] + 2[(\boldsymbol{p}'_1+\boldsymbol{p}_1) + i(1+\chi_V)\boldsymbol{\sigma}_1\times\boldsymbol{q}_1]\boldsymbol{\sigma}_2\cdot\boldsymbol{k} + 2[\boldsymbol{\sigma}_2]\{2m_N k_0 - \boldsymbol{k}\cdot[(\boldsymbol{p}'_1 + \boldsymbol{p}_1) + i(1+\chi_V)\boldsymbol{\sigma}_1\times\boldsymbol{q}_1]\})\frac{(2\pi)^3\delta^{(3)}(\cdots)}{(\boldsymbol{q}_1^2+m_\rho^2)(\boldsymbol{q}_2^2+m_\rho^2)} + (1\leftrightarrow 2), \tag{21}$$

$$\mathbf{j}^{\rho\pi}_{mesonic} = \frac{eg_{\rho NN}g_{\rho\pi\gamma}h^1_\pi}{\sqrt{2}m_\rho}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z[\boldsymbol{q}_1\times\boldsymbol{q}_2]\frac{(2\pi)^3\delta^{(3)}(\cdots)}{(\boldsymbol{q}_1^2+m_\rho^2)(\boldsymbol{q}_2^2+m_\pi^2)} + (1\leftrightarrow 2), \tag{22}$$

$$\mathbf{j}^{\omega\pi}_{mesonic}\approx 0, \tag{23}$$

where $k_0=E_1+E_2-E'_1-E'_2=E_i-E_f$. Specially note that there is a mesonic contribution to the charge density at the same order as the nucleon charge:

$$\rho^{\rho}_{mesonic} = -2eg_{\rho NN}\left(h^0_\rho - \frac{h^2_\rho}{2\sqrt{6}}\right) i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z\boldsymbol{\sigma}_2\cdot\boldsymbol{k}\frac{(2\pi)^3\delta^{(3)}(\cdots)}{(\boldsymbol{q}_1^2+m_\rho^2)(\boldsymbol{q}_2^2+m_\rho^2)} + (1\leftrightarrow 2). \tag{24}$$

C. PNC *NN* interaction and current conservation

The DDH potential in momentum space could be expressed as

$$V^{\pi}_{\rm PNC} = \frac{g_{\pi NN}h^1_\pi}{2\sqrt{2}m_N} i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{u}_\pi, \tag{25}$$

$$V^{\rho}_{\rm PNC} = \frac{-g_{\rho NN}}{m_N}\left[\left(h^0_\rho\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + \frac{h^1_\rho}{2}(\tau^z_1+\tau^z_2) + \frac{h^2_\rho}{2\sqrt{6}}(3\tau^z_1\tau^z_2 - \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\right)[i(1+\chi_V)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{u}_\rho + (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{v}_\rho] - \frac{h^1_\rho}{2}(\tau^z_1-\tau^z_2)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{v}_\rho\right], \tag{26}$$

$$V^{\omega}_{\rm PNC} = \frac{-g_{\omega NN}}{m_N}\left[\left(h^0_\omega + \frac{h^1_\omega}{2}(\tau^z_1+\tau^z_2)\right)[i(1+\chi_S)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot u_\omega + (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{v}_\omega] + \frac{h^1_\omega}{2}(\tau^z_1-\tau^z_2)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{v}_\omega\right], \tag{27}$$

where

$$\boldsymbol{u}_X = \frac{\boldsymbol{q}}{\boldsymbol{q}^2+m_X^2},\quad \boldsymbol{v}_X = \frac{(\boldsymbol{p}'_1+\boldsymbol{p}_1)-(\boldsymbol{p}'_2+\boldsymbol{p}_2)}{2(\boldsymbol{q}^2+m_X^2)},$$

and $\boldsymbol{q}$ denotes the meson 3-momentum.

To prove the conservation of these PNC currents at the operator level, we showed explicitly the following matrix element identities (with bra $\langle\boldsymbol{p}'_1,\boldsymbol{p}'_2|$ and ket $|\boldsymbol{p}_1,\boldsymbol{p}_2\rangle$):

$$\langle[\rho^{(1)}, V^{\pi}_{\rm PNC}]\rangle = \boldsymbol{k}\cdot\langle\boldsymbol{j}^{\pi}_{pair} + \boldsymbol{j}^{\pi}_{mesonic}\rangle, \tag{28}$$

$$\langle[\rho^{(1)}, V^{\rho}_{\rm PNC}]\rangle = \boldsymbol{k}\cdot\langle\boldsymbol{j}^{\rho}_{pair+KR} + \boldsymbol{j}^{\rho(\rm I)}_{mesonic}\rangle, \tag{29}$$

$$\langle[\rho^{(1)}, V^{\omega}_{\rm PNC}]\rangle = \boldsymbol{k}\cdot\langle\boldsymbol{j}^{\omega}_{pair}\rangle, \tag{30}$$

$$\langle[\rho^{\rho}_{mesonic}, H]\rangle = \boldsymbol{k}\cdot\langle\boldsymbol{j}^{\rho(\rm II)}_{mesonic}\rangle, \tag{31}$$

$$0 = \boldsymbol{k}\cdot\langle\boldsymbol{j}^{\rho\pi}_{mesonic} + \boldsymbol{j}^{\omega\pi}_{mesonic}\rangle, \tag{32}$$

where H is the total Hamiltonian, which is the sum of the kinetic energy (T) and both the PC and PNC potentials ($V_{\rm PC}$ and $V_{\rm PNC}$); and the ρ mesonic current, Eq. (21), is separated into two parts: (I) is proportional to the vector $(\boldsymbol{q}_1-\boldsymbol{q}_2)$, and (II) contains the rest. The continuity equation Eq. (31) indicates that $(\rho^{\rho}_{mesonic},\boldsymbol{j}^{\rho(\rm II)}_{mesonic})$ forms a conserved current not constrained by the DDH potential, which results from the self-conserved $\rho\rho\gamma$ vertex mentioned above. The last equality, Eq. (32), shows the transversality of NB currents. Obviously, the total PNC EC $(\rho^{(2)}_{\rm PNC},\boldsymbol{j}^{(2)}_{\rm PNC})$, the sum of Eqs. (17)–(24), satisfies the total current conservation condition

$$[\rho^{(1)}+\rho^{(2)}_{\rm PNC}, T+V_{\rm PC}+V_{\rm PNC}] = \boldsymbol{k}\cdot(\boldsymbol{j}^{(1)}+\boldsymbol{j}^{(2)}_{\rm PC}+\boldsymbol{j}^{(2)}_{\rm PNC}),$$

as long as the two-body PC EC, $(\rho^{(2)}_{\rm PC},\boldsymbol{j}^{(2)}_{\rm PC})$, is conserved, i.e.,

$$[\rho^{(1)}, V_{\rm PC}] = \boldsymbol{k}\cdot\boldsymbol{j}^{(2)}_{PC}$$

($\rho^{(2)}_{\rm PC}$ is higher order in $1/m_N$). Therefore, at least for the PNC part, we have every conservation condition met.

For the AM calculation, we have to use both the DDH potential and current operators in coordinate space. These expressions could be found in the Appendix.

III. DEUTERON ANAPOLE MOMENT

A. Determination of the deuteron wave function

Due to the PNC *NN* interaction, the deuteron wave function, mainly a ${}^3S_1$ state with some fraction of ${}^3D_1$ component, could have parity admixtures in ${}^3\tilde{P}_1$ and ${}^1\tilde{P}_1$ channels. The former channel, induced by the isovector part of the DDH potential, is dominated by the π exchange, while the latter one, resulting from the isoscalar interaction, only arises from the heavy-meson exchange. Therefore, we express the full parity-admixed deuteron wave function as

$$\Psi(\boldsymbol{r}) = \frac{1}{\sqrt{4\pi}r}\Bigg[\left(u(r)+\frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}w(r)\right)\zeta_{00} - i\sqrt{\frac{3}{8}}\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}\,\tilde{v}_{3p1}(r)\,\zeta_{10} + i\frac{\sqrt{3}}{2}\,(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}\,\tilde{v}_{1p1}(r)\,\zeta_{00}\Bigg]\chi_{1J_z}, \tag{33}$$

where $S_{12}(\hat{\boldsymbol{r}}) \equiv 3\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}}\ \boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, and χ and ζ represent spinor and isospinor wave functions, respectively.

The Schrödinger equation in the center of mass frame is

$$H\,\Psi(\boldsymbol{r}) = \Bigg[-\frac{1}{m_N}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}\,r - \frac{l(l+1)}{r^2}\right) + V_C(r) + V_T(r)\,S_{12}(\hat{\boldsymbol{r}}) + V_{\rm PNC}(\boldsymbol{r})\Bigg]\,\Psi(\boldsymbol{r}) = E\,\Psi(\boldsymbol{r}), \tag{34}$$

where $V_C(r)$ and $V_T(r)$ are the central and tensor parts of the strong potential, respectively, and $V_{\rm PNC}(\boldsymbol{r})$ is the sum of the PNC potentials, Eqs. (25), (26), and (27), given above. Up to the first order in the PNC NN interaction, radial wave functions satisfy the following differential equations

$$u''(r) + m_N\,[E - V_C(r)]u(r) = \sqrt{8}m_N V_T(r)w(r), \tag{35}$$

$$w''(r) - \frac{6}{r^2}w(r) + m_N[E - V_C(r) + 2V_T(r)]w(r) = \sqrt{8}m_N V_T(r)u(r), \tag{36}$$

$$\begin{aligned}\tilde{v}''_{3p1}(r) - \frac{2}{r^2}\tilde{v}_{3p1}(r) + m_N[E - V_C(r) - 2V_T(r)]\tilde{v}_{3p1}(r) = \frac{2}{\sqrt{3}}\Bigg[&\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right)\frac{\partial}{\partial r}[F^1_\pi(r) + \sqrt{2}F^1_\rho(r) - \sqrt{2}F^1_\omega(r)] + 2\sqrt{2}[F^1_\rho(r) - F^1_\omega(r)]\\ &\times\frac{\partial}{\partial r}\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right) - \frac{2\sqrt{2}}{r}[F^1_\rho(r) - F^1_\omega(r)][u(r) - \sqrt{2}w(r)]\Bigg],\end{aligned} \tag{37}$$

$$\begin{aligned}\tilde{v}''_{1p1} - \frac{2}{r^2}\tilde{v}_{1p1}(r) + m_N[E - V_C(r)]\tilde{v}_{1p1}(r) = \frac{2}{\sqrt{3}}\Bigg[&[u(r) - \sqrt{2}w(r)]\frac{\partial}{\partial r}[3\chi_V F^0_\rho(r) - \chi_S F^0_\omega(r)] - 2[3F^0_\rho(r) - F^0_\omega(r)]\frac{\partial}{\partial r}[u(r) - \sqrt{2}w(r)]\\ &+ \frac{2}{r}[3F^0_\rho(r) - F^0_\omega(r)][u(r) + 2\sqrt{2}w(r)]\Bigg],\end{aligned} \tag{38}$$

where $F^1_\pi(r) \equiv g_{\pi NN}h^1_\pi f_\pi(r)$, $F^0_\rho(r) \equiv g_{\rho NN}h^0_\rho f_\rho(r)$, $F^1_\rho(r) \equiv g_{\rho NN}h^1_\rho f_\rho(r)$, $F^0_\omega(r) \equiv g_{\omega NN}h^0_\omega f_\omega(r)$, and $F^1_\omega(r) \equiv g_{\omega NN}h^1_\omega f_\omega(r)$. In our numerical calculations, $g_{\pi NN} = 13.45$, $g_{\rho NN} = 2.79$, $g_{\omega NN} = 8.37$, as well as DDH best values (in units of 10^{-7}) $h^1_\pi = 4.6$, $h^0_\rho = -11.4$, $h^1_\rho = -0.2$, $h^2_\rho = -9.5$, $h^0_\omega = -1.9$, and $h^1_\omega = -1.1$, are assumed.

B. Anapole moment: Expressions for matrix elements

The anapole operator we use takes the form

$$\boldsymbol{a} \equiv \frac{2\pi}{3}\int d\boldsymbol{x}\ \boldsymbol{x}\times(\boldsymbol{x}\times\boldsymbol{j}(\boldsymbol{x})), \tag{39}$$

where $\boldsymbol{j}(\boldsymbol{x})$ contains all the one-body and PNC exchange currents discussed in Sec. II. Note that this form is equivalent to what has been recommended in Refs. [27,9], which is a result from implementing the extended Siegert's theorem [28].

With the deuteron wave function, Eq. (33), we obtain the anapole moment from the spin term,

$$a_{spin} = -\frac{\pi}{\sqrt{6}\ m_N}\Bigg[\mu_V\int dr\ r[u(r) - \sqrt{2}w(r)]\tilde{v}_{3p1}(r) - \sqrt{2}\ \mu_S\int dr\ r\left(u(r) + \frac{1}{\sqrt{2}}w(r)\right)\tilde{v}_{1p1}(r)\Bigg]\ e\ \boldsymbol{I}, \tag{40}$$

where $\boldsymbol{I} \equiv 1/2\,\chi^\dagger_{1J_z}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\chi_{1J_z}$ is the intrinsic spin taken in the spinor basis, and we note that this is equivalent to the total angular momentum taken in the total angular momentum basis, i.e., $\boldsymbol{I} = \langle J=1, J_z|\boldsymbol{J}|J=1, J_z\rangle$ [29].

The matrix element of the convection current is written as

$$\boldsymbol{j}_{conv}(\boldsymbol{x}) = \boldsymbol{j}^+_{conv}(\boldsymbol{x}) + \boldsymbol{j}^-_{conv}(\boldsymbol{x}),$$

$$\boldsymbol{j}^+_{conv}(\boldsymbol{x}) \equiv \frac{e}{4m_N}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\frac{1}{\sqrt{4\pi}r}\chi^\dagger_{1J_z}\left(u(r) + \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}w(\boldsymbol{r}) - i\frac{\sqrt{3}}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}\tilde{v}_{1p1}(r)\right)(\boldsymbol{p}_1, \boldsymbol{p}_2)^+\left(u(r) + \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}w(r) + i\frac{\sqrt{3}}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}\tilde{v}_{1p1}(r)\right)\frac{1}{\sqrt{4\pi}r}\chi_{1J_z}, \tag{41}$$

$$\begin{aligned}\boldsymbol{j}^-_{conv}(\boldsymbol{x}) \equiv \frac{e}{4m_N}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\frac{1}{\sqrt{4\pi}r}\chi^\dagger_{1J_z}\Bigg[&\left(u(r) + \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}w(r)\right)\times(\boldsymbol{p}_1, \boldsymbol{p}_2)^-\left(-i\sqrt{\frac{3}{8}}\right)(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}\tilde{v}_{3p1}(r)\\ &+ i\sqrt{\frac{3}{8}}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}\tilde{v}_{3\boldsymbol{p}1}(r)(\boldsymbol{p}_1, \boldsymbol{p}_2)^-\left(u(r) + \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}w(r)\right)\Bigg]\frac{1}{\sqrt{4\pi}r}\chi_{1J_z},\end{aligned} \tag{42}$$

where

$$(\boldsymbol{p}_1,\boldsymbol{p}_2)^{\pm} \equiv \{\boldsymbol{p}_1, \delta^3(\boldsymbol{x}-\boldsymbol{r}_1)\} \pm \{\boldsymbol{p}_2, \delta^3(\boldsymbol{x}-\boldsymbol{r}_2)\}. \quad (43)$$

It can be shown that the matrix element of the operator $(\boldsymbol{p}_1,\boldsymbol{p}_2)^+$ is proportional to

$$-2i\boldsymbol{R}+\boldsymbol{R}(\boldsymbol{R}\cdot\boldsymbol{P})+\boldsymbol{r}\left(\boldsymbol{R}\cdot\boldsymbol{p}+\tfrac{1}{4}\boldsymbol{r}\cdot\boldsymbol{P}\right)-\left(\boldsymbol{R}^2+\tfrac{1}{4}\boldsymbol{r}^2\right)\boldsymbol{P}-2\boldsymbol{R}\cdot\boldsymbol{r}\boldsymbol{p},$$

where $\boldsymbol{R}=(\boldsymbol{r}_1+\boldsymbol{r}_2)/2$ is the coordinate of the center of mass and $\boldsymbol{P}=\boldsymbol{p}_1+\boldsymbol{p}_2$ is the conjugate momentum. In the center of mass frame where only the relative coordinate and momentum are relevant, the terms proportional to $\boldsymbol{R}$ or $\boldsymbol{P}$ can be discarded. Therefore, the "true" internal convection current is determined solely by Eq. (42). Using the result in Ref. [8],

$$\frac{2\pi}{3}\int d\boldsymbol{x}\boldsymbol{x}\times[\boldsymbol{x}\times\boldsymbol{j}^{-}_{conv}(\boldsymbol{x})]=\left\langle -i\frac{\pi e}{12m_N}[\boldsymbol{l}^2,\boldsymbol{r}]\right\rangle, \quad (44)$$

where $\boldsymbol{l}=\boldsymbol{r}\times\boldsymbol{p}$ [30], the anapole moment from the convection term then reads

$$\boldsymbol{a}_{conv}=\frac{1}{3}\frac{\pi}{\sqrt{6}\,m_N}\int dr\, r\,(u(r)-\sqrt{2}w(r))\tilde{v}_{3p1}(r)e\,\boldsymbol{I}. \quad (45)$$

Contributions from the PNC exchange currents are evaluated with the parity-even channels ($^3S_1$ and $^3D_1$) in the initial and final states. Because these channels are spin triplet ($S=1$) and isospin singlet ($T=0$), the spin and isospin selection rules

$$\langle S=1||(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)||S=1\rangle=\langle S=1||(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)||S=1\rangle=0, \quad (46)$$

$$\langle T=0||(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z||T=0\rangle=0, \quad (47)$$

where the double bar $||$ means the reduced matrix element greatly simplifies the calculations: the matrix elements of $\boldsymbol{j}^{\rho^{\pm}}_{pair+KR}$, $\boldsymbol{j}^{\rho^{\pm}}_{mesonic}$, $\boldsymbol{j}^{\rho\pi}_{mesonic}$, and the parts proportional to $\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2$ in $\boldsymbol{j}^{\rho^0}_{pair}$ and $\boldsymbol{j}^{\omega}_{pair}$ all vanish. Only the π pair, π mesonic terms, and the parts proportional to $\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2$ in ρ^0 pair and ω pair ones have to be considered. Pair and mesonic pion contributions had already been calculated in [8]

$$\boldsymbol{a}^{\pi}_{pair}=-\frac{\sqrt{2}\pi g_{\pi NN}}{9m_N}\int dr\, r^2\, f_{\pi}(r)\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right)[u(r)-\sqrt{2}w(r)]e\,\boldsymbol{I}\,h^1_{\pi}, \quad (48)$$

$$\boldsymbol{a}^{\pi}_{mesonic}=\frac{\sqrt{2}\pi g_{\pi NN}}{3m_N m_{\pi}}\int dr\, r\, f_{\pi}(r)\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right)\left[u(r)\left(1-\frac{1}{3}m_{\pi}r\right)-\frac{1}{\sqrt{2}}w(r)\left(1+\frac{1}{3}m_{\pi}r\right)\right]e\,\boldsymbol{I}\,h^1_{\pi}, \quad (49)$$

and the heavy-meson contributions are now found to be

$$\boldsymbol{a}^{\rho^0}_{pair}=-\frac{2\pi\, g_{\rho NN}}{9\, m_N}\int dr\, r^2\, f_{\rho}(r)\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right)[u(r)-\sqrt{2}w(r)]e\,\boldsymbol{I}\,h^1_{\rho}, \quad (50)$$

$$\boldsymbol{a}^{\omega}_{pair}=\frac{2\pi\, g_{\omega NN}}{9\, m_N}\int dr\, r^2\, f_{\omega}(r)\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right)[u(r)-\sqrt{2}w(r)]e\,\boldsymbol{I}h^1_{\omega}. \quad (51)$$

C. Numerical results for the Argonne v_{18} NN interaction model

Using the Av_{18} model, the numerical results (in units of fm^2) are

$$\boldsymbol{a}_{spin}=-0.547\,h^1_{\pi}\,e\,\boldsymbol{I}+(-3.7\,h^1_{\rho}+10.8\,h^1_{\omega}+7.3\,h^0_{\rho}+3.7\,h^0_{\omega})\times10^{-3}\,e\,\boldsymbol{I}, \quad (52)$$

$$\boldsymbol{a}_{conv}=0.039\,h^1_{\pi}\,e\,\boldsymbol{I}+(2.7\,h^1_{\rho}-7.6\,h^1_{\omega})\times10^{-4}\,e\,\boldsymbol{I}, \quad (53)$$

$$\boldsymbol{a}^{\pi}_{ex}=\boldsymbol{a}^{\pi}_{pair}+\boldsymbol{a}^{\pi}_{mesonic}=(-0.027+0.028)\,h^1_{\pi}\,e\,\boldsymbol{I}, \quad (54)$$

$$\boldsymbol{a}^{\rho}_{ex}=\boldsymbol{a}^{\rho^0}_{pair}=-0.7\times10^{-4}\,h^1_{\rho}\,e\,\boldsymbol{I}, \quad (55)$$

$$\boldsymbol{a}^{\omega}_{ex}=\boldsymbol{a}^{\omega}_{pair}=1.8\times10^{-4}\,h^1_{\omega}\,e\,\boldsymbol{I}. \quad (56)$$

Nevertheless, in order to complete the calculation of the deuteron AM, we have to estimate the contributions from nucleonic AM and PC ECs.

Because the deuteron is isosinglet, the nucleonic contribution to the deuteron AM, a_N, only arises from the isoscalar component, i.e.,

$$\boldsymbol{a}_N=\langle d|\sum_{i=1}^{2}(a^{(1)}_S+a^{(1)}_V\tau^z_i)\,\boldsymbol{\sigma}_i\,|d\rangle=2a^{(1)}_S\left(1-\frac{3}{2}P_D\right)\boldsymbol{I}, \quad (57)$$

where $a^{(1)}_{S,V}$ denote the isoscalar and isovector nucleonic AMs, and P_D is the deuteron D-state probability. Several theoretical estimates for the nucleonic AM exist [27,31–35], and here we use the result of Ref. [35] because it is the most recent one which includes the full DDH interaction at the nucleon level.

For the pion sector, Ref. [35] gave

$$a^{\pi(1)}_S=-\frac{g_A h^1_{\pi}}{12\sqrt{2}f_{\pi}m_{\pi}}\frac{\Lambda^2_{\chi}}{m^2_N}\left(1-\frac{6}{\pi}\frac{m_{\pi}}{m_N}\ln\frac{\Lambda_{\chi}}{m_{\pi}}\right),$$

where Λ_{χ} is the chiral symmetry breaking scale (the authors chose it to be $4\pi f_{\pi}$) [40]. When setting $\Lambda_{\chi}=m_N$, the leading term is equivalent to what has been used in Refs. [4,5,8] for the deuteron AM calculations, while the full result is the same as Ref. [7]. By including the heavy mesons, the numerical result is [35]

$$a^{(1)}_S=-0.274\,h^1_{\pi}-0.419\,h^1_{\rho}-0.129\,h^0_{\omega}, \quad (58)$$

and this leads to the nucleonic contribution

$$\boldsymbol{a}_N=(-0.250\,h^1_{\pi}-0.383\,h^1_{\rho}-0.118h^0_{\omega})\,e\,\boldsymbol{I}. \quad (59)$$

As for the PC ECs, we try to approximate them by the ECs originated from one-pion exchange diagrams (corresponding currents in the configuration space are given in the Appendix). These PC EC contributions of pair and mesonic types are

$$\boldsymbol{a}_{pair}^{\mathrm{PC}} = \frac{1}{3\sqrt{6}}\left(\frac{g_{\pi NN}}{2m_N}\right)^2 \int dr\ e^{-m_\pi r}(1+m_\pi r)\ \tilde{v}_{3p1}(r)[u(r) - \sqrt{2}w(r)]\ e\,\boldsymbol{I}, \tag{60}$$

$$\boldsymbol{a}_{mesonic}^{\mathrm{PC}} = -\frac{1}{3\sqrt{6}}\left(\frac{g_{\pi NN}}{2m_N}\right)^2 \int dr\ e^{-m_\pi r}\ \tilde{v}_{3p1}(r)\left[3u(r) - m_\pi\, r\left(u(r) + \frac{1}{\sqrt{2}}w(r)\right)\right] e\,\boldsymbol{I}. \tag{61}$$

Numerically we obtain

$$\boldsymbol{a}_{ex}^{\mathrm{PC}} = \boldsymbol{a}_{pair}^{\mathrm{PC}} + \boldsymbol{a}_{mesonic}^{\mathrm{PC}} = -(7.8\ \ h_\pi^1 + 0.4\ \ h_\rho^1 - 1.1 h_\omega^1) \times 10^{-4}\ e\,\boldsymbol{I}. \tag{62}$$

In the following section, we will discuss if this is a reasonable approximation and to what extent the current conservation is broken for not being fully consistent with Av_{18}.

Finally, the full deuteron AM could be expressed as

$$\boldsymbol{a}_d = \boldsymbol{a}_{spin} + \boldsymbol{a}_{conv} + \boldsymbol{a}_{ex}^{\mathrm{PNC}} + \boldsymbol{a}_N + \boldsymbol{a}_{ex}^{\mathrm{PC}} = (-0.756\ h_\pi^1 - 0.387\ h_\rho^1 + 0.010 h_\omega^1 + 0.007\ h_\rho^0 - 0.114\ h_\omega^0) e\boldsymbol{I}. \tag{63}$$

IV. DISCUSSIONS

The contributions of the heavy mesons to the nuclear part (spin, convection, PNC EC, and PC EC) are smaller than those of the pion by two or three orders of magnitude. This suppression can be understood by investigating the PNC pair terms of π, ρ, and ω mesons, which correspond to Eqs. (48), (50), and (51), respectively. Aside from the weak coupling constants, they differ only by the Yukawa function $f_\pi(r)$ or $f_{\rho,\omega}(r)$ in the integrand. At small r, the deuteron wave function is proportional to r (for simplicity, we neglect the D-state component) and increases very fast up to $r \sim 2$ fm where the maximum is reached. Afterwards, the wave function decreases very slowly and converges to zero. The factor r^2 multiplied by $u^2(r)$ gives rise to the suppression at short range. The top panel of Fig. 3 shows this behavior. One can expect that this r^4 behavior at short distances makes the contribution from $r \leqslant 1$ fm quite small. On the other hand, the Yukawa function which behaves like $1/r$ at small r is short-range peaked and its curvature in the intermediate range depends strongly on the mass of the meson. A comparison of $f_\pi(r)$ and $f_\rho(r)$ is given in the central panel of Fig. 3. The quantity $f_\rho(r)$ is not negligible for $r \leqslant 0.5$ fm but the remaining part of the integrand, approximately $r^2\ u^2(r)$, is small in this region. Consequently, the heavy-meson contribution will be suppressed substantially compared to that of the pions. The bottom panel of Fig. 3 shows the behavior of the total integrands in Eqs. (48) and (50). If we approximate the integrand as $r^3\exp(-m_X r)$, the maximum of the integrand reaches at $r \sim 3/m_X$. Even though the approximation is crude for the pion, this can provide a semiquantitative explanation for the suppression of the heavy-meson contributions. At the maximum, the integrand has a value proportional to $1/m_X^3$. If comparison is made for π and ρ mesons, the maximum value for the latter is smaller than for the former by a factor of 1/200, which can partially account for the suppression of two orders of magnitude in the heavy-meson contributions.

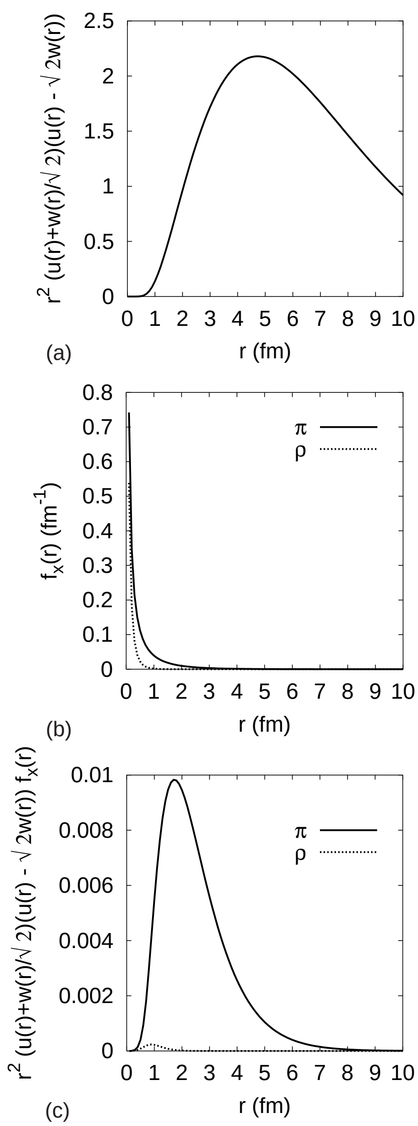

FIG. 3. The top panel shows the behavior of $r^2[u(r)+w(r)/\sqrt{2}] \times [u(r)-\sqrt{2}w(r)]$. The central panel compares the Yukawa functions of the pion (solid line) and ρ meson (dotted line). The bottom panel shows the behaviors of the integrands for the π- and ρ-pair terms in Eq. (48) and (50), respectively.

To a lesser extent, a similar argument can be applied to the qualitative understanding of the heavy-meson suppression of spin and convection terms. The right hand sides of Eqs. (37) and (38), which are the sources of parity admixed P states, also contain the Yukawa functions. When the equations are solved, i.e., integrated with respect to r, one can expect some amount of suppression for the heavy mesons as

TABLE I. Comparison of the heavy meson with the pion contribution to the deuteron anapole moment, term by term. The ratios are given in %.

Term	Spin	Conv.	PNC EC	PC EC	Nucleonic	Total
$(\rho+\omega)/\pi$	4.0	0.6	−2.0	−3.1	−26.2	−5.9

was discussed in the analysis of pair ECs. In Table I, we summarize the ratio of the heavy meson contribution to the pion one with the weak coupling constants given by DDH best values. The magnitudes of the heavy-meson terms are suppressed by two orders of magnitude compared to the pion terms in common. Thus, as far as the nuclear part is concerned, the π contribution is the most dominant one. If the contribution of the nuclear part can be disentangled from the total deuteron anapole moment, this information helps to determine the magnitude of h_π^1 with high accuracy. As a side remark, it is noticed that the combination of the isovector couplings, h_π^1, h_ρ^1, and h_ω^1, entering the spin and convection contributions is close to the one that determines low-energy PNC nucleon-nucleon scattering, roughly $3h_\pi^1+0.02\, h_\rho^1 -0.06\, h_\omega^1$ [37]. This lets us think that Danilov's approach [38], whose application for the deuteron anapole moment was proposed later on by Savage [6], should provide a reasonable estimate for this contribution.

Contrary to the nuclear part, the nucleonic one has sizable contributions from the heavy mesons. According to the nucleonic anapole moment estimated in Ref. [35], the ratio of the heavy-meson contribution to the pion one is about 26%. However, we should also note that these authors considered non-DDH type couplings such as non-Yukawa type πNN couplings and the inclusion of hyperons as well. So far, no detailed knowledge about these exotic couplings exists, and the theoretical uncertainty could be huge. Study of these terms and their implication for two-body nuclear contributions will be an interesting topic for further exploration.

Another issue that should be addressed is the gauge invariance of the results. We showed in Sec. II that the PNC ECs satisfy gauge invariance with the DDH potential. However, with the phenomenological strong interaction models like the one adopted in this work, gauge invariance may not be satisfied if the ECs are not consistent with the phenomenological potentials. The Av_{18} model has 18 types of operators, but since we took into account the dominant ECs only, it is natural to expect the breakdown of gauge invariance. Investigating the extent to which gauge invariance is broken provides an estimate of the error.

Most phenomenological potentials are very complicated and the analytic analysis of gauge invariance is a formidable task. However, as suggested in Refs. [39,8], one can estimate the amount of gauge-invariance breaking by comparing the results obtained with two different formulations of the anapole operator: Eq. (39), which we use for Sec. III, and an alternative one

$$\boldsymbol{a} = -\pi \int d\boldsymbol{x}\, x^2 \boldsymbol{j}(\boldsymbol{x}), \tag{64}$$

which can be obtained from Eq. (39) with the assumption

$$\nabla \cdot \boldsymbol{j}(\boldsymbol{x}) = 0. \tag{65}$$

In Ref. [8], it has been shown that the inclusion of the PC ECs related to the π meson removes almost all the inconsistency. In this work, we follow the same procedure.

The spin current obviously satisfies Eq. (65), so both anapole operators give the same result. For the convection, pair, and mesonic contributions, the alternative anapole operator, Eq. (64), gives

$$\boldsymbol{a}_{conv}^{CC} = \frac{\pi}{2\sqrt{6}\, m_N} \int dr\, r^2 \Bigg[u(r)\left(\tilde{v}'_{3p1}(r) + 2\frac{\tilde{v}_{3p1}(r)}{r}\right) + \frac{w(r)}{\sqrt{2}}\left(\tilde{v}'_{3p1}(r) - \frac{\tilde{v}_{3p1}(r)}{r}\right)\Bigg]\, e\,\boldsymbol{I}, \tag{66}$$

$$\boldsymbol{a}_{pair}^{\pi,CC} = -\frac{\pi\, g_{\pi NN}}{2\sqrt{2}\, m_N}\int dr\, r^2\, f_\pi(r)\left(u^2(r) - \frac{w^2(r)}{2}\right) h_\pi^1\, e\,\boldsymbol{I}, \tag{67}$$

$$\boldsymbol{a}_{mesonic}^{\pi,CC} = \frac{g_{\pi NN}}{24\sqrt{2}\, m_N\, m_\pi}\int dr\, e^{-m_\pi r}\Bigg[\left(u^2(r) - \frac{w^2(r)}{2}\right)(m_\pi r + 4) - \frac{1}{3}\left(u(r) + \frac{w(r)}{\sqrt{2}}\right)^2 m_\pi r(m_\pi r + 3)\Bigg] h_\pi^1\, e\,\boldsymbol{I}, \tag{68}$$

$$\boldsymbol{a}_{pair}^{\rho^0,CC} = -\frac{\pi\, g_{\rho NN}}{2\, m_N}\int dr\, r^2\, f_\rho(r)\left(u^2(r) - \frac{w^2(r)}{2}\right) h_\rho^1\, e\,\boldsymbol{I}, \tag{69}$$

$$\boldsymbol{a}_{pair}^{\omega,CC} = \frac{\pi\, g_{\omega NN}}{2\, m_N}\int dr\, r^2\, f_\omega(r)\left(u^2(r) - \frac{w^2(r)}{2}\right) h_\omega^1\, e\,\boldsymbol{I}, \tag{70}$$

$$\boldsymbol{a}_{pair}^{\mathrm{PC},CC} = \frac{1}{\sqrt{6}}\left(\frac{g_{\pi NN}}{2\, m_N}\right)^2 \int dr\, e^{-m_\pi r}(1 + m_\pi r)\tilde{v}_{3p1}(r)\left(u(r) - \frac{1}{\sqrt{8}}\, w(r)\right) e\,\boldsymbol{I}, \tag{71}$$

$$\boldsymbol{a}_{mesonic}^{\mathrm{PC},CC} = -\frac{1}{12\sqrt{6}}\left(\frac{g_{\pi NN}}{2\, m_N}\right)^2 \int dr\, e^{-m_\pi r}\, \tilde{v}_{3p1}(r)\Bigg[u(r)(18 + 2m_\pi r - m_\pi^2 r^2) - \frac{w(r)}{\sqrt{2}}\, m_\pi r\,(4 + m_\pi r)\Bigg]\, e\,\boldsymbol{I}, \tag{72}$$

where the superscript CC denotes the quantity calculated with Eq. (64). Numerical results are summarized in Table II.

The "Total" column in Table II shows that the two definitions of the anapole operator differ in results only by 3%, 6%, and 4% for the π, ρ, and ω contributions, respectively; this means that our results satisfy current conservation very

TABLE II. Coefficients of the weak coupling constants for given terms with different definitions of the anapole operator, Eq. (39) and (64).

	Eq. (39)			Eq. (64)		
	h_π^1	h_ρ^1	h_ω^1	h_π^1	h_ρ^1	h_ω^1
Conv.	0.03925	0.00027	−0.00076	0.05348	0.00024	−0.00066
PNC π pair	−0.02668			−0.07895		
PNC π mesonic	0.02830			0.04706		
PNC ρ pair		−0.00007			−0.00023	
PNC ω pair			0.00018			0.00061
PC π pair	0.01400	0.00013	−0.00038	0.06325	0.00062	−0.00177
PC π mesonic	−0.01478	−0.00017	0.00049	−0.04366	−0.00047	0.00133
Total w/o PC	0.04087	0.00020	−0.00058	0.02159	0.00001	−0.00005
Total	0.04010	0.00016	−0.00047	0.04117	0.00017	−0.00049

well. This is quite surprising because only the PC one-pion exchange current—not fully consistent with the adopted strong potential, Av_{18}—is included in our calculation. The reason can be found in that the AM is a r^2 weighted moment and that the deuteron wave function peaks around 2 fm with a long tail; therefore the long-range physics, which is dominated by the one-pion exchange included in the case of Av_{18}, becomes much more important. This can also explain partially why our observation about the role of ECs differs from the one found in Ref. [13] where undetermined ECs have large effects. Their observation was concerning a different operator ($E1$ transition) which has a relatively shorter range than the anapole operator considered in this work.

Furthermore, one can observe that for the contributions from exchange currents, Eq. (39) always gives smaller values than Eq. (64). This implies that the calculation using Eq. (39) suffers less from the incomplete knowledge or uncertainty of exchange currents which causes the breaking of current conservation. For example, if the PC pion ECs are left out in our calculation, by comparing the "Total" and "Total w/o PC" columns in Table II, the error is 2%, 25%, and 23% for π, ρ, and ω components, respectively, when Eq. (39) is used; however, the error becomes −46%, −94%, and −90%, for Eq. (64). The reason for the small contribution of ECs in the former case may be looked for in the proportionality of the dominant ECs to the position vector $\boldsymbol{x}$, which readily gives zero when inserted into the corresponding anapole operator, Eq. (39). Therefore the use of Eq. (39) is preferred at least for the deuteron case.

In conclusion, we have constructed the PNC ECs due to one π, ρ, and ω exchanges, and showed that they satisfy current conservation—so the consistency with the adopted DDH PNC potential is checked. An application was made to the calculation of the deuteron anapole moment. We observed that, for the nuclear part, the contribution of heavy mesons is suppressed by two orders of magnitude compared to the pion one, a result consistent with the similar work by Blunden [40]. Consistency with Av_{18} was also checked. We found that the approximation of using only the PC one-pion exchange current is reasonable—the breaking of current conservation only amounts to a few percent and this is supposed to be fixed by using the PC exchange current fully consistent with Av_{18}. Therefore, the contribution from the nuclear part to the deuteron anapole moment can be determined with an error less than 5%, while the major uncertainty should come from the nucleonic anapole moment instead.

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APPENDIX: PNC *NN* POTENTIAL AND EXCHANGE CURRENTS IN COORDINATE SPACE

The DDH potential in coordinate space could easily be obtained by applying the following transformation rules to Eqs. [25–27]:

$$\boldsymbol{u}_X \to \boldsymbol{u}_X(\boldsymbol{r}) = [\boldsymbol{p}, f_X(r)],$$

$$\boldsymbol{v}_X \to \boldsymbol{v}_X(\boldsymbol{r}) = \{\boldsymbol{p}, f_X(r)\},$$

where $\boldsymbol{r} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2$; $r = |\boldsymbol{r}|$; $\boldsymbol{p} \equiv (\boldsymbol{p}_1 - \boldsymbol{p}_2)/2 = -i\boldsymbol{\nabla}_r$; and the Yukawa functions $f_X(r)$ are defined as

$$f_X(r) = \frac{e^{-m_X r}}{4\pi r}.$$

For the PNC ECs, we list all the leading-order, $O(1/m_N)$, 3-currents, which are relevant for the AM calculation:

$$\boldsymbol{j}^{\pi}_{pair}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\pi NN}\, h_\pi^1}{2\sqrt{2}\, m_N} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \tau_1^z \tau_2^z)\, f_\pi(r) \times \sum_{i=1}^{2} \delta^{(3)}(\boldsymbol{x} - \boldsymbol{r}_i)\, \boldsymbol{\sigma}_i, \tag{A1}$$

$$\boldsymbol{j}^{\pi}_{mesonic}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\pi NN}\, h_\pi^1}{2\sqrt{2}\, m_N} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \tau_1^z \tau_2^z)(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2) \times [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla}_1 - \boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla}_2), f_\pi(r_{x1})\, f_\pi(r_{x2})], \tag{A2}$$

$$j^{\rho^\pm}_{pair+KR}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\rho NN}}{2\, m_N}\, f_\rho(r)\left(h^0_\rho - \frac{1}{2\sqrt{6}}\, h^2_\rho\right)\Big((\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \tau^z_1\tau^z_2)\,(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)[\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_1)-\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_2)] + (1+\chi_V)\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z\,(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\sum_i \delta^{(3)}(\boldsymbol{x}-\boldsymbol{r})_i\Big), \tag{A3}$$

$$\boldsymbol{j}^{\rho^0}_{pair}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\rho NN}}{2\, m_N}\, f_\rho(r)\,\tau^z_1\,\tau^z_2\left[\left(h^0_\rho + \frac{1}{2}\,h^1_\rho\,(\tau^z_1+\tau^z_2) + \frac{1}{\sqrt{6}}\,h^2_\rho\right)(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) + \frac{1}{2}\,h^1_\rho\,(\tau^z_1-\tau^z_2)\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\right]\Big((1+\tau^z_1)\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_1) - (1+\tau^z_2)\times\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_2)\Big), \tag{A4}$$

$$\boldsymbol{j}^{\rho^\pm}_{mesonic}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\rho NN}}{2\, m_N}\left(h^0_\rho - \frac{1}{2\sqrt{6}}\,h^2_\rho\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z\Big\{(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)\,\big(-i(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2), f_\rho(r_{x1})\, f_\rho(r_{x2})\} + (1+\chi_V)\,(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot[(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2), f_\rho(r_{x1})\, f_\rho(r_{x2})]\big) + 2\,\nabla^a_x\,(i\,\{\nabla^a_1\,\boldsymbol{\sigma}_2 - \nabla^a_2\,\boldsymbol{\sigma}_1 + \sigma^a_1\,\boldsymbol{\nabla}_2 - \sigma^a_2\,\boldsymbol{\nabla}_1, f_\rho(r_{x1})\, f_\rho(r_{x2})\} - (1+\chi_V)\,[(\sigma_1\times\nabla_1)^a\boldsymbol{\sigma}_2 - (\sigma_2\times\nabla_2)^a\,\boldsymbol{\sigma}_1 + \sigma^a_1\,\boldsymbol{\sigma}_2\times\boldsymbol{\nabla}_2 - \sigma^a_2\boldsymbol{\sigma}_1\times\boldsymbol{\nabla}_1, f_\rho(r_{x1}) f_\rho(r_{x2})]) + 4\, i\, m_N\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)[H, f_\rho(r_{x1})\, f_\rho(r_{x2})]\Big\}, \tag{A5}$$

$$\boldsymbol{j}^{\omega}_{pair}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\omega NN}}{2\, m_N}\, f_\omega(r)\left[\left(h^0_\omega + \frac{1}{2}\,h^1_\omega\,(\tau^z_1+\tau^z_2)\right)(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) + \frac{1}{2}\,h^1_\omega\,(\tau^z_1-\tau^z_2)\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\right][(1+\tau^z_1)\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_1) - (1+\tau^z_2)\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_2)], \tag{A6}$$

$$\boldsymbol{j}^{\rho\pi}_{mesonic}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{e\, g_{\rho NN} g_{\rho\pi\gamma}\, h^1_\pi}{\sqrt{2}\, m_\rho}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z(\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2)\times[f_\rho(r_{x1})\, f_\pi(r_{x2}) + f_\pi(r_{x1})\, f_\rho(r_{x2})], \tag{A7}$$

where $r_{xi}\equiv|\boldsymbol{x}-\boldsymbol{r}_i|$; $\boldsymbol{\nabla}_i$ and $\boldsymbol{\nabla}_x$ act on the source point $\boldsymbol{r}_i$ and the field point $\boldsymbol{x}$, respectively; and the superscript a is the index to be summed from 1 to 3. Note that we separate charged and neutral ρ mesons according to their isospin structure; and the last term of Eq. (A5) should be combined with a charge density corresponding to Eq. (24) in order to ensure the current conservation.

PC one-pion ECs of the pair and mesonic types that contribute to the AM are

$$\boldsymbol{j}^{\rm PC}_{pair}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = \left(\frac{g_{\pi NN}}{2m_N}\right)^2(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z[\boldsymbol{\sigma}_2\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_2)\boldsymbol{\sigma}_1\cdot\nabla + \boldsymbol{\sigma}_1\delta^{(3)}(\boldsymbol{x}-\boldsymbol{r}_1)\boldsymbol{\sigma}_2\cdot\nabla]f_\pi(r), \tag{A8}$$

$$\boldsymbol{j}^{\rm PC}_{mesonic}(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) = -\left(\frac{g_{\pi NN}}{2m_N}\right)^2(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z(\boldsymbol{\sigma}_1\cdot\nabla_1)(\boldsymbol{\sigma}_2\cdot\nabla_2)(\nabla_1-\nabla_2)f_\pi(r_{x1})f_\pi(r_{x2}). \tag{A9}$$

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