

Description of single and double analog states in the $f_{7/2}$ shell: The Ti isotopesA. A. Raduta,¹ L. Zamick,² E. Moya de Guerra,³ Amand Faessler,⁴ and P. Sarriguren³¹*Institute of Physics and Nuclear Engineering, Bucharest, P.O. Box MG6, Romania*
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The excitation energies of single analog states in even-odd Ti isotopes and double analog states in even-even Ti isotopes are microscopically described in a single j -shell formalism. A projection procedure for generalized BCS states has been used. As an alternative description a particle-core formalism is proposed. The latter picture provides a two-parameter expression for excitation energies, which describes fairly well the data in four odd and three even isotopes of Ti.

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I. INTRODUCTION

In a series of papers [1–5], Zamick and his collaborators investigated the spectra of even-even and even-odd nuclei in the pf region. The authors used the shell model calculations in a single open shell for both protons and neutrons. They remarked that for some even-even nuclei, the $J^\pi=0^+$ excited states were at twice the energies of $J=j$ excited states in the neighboring even-odd nuclei, in a single j -shell calculation for any isospin conserving interaction. Examples of this type are the pairs (^{44}Ti , ^{43}Sc), (^{44}Ti , ^{43}Ti), (^{48}Ti , ^{49}Ti), (^{48}Ti , ^{47}Sc). In order that this relationship holds it is necessary that the diagonalization spaces for the odd and even systems are equal to each other. For the nuclei mentioned above the proof was analytically given in Ref. [2]. The states mentioned above for the even-odd and even-even isotopes can be viewed as single analog (SA) and double analog (DA) states, respectively.

For the combination (^{46}Ti , ^{45}Sc) and the cross conjugate pair (^{50}Cr , ^{51}Cr) one almost gets a 2 to 1 relation, which becomes precisely 2 to 1 if one does not allow admixtures of seniority $\nu=4$ states in the ground states of the even-even nuclei. In Ref. [4] the excitation energies of higher isospin states were parametrized by linear relations

$$E_{T+1}^* = b(T+X) \text{ for } J=j, \text{ SA state,}$$

$$E_{T+2}^* = 2b\left(T+X+\frac{1}{2}\right) \text{ for } J=0, \text{ DA state.} \quad (1.1)$$

These are equivalent to a quadratic expression for the binding energies $BE = -2bT(T+Y)$, where $Y=2X-1$. For Wigner SU(4) model Y is equal to 4 and $X=2.5$.

In the present paper we revisit the above problem for the isotopes of Ti. We suppose that the states of interest are mainly determined by the valence nucleons in the $f_{7/2}$ shell, which interact among themselves through an isospin invariant pairing force.

The SA states are populated in a (p, n) reaction process [6,7]. Thus, the ground state ($T, T_3=T$) of the target nucleus (N, Z) (often called mother nucleus) is transformed by the

isospin raising operator¹ T_+ to the state $(T, T-1)$ describing the daughter nucleus $(N-1, Z+1)$. This state is certainly an excited state, the daughter nucleus ground state being a $(T-1, T-1)$ state. The SA state has the same quantum numbers as the ground state of the mother nucleus except for T_3 which is by one unit smaller than the initial value. This state can be described either by a neutron hole-proton particle Tamm-Dancoff approximation (TDA) applied to the mother system or by diagonalizing a many body Hamiltonian associated with the daughter nucleus within a $|\alpha TT_3\rangle$ basis. Considering now the daughter nucleus in the SA state as a target, one may define again, by a (p, n) process, a new SA state. This is a DA state with respect to the initial mother nucleus and has the isospin quantum numbers $(T, T-2)$. According to the above considerations this state can be obtained by two successive TDA phonon excitations of the ground state of the mother nucleus. Roughly speaking the SA energy is about the shift of proton single-particle energy due to the Coulomb interaction and symmetry energy, the deviation being caused by the proton-neutron residual interaction. The remark of Zamick and his collaborators is that in some Ti even isotopes the excitation energy of the DA states is twice as much as the excitation energy of SA in the neighboring even-odd Ti isotopes. For illustration, an example is given in Figs. 1 and 2. The question is, of course, whether this is a general feature or is mainly determined by the interaction regime (strong or weak) between the odd nucleons and the core on one side and between odd nucleons on other side.

The states whose energies are suspected to provide a ratio of 2 to 1 are considered as approximate eigenstates of the model Hamiltonian, which will be treated within a generalized BCS formalism. The corresponding energies are obtained as average values of the many body Hamiltonian on N, T, T_3 projected states, generated by the generalized BCS ground state. As in a previous publication [8], we restrict the space of single-particle states to a proton single j_p and a neutron single j_n shell with $j_p=j_n$.

¹Throughout this paper we consider that protons and neutrons have T_3 equal to $\frac{1}{2}, -\frac{1}{2}$, respectively.

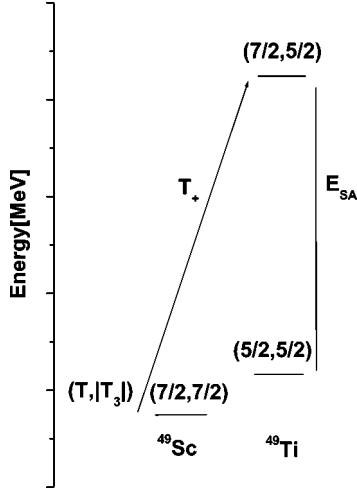


FIG. 1. The SA state in the daughter nucleus ^{49}Ti and the ground state of the mother nucleus ^{49}Sc are schematically presented.

The results are presented as follows. In Sec. II, the even-even system is presented. Since the results for such systems were earlier described in Ref. [8], here we collect only the main results in order to facilitate a self-consistent picture. Section III is devoted to the even-odd system. Numerical results are commented in Sec. IV and the final conclusions are given in Sec. V.

II. EVEN-EVEN SYSTEM

The many body systems for mother and daughter nuclei are described by the following Hamiltonian:

$$H = \epsilon(\hat{N}_p + \hat{N}_n) - \frac{G}{4} \sum_{j,j'} [P_p^\dagger(j)P_p(j') + P_n^\dagger(j)P_n(j') + 2P_{pn}^\dagger(j)P_{pn}(j')], \quad (2.1)$$

where the following notations have been used:

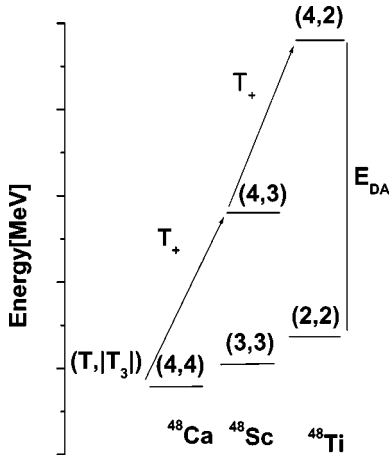


FIG. 2. The DA state in the final nucleus ^{48}Ti and the ground state of the initial nucleus ^{48}Ca are schematically presented.

TABLE I. Excitation energies of single and double analog states in Ti isotopes (MeV).

Even-odd		Single j shell	Experiment
		E (MeV)	E (MeV)
^{43}Ti	1/2 3/2	4.340	4.338
^{45}Ti	1/2 3/2	4.178	4.176
		4.725	
^{47}Ti	3/2 5/2	7.191	7.187
^{49}Ti	5/2 7/2	8.725	8.724
Even-even			
^{44}Ti	0 2	9.345	9.340
^{46}Ti	1 3	14.150	14.153
^{48}Ti	2 4	17.395	17.398

$$P_\tau^\dagger(j) = \sum_m c_{\tau jm}^\dagger c_{\tau jm}^\dagger, \quad \tau = p, n,$$

$$P_{pn}^\dagger(j) = \sum_m c_{pjm}^\dagger c_{njm}^\dagger. \quad (2.2)$$

In a previous paper, two of us (A.A.R. and E.M.G.) approached the generalized BCS ground state by a wave function obtained by rotating the ground state of a proton-neutron pairing Hamiltonian in the isospin space. Rotation angles depend on the quantum numbers defining the proton-neutron pair:

$$|BCS\rangle = \prod_{jm} \hat{R}(\Omega_0^{(j)})(U_j + V_j^* C_{pjm}^\dagger c_{njm}^\dagger)|0\rangle \equiv \hat{R}(\Omega_0)|BCS\rangle_{pn}, \quad (2.3)$$

where the standard notation for the BCS ground state of the p - n pairing Hamiltonian, $|BCS\rangle_{pn}$, and the rotation in the space of the many body states,

$$\hat{R}(\Omega_0) = \otimes_j \hat{R}(\Omega_0^{(j)}), \quad (2.4)$$

have been used. The transformation $\hat{R}(\Omega_0)$ has the meaning of a collective rotation which fixes the orientation of the intrinsiclike frame in the space of isospin. Thus $|BCS\rangle$ plays the role of an intrinsic state while $|BCS\rangle_{pn}$ of an auxiliary intrinsic state. Clearly the function $|BCS\rangle$ breaks the gauge and isospin symmetries. These symmetries can be restored by a projection procedure, and the resulting states are

$$|NTMK\rangle = \frac{2T+1}{16\pi^3} \mathcal{N}_{NT} \int D_{MK}^T * e^{i(\hat{N}-N)\Phi} \hat{R}(\Omega) \hat{R}(\Omega_0) \times |BCS\rangle_{pn} d\Omega d\Phi. \quad (2.5)$$

Here N stands for the total number of valence nucleons, T is the total isospin, K is the third component of the total isospin (equal to minus the half of the neutron excess). As we already mentioned we suppose that the excitation energies are determined by the valence nucleons. In the case of Ti isotopes, the open shells for protons and neutrons are $f_{7/2}$. In what follows we give, therefore, the useful results for the case of a single j -shell. To simplify the notations we omit the index j specifying the shell. Semidegeneracy

of the j shell is denoted by $\Omega=(2j+1)/2$.

The norm for the state $|NTMK\rangle$ is

$$\begin{aligned} \mathcal{N}_{NT}^{-2} &= |D_{K_0}^T(\Omega_0)|^2 (N_{pn})^2 \left(\frac{|V|}{U}\right)^N \sum_p (-2)^p \binom{\Omega}{p} \\ &\times \binom{2\Omega-2p}{\frac{N}{2}-2p} Z\left(\frac{N}{2}, p\right). \end{aligned} \quad (2.6)$$

The projected state energy characterizing the even-even system of valence nucleons is analytically given by

$$\begin{aligned} \langle NTMK|H|NTMK\rangle &= \epsilon N - G \frac{N}{4} \left(2\Omega - \frac{N}{2} + 1\right) \\ &- \frac{G}{2} \mathcal{N}_{NT}^2 N_{pn}^2 |D_{K_0}^T(\Omega_0)|^2 \sum_p S_p Z\left(\frac{N}{2}, p\right) \end{aligned} \quad (2.7)$$

with the notations

$$\begin{aligned} S_p &= \left(\frac{|V|}{U}\right)^N \times \left[4p \binom{\Omega - \frac{N}{2} + p + 1}{p} (-2)^p \binom{\Omega}{p} \binom{2\Omega - 2p}{\frac{N}{2} - 2p} \right. \\ &\left. + 4(-2)^p (p+1)^2 \binom{\Omega}{p+1} \binom{2\Omega - 2(p+1)}{\frac{N}{2} - 2(p+1)} \right]. \end{aligned} \quad (2.8)$$

The function $Z(N/2, p)$ is defined in the Appendix.²

III. EVEN-ODD SYSTEM

A. Extension of projection formalism

The projected state describing the even-odd system is defined by

$$\begin{aligned} &|N+1, T_o M_o K_o\rangle \\ &= \mathcal{N}_{N+1, T_o} \frac{2T_o + 1}{16\pi^3} \int e^{i[\hat{N} - (N+1)]\phi} D_{M_o K_o}^{T_o}{}^* \\ &\times \hat{R}(\Omega) \hat{R}(\Omega_0) c_{njml}^\dagger |BCS\rangle_{pn} d\Omega d\phi. \end{aligned} \quad (3.1)$$

The norm of the state $|N+1 T_o M_o K_o\rangle$ has the expression

$$\begin{aligned} \mathcal{N}_{N+1, T_o}^{-2} &= |D_{K_o, -1/2}^{T_o}(\Omega_0)|^2 (N_{pn})^2 \left(\frac{|V|}{U}\right)^N \sum_p (-2)^p \binom{\Omega-1}{p} \\ &\times \left[\binom{2\Omega-2p-1}{\frac{N}{2}-2p} Z^{(\nu)}\left(\frac{N}{2}, p\right) \right. \\ &\left. - \sqrt{2} \binom{2\Omega-2p-2}{\frac{N}{2}-2p-1} Z^{(\pi)}\left(\frac{N}{2}, p\right) \right], \quad N = \text{even}, \end{aligned} \quad (3.2)$$

where the functions $Z^{(\pi)}(N/2, p)$ are those defined in the Appendix. Alternatively, the norm of the projected state describing an odd number of nucleons can be expressed in terms of norms associated with the even system:

$$\mathcal{N}_{N+1, T_o}^{-2} = U^2 |D_{K_o, -1/2}^{T_o}(\Omega_0)|^2 \sum_{T_e} \mathcal{N}_{NT_e}^{-2} (C_{0, 1/2, 1/2}^{T_e})^2. \quad (3.3)$$

Following the procedure described in Ref. [8], one can express the energy for the even-odd system as

$$\begin{aligned} &\langle N+1 T_o M_o K_o | H | N+1 T_o M_o K_o \rangle \\ &= \epsilon(N+1) - \frac{GN}{2} \left(2\Omega - \frac{N}{2}\right) - \frac{G}{4} \mathcal{N}_{N+1, T_o}^2 N_{pn}^2 \left(\frac{|V|}{U}\right)^N |D_{K_o, -1/2}^{T_o}(\Omega_0)|^2 \\ &\times \sum_p \left\{ Z^{(\nu)}\left(\frac{N}{2}, p\right) 4(-2)^p \left[p(2\Omega - N + 2p + 1) \binom{\Omega-1}{p} \binom{2\Omega-2p-1}{\frac{N}{2}-2p} + 2(p+1)^2 \binom{\Omega-1}{p+1} \binom{2\Omega-2(p+1)-1}{\frac{N}{2}-2(p+1)} \right. \right. \\ &\left. \left. + (p+1) \binom{\Omega-1}{p} \binom{2\Omega-2p-2}{\frac{N}{2}-2p-1} \right] + Z^{(\pi)}\left(\frac{N}{2}, p\right) \frac{4(-2)^{p+1}}{\sqrt{2}} \left[(2p+1) \binom{\Omega - \frac{N}{2} + p + 1}{p} \binom{\Omega-1}{p} \binom{2\Omega-2p-2}{\frac{N}{2}-2p-1} \right. \right. \\ &\left. \left. + 2(p+1)(p+2) \binom{\Omega-1}{p+1} \binom{2\Omega-2(p+1)-2}{\frac{N}{2}-2(p+1)-1} - (p+1) \binom{\Omega-1}{p+1} \binom{2\Omega-2(p+1)-1}{\frac{N}{2}-2(p+1)} \right] \right\} \end{aligned} \quad (3.4)$$

²In Ref. [8] the second term in Eq. (2.8) has, by error, a factor 2 instead of 4. For the angular momentum considered there the deviation from the correct result is however very small.

with the factors Z defined in the Appendix. Also the notation Ω for the state semidegeneracy is used.

B. Particle-core interaction

The odd system can be described within a particle-core coupled basis:

$$|N + 1T_o M_o K_o\rangle = \sum_{M_e, \mu} C_{M_e \mu M_o}^{T_o 1/2 T_o} |NT_e M_e K_e\rangle \chi_\mu, \quad (3.5)$$

where $|NT_e M_e K_e\rangle$ describes the even-even system and is given by Eq. (2.5) while χ_μ stands for the odd neutron wave function in the laboratory frame. Of course the isospin projections in the intrinsic frame satisfy the equation

$$K_o = K_e - \frac{1}{2}. \quad (3.6)$$

Since the even system states are $J=0$ states, it results that the odd system states are $J=j$ states.

In order to treat the many body Hamiltonian (2.1) in the basis defined above it is convenient to separate it into three parts describing the two components of the system as well as their mutual interaction:

$$H = H_{sp} + H_{core} + H_{coup}. \quad (3.7)$$

We assume that the coupling term is simulated by a scalar operator with respect to isospin rotations:

$$H_{coup} = 2F \vec{t} \vec{T}, \quad (3.8)$$

where \vec{t} and \vec{T} are the isospin operators acting on the state describing the odd particle and the even-even core, respectively.

As can be seen from Table I, each state of the odd system can be obtained uniquely by coupling the odd neutron isospin to a single isospin state from the core space. For example, the isospin $3/2$ of ^{47}Ti can be obtained by coupling the odd neutron isospin to the state of the even core with $T=1$ while the state $5/2$ originates from the coupling of the odd particle to the $T=3$ core state. Another remark which is pertinent is that the low isospin state in the odd system is obtained by aligning the odd neutron isospin to the core isospin. On the contrary for the higher isospin state an antialignment of the two isospin takes place. This property holds for all even-odd pairs of isotopes considered here. As a consequence the particle core interaction is repulsive in lowest isospin states and attractive in the higher isospin ones. For particle-core coupling models it is customary to adopt the approximations

$$\begin{aligned} \langle N + 1T_o M_o K_o | H_{core} | N + 1T_o M_o K_o \rangle \\ = \langle NT_e M_e K_e | H_{core} | NT_e M_e K_e \rangle, \end{aligned} \quad (3.9)$$

$$\langle N + 1T_o M_o K_o | H_{sp} | N + 1T_o M_o K_o \rangle = \epsilon.$$

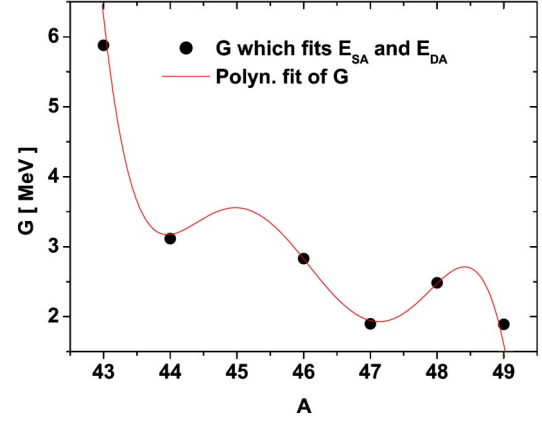


FIG. 3. (Color online) The pairing strength as function of the atomic mass A . The values of G are obtained by fitting the single and double analog resonances, respectively, and represented by black circles. These points may be interpolated by a polynomial of fifth order in A . The resulting polynomial is given by the full line.

The average for the core Hamiltonian has been calculated in Sec. II, being given by Eq. (2.7). In this way the excitation energies in odd isotopes are related to those in the neighboring even-even ones by

$$\Delta E\left(\frac{7}{2}, \frac{5}{2}\right) = \Delta E(4, 2) - 7F,$$

$$\Delta E\left(\frac{5}{2}, \frac{3}{2}\right) = \Delta E(3, 1) - 5F,$$

$$\Delta E\left(\frac{3}{2}, \frac{1}{2}\right) = \Delta E(2, 0) - 3F.$$

IV. NUMERICAL RESULTS

Here we consider the SA states in odd and DA states in even-mass isotopes of Ti. The energies for the lowest isospin states in each case are obtained as expectation values of the many body Hamiltonian on the particle number and isospin projected states. Excitation energies depend on the pairing strength G . This is fixed so that the experimental values of the excitation energies are reproduced. In this way we want to explore effects such as the dependence of G on the atomic mass number, the blocking due to the odd particle, as well as whether the $\nu=0$ (ν denotes seniority quantum number) description suffices for both even-even and even-odd nuclei. As we stated from the beginning we restrict the space of single-particle states to a single j shell, i.e., $f_{7/2}$. The excitation energies obtained in this way are given in Table I. The values of G obtained through the fitting procedure mentioned above are plotted in Figs. 3–6. as function of A . In the first figures we omitted the value of G for ^{45}Ti which is quite large (≈ 8.35 MeV). The large values of G for the lowest mass odd isotopes might be caused by the fact that the restriction of the many body states to the $\nu=0$ subspace is unrealistic and the neglected contribution is compensated through an artificial increase of the pairing strength. Also the blocking effect of the odd particle, which as a matter of fact is neglected here, is suspected to be large. For the last two pairs of isotopes (^{46}Ti , ^{47}Ti) and (^{48}Ti , ^{49}Ti), the pairing strengths are fairly

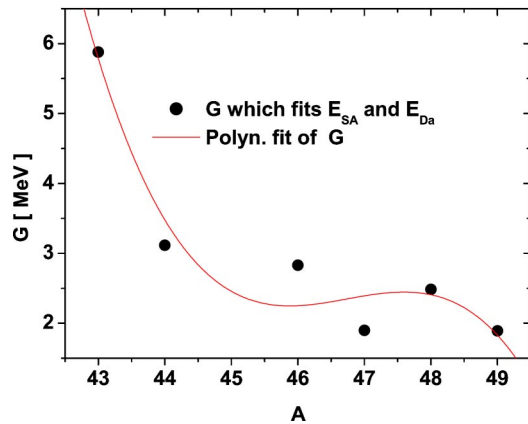


FIG. 4. (Color online) The pairing strength as function of the atomic mass A . The values of G are obtained by fitting the single and double analog resonances, respectively, and represented by black circles. These points may be interpolated by a polynomial of third order in A . The resulting polynomial is given by the full line.

close to each other. Moreover the ordering of the G value for even-even and even-odd isotopes is opposite to that of the case of light isotopes. As shown in Fig. 4 the pairing strength falls on a third-degree polynomial in A . The A dependence of G is plotted separately for even-even and even-odd isotopes in Figs. 5 and 6, respectively. While in the even A case the points are almost collinear, for odd isotopes the pairing strengths lie on a Gaussian curve.

It is worth mentioning that the isospin invariant pairing Hamiltonian (H_{pair}) considered for a single j shell is exactly solvable. Indeed, H_{pair} can be expressed in terms of the quadratic Casimir operator of the group $O(5)$ generated by the proton-proton, neutron-neutron, and proton-neutron quasispin operators [9]. Therefore, its eigenvalues have simple expressions in terms of the highest weight of the irreducible representations of the $O(5)$ group. These weights are determined by the reduced isospin t [10] and the seniority quantum number v ,

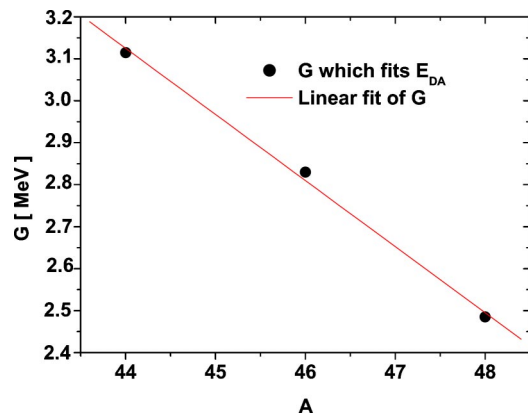


FIG. 5. (Color online) The pairing strength as function of the atomic mass A for even-mass isotopes of Ti. The values of G are obtained by fitting the double analog resonance energies, respectively, and represented by black circles. These points may be interpolated by a linear polynomial in A . The resulting polynomial is given by the full line.

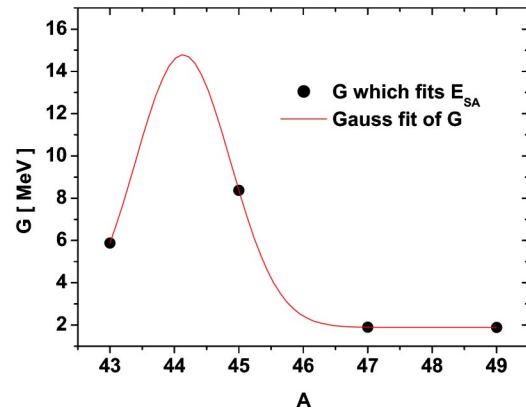


FIG. 6. (Color online) The pairing strength as function of the atomic mass A for odd-mass isotopes of Ti. The values of G are obtained by fitting the single analog resonance energies, respectively, and represented by black circles. These points may be interpolated by a Gauss function of A . The resulting function is represented by a full line.

$$H_{pair} = -\frac{G}{2} \left(\frac{1}{4} (N-v)(4\Omega - N - v + 6) + t(t+1) - T(T+1) \right). \quad (4.1)$$

The notation Ω stands for the state semidegeneracy. In the BCS treatment adopted here the projected states have vanishing seniority and reduced isospin. Therefore for the even-even isotopes one obtains a close formula expressing the energy of a system of N valence nucleons:

$$E_T = \epsilon N - \frac{G}{2} \left(\frac{N}{4} (4\Omega - N + 6) - T(T+1) \right). \quad (4.2)$$

Using this formula with G values obtained in the manner specified above one obtains for excitation energies exactly the same energies as given in Table I. One may conclude that the projection procedure yields for single j energy the same value as the group theory method.

Now let us turn our attention to the particle-core formalism. We applied Eq. (3.9) for odd isotopes in the following way. The strength of the particle-core interaction was fixed so that the excitation energies for the SA states are reproduced. The values of F , obtained in this way, are plotted in Fig. 7. They are also listed in Table II. From Fig. 7 one sees that apart from F of ^{45}Ti the other three points lie on a straight line. However the point for ^{45}Ti is not far from the straight line. Indeed, taking the value of F which falls on the straight line (this value is equal to 1.54 MeV and is given in Table II in brackets) and calculating the corresponding excitation energy for the state with $T=\frac{3}{2}$, one obtains a value equal to 4.725 MeV which has to be compared with the experimental value of 4.176 MeV.

Comparing the values of F and G given in Table II one remarks that F is almost half the value of G .

Coming back to Eq. (3.9) and using $F=G/2$ one obtains for the excitation energies in the odd isotopes the values 4.672, 4.672, 7.104, and 7.698 MeV, respectively. We remark that the last two values listed above are close to the

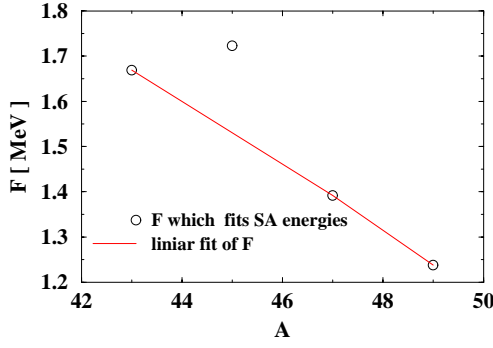


FIG. 7. (Color online) The particle-core coupling strength as function of the atomic mass A for odd-mass isotopes of Ti. The values of F are obtained by fitting the single analog resonances energies, respectively, by using Eq. (3.9), and represented by white circles. Three points lie on a straight line represented by a full line.

experimental values in ^{47}Ti and ^{49}Ti , respectively, while for ^{43}Ti and ^{45}Ti the deviations are equal to 334 and 496 keV, respectively. Thus one could state that we obtained a quite reasonable description of the excitation energies of SA and DA states in Ti isotopes using only one parameter, the pairing strength, which is depending linearly on A .

Denoting by $T=|N-Z|/2$ the ground state isospin, one arrives at very simple expressions for SA and DA excitation energies in terms of isospin:

$$E_{\text{DA}} = G(2T + 3),$$

$$E_{\text{SA}} = G(T + 1),$$

$$G = -0.156A + 9.979 \text{ (MeV)}, \quad A = \text{even}. \quad (4.3)$$

For even-odd isotopes one should take in the expression of G the value of A corresponding to the neighboring even-even core. If the small deviation of F from $G/2$ is not ignored then the SA excitation energy is given by

TABLE II. The values of pairing (second column) and particle-core strengths (fourth column) are given in units of MeV.

Even-even	G (MeV)	Even-odd	F (MeV)
^{44}Ti	3.115	^{43}Ti	1.669
		^{45}Ti	1.723 (1.54)
^{46}Ti	2.830	^{47}Ti	1.392
^{48}Ti	2.485	^{49}Ti	1.239

$$S_{\text{SA}} = 2(G - F)(T + 1),$$

$$F = -0.0717A + 1.9773 \text{ (MeV)}, \quad A = \text{odd}. \quad (4.4)$$

Recalling that $A=2Z\pm 2T$ where the minus sign holds for ^{43}Ti , it is clear that the above equations provide quadratic expressions of T for SA and DA excitation energies.

V. CONCLUSIONS

Excitation energies for SA (single analog) and DA (double analog) states in even-odd and even-even isotopes of Ti, respectively, have been described with a many body isospin invariant Hamiltonian in a single j shell. The states describing the neighboring even-even and even-odd isotopes are obtained through an N, T projection procedure from a generalized proton-neutron BCS wave function. The pairing strength for even-even systems depends linearly on the atomic mass number A . It is worth mentioning that the group theory formula produces identical results for excitation energies as the projection procedure. Alternatively the even-odd systems are described in a particle-core coupling scheme where the coupling interaction is an isospin invariant operator acting on both odd particle and core degrees of freedom. The fitted strength for the coupling term is almost half the pairing strength. Taking into account that the excitation energy for the even system is analytically given by a very simple formula, it results that the present paper provides also a compact expression for the excitation energy in even-odd isotopes. Since G depends linearly on A , one may conclude that the excitation energies in the seven isotopes considered here are fairly well obtained by fixing only two free parameters. Finally, analytical expressions for a quadratic dependence on T for SA and DA excitation energies are obtained.

APPENDIX

Here we give the analytical expressions for the overlap integrals which enter the formulas determining the energies for even-even and even-odd systems:

$$\begin{aligned} Z\left(\frac{N}{2}, p\right) &\equiv \frac{2T+1}{2} \int d_{00}^T (d_{00}^1)^{N/2-2p} (d_{10}^1 d_{-10}^1)^p \sin\beta d\beta \\ &= \sum_{k,m,n} (-)^{p+k+n} \frac{(2T+1)2^{-T-p}}{m+n+s+1} \binom{T}{k}^2 \binom{T+p-k}{m} \binom{k+p}{n}, \quad m+n+s = \text{even}; \quad s = \frac{N}{2} - 2p, \end{aligned} \quad (\text{A1})$$

$$\begin{aligned}
Z^{(p)}\left(\frac{N}{2}, p\right) &\equiv \frac{2T_o + 1}{2} \int d_{-1/2, -1/2}^{T_o} (d_{00}^1)^{N/2-2p} (d_{10}^1 d_{-10}^1)^p d_{-1/2, -1/2}^{1/2} \sin\beta d\beta \\
&= \sum_{k,m,l} (-)^{p+k+m} \frac{(2T_o + 1) 2^{N/2-T_o-(5/2)p+l+m+1}}{\frac{N}{2} - 2p + l + m + 1} \begin{pmatrix} T_o + \frac{1}{2} \\ k \end{pmatrix} \begin{pmatrix} T_o - \frac{1}{2} \\ k \end{pmatrix} \begin{pmatrix} T_o + p - k + \frac{1}{2} \\ l \end{pmatrix} \begin{pmatrix} k + p \\ m \end{pmatrix}, \\
\frac{N}{2} + l + m &= \text{even};
\end{aligned} \tag{A2}$$

$$\begin{aligned}
Z^{(m)}\left(\frac{N}{2}, p\right) &\equiv \frac{2T_o + 1}{2} \int d_{-1/2, -1/2}^{T_o} (d_{00}^1)^{N/2-2p-1} (d_{10}^1 d_{-10}^1)^p d_{-10}^1 d_{1/2, -1/2}^{1/2} \sin\beta d\beta \\
&= \sum_{k,m,l} (-)^{p+k+m} \frac{(2T_o + 1) 2^{N/2-T_o-(5/2)p+l+m-1}}{\frac{N}{2} - 2p + l + m} \begin{pmatrix} T_o + \frac{1}{2} \\ k \end{pmatrix} \begin{pmatrix} T_o - \frac{1}{2} \\ k \end{pmatrix} \begin{pmatrix} T_o + p - k + \frac{1}{2} \\ l \end{pmatrix} \begin{pmatrix} k + p + 1 \\ m \end{pmatrix}. \\
\frac{N}{2} + l + m &= \text{odd}.
\end{aligned}$$

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- [1] J. D. McCullen, B. F. Baymann, and L. Zamick, Phys. Rev. **134**, B515 (1964); Technical Report No. NYO-9891.
- [2] L. Zamick and Y. D. Devi, Phys. Rev. C **60**, 054317 (1999).
- [3] L. Zamick and D. C. Zheng, Phys. Rev. C **46**, 815 (1992).
- [4] Y. Durga Devi, Shadow Robinson, and Larry Zamick, Phys. Rev. C **61**, 037305 (2000).
- [5] L. Zamick, S. J. Q. Robinson, E. Moya de Guerra, and A. A. Raduta, Phys. Rev. C (unpublished).
- [6] N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. **44**, 48 (1972).
- [7] P. Ring and P. Schuck, The Nuclear Many-Body Problem, p. 297.
- [8] A. A. Raduta and E. Moya de Guerra, Ann. Phys. (N.Y.) **284**, 134 (2000).
- [9] Y. Takahashi, Prog. Theor. Phys. **46**, 263 (1971).
- [10] B. H. Flowers, Proc. R. Soc. London, Ser. A **212**, 248 (1952).