α radioactivity of superheavy nuclei

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Calculations based on the relativistic mean field framework have been carried out for the ground state properties of the relevant nuclei appearing in the α-emission chain of superheavy element $Z=112$. The calculations compare well with the experiment. The calculated densities along with the energy and the density dependent M3Y effective nucleon-nucleon interaction are used in the double folding model to compute the respective total interaction energies. These, in turn are used to calculate the half-lives of the parent nuclei against the split into an α and the daughter in the WKB approximation. The calculations are repeated for the nuclei appearing in the $4n+2$ α chain for a comparative study. The sensitivity of the calculated half-lives on the input information, especially the Q values are quantitatively investigated.

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I. INTRODUCTION

The production, identification and study of Super Heavy Elements is a challenging task and has been the focus of extensive research activity. With every integral increment in Z, the level of experimental difficulty increases manyfold. The cold (hot) fusion with Pb-/Bi- (actinide: ^{235}U/^{244}Pu) targets and suitable projectiles ^{64}Ni/^{70}Zn (^{48}Ca) have been successfully used for the production of superheavy elements with $Z=110,111,112$ (114,116) at GSI, Germany (Dubna, Russia). Out of these the element $Z=112$ is unambiguously identified using α-α parent-daughter correlation technique. It decays with a half-life of 280 μs followed by a long α-decay chain. Thus its identification relies on the known isotopes of elements with $Z=106$ and below. Unfortunately, the measured α energies (Q values) and half-lives still have large inherent uncertainties in their measurements. The nuclei in this range ($Z=\ldots,110,\ldots$) bridge the gap between the known actinides and the unknown superheavy nuclei, and therefore are the focus of the current and future nuclear structure research.

The theoretical macroscopic-microscopic and the self-consistent mean field (both relativistic and nonrelativistic) studies have also been carried out with the primary aim to predict the combination of neutron number (N) and the proton number (Z), where the spherical shell closure may occur. In the self-consistent models, the occurrence of a spherical proton (neutron) shell closure with given $Z(N)$ can change with varying neutron number N (proton number Z). In such studies the pairing and deformation may play an important role. An "island of stability" has been predicted around the hypothetical doubly magic ^{298}114 ($N=184$) about 30 years ago [1,2]. Nuclei in the vicinity are expected to be spherical or nearly so with longer half-lives.

The neutron number $N=162$ has been predicted [3] to exhibit shell closure at around $Z=108-110$. This is consistent with the findings of Ref. [4] about the nucleus ^{273}110. In addition, the calculated shell corrections (see Table 5, p. 222 of Ref. [3]) peak at $N=166$ for $Z=112$ indicating the stable structure of $Z=112$ around N=166. This is consistent with the observation of the superheavy nucleus ^{277}112.

Recently, an extensive article "Microscopic Theory of Cluster Radioactivity" has appeared in Ref. [5]. There, the authors employ the R-matrix theory for the calculation of the decay widths using the required input information obtained from the shell/cluster + BCS models.

In this short paper we report the results based on the self-consistent relativistic mean field (RMF) models for the ground state properties of relevant nuclei appearing in the α-emission chain of superheavy element $Z=112$ and those in the known $4n+2$ α-decay chain. The calculated densities along with the energy and density dependent M3Y effective nucleon-nucleon interaction (DDM3Y) is used in the double folding model ($t\rho\rho$ approximation), to compute the respective interaction energies between the α daughter system. These, in turn are used to calculate the half-lives of the parent nuclei for the α emission in the Super-Asymmetric Fission Model (SAFM) using the WKB approximation. A short sketch of the RMF formulation relevant to the present study now follows.

II. ESSENTIALS OF RMF

The relativistic mean field models start with an effective Lagrangian describing the nucleons interacting only via the electromagnetic (e.m.) and the effective meson fields. Usually, the isoscalar-scalar σ, isoscalar-vector ω, and isovector-vector ρ mesons are considered in such Lagrangians. We use the standard nonlinear (σ,ω,ρ) interaction Lagrangian developed for and widely used in the nuclear structure applications [6,7]. The Euler-Lagrange variational principle yields the equations of motion. By replacing the field operators by their expectation values, one ends up with a set of coupled equations: the Dirac equation with potential terms involving meson and e.m. fields describing the nucleon dynamics and a

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set of Klein-Gordon type equations with sources involving nucleonic currents and densities for the mesons and the photon. This set of RMF equations is to be solved self-consistently.

The pairing correlations are incorporated either by using simple BCS prescription (constant gap) or self-consistently, through the Bogoliubov transformation. The latter leads to the Relativistic Hartree Bogoliubov (RHB) equations [7,8]:

$$\begin{pmatrix} h_D - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -h_D^* + \lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_k = E_k \begin{pmatrix} U \\ V \end{pmatrix}_k . \tag{1}$$

Here, λ is the Lagrange multiplier, E_k is the quasiparticle energy, and U_k and V_k are four-dimensional Dirac superspinors, correctly normalized. h_D is the usual Dirac Hamiltonian

$$h_D = -\imath\alpha \cdot \nabla + \beta(M + g_\sigma\sigma) + g_\omega\omega^o + g_\rho\tau_3\rho_3^o + e\frac{1-\tau_3}{2}A^o, \tag{2}$$

where M is the nucleon mass and σ, ω^o, ρ_3^o, and A^o are the meson and e.m. fields. These fields are to be determined self-consistently from the corresponding Klein-Gordon equations [7] with sources (nuclear currents and densities) involving superspinors $[U(V)]$ [7,8].

The RHB equations have two distinct parts: the self-consistent field (h_D) that describes the long range particle-hole correlations and the pairing field ($\hat{\Delta}$) that accounts for the correlations in the particle-particle (pp) channel. The pairing field $\hat{\Delta}$ is expressed in terms of the matrix elements of the two body nuclear potential (V^{pp}) in the pp channel and the pairing tensor involving the superspinors (U,V). In the case of the constant gap, $\hat{\Delta}_a$ ($\equiv\Delta$) becomes diagonal and decouples into a set of 2×2 diagonal matrices resulting in the BCS type expressions for the occupation probabilities (U,V). As a result, the RHB equations reduce to the RMF equations with constant gap. Reliable and satisfactory derivation of V^{pp} is not yet available in RMF (see Refs. [7,9]). In practical calculations, therefore, one often uses the finite range Gogny D1S [10,11] interaction for V^{pp} while solving the RHB equations

$$V(\mathbf{r}_1,\mathbf{r}_2) = \sum_{i=1,2} e^{-\{(\mathbf{r}_1-\mathbf{r}_2)/\mu_i\}^2}(W_i + B_iP^\sigma - H_iP^\tau - M_iP^\sigma P^\tau). \tag{3}$$

The factors μ_i, W_i, B_i, H_i, and M_i $(i=1,2)$ are parameters of the interaction.

A. Ground state properties

The RHB equations are solved employing the spherical basis (isotropic harmonic oscillator) expansion method and using the Lagrangian parameter set NL3 [12] along with the finite range Gogny D1S interaction in the pp (pairing) channel. The results obtained are labeled as RHB(ob). The RHB equations are again solved in the constant gap approximation with a cutoff ($2\hbar\omega$), where the neutron and the proton gaps are adjusted so as to reproduce the respective neutron and proton pairing energies of RHB(ob) calculations. These gaps along with the cutoff are now used to solve the RHB equations for the deformed case. These results are designated by DEF. For odd-A nuclei, the last odd nucleon does not have a partner to occupy its time reversed state. As a result, the mean field ground state wave function does not have time reversal symmetry. For this purpose, we follow the well tested tagged Hartree-Fock procedure, frequently used in the nonrelativistic calculations.

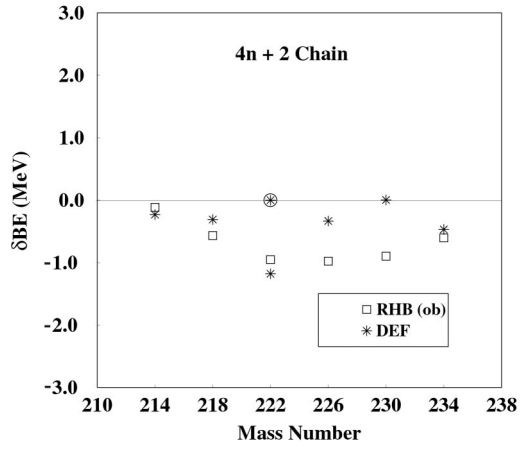

FIG. 1. Difference between the calculated and the corresponding experimental [13] binding energies for $4n+2$ α-decay chain.

The difference between the calculated and the corresponding experimental [13] binding energies are displayed in Fig. 1 for the $4n+2$ α-decay chain. Clearly, the calculations agree well with the experimental values. The inclusion of deformation brings the calculations closer to the experiment. The maximum deviation ($\approx$1 MeV) appears for ^{222}Rn. The calculated binding energies for the $Z=112$ α-decay chain are shown in Fig. 2 along with the corresponding values of Möller and Nix (MN) [14] and of Audi and Wapstra (Audi) [13]. All reasonably agree among themselves.

The calculated (DEF) deformation parameters β shown in Figs. 3 and 4 indicate that all nuclei mostly have prolate

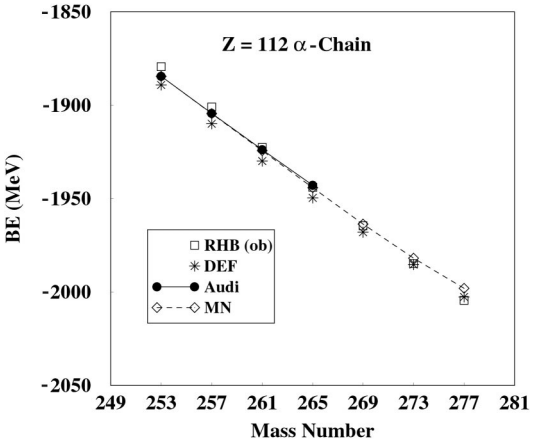

FIG. 2. The calculated and the corresponding Möller-Nix binding energies [14] for $Z=112$ α-decay chain.

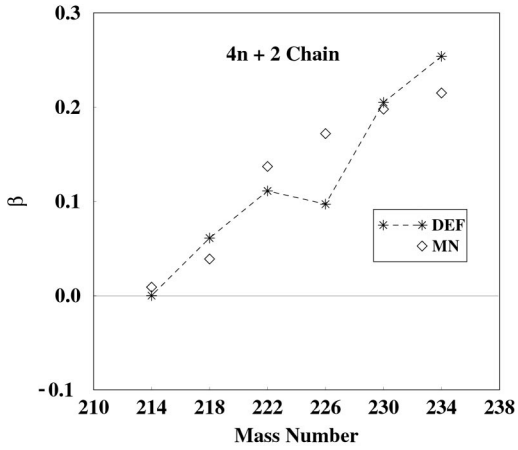

FIG. 3. The calculated (DEF) quadrupole deformation parameters (β) for $4n+2$ α-decay chain. The corresponding Möller-Nix values (MN) [15] are also shown for comparison.

shape ($\beta>0$). This observation agrees with the findings of Möller and Nix [15], though the calculated β's are slightly higher for the $Z=112$ α chain (note the enlarged scale in Fig. 4) than that of MN.

The calculated Q values for the respective α-decays are shown in Fig. 5(a) [5(b)] for $4n+2$ ($Z=112$) α chains, along with the available experimental [13,16] values. The corresponding values of MN [14] are also shown for comparison. The calculations agree reasonably well with the experiment. The DEF results seem to be relatively better and agree well with that of MN. As remarked earlier, the maximum difference ($\approx$1 MeV) between the calculated (DEF) and the experimental binding energy is noticed for ^{222}Rn nucleus. The binding energy of ^{222}Rn appears in the calculation of the α-decay Q values for ^{222}Rn and also for ^{226}Ra. As a result, the calculated Q values [shown by arrows in Fig. 5(a)] for these nuclei though are close to the experimental values, deviate by as much as 1 MeV. Therefore, we have used the experimental value of binding energy of ^{222}Rn in the calculation of Q values. This is crucial for the calculations of half-lives.

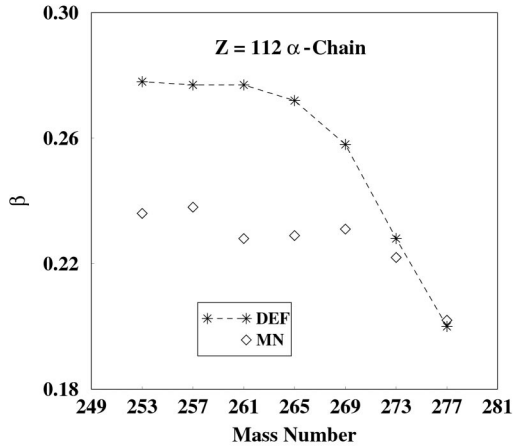

FIG. 4. The calculated (DEF) quadrupole deformation parameters (β) for $Z=112$ α-decay chain. The corresponding Möller-Nix values (MN) [15] are also shown for comparison.

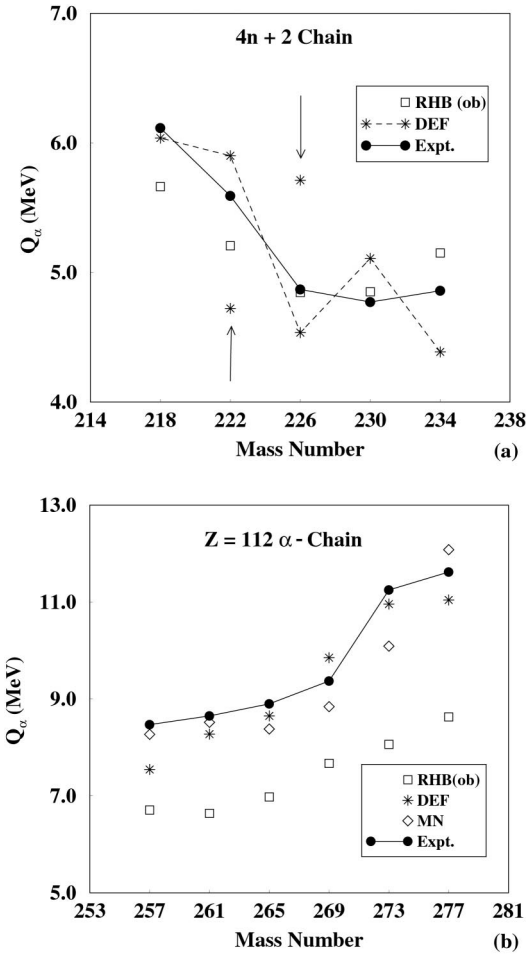

FIG. 5. a (b) The calculated Q values for α-decay of the nuclei belonging to the $4n+2$ ($Z=112$) chain. The corresponding experimental values [13] ([16]) and the Möller-Nix values (MN) [14] are also shown for comparison. The arrows in $4n+2$ indicate the values obtained by using the calculated (DEF) binding energy for ^{222}Rn.

To convey the structure of the single particle levels near the Fermi surface, we display these levels (starting from the last occupied level) in Fig. 6 for all the nuclei appearing in the $4n+2$ and $Z=112$ α chains. These are reasonable and are as expected.

III. DOUBLE FOLDING ($t\rho\rho$) MODEL

The calculated densities along with an effective nucleon-nucleon interaction are used to obtain the total interaction energy of the α-daughter system in the double folding model [17,18]. The geometry of the model is depicted in Fig. 7. The model helps to compute a reasonable nucleus-nucleus potential, starting from an effective nucleon-nucleon interaction. In general, the double folding ($t\rho\rho$ approximation) potential contains both the direct and the exchange terms. The latter is considerably more difficult to handle in practice. Thus, for some of the applications, the exchange term is simulated by a delta function pseudopotential [19]. The density dependent M3Y nucleon-nucleon interaction with pseudopotential (DDM3Y) used in the present work is given by

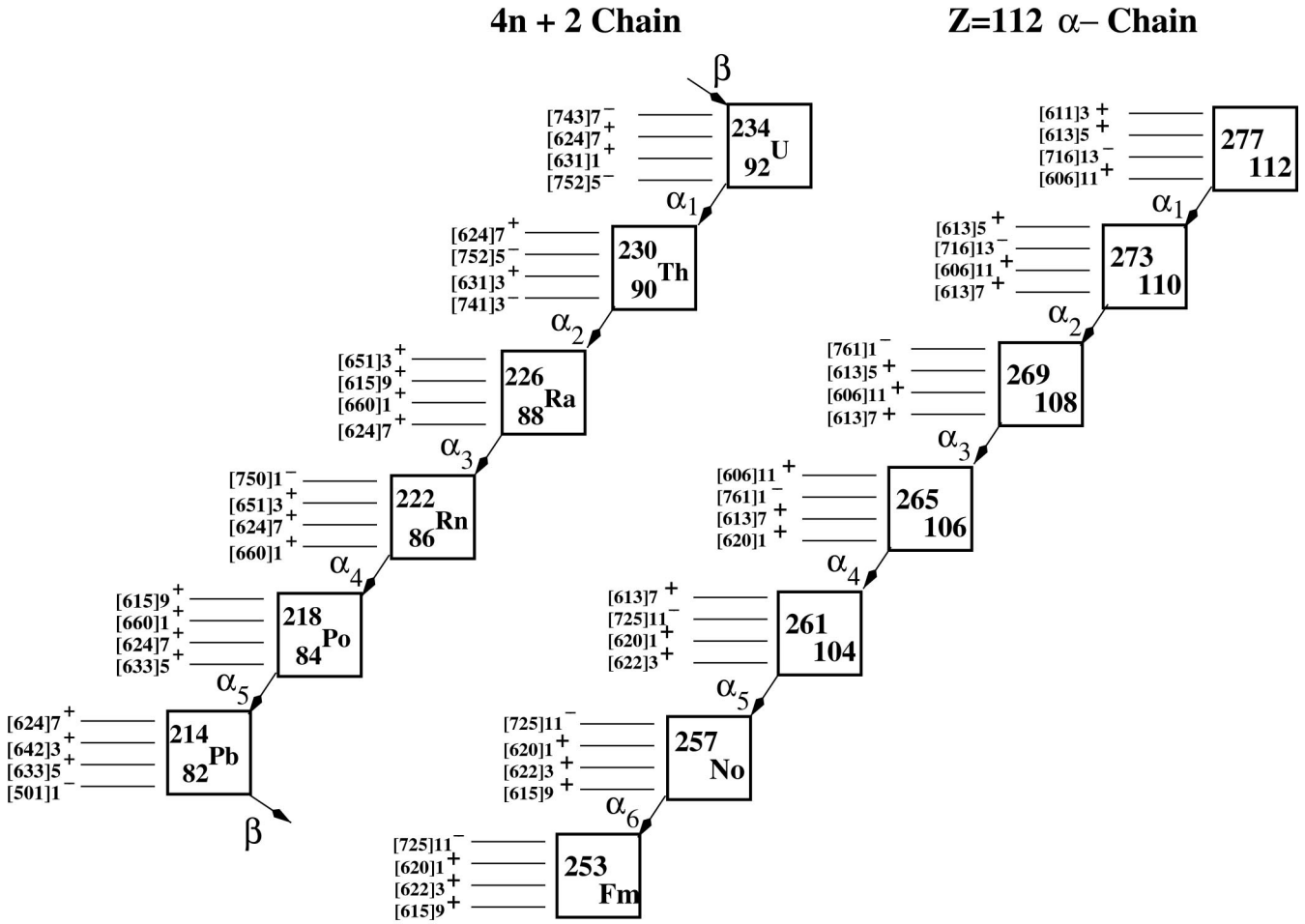

FIG. 6. The deformed single particle levels for $4n+2$ and $Z=112$ α-decay chains obtained from DEF calculations near the Fermi surface.

$$v(s) = \left(7999.0\frac{e^{-4s}}{4s} - 2134.25\frac{e^{-2.5s}}{2.5s} - 276\delta(s)\right) \times (1-\beta\rho_P^{2/3})(1-\beta\rho_T^{2/3}); \quad (4)$$

where, $\beta=1.6$. The density dependence is supposed to take into account the higher order exchange effects and the Pauli Blocking. The total double folding potential between the nucleus-nucleus (Projectile: P; Target: T) system is given by:

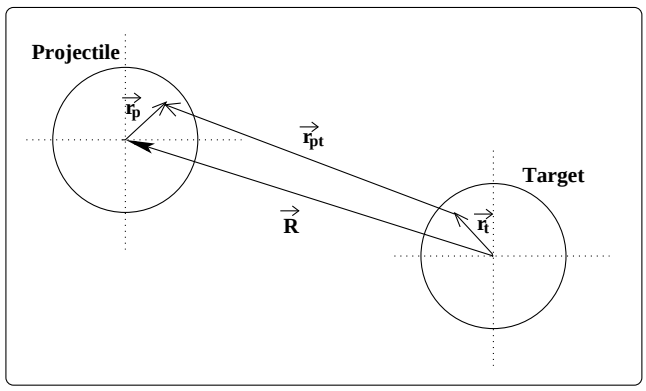

FIG. 7. Geometry of the Double Folding Model.

$$V_{PT}(\vec{R}) = \int \rho_P(\vec{r}_p)\rho_T(\vec{r}_t)v(\vec{r}_p - \vec{r}_t + \vec{R} \equiv \vec{s})d^3\vec{r}_p d^3\vec{r}_t. \quad (5)$$

IV. HALF-LIFE: CALCULATION AND RESULTS

The half-life time of the nucleus (parent) against the α-decay in the Super-Asymmetric Fission Model [20] is given by

$$T_{1/2} = \frac{h\ln(2)}{2E_\nu}(1+e^K); \quad (6)$$

where, within the WKB approximation, the action integral K appearing in Eq. (6) reads

$$K = \frac{2}{\hbar}\int_{R_a}^{R_b}\{2\mu[V_{PT}(R) + V_C(R) - E_\nu - Q]\}^{1/2}dR. \quad (7)$$

$V_C(R)$ is the usual Coulomb potential for the α-daughter system. The Q value for the α-decay can be obtained either from the kinetic energy (corrected for the recoil) of the emerging α particle or from the binding energies of the α nuclei, daughter nuclei and the parent nuclei. E_ν is the zero point vibrational energy and is the parameter appearing in SAFM. For the parent even N-even Z (odd

N-even Z) nucleus, E_ν is taken to be 0.1045 Q(0.0907Q) [21]. R_a and R_b appearing in Eq. (7) are the lower and upper turning points, respectively, determined by the condition such that the integrand in Eq. (7) vanishes. In this study, the calculated turning points turn out to be in the range $R_a \approx 7-8$ fm and $R_b \approx 35-48$ fm.

The Viola-Seaborg (VS) empirical formula [22,23] gives the half-life for α-decay. Here we use the modified version [3,23] of the VS formula, given by

$$\log_{10} T_{1/2} = (aZ + b)/Q_\alpha^{1/2} + (cZ + d), \tag{8}$$

where Z is the atomic number of the parent nucleus, Q_α is the Q value for α-decay, and a, b, c, and d are the fitted parameters: $a=1.66175$, $b=-8.5166$, $c=-0.20228$, and $d=-33.9069$.

For further calculations, the deformed (DEF) densities are expanded in terms of multipoles and the $L=0$ component is projected and renormalized. These spherical ($L=0$) densities along with the density dependent nucleon-nucleon interaction (DDM3Y) are then used to obtain the interaction energy between α and the daughter nucleus through Eq. (5). This along with the calculated Q_α and the Coulomb potential are used to calculate the half-life [Eq. (6)] for the α-decay of the parent nuclei appearing in the $4n+2$ and $Z=112$ α chains. The respective results (DEF+WKB) are shown in Figs. 8(a) and 8(b) along with the corresponding experimental values and similar results are obtained by using the experimental Q_α's in place of the calculated Q_α (the results are labeled as $Q_{expt.}$+WKB). The action K [Eq. (7)] is multiplied by an overall factor 1.049 for the $4n+2$ α-decay chain, whereas for the experimental Q values of $Z=112$ α-decay chain, the factor 1.13 is used. The $T_{1/2}$ obtained by the modified VS formula [Eq. (8)] (the results marked as $Q_{expt.}$+MVS) are also shown in Fig. 8(a) and 8(b). The comparison between the DEF+WKB and $Q_{expt.}$+WKB reveals the hypersensitivity of $T_{1/2}$ on the Q_α values used in the calculation. Even a small variation of 0.1 MeV may result in a considerable (order of magnitude) change in $T_{1/2}$. This is known and is expected because of very broad barrier ($R_a - R_b \approx 30$ fm). For example, a variation of 0.1 MeV in Q_α may get accumulated and may change the integral [Eq. (7)] and subsequently result in a big change (order of magnitude) in $T_{1/2}$. The corresponding half-lives for ^{222}Rn and ^{226}Ra obtained by using the calculated (DEF) Q values are also shown by arrows in Fig. 8(a).

It is to be mentioned that though the Q values obtained in the relativistic mean field framework are reasonably close (within 1–2 MeV) to the experiment, these may not be suitable for the calculation of half-lives in SAFM because of the hypersensitivity of the half-lives on Q values. On the other hand, the various semiempirical mass formulas are designed to yield more reliable Q values. Therefore, it is to be stressed that the intention here is not to replace the semiempirical mass formulas by the microscopic RMF calculations, particularly for the determination of half-lives. Both the microscopic calculations and the semiempirical mass formulas have their own virtues.

It is observed that $Q_{expt.}$+WKB agree well with the experiment as well as with $Q_{expt.}$+MVS. This indicates that the microscopic calculation of the interaction energy in the $t\rho\rho$ approximation, which also corresponds to the real part of the optical potential between the α and the daughter nucleus, is reliable. Therefore, this potential can be used with confidence in the reaction calculations.

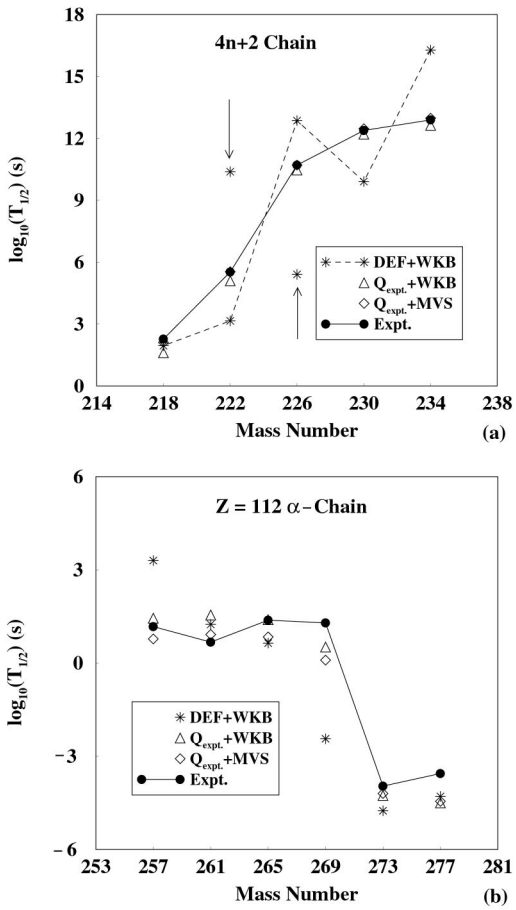

FIG. 8. a (b) The calculated $\log_{10} T_{1/2}$ values for $4n+2$ ($Z=112$) α-decay chain. For arrows, see caption of Fig. 5).

Similar calculations have been carried out for cluster (heavier than α-particle) radioactivity. As an illustration, we present the results for the ^{24}Ne cluster emission from ^{234}U:

$$^{234}\mathrm{U} \rightarrow {}^{24}\mathrm{Ne} + {}^{210}\mathrm{Pb}$$

The results are summarized in Table I. As expected, the RMF successfully describes the ground state properties like the binding energies (BE), charge radii (r_c), etc. The Q values are also in good agreement with the experiment. The calculated half-life is closer to the experiment. The reason here is that the barrier is narrower (turning points are $\approx$9.4 and $\approx$19.1 fm), as compared to the case of α-radioactivity.

The results obtained using the other successful Lagrangian parameter sets (e.g., NL1 [6], NL-SH [28], NL-Z2 [29]) exhibit identical systematics. Therefore, the conclusions/inferences drawn here will be general and will also hold for these parameter sets.

V. SUMMARY AND CONCLUSIONS

In summary, the results for the α-radioactivity and cluster radioactivity of the actinide and the superheavy nuclei are

TABLE I. The calculated ground state properties (BE, r_c, and β) of the nuclei appearing in the cluster (^{24}Ne) decay of ^{234}U, along with the corresponding experimental values. The Möller-Nix values [15] (for β) are also included (in parentheses) for comparison.

	$^{234}_{92}$U			$^{210}_{82}$Pb			$^{24}_{10}$Ne	
BE	r_c	β	BE	r_c	β	BE	r_c	β
−1779.048	5.84	0.25	−1648.259	5.54	0.00	−189.337	2.90	−0.23
(−1778.573)	(5.83)[a]	(0.22)[b]	(−1645.567)	(5.50)[c]	(0.00)[b]	(−191.836)	(2.89)[d]	(−0.22)[b]
	Q-value :58.55 (58.83)[e]						log$T_{1/2}$(s): 25.65 (25.25)[f]	

[a]Reference [24].
[b]Reference [15].
[c]Reference [25].
[d]References [25,26].
[e]Reference [13].
[f]Reference [27].

presented. The procedure consists of three independent steps. In the first step, the ground state properties of the concerned nuclei are successfully described in the RMF framework. The calculated mean field densities are used along with the DDM3Y nucleon-nucleon interaction potential to compute the internuclear potential between α/cluster and the daughter nucleus in the double folding ($t\rho\rho$ approximation) procedure. This interaction potential along with the calculated and also the experimental Q values are used in the WKB approximation to obtain the half-lives of these decays. The calculated $T_{1/2}$ is shown to be very sensitive to the Q values. However, the calculated interaction potential seems to be reliable and can be used as the real part of the optical potential in the reaction studies involving these (α/cluster and daughter) nuclei.

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