Uncertainty in the $0\nu\beta\beta$ decay nuclear matrix elements

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The nuclear matrix elements $M^{0\nu}$ of the neutrinoless double-beta decay $(0\nu\beta\beta)$ are evaluated for ^{76}Ge, ^{100}Mo, ^{130}Te, and ^{136}Xe within the renormalized quasiparticle random phase approximation (RQRPA) and the simple QRPA. Three sets of single particle level schemes are used, ranging in size from 9 to 23 orbits. When the strength of the particle-particle interaction is adjusted so that the $2\nu\beta\beta$ decay rate is correctly reproduced, the resulting $M^{0\nu}$ values become essentially independent of the size of the basis, and of the form of different realistic nucleon-nucleon potentials. Thus, one of the main reasons for variability of the calculated $M^{0\nu}$ within these methods is eliminated.

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The discovery of oscillations of atmospheric [1], solar [2], and reactor [3] neutrinos shows that neutrinos have a nonvanishing rest mass. However, since the study of oscillations provides only information on the mass-squared differences, it cannot by itself determine the absolute values of the masses (or even, at present, the sign of the mass-squared differences). Moreover, flavor oscillations are insensitive to the charge conjugation properties of the neutrinos, i.e., whether massive neutrinos are Dirac or Majorana particles. Study of the neutrinoless double-beta decay $(0\nu\beta\beta)$ is the best potential source of this crucial information on the Majorana nature of the neutrinos and on the absolute mass scale.

The search for $0\nu\beta\beta$ decay has a long history. So far, the decay has not been seen, but impressive limits on its half-life have been established (for the review of the field, see, e.g., Ref. [4] or Ref. [5]). With the progress in oscillation studies, which now established the mass scale $m_{scale} \approx \sqrt{\Delta m^2}$, a wave of enthusiasm emerged in the community to develop a new generation of experiments, sensitive to such mass scale. In that context it is crucial to develop in parallel methods capable of reliably evaluating the nuclear matrix elements, and realistically asses their uncertainties. The goal of the present work is to make a contribution to that endeavor.

Clearly, the determination of the effective Majorana mass $\langle m_\nu \rangle$ can be only as good as the knowledge of the nuclear matrix elements, since the half-life of the $0\nu\beta\beta$ decay and $\langle m_\nu \rangle$ are related by

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M^{0\nu}|^2|\langle m_\nu \rangle|^2, \tag{1}$$

where $G^{0\nu}(E_0,Z)$ is the precisely calculable phase-space factor, and $M^{0\nu}$ is the corresponding nuclear matrix element. Thus, obviously, any uncertainty in $M^{0\nu}$ makes the value of $\langle m_\nu \rangle$ equally uncertain. In turn,

$$\langle m_\nu \rangle = \sum_i^N |U_{ei}|^2 e^{i\alpha_i} m_i \quad (\text{all } m_i \geqslant 0), \tag{2}$$

where α_i are the unknown Majorana phases. The elements of the mixing matrix $|U_{ei}|^2$ and the mass-squared differences Δm^2 can be determined in the oscillation experiments. Using the present knowledge of these quantities, (see, e.g., Ref. [6]) and limits on $T_{1/2}$, one could decide whether the neutrino mass pattern is degenerate, or follows the inverse or normal hierarchies. If, on the other hand, the existence of the $0\nu\beta\beta$ decay is proven and the value of $T_{1/2}$ is found, a relatively narrow range of absolute neutrino mass scale can be determined, independently of the phases α_i in most situations [4,6]. However, such an important insight is possible only if the nuclear matrix elements are accurately known.

The nuclear matrix element $M^{0\nu}$ is defined as

$$M^{0\nu} = \langle f | -\frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu} | i \rangle \tag{3}$$

where $|i\rangle(|f\rangle)$ are the wave functions of the ground states of the initial (final) nuclei. The explicit forms of the operators $M_F^{0\nu}$, $M_{GT}^{0\nu}$, and $M_T^{0\nu}$ are given in Ref. [7]. In comparison with most of previous $0\nu\beta\beta$ decay studies [8–11] the higher order terms of the nucleon current are also included in the present calculation, resulting in a suppression of the nuclear matrix element by about 30% [7]. Note that in the numerical calculation of $M^{0\nu}$ here the closure approximation is avoided and the unquenched values $g_A = 1.25$, $g_V = 1.0$ are used.

Two basic methods are used in the evaluation of $M^{0\nu}$, the quasiparticle random phase approximation (QRPA) with its various modifications and the nuclear shell model (NSM). These two approaches represent in some sense opposite extremes.

In the QRPA one can include essentially unlimited set of single-particle states, but only a limited subset of configurations (iterations of the particle-hole, respectively two-quasiparticle configurations). In the context of QRPA several issues have been raised, and deserve a systematic study.

(1) For realistic interactions the QRPA solutions are near the point of the so-called collapse and thus its applicability is questionable. Numerous attempts have been made to extend

the method's range of validity by partially avoiding the violation of the Pauli principle. Here we consider the simplest of them, the renormalized QRPA (RQRPA) [8,12]. We compare the results of RQRPA with those of the standard QRPA [9–11].

(2) The choice of the size of single-particle (s.p.) space is to some extent arbitrary, often dictated by convenience. What effect this choice has on the $M^{0\nu}$ values is the main thrust of the present work.

(3) There are various forms of the nucleon-nucleon potential that lead to somewhat different forms of the resulting G-matrix.[1] By comparing the results obtained with three such potentials (charge dependent Bonn [16], Argonne [17], and Nijmegen [18]) we show that the resulting $M^{0\nu}$ are essentially identical, and independent of the choice of the realistic nucleon-nucleon potential.

In contrast, in the NSM one chooses a limited set of single-particle states in the vicinity of the Fermi level, and includes all (or most) configurations of the valence nucleons on these orbits in the evaluation of $M^{0\nu}$. The main open question in this approach is to determine the effects of the neglected single-particle states further away from the Fermi level. As shown below, we have also performed QRPA and RQRPA calculations with the set of s.p. states used usually in the NSM. It appears that these methods, at least with the nucleon-nucleon interaction we used, are not applicable for such small s.p. bases.

In what follows the $0\nu\beta\beta$ decay nuclear matrix elements $M^{0\nu}$ for ^{76}Ge, ^{100}Mo, ^{130}Te, and ^{136}Xe are evaluated. These nuclei are most often considered as candidate sources for the next generation of the experimental search for $0\nu\beta\beta$ decay. For each of them three choices of the s.p. basis are considered. The smallest one has 9 levels (oscillator shells $N=3,4$) for ^{76}Ge, and 13 levels (oscillator shells $N=3,4$ plus the $f+h$ orbits from $N=5$) for ^{100}Mo, ^{130}Te, and ^{136}Xe. For the intermediate size s.p. base the $N=2$ shells in ^{76}Ge and ^{100}Mo are added, and for ^{130}Te and ^{136}Xe also the p orbits from $N=5$. Finally, the largest s.p. space [8] contains 21 levels for ^{76}Ge and ^{100}Mo (all states from shells $N=1$–5), and 23 levels for ^{130}Te and ^{136}Xe ($N=1$–5 and i orbits from $N=6$). Thus the smallest set corresponds to $1\hbar\omega$ particle-hole excitations, and the largest to about $4\hbar\omega$ excitations. The s.p. energies have been calculated with the Coulomb corrected Woods-Saxon potential.

It is well known that the residual interaction is an effective interaction that depends on the size of the single-particle basis. Hence, when the basis is changed, the interaction should be modified as well. Here we propose a rather simple way to accomplish the needed renormalization.

There are three important ingredients in QRPA and RQRPA. First, the pairing interaction has to be included by solving the corresponding gap equations. Within the BCS method the strength of the pairing interaction depends on the size of the s.p. basis. As usual, we multiply the pairing part of the interaction by a factor g_{pair} whose magnitude is adjusted, for both protons and neutrons, such that the pairing gap is correctly reproduced, separately for the initial and final nuclei.

Second, QRPA equations of motion contain a block corresponding to the particle-hole interaction, renormalized by an overall strength parameter g_{ph}. That parameter is typically adjusted by requiring that the energy of some chosen collective state, often the giant Gamow-Teller (GT) resonance, is correctly reproduced. We find that the calculated energy of the giant GT state is almost independent of the size of the s.p. basis and is well reproduced with $g_{ph}\approx 1$. Accordingly, we use $g_{ph}=1$ throughout, without adjustment.

Finally, QRPA equations of motion contain a block corresponding to the particle-particle interaction, renormalized by an overall strength parameter g_{pp}. (The importance of the particle-particle interaction for the β strength was recognized first in Ref. [19], and for the $\beta\beta$ decay in Ref. [9].) It is well known that the decay rate for both modes of $\beta\beta$ decay depends sensitively on the value of g_{pp}. In the following we use this property to find the value of g_{pp} for each of the possible s.p. bases. The value of the parameter g_{pp} is fixed in each case so that the known half-life of the $2\nu\beta\beta$ decay is correctly reproduced. The $2\nu\beta\beta$ half-lives and average matrix elements collected in Table 1 of Ref. [4] are used, where the original references to the corresponding experiments can be found. A similar compilation of the $2\nu\beta\beta$ data can be found in Ref. [20]. The resulting adjusted values of g_{pp} are shown in Table I for both the RQRPA and QRPA methods. One can see that as the basis increases, the effective g_{pp} decreases[2] as expected.

The adjustment of g_{pp} is a crucial point of the present work. Several studies of the sensitivity of the nuclear matrix elements $M^{0\nu}$ to various modifications of the QRPA method as well as to the number of s.p. states and values of g_{pp} were made in the recent past [8,21,22]. Typically, these studies concluded that the values of $M^{0\nu}$ vary substantially depending on all of these things. While our calculations show a similar trend, we demonstrate in this work that by requiring that the known $2\nu\beta\beta$ decay half-life is correctly reproduced, and adjusting the parameter g_{pp} accordingly, we remove much of the sensitivity on the number of single-particle states, on the NN potential employed, and even on whether RQRPA or just simple QRPA methods are used.

Clearly, the chosen procedure of finding the effective interaction is rather crude. Ideally, properly evaluated effective Hamiltonian as well as the corresponding effective $\beta\beta$ operator should be used in each case. However, as shown below, the chosen procedure appears to be sufficient to stabilize the values of the nuclear matrix elements $M^{0\nu}$. Note that we did not use the quenched value of the axial current coupling constant g_A, as is often done in studies of ordinary β decay. We are convinced that with the quenched g_A the basic conclusion of our work will be similar, even though the values of g_{pp} in Table I would be of course different.

[1]Brueckner reaction matrix elements with these nucleon-nucleon potentials were calculated and used in Refs. [13–15].

[2]For ^{136}Xe the 2ν half-life is unknown. We bracket the possible values of g_{pp} by using the experimental lower half-life limit (the upper line for ^{136}Xe) and a vanishing 2ν matrix element (i.e., an infinitely long half-life) for the lower lines.

TABLE I. Values of the effective particle-particle strength parameter g_{pp} for the different nuclei and different sizes of the single-particle space. The values in column 3 correspond to the minimal s.p. space, in column 4 to the intermediate one, and in column 5 to the largest s.p. space. In each of these there are three entries corresponding to the G matrix based on the Bonn, Argonne, and Nijmegen potentials (in that order). For every considered nucleus the upper line was obtained in RQRPA and the lower one in the simple QRPA. In column 2 we give the corresponding half-life of the $2\nu\beta\beta$ decay used in the adjustment (for ^{136}Xe see footnote 2 for explanation).

Nucleus	$T_{1/2}^{2\nu}$ (in 10^{20} yr)	min. s.p. space	interm. s.p. space	largest s.p. space
^{76}Ge	13.0	0.99 1.12 1.07	0.88 1.00 0.95	0.79 0.88 0.84
		0.85 0.96 0.91	0.75 0.85 0.81	0.65 0.72 0.69
^{100}Mo	0.08	1.21 1.35 1.30	1.09 1.21 1.17	1.00 1.10 1.07
		0.94 1.03 1.00	0.85 0.93 0.90	0.78 0.85 0.83
^{130}Te	27.0	0.97 1.10 1.05	0.90 1.01 0.97	0.84 0.94 0.90
		0.84 0.94 0.90	0.78 0.86 0.83	0.71 0.78 0.76
^{136}Xe	8.1	0.82 0.93 0.89	0.77 0.87 0.83	0.72 0.82 0.78
		0.73 0.83 0.79	0.68 0.77 0.74	0.62 0.70 0.67
	∞	0.88 1.00 0.95	0.82 0.93 0.89	0.77 0.87 0.83
		0.78 0.88 0.85	0.72 0.81 0.78	0.66 0.74 0.71

Alternative procedures of adjusting the parameters, by requiring a good agreement with ordinary β decay, have been used for a long time (see, e.g., Ref. [23,24]). We believe that the 2ν decay rate, involving the same initial and final states as the 0ν decay, is particularly suitable for such adjustment. Naturally, in an ideal, but as far as the considered methods' unrealistic situation, the nuclear model should reproduce many spectroscopic properties (energy levels and transition strengths) of the involved nuclei.

Having fixed the parameters of the effective Hamiltonian we can proceed and evaluate the $0\nu\beta\beta$ nuclear matrix elements $M^{0\nu}$ and then the corresponding half-life (we list the phase-space factors in Table II in units of 10^{-25} yr for $\langle m_\nu\rangle$ in eV.) Since short-range nucleon-nucleon repulsion is not explicitly included in our method, we take it into account in the standard way, i.e., by multiplying the operators with the square of the correlation Jastrow-like function [25],

$$f(r) = 1 - e^{-ar^2}(1 - br^2), \quad a = 1.1\ \mathrm{fm}^{-2}, \quad b = 0.68\ \mathrm{fm}^{-2}. \tag{4}$$

Thus, we effectively prevent the two participating nucleons from being very close to each other. Note that the effect of short-range correlations reduces the matrix elements $M^{0\nu}$ by a factor of about 2, in agreement with other evaluations.

As pointed out earlier, in the nuclear shell model an even smaller set of single-particle states is used corresponding to $0\hbar\omega$. This choice reflects the practical computational limitations in handling the extremely large number of possible configurations, while it seems to be sufficient to describe the spectroscopy of low-lying nuclear states. In the NSM evaluation of the $\beta\beta$ decay rates [26] four s.p. orbits $(f_{5/2}, p_{3/2}, p_{1/2}, g_{9/2})$ were used for ^{76}Ge and five orbits $(d_{5/2}, d_{3/2}, s_{1/2}, g_{7/2}, h_{11/2})$ for ^{130}Te and ^{136}Xe. These s.p. sets are free of the spurious center-of-mass states, but obviously miss a large part of the GT strength as well as of the strength corresponding to the higher multipoles. In order to describe GT transitions between low-lying states in the NSM, it is necessary to quench the corresponding strength. This is most conveniently formally achieved by using g_A=1.0 instead of the free nucleon value of g_A=1.25. We follow this prescription in our attempt to use this smallest s.p. space, and only there.

TABLE II. Values of the nuclear matrix element $|M^{0\nu}|$ for the different nuclei and different sizes of the single-particle space. The phase space factors $G^{0\nu}$ are listed in column 2. For the explanation of the notation in columns 3–5 see caption of Table I.

Nucleus	$G^{0\nu}$ (in 10^{-25} yr eV^{-2})	min. s.p. space	interm. s.p. space	largest s.p. space
^{76}Ge	0.30	2.41 2.37 2.35	2.52 2.44 2.47	2.32 2.34 2.35
		2.68 2.62 2.65	2.81 2.74 2.72	2.62 2.64 2.64
^{100}Mo	2.19	1.08 1.08 1.05	1.12 1.14 1.08	1.28 1.34 1.27
		1.19 1.22 1.19	1.25 1.28 1.20	1.38 1.41 1.40
^{130}Te	2.12	1.42 1.32 1.34	1.40 1.33 1.32	1.17 1.13 1.14
		1.50 1.42 1.40	1.46 1.42 1.39	1.22 1.17 1.15
^{136}Xe	2.26	1.05 0.99 0.99	1.09 1.03 1.04	0.92 0.83 0.86
		1.11 1.02 1.05	1.15 1.06 1.06	0.98 0.90 0.91
		0.82 0.74 0.77	0.86 0.77 0.78	0.66 0.59 0.61
		0.86 0.78 0.75	0.92 0.84 0.83	0.70 0.63 0.64

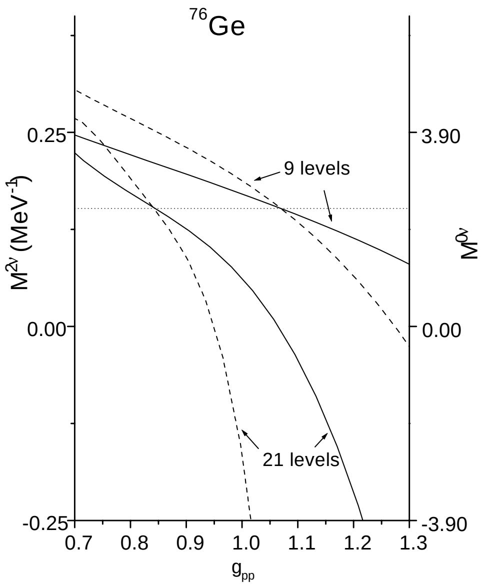

FIG. 1. Dependence of the matrix elements $M_{2\nu\beta\beta}$ (left scale, dashed lines) and $M_{0\nu\beta\beta}$ (right scale, full lines) on the parameter g_{pp}. Calculations were performed for 9 and 21 s.p. levels for ^{76}Ge as indicated; the Nijmegen potential and RQRPA method were used. The thin dotted horizontal line indicates that by fixing g_{pp} to reproduce the experimental value $M_{2\nu\beta\beta}$=0.15 MeV^{-1} the value of $M_{0\nu\beta\beta}$ is also stabilized.

It appears that it is impossible to describe the $2\nu\beta\beta$ decay in such s.p. space using QRPA or RQRPA, and the nucleon-nucleon potentials employed in this work. One would have to renormalize the particle-particle block too much, with $g_{pp}\sim 2.0$, unlike the rather modest renormalization shown in Table I. With such large value of g_{pp} the interaction is too far removed from the G matrix used in the rest of this work. Therefore, one cannot expect to obtain sensible 0ν matrix elements. In fact, we obtained very small matrix elements in this case for ^{130}Te and ^{136}Xe, while, perhaps accidentally, for ^{76}Ge they are in a crude agreement with the NSM result [26].

We list the results with the three larger single-particle bases in Table II which represents the most significant part of the present work. As one can see by inspecting the entries, one can draw two important conclusions.

(1) The resulting $M^{0\nu}$ do not depend noticeably on the form of the nucleon-nucleon potential used. That is not an unexpected result.

(2) Even more importantly, with our choice of g_{pp} the results are also essentially independent of the size of the s.p. basis. This is a much less obvious and rather pleasing conclusion. It can be contrasted with the result one would get for a constant g_{pp} independent of the size of the s.p. basis. The values of $M^{0\nu}$ then differ between the small and large bases by a factor of 2 or more.

The effect of the g_{pp} adjustment is illustrated in Fig. 1, showing that our procedure leads to almost constant $M^{0\nu}$ matrix elements. On the other hand, by choosing a fixed value of g_{pp} (e.g., g_{pp}=1.0) the resulting $M^{0\nu}$ matrix elements for 9 and 21 s.p. levels would differ substantially (by a factor of 2.8).

One can qualitatively understand why our chosen procedure stabilizes the $M^{0\nu}$ matrix elements as follows: The $M^{2\nu}$ matrix elements involve only the 1^+ (virtual) states in the intermediate odd-odd nucleus. The nuclear interaction is such that the Gamow-Teller correlations (spin 1, isospin 0 pairs; after all the deuteron is bound and the dineutron is not) are very near the corresponding phase transition in the 1^+ channel (corresponding to the collapse of the QRPA equations of motion). The contributions of the 1^+ multipole for both modes of the $\beta\beta$ decay (2ν and 0ν) depend therefore very sensitively on the strength of the particle-particle force, parametrized by the g_{pp}. On the other hand, the $M^{0\nu}$ matrix element, due to the presence of the neutrino propagator, depends on many multipole states of the virtual intermediate odd-odd nucleus. These other multipoles (other than 1^+) correspond to small amplitudes of the collective motion; there is no instability. Hence, they are insensitive to the value of g_{pp}. By making sure that the contribution of the 1^+ multipole is fixed, we stabilize the $M^{0\nu}$ value. The fact that RQRPA essentially removes the instability becomes then essentially irrelevant thanks to the adjustment of g_{pp}.

The entries in Table II are relatively close to each other. To emphasize this feature, each calculated value is treated as an independent determination and for each nucleus the corresponding average $\langle M^{0\nu}\rangle$ matrix element (averaged over the three potentials and the three choices of the s.p. space) is evaluated, as well as its variance σ:

$$\sigma^2=\frac{1}{N-1}\sum_{i=1}^{N}(M_i^{0\nu}-\langle M^{0\nu}\rangle)^2 \qquad (N=9). \tag{5}$$

These quantities (with the value of σ in parentheses) are shown in Table III. Not only is the variance substantially

TABLE III. Averaged $0\nu\beta\beta$ nuclear matrix elements $\langle M^{0\nu}\rangle$ and their variance σ (in parentheses) evaluated in the RQRPA and QRPA. In column 4 the $0\nu\beta\beta$ half-lives evaluated with the RQRPA average nuclear matrix element and for the $\langle m_\nu\rangle$=50 meV are shown. The two entries for ^{136}Xe correspond to the two extremes of adjustment for the unknown half-life of the $2\nu\beta\beta$ decay (see text).

Nucleus	RQRPA	QRPA	$T_{1/2}$ (in 10^{27} yr for $\langle m_\nu\rangle$=50 meV)
^{76}Ge	2.40(0.07)	2.68(0.06)	2.3
^{100}Mo	1.16(0.11)	1.28(0.09)	1.4
^{130}Te	1.29(0.11)	1.35(0.13)	1.1
^{136}Xe	0.98(0.09)	1.03(0.08)	1.9
	0.73(0.09)	0.77(0.10)	3.2

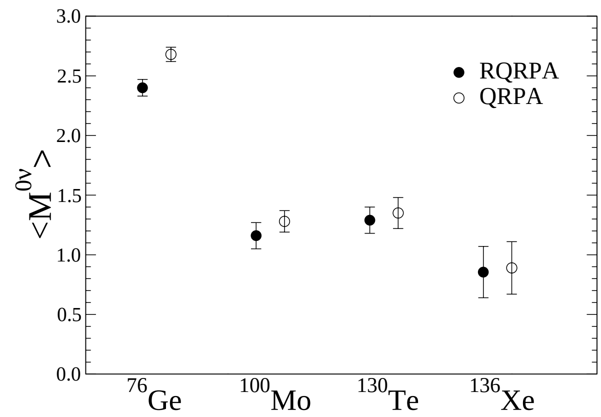

FIG. 2. Average nuclear matrix elements $\langle M^{0\nu}\rangle$ and their variance for both methods and for the four considered nuclei. For ^{136}Xe the error bars encompass the whole interval related to the unknown rate of the $2\nu\beta\beta$ decay.

less than the average value, but the results of QRPA, albeit slightly larger, are quite close to the RQRPA values. The averaged nuclear matrix elements for both methods and their variance are shown in Fig. 2.

Combining the average $\langle M^{0\nu}\rangle$ with the phase-space factors listed in Table II the expected half-lives (for RQRPA and $\langle m_\nu\rangle$=50 meV, the scale of neutrino masses suggested by oscillation experiments) are also shown in Table III. These predicted half-lives are a bit longer (particularly for the last three nuclei on our list) than various QRPA calculations usually predict. They are faster, however, than the shell model results of Ref. [26].

Given the average nuclear matrix elements in Table III and the phase-space factors in Table II one can find a limit (or actual value) of the effective neutrino mass $\langle m_\nu\rangle$ from any limit (or value) of $T_{1/2}$ from

$$\langle m_\nu\rangle(\mathrm{eV}) = (T_{1/2}G^{0\nu})^{-1/2}\frac{1}{\langle M^{0\nu}\rangle}. \tag{6}$$

In conclusion, we have developed a "practical" way of stabilizing the values of the $0\nu\beta\beta$ nuclear matrix elements against their variation caused by the modification of the nucleon-nucleon interaction potential and the chosen size of the single-particle space. We have also shown that the procedure yields very similar matrix elements for the QRPA and RQRPA variants of the basic method. Even though we cannot guarantee that this basic method is trustworthy, we have eliminated, or at least greatly reduced, the arbitrariness commonly present in the published calculations.

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