Valence quark distribution in $A=3$ nuclei

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We calculate the quark distribution function for ^{3}He/^{3}H in a relativistic quark model of nuclear structure which adequately reproduces the nucleon approximation, nuclear binding energies, and nuclear sizes for small nuclei. The results show a clear distortion from the quark distribution function for individual nucleons (EMC effect) arising dominantly from a combination of recoil and quark tunneling effects. Antisymmetrization (Pauli) effects are found to be small due to limited spatial overlaps. We compare our predictions with a published parametrization of the nuclear valence quark distributions and find significant agreement.

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I. INTRODUCTION

The validity of the nucleon approximation—the approximation that atomic nuclei are accurately viewed as bound agglomerations of protons and neutrons—was recognized as essential to nuclear physics from its very beginning [1]. The development of the meson-exchange picture of hadronic interactions did not impact this question due to difficulties with field theory self-consistencies (e.g., physical interpretation of off-shell quantities) although the distortion in the nuclear medium of the meson (or at least pion) cloud surrounding a nucleon was also recognized [2]. The validity of the approximation remained assumed after the advent of quantum chromodynamics (QCD) allowed dynamical examination of the quark and gluon substructure of nucleons, despite the natural question as to why these strongly interacting, composite objects do not distort each other's internal structure beyond recognition when brought into close proximity [3,4], as in a nucleus.

It was therefore disturbing when the EMC Collaboration first showed that the structure function of a nucleus containing A nucleons, as measured in deep inelastic lepton scattering is not a simple multiple (A) of the corresponding structure function of a free nucleon [5], although it was clear that recoil effects (as in a Fermi gas model, see Ref. [6]) would produce nonzero response beyond the kinematic limit of Bjorken-x ($x_{Bj}=1$) for a nucleon with concomitant implications for the $x_{Bj}\leqslant 1$ region. The experimental result has become steadily clearer with time [7]. A number of heuristic approaches (nucleon swelling, binding energy rescaling, and meson convolution models) produce reasonable agreement with some of the data [8] without providing any deep, dynamical understanding. Indeed, Miller [9] has recently remarked that the situation remains unresolved in terms of a satisfactory understanding of the phenomenon: Nothing short of changing the structure of nucleons in the nuclear medium seems to be called for. Such a change occurs completely naturally in the quark nuclear model used here [10], and may be compared with the work of others [11,12], who have considered the contributions to the EMC effect from quark Pauli effects (the minimal, quantum mechanically required change in the structure of nucleons in the nuclear medium).

We have approached the problem from a different direction, building on a description of nuclei directly in terms of relativistic, four-component Dirac quark wave functions [10]. The physics of that model provides an immediate, qualitatively clear, justification of the nucleon approximation. The tendency for quark wave functions to reduce their kinetic energy by spreading out between nucleons in close proximity (delocalization) is countered by a reduction, engendered by this delocalization, in the amount of the attractive interaction energy due to the color magnetic hyperfine interaction between the correlated quarks in a nucleon [13]. Detailed quantitative calculation confirms this insight [10,14], and shows that the probability of finding a quark in a location other than what would be expected if its wave function were unaltered from that of an isolated nucleon is limited to the level of a few percent.

The results of the variational calculations in the model also reproduce qualitatively (and systematically) correct binding energies in the $A=3$ and $A=4$ systems [15], as well as overall matter distributions consistent with those experimentally inferred from low energy electron scattering and other measures [16]. To be explicit, we obtain a binding energy of (20 ± 4)MeV and rms matter (not charge) size of 1.34 fm for ^{4}He [10] (cf. 1.42 fm in an *ab initio* calculation using a realistic nucleon-nucleon potential [17]) and a binding energy of (4 ± 2)MeV for ^{3}He/^{3}H [14] with a slightly smaller rms size. Although not comparable to shell model accuracies, and still not as good as the results of the *ab initio* calculations using pairwise nucleon interaction potentials [18] noted above, these results are obtained with no free parameters beyond those in the quark model used to fit the nucleon and Δ-baryon masses, and represent a fraction of a

percent accuracy in the energy calculated per quark.

All these features depend on the model introduction of a geometrically complex, mean field describing the overall confining potential encountered by a quark in a nucleus. The potential is composed of individual, linearly confining (nucleonlike) potential wells, set out in a regular array (equilateral triangle for $A=3$ and tetrahedron for $A=4$) in a body-fixed frame, separated by a variationally determined distance scale d (common side length) and truncated on the midplanes between each pair of wells. The structure of this "egg-crate" potential, if extended to larger A, may be relevant to the observations of Cook and others [19].

The quark delocalization is described by the variationally determined parameter ϵ, which is, roughly speaking, the probability amplitude for a quark to appear in the interior of a nucleonlike well other than the one in which it originates. We find a value of $\epsilon=0.136$ for ^{4}He and $\epsilon=0.104$ for ^{3}He/^{3}H. The corresponding values of the distance scale are $d=1.75$ fm for ^{4}He and $d=1.80$ fm for ^{3}He/^{3}H.

Both these parameters are jointly determined variationally by minimizing the overall (one-body kinetic and potential plus two-body color magnetic interaction) energy of the quark nuclear configuration in each nucleus. Thus, this approach allows for no new free parameters and predicts the quark nuclear structure within the assumptions of the model and approximations made to carry out the calculations.

In the following section, we describe the structure of the wave functions for the model as applied to ^{3}He/^{3}H. In Sec. III we describe the method used for calculating the quark distribution functions from these wave functions, including the effects of Pauli antisymmetrization. We present our results in Sec.IV and discuss these results in Sec. V, comparing them with a phenomenological extraction of the valence quark distributions in nuclei from Ref. [20], and concluding with a review of additional elements not yet included in the model.

II. WAVE FUNCTIONS

The variational wave functions of the model are built on full, four-component, Dirac solutions of the quark wave functions in an isolated, Lorentz scalar, confining potential well [10]. The Lorentz character of confinement remains a subject of dispute [21], but there is little difference between the spatial wave functions found for linear confining potentials of either scalar or vector character. If the absence of spin-orbit effects in the hadron spectrum [22] does reflect the character of the confining potential [23], then an equal admixture is to be expected, further weakening the distinction with the scalar presumption. The potential used is

$$V_c(r) = k(r - r_0), \tag{1}$$

where $k=0.9$ GeV/fm is the conventional slope and $r_0=0.57$ fm has been chosen to remain within the error band of phenomenological fits to spectral data [24]. The negative region near the origin is expected to account, albeit crudely, for the (undoubtedly vector) color Coulomb potential at short distances. More modern analyses that explicitly separate out the short-distance color Coulomb component do not markedly affect the value of the slope used here.

The basic wave function $\psi_0(\vec{r})$ is found by solving the Dirac equation for the potential in Eq. (1) for a massless (negligible mass) quark. The quarks are thus "current" (not constituent) u and d quarks so that there are no complications in determining the photon coupling to them needed for our deep inelastic scattering (DIS) calculations below.

The variational spatial wave function $\psi_j(\vec{r},\epsilon)$ for each quark in the nucleus is composed as

$$\psi_j(\vec{r},\epsilon) = \frac{1}{N(\epsilon)}[\psi_0(\vec{r}-\vec{R}_j) + \epsilon\sum_{i\neq j}\psi_0(\vec{r}-\vec{R}_i)], \tag{2}$$

where the color and flavor indices have been suppressed and $N(\epsilon)$ is a normalization factor. The quantities $\vec{R}_i$ describe the three origins of potentials, each of the form $V_c(r)$, but truncated on the midplanes between each (i,j) pair, corresponding to the average location of a nucleon in a body-fixed frame for ^{3}He/^{3}H. By symmetry, these average locations define an equilateral triangle for the minimum energy configuration of the nucleus as a whole. The scale of this triangle is given by the variational parameter d, which has a value of $d=1.80$ fm, while the other variational parameter attains the value $\epsilon=0.104$ at the variational minimum.

The three quarks with spatial wave functions $\psi_j(\vec{r},\epsilon)$ are antisymmetrized in color and appropriately coupled in spin flavor to nucleon quantum numbers. These are, in turn, antisymmetrized over the three well locations. Beyond this, the final wave function includes all antisymmetrizations (pairwise, triples, etc.) between all nine quarks. (Note that these nucleons are *not* sharply localized; this is true only for the origins of the potentials, so that the geometrically complex, confining, mean-field potential is well defined.)

In order to perform the calculations of quark distributions that follow, the wave function is written in Foch space as

$$|A\rangle = \mathcal{N}\int d^3r_{CM}\int d\Omega\chi^{abc}N_a^\dagger(\vec{R}_1)N_b^\dagger(\vec{R}_2)N_c^\dagger(\vec{R}_3)|EB\{R_i\}\rangle, \tag{3}$$

with $\mathcal{N}$ a normalization factor and χ^{abc} the nuclear spin-isospin wave function. The effective creation operator for a nucleon of spin-isospin a is given by

$$N_a^\dagger(\vec{R}_i) = T_a^{\alpha\beta\gamma}b_{0\alpha}^\dagger(\vec{R}_i)b_{0\beta}^\dagger(\vec{R}_i)b_{0\gamma}^\dagger(\vec{R}_i), \tag{4}$$

where $b_{0\alpha}^\dagger(\vec{R}_i)$ creates a quark of spin-isospin-color α with wave function $\psi_0(\vec{r}-\vec{R}_i)$ centered on the ith well, and $T_a^{\alpha\beta\gamma}$ is the quark spin-isospin-color wave function for a nucleon of spin-isospin a.

In $|A\rangle$, the integration over $\vec{r}_{CM}=\frac{1}{3}\Sigma\vec{R}_i$, the center of the triangle, projects onto a state of zero momentum, while the integration over the three Euler angles denoted by $d\Omega$ projects onto zero orbital angular momentum. Finally, for nonzero values of ϵ, the individual quark creation operators are replaced by

$$b^\dagger_\alpha(\vec{R}_i,\epsilon) = b^\dagger_{0\alpha}(\vec{R}_i) + \epsilon\sum_{j\neq i} b^\dagger_{0\alpha}(\vec{R}_j). \quad (5)$$

It is useful to note that the wave function contains no D-wave components, and that the wave function is completely antisymmetrized at both the quark and nucleon levels. As a result, quarks within a given quasinucleon are indistinguishable from one another, and, as a result of the summation over the well locations and integrations over nuclear orientations, the individual nucleons are also indistinguishable from one another.

III. CALCULATION

The calculation of valence quark distributions here parallels previous work on quark distributions in the nucleon [25] and for the color-hyperfine-induced flavor distortions between u and d valence distributions [26]. The valence distributions are calculated at a low momentum scale $\leqslant 1$ GeV^2, but this scale is still sufficiently large so that it is reasonable to conceive of a nucleon/nucleus as a simple object that may be described by a quark model. At this scale, the twist-two contribution to the structure functions of the target are projected out by taking the Bjorken limit on the momentum transfer.

For unpolarized scattering, the relevant matrix elements for a quark distributions in nucleus A are given by [27]

$$q_\alpha(x) = \frac{1}{4\pi}\int d\xi^- e^{iq^+\xi^-}\langle A|\bar{\psi}_\alpha(\xi^-)\gamma^+\psi_\alpha(0)|A\rangle|_{LC},$$

$$\bar{q}_\alpha(x) = -\frac{1}{4\pi}\int d\xi^- e^{iq^+\xi^-}\langle A|\bar{\psi}_\alpha(0)\gamma^+\psi_\alpha(\xi^-)|A\rangle|_{LC}, \quad (6)$$

where $q^+ = -Mx/\sqrt{2}$ (with $x\equiv x_{Bj}$ is the Bjorken scaling variable), $\psi_\alpha(\bar{\psi}_\alpha)$ are field operators for quarks of flavor α, γ^+ is the light cone projected (0+3 component) Dirac gamma matrix, and the subscript LC indicates a light cone condition on ξ, namely, $\xi^+ = \vec{\xi}_\perp = 0$.

The approach we adopt consists of a straightforward evaluation of the matrix elements of Eq. (6) in a Peierls-Yoccoz projected momentum eigenstate, assuming that the time dependence of the field operator is dominated by the lowest eigenvalue of the Dirac equation used to obtain the wave functions of the struck quark. The details of this procedure are described in Ref. [25], where the valence quark distributions for flavor i in a free nucleon are shown to be given by

$$xq^i_V(x) = \frac{MxN_i}{\pi V}\left\{\left[\int_{|k_-|}^\infty kdkG(k)\left(t_0^2(k)+t_1^2(k) + 2\frac{k_-}{k}t_0(k)t_1(k)\right)\right] + [k_- \to k_+]\right\}, \quad (7)$$

where

$$G(k) = \int \frac{d^3r}{4\pi}e^{i\vec{k}\cdot\vec{r}}[\Delta(r)]^2 EB(r),$$

$$V = \int \frac{d^3r}{4\pi}[\Delta(r)]^3 EB(r),$$

$$t_0(k) = \int r^2 dr j_0(kr)u(r),$$

$$t_1(k) = \int r^2 dr j_1(kr)v(r),$$

$$\Delta(r) = \int d^3z\psi_0^\dagger(\vec{z}-\vec{r})\psi_0(\vec{z}), \quad (8)$$

with N_i the number of valence quarks of flavor i in the nucleon, $\psi_0(\vec{r})$ the ground state valence quark wave function of Eq. (2), with upper and lower components $u(r)$ and $i\vec{\sigma}\cdot\vec{r}v(r)/r$ (both times a fixed spinor), respectively, and $k_\pm = \omega \pm Mx$, with ω the ground state struck quark energy eigenvalue. Here, and in the nucleus, ω is taken to be the one-body quark energy.

Also, $EB(r)\equiv\langle EB,\vec{R}_{CM}=\vec{r}|EB,\vec{R}_{CM}=\vec{0}\rangle$ is the overlap function for two "empty bags" (potential wells without the quarks in them) separated by a distance r, which accounts for the dynamics of the confining degrees of freedom. In this paper, we assume that the function $EB(r)$ is a constant for both the nucleon and the nucleus [28]. We will discuss below some effects of this assumption.

Note that the function $\Delta(r)$ measures the overlap of two (bra and ket) quark wave functions centered in wells separated by a distance r, and that the recoil function $G(k)$ measures the probability of finding all the spectator degrees of freedom of the unprojected state carrying a net momentum opposite to that of the struck quark. It is also useful to note that $G(k)$ is the Fourier transform, over the separation between the centers of the struck quark wave functions, of the spatial overlap of the spectator degrees of freedom in the unprojected state.

Remarkably, generalizing this expression to nuclei only requires modification of $G(k)$ and the normalization volume V. To see this, recall, as we have already noted, that in the momentum and angular momentum projected wave functions of the nucleus, the individual nucleons are indistinguishable, so that we may always consider the struck quark in the bra and the ket to originate in the *same* well. Consequently, after a change of integration variables from the center of mass position to the separation of the centers of the struck wells, all of the complexity of the nuclear wave function, including exchange effects and tunneling, can be incorporated into a redefinition of the recoil function and of the requisite normalization volume. Explicitly,

$$G(k,\epsilon) = \int d\Omega'\int d\Omega\int\frac{d^3r}{4\pi}e^{i\vec{k}\cdot\vec{r}}\sum_{m=0}^{18}\left(\sum_{\{n_{mjk}\}}PF(\{n_{mjk}\})\epsilon^m \times\prod_{j,k=1..3}[\Delta(\vec{R}_j-\vec{R}'_k)]^{n_{mjk}}EB(\{\vec{R}_\alpha,\vec{R}'_\alpha\},\Omega,\Omega')\right),$$

$$V(\epsilon) = \int d\Omega' \int d\Omega \int \frac{d^3r}{4\pi}\Delta(r) \sum_{m=0}^{18} \left(\sum_{\{n_{mjk}\}} PF(\{n_{mjk}\})\epsilon^m \times \prod_{j,k=1..3} [\Delta(\vec{R}_j - \vec{R}'_k)]^{n_{mjk}} EB(\{\vec{R}_\alpha, \vec{R}'_\alpha\}, \Omega, \Omega') \right). \quad (9)$$

In this expression, the integer m indicates the total number of quarks in the bra and ket that have tunneled. Since there are many possible configurations with the same m, it is necessary to characterize each configuration by a set of nine integers n_{mjk} which indicates the number of quarks from well j in the bra which are contracted with quarks from well k in the ket. The factor $PF(\{n_{mjk}\})$ is the permutation-flavor-color-spin (PerFlaCS) factor that goes with the exchange/tunneling term associated with $\{n_{mjk}\}$. Finally, the empty-bag overlaps will, in general, depend on the locations of the well centers in the bra ($\{\vec{R}_\alpha\}$) and ket ($\{\vec{R}'_\alpha\}$) and on their relative orientations.

The calculation is simplified by the existence of two small parameters, $\epsilon \approx \Delta(d) \approx 0.1$, which allow for a consistent truncation of the sums appearing in $G(k,\epsilon)$. Since at least two quarks must be exchanged between different wells, and each such exchange requires an overlap between quarks in different wells, we expect Pauli effects are of the order of $\Delta^2(d) \approx 0.01$ or smaller. Further suppression of exchange effects comes from the PerFlaCS factors, which are smaller for the exchange terms than for the direct term. Similarly, if a single quark tunnels to a different well, it must overlap with a quark from the original well, so tunneling effects are of the order of $\epsilon\Delta(d) \approx 0.01$ or smaller.

In contrast to the exchange effects, tunneling is not suppressed by a PerFlaCS factor, but rather tends to be enhanced due to combinatorics. Our expectation, then, is that quark tunneling will play a larger role than exchange effects in modifying the shape of the valence distribution. In the following, we keep only the leading contributions from the latter.

The dominant effect, analogous to Fermi motion, comes from the modification of $G(k)$. Since $G(k)$ measures the probability that the spectator quarks in the unprojected state have a total momentum equal and opposite to that of the struck quark, the naive expectation is that systems with more spectators will tend to have larger contributions from large momentum states, resulting in an enhancement of the valence distribution at large x. In the absence of the rotational projection, one expects that the overlap function $G(k)$ falls off roughly as the Fourier transform of $\Delta^{n_s}(r)$ where n_s is the number of spectator quarks. In the limit of a large number of spectators, $\Delta^{n_s}(r)$ is very sharply peaked, so that $G(k)$ becomes essentially constant and, as expected for a large system, the momentum projection is replaced by an incoherent average over center of mass positions. When additional collective effects, such as rotations, are included, the correlations between the motions of quarks in different wells is weakened, yielding spectator overlap functions that approach the limiting case more slowly, as the additional degrees of freedom allow the (bra and ket) spectator wells to be closer to each other than the struck quark wells are in some orientations of the system.

The integrations over orientations were calculated using Gauss-Laguerre quadrature over the five independent orientation (Euler) angles defined by the two planes of the equilateral triangles for the bra and ket state nuclei, and over the separation between the $\vec{R}_j$, i.e., locations, of the origins of the confining potential wells for the struck quarks. Exchange terms are calculated to all orders in $\Delta(d)$ and tunneling terms are included to the order of $\epsilon\Delta(d)$.

We note in passing that the ^{3}He/^{3}H calculation carried out here has been additionally checked by restricting the calculation to the case of a quasideuteron (two wells with their nucleons coupled to $I=0, J=1$, with values for ϵ and d similar to those used for ^{3}He/^{3}H, even though these do not correspond to realistic values for the actual deuteron). The results of our earlier calculation [29] of this case, which was carried out with a completely independent (and differently structured) code for the integrations, are accurately reproduced.

IV. RESULTS

We show our d-quark valence distributions in Fig. 1 for $d=1.8$ fm and three values of ϵ, with and without the Pauli antisymmetrization corrections. As has been noted above, the Pauli effects are negligible for these distributions. This conclusion differs from that derived in Ref. [11], but is in agreement with Ref. [12]. In both instances, this is due to the smaller overlap we find between quarks from different quasinucleons than is the case for the nuclear wave functions used in Ref. [11], which allow nucleons to come into closer proximity to one another than is the case for more conventional nuclear wave functions [12].

In our earlier quasideuteron calculations [29], we also obtained results that showed a very small effect from Pauli statistics—the results for $I=0, J=1$ and $I=1, J=0$ states were very similar to each other. Here we understand the result as being due to the fact that, in the absence of tunneling, at least two additional powers of the overlap, $\Delta(d)$, are required for every pairwise antisymmetrization of a ket wave function relative to a bra wave function and vice versa. Since our calculations find that $\Delta(d) \sim 0.1$ at best, this leads to, at a minimum, a two order of magnitude suppression of quark statistics effects.

Furthermore, there is an additional suppression due to the combination of spin, color, and isospin factors associated with the Pauli exchanges. Since the spin, color, and flavor factors do not change for the tunneling terms, and since it is possible for a quark to tunnel into the same well as the quark with which it is being exchanged, the tunneling and exchange terms may partially compensate for each other. Thus, since there are simply more possible tunnelings, we expect, and find, very small effects from statistics alone, but significant effects from tunneling.

In Fig. 2, the isospin averaged ratio of the valence quark distribution in ^{3}He to that in a free nucleon is shown for $d=1.8$ fm and several values of ϵ. For comparison, we also show a parametrization of this ratio [20] drawn from data.

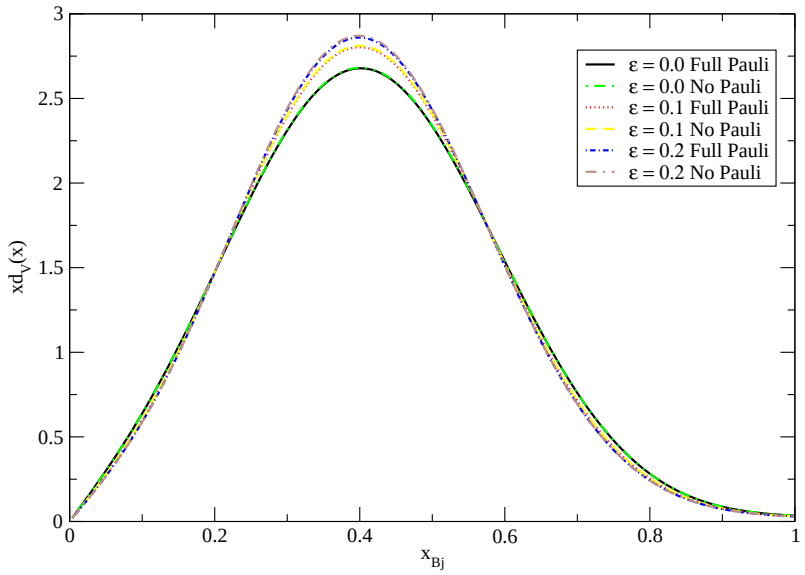

FIG. 1. (Color online) A=3 d-quark distributions with and without Pauli terms for ϵ =0.0,0.1, and 0.2 and d=1.8 fm. See text for an explanation of why the absence of discernable differences due to the inclusion of the Pauli antisymmetrization is to be expected.

Even in the absence of tunneling, we obtain a qualitatively correct EMC effect due to the modification of $G(k)$ caused by the larger number of spectators available to share momentum with. Such an effect is apparent in even semirealistic nuclear models, such as a Fermi gas picture of a nucleus [6]. The large x behavior has been noted before, and, as has been argued by West [30], the normalization constraint on the valence distribution then also requires the rise at x near zero. Since we have already shown that Pauli effects are negligible, the ϵ=0 distribution may be interpreted as being analogous to the effect of Fermi motion within the nucleus.

We note also that the ϵ=0 term significantly overpredicts the size of the effect, but that the ratio softens considerably when the tunneling corrections are included. Physically, this is because the quark tunneling terms weaken the correlation between the motions of the quarks in the individual wells, enhancing the probability of finding quarks with low momenta. We find this result to be extremely encouraging, as it indicates that quark tunneling, which plays a critical role in producing the correct binding in this model, is equally important to the EMC ratio.

Even with the tunneling terms, however, the calculated ratio still rises too rapidly at large x. We attribute this to the strong correlation between the well centers and consequently the quarks within the wells. We have investigated the effect on the nuclear wave function of including additional, collective oscillations/excitations of the well centers; in the case of the deuteron with no tunneling, it can be shown analytically that these collective motions tend to decrease the valence ratio at large x until ultimately the free-nucleon response is restored in the limit of uncorrelated wells. We expect a similar softening in the ^{3}He ratio when collective effects are included, which will improve the agreement with data in both the high and low x regions.

In Fig. 3, the dependence of the ratio of valence distributions is shown as a function of the separation of the wells for fixed ϵ. In this case, the ratio is relatively insensitive to the distance between the wells. For large separations, we see an

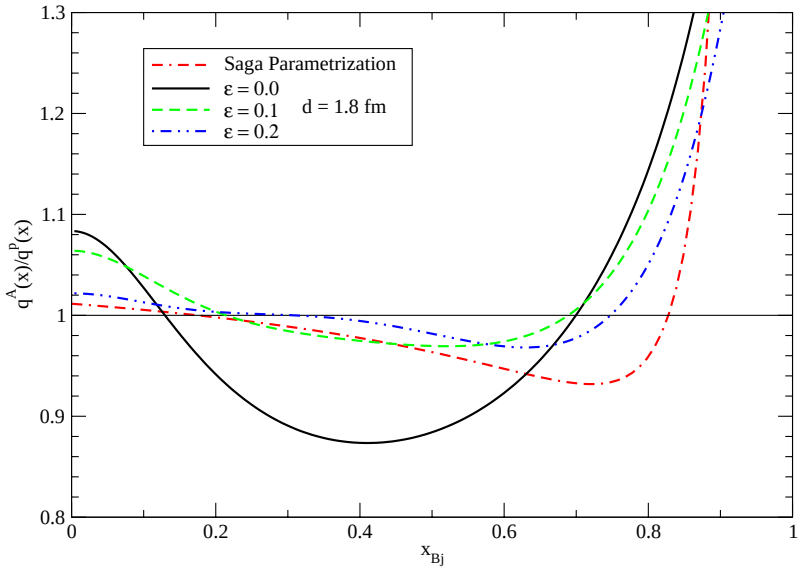

FIG. 2. (Color online) The ^{3}He/^{3}H EMC ratio for valence quarks for three values of ϵ at fixed d=1.8 fm. The parametrization of the data in Ref. [20] is also plotted for comparison.

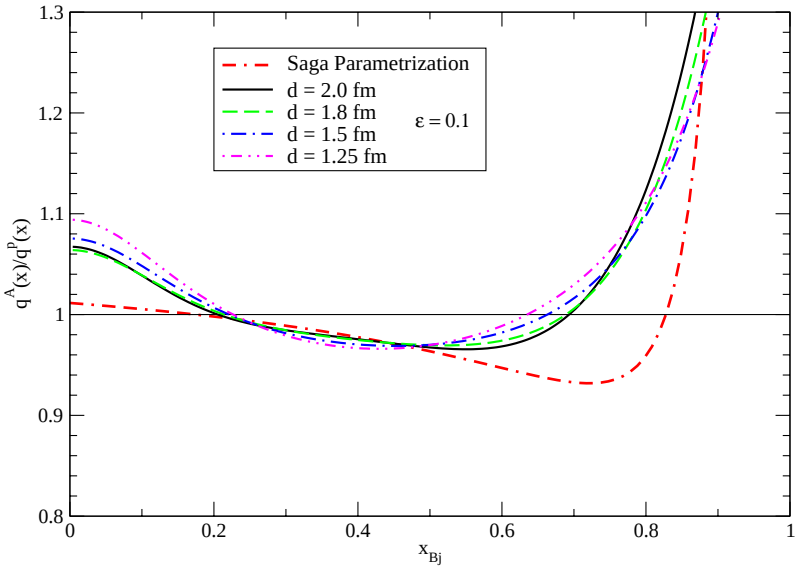

FIG. 3. (Color online) The ^{3}He/nucleon valence ratio for five values of d at fixed ϵ=0.1. The parametrization of the data in Ref. [20] is also plotted for comparison.

increase in the enhancement at large x. As the separation between the wells increases, the overlap of the bra and ket decreases more rapidly as their relative orientations change, which in turn decreases the coherence of angular projection. Since, as we have already noted, the coherent averaging over relative orientations softens the valence distribution, the valence distributions harden as the size of the system increases. For smaller d values, we also see an enhancement of the valence distribution at small x and a corresponding decrease at larger x.

Since the values of d and ϵ are strongly correlated in the quark nuclear model [14,10], the valence distribution ratio should not be viewed as a function of either variable alone. In Fig. 4, we show the valence distribution ratio for correlated values of d and ϵ chosen to lie approximately on the line of minimum energy in a plot of energy versus d and ϵ obtained from the model. This plot gives a better description of how the ratio is expected to change as the separation between wells is varied, and, due to the slow motion of the well centers compared to quarks, it is also relevant for the consideration of possible collective effects through a Born-Oppenheimer approximation.

V. DISCUSSION

The qualitative agreement of these calculations with the expected ratio of valence distributions demonstrates that it is possible to formulate a quark-based description of nuclei which correctly reproduces both their observed low energy properties (binding energy, rms size) and, *without introducing any additional parameters*, also produces a shift in quark distributions comparable, both systematically and in magnitude, to the EMC data. Compellingly, the delocalization of quarks produced by tunneling, which plays a crucial role in

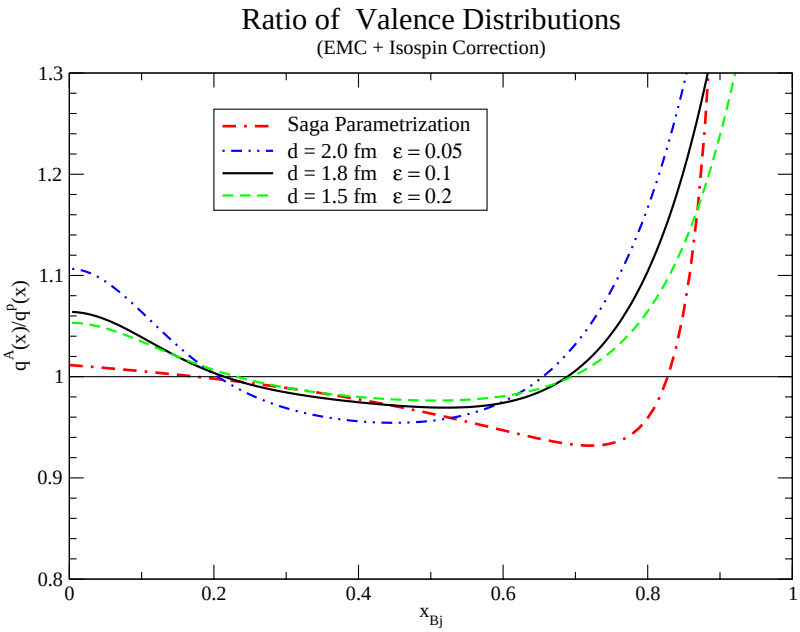

FIG. 4. (Color online) The ^{3}He/nucleon valence ratio for correlated values of d and ϵ chosen to minimize the energy of the nucleus. The parametrization of the data in Ref. [20] is again plotted for comparison.

generating phenomenologically suitable binding energies, is also critical for producing the right magnitude of changes in the valence distributions.

It should be noted that it is precisely this delocalization of quarks which the model employs to provide for the physics conventionally described by meson exchanges between nucleons, and that it does so without introducing any additional antiquark amplitude. The latter, predicted in pion excess models [31] that have attempted to account for the EMC effect based on the meson-exchange picture of nuclear binding, is inconsistent with experimental measurements, using Drell-Yan techniques [32], of the antiquark amplitude in nuclei. We are, therefore, encouraged in the belief that quark tunneling plays an essential role in the modification of the structure of the nucleon within a nucleus and the concomitant nuclear binding.

Despite the qualitative success of the model as described here, there are a number of avenues for quantitative improvements and extensions that should be pursued.

Collective degrees of freedom. In the current incarnation of the model, the positions of the potential wells are dictated by a rigid geometrical structure which may be rotated and translated, but neither the size nor the shape of this structure varies. This strong correlation between the well positions, and consequently between the quark wave functions defined with respect to them, leads to strong enhancement of the valence distribution at high x, overpredicting the data in that region. Allowing the centers of the wells to oscillate around their equilibrium positions decreases this correlation, which leads to softer distributions more consistent with the data and to an overall picture of the nucleus that is in better accord with standard nuclear physics. (In the case of the quasideuteron, where the calculation is simpler, we have shown analytically that allowing the wells to move relative to one another softens the quark distributions, ultimately recovering the free nucleon distribution in the free-particle limit.)

Convergence of the $\epsilon-\Delta$ expansion. In the present calculation, we have included only the leading contributions in ϵ. Since $\epsilon \approx 0.1$, and since most tunnelings produce additional off-diagonal overlap factors [$\Delta(d)$'s], our expectation is that the calculation converges rapidly. In the case of the quasideuteron calculations, where it is possible to calculate all possible tunnelings, we have explicitly verified that this is a good approximation. Due to increasing combinatoric factors, the validity of this approximation is less certain here, and the question of higher order corrections should be explored.

Self-consistency of the confining potential. Here, the confining potential has been fixed by fiat, rather than determined by a self-consistent generation due to the quark density distribution. We have in mind an analog of the view that confinement is related to the formation of a gluonic condensate in the QCD vacuum, whereas the perturbative vacuum is (at least partially) restored in the presence of a non-negligible quark matter density. In the tunneling regions between quasinucleons, the density is somewhat higher than that near the (fuzzy) matter surface of an isolated nucleon, which will further reduce resistance to tunneling. Still not investigated is the question of whether iteration along these lines leads to convergence or to complete breakdown of the barrier. If the latter were to occur, the justification found in the model for the nucleon approximation would disappear.

A dynamically generated potential also allows relaxation of the assumption that the function $EB(r)$ is a constant. This will in turn produce softer free-nucleon quark distributions that would again improve agreement with those observed experimentally. Additionally, such a model allows for calculation of changes in the parton distribution due to changes in the confining degrees of freedom.

Contributions to the binding from long range pion exchange between quarks. In a nonrelativistic version of this quark nuclear model, the introduction of pion exchange between pairs of quarks beyond a minimum separation (short-distance cutoff) has only small effects on compact objects and is only significant for the delicate binding of the deuteron [15]. Nonetheless, we expect inclusion of this physics to have some effect on the precise values of d and ϵ.

Large A nuclei. As the number of spectator degrees of freedom in the system grows, the number of wave function overlap factors, $\Delta(r)$, grows, so that the total overlap of the system becomes more sharply peaked in coordinate space. Eventually, this trend will "lock down" the center of mass and rotational degrees of freedom due to increasingly stringent alignment requirements on the bra and ket states. A comparison of locked bra and ket configurations with the calculated result would be of considerable interest. Such locking would considerably simplify calculations for large A nuclei and invite consideration of the possibility of effects analogous to those in powder diffraction. X-ray experiments on solids being observable in DIS.

While these items may well be significant amendments to the model and the calculational result, they appear to us to be both of an offsetting character among themselves, and qualitatively similar to the effects already examined. We are therefore strongly encouraged by the qualitative similarity to the observed EMC effect, in size and shape, already evidenced in our model calculation.

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