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Probing mean field of neutron rich nuclei by cold fission

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We propose the cold (neutronless) fission process as a tool for probing mean field properties of exotic neutron rich nuclei. The fissioning state is described as a resonance in the potential well between the emitted fragments. Coupled channels analysis shows that the double fine structure of the binary cold fission is very sensitive to the density distribution. In this way cold fission can be used to investigate the density profile of neutron rich fragments.

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One of the most active fields in today's nuclear physics is the study of nuclei far from β-stability line using radioactive beams. A very powerful tool to investigate proton rich nuclei is proton emission, because the unbound particle is kept for enough time inside the Coulomb barrier. On the other extreme the probing of density profiles in neutron rich nuclei still remains a dream. The only experimental information coming from radioactive beams is γ spectroscopy but the information based on electromagnetic transitions is not very sensitive to the nuclear density tail. Therefore until now it was possible to make only some theoretical predictions concerning the mean field of nuclei close to the neutron drip line. In the last decade many papers were devoted to the prediction of single particle levels [1–3] and density profiles of such nuclei [4]. For a review of such calculations see, for instance, Ref. [5].

On the other hand, in the last years an intense experimental activity was performed in order to investigate cold (neutronless) binary and ternary fission process of ^{252}Cf [6–14]. It involved modern facilities, as the Gammasphere and Eurogam, which were able to identify this process using the triple γ-rays coincidence technique. The emitted fragments are neutron rich but until now the opportunity to analyze their structure was hindered by the fact that only total relative yields were available.

Recently the relative yields of rotational states were extracted from intensities of γ rays emitted in coincidence, for ^{104}Mo-^{148}Ba and ^{106}Mo-^{146}Ba [7]. It was shown that the cold fission population is centered around the low-lying 2^+ and 4^+ states and the states higher than 6^+ are practically not populated. This proves the assumption concerning the cold rearrangement of nucleons during the cold fission [15,16]. A very convincing theoretical evidence that it has a sub-barrier character was the WKB penetration calculation, using a double folding potential with M3Y plus Coulomb nucleon-nucleon forces. This simple estimate was able to reproduce the gross features of the binary cold fragmentation isotopic yields from ^{252}Cf [17].

In two recent papers [18,19] we analyzed the double fine structure of emitted fragments within the stationary scattering formalism. The fissioning state was identified with a resonance in the interfragment potential, calculated using the double folding procedure. In order to simplify the description we used a planar harmonic oscillator basis in the overlapping region.

The aim of this paper is to analyze the influence of neutron densities on relative low-lying rotational yields of emitted fragments. To this purpose we generalized our previous version of the coupled channels procedure by using a rotational basis. This allows us to take into account not only planar, but also torsional degrees of freedom.

The fission process is described by pure outgoing solutions of the stationary Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}\nabla_R^2 + H_1(\Omega_1) + H_2(\Omega_2) + V(\mathbf{R},\Omega_1,\Omega_2)\right]\Psi(\mathbf{R},\Omega_1,\Omega_2) = E\Psi(\mathbf{R},\Omega_1,\Omega_2), \tag{1}$$

where μ is the reduced mass of the dinuclear system and H_k are the Hamiltonians describing the rotation of fragments. Here $\mathbf{R}=(R,\Omega)$ denotes the distance between the centers of two deformed nuclei. The orientation of their major axes in the laboratory system is given by Euler angles $\Omega_k=(\varphi_k,\theta_k,0), k=1,2$.

We estimate the fission barrier in terms of the double folding between the nuclear densities [20] by using the M3Y nucleon-nucleon [21] plus Coulomb force

$$V(\mathbf{R},\Omega_1,\Omega_2) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho^{(1)}(\mathbf{r}_1)\rho^{(2)}(\mathbf{r}_2) v(\mathbf{R}+\mathbf{r}_1-\mathbf{r}_2). \tag{2}$$

We suppose that the emitted fragments are axially deformed. Their nuclear densities are parametrized in the intrinsic system of coordinates by deformed Fermi distributions as follows:

$$\rho^{(k)}(\mathbf{r}_k) = \frac{\rho_0^{(k)}}{1+\exp[(r_k-c^{(k)})/a^{(k)}]}, \quad k=1,2, \tag{3}$$

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$$c^{(k)} = c_0^{(k)} \left[1 + \sum_{\lambda \geq 2} \beta_\lambda^{(k)} Y_{\lambda 0}(\Omega'_k) \right],$$

where the nuclear radius is given by

$$c_0^{(k)} = (r_0 + w^{(k)}) A^{1/3}, \tag{4}$$

$$r_0 = (d - 1/d) A^{-1/3}, \quad d = 1.28 A^{1/3} + 0.8 A^{-1/3} - 0.76.$$

Here we dropped the isospin index. For $w^{(k)}=0$ one obtains the standard liquid drop expression. The energy of the system is adjusted by using an internal repulsive core, depending on some strength parameter as in Ref. [19]. We expand the interaction and the wave function in the laboratory system

$$V(\mathbf{R}, \Omega_1, \Omega_2) = \sum_{\lambda_0 \lambda_1 \lambda_2} V_{\lambda_0 \lambda_1 \lambda_2}(R) \mathcal{Y}_{\lambda_0 \lambda_1 \lambda_2}(\Omega, \Omega_1, \Omega_2), \tag{5}$$

$$\Psi(\mathbf{R}, \Omega_1, \Omega_2,) = \frac{1}{R} \sum_{l I_1 I_2} f_{l I_1 I_2}(R) \mathcal{Y}_{l I_1 I_2}(\Omega, \Omega_1, \Omega_2).$$

Here the angular part has the following ansatz

$$\mathcal{Y}_{\lambda_0 \lambda_1 \lambda_2}(\Omega, \Omega_1, \Omega_2) = \{ Y_{\lambda_0}(\Omega) \otimes [Y_{\lambda_1}(\Omega_1) \otimes Y_{\lambda_2}(\Omega_2)]_{\lambda_0} \}_0. \tag{6}$$

Thus, we suppose that rotational states of fragments belong to the ground band. The above expression does not depend on the whole orientation of the system in space, given by Ω.

By using the orthonormality of angular functions one obtains in a standard way the coupled system of differential equations for radial components as follows:

$$\begin{aligned} \frac{d^2 f_{l I_1 I_2}(R)}{dR^2} = & \left\{ \frac{l(l+1)}{R^2} + \frac{2\mu}{\hbar^2} \right. \\ & \left. \times [V_0(R) - (E - E_{I_1} - E_{I_2})] \right\} \delta_{ll'} \delta_{I_1 I'_1} \delta_{I_2 I'_2} \\ & + \frac{2\mu}{\hbar^2} \sum_{l', I'_1, I'_2 \neq 0,0,0} \langle \mathcal{Y}_{l I_1 I_2} | V(R) | \mathcal{Y}_{l' I'_1 I'_2} \rangle \, f_{l' I'_1 I'_2}(R). \end{aligned} \tag{7}$$

Here $V_0(R)$ is the spherical part of the interaction and E_{I_1}, E_{I_2} are the ground band energies of emitted fragments.

In order to find resonant states we compute the matrix of internal fundamental solutions $\mathcal{R}(R)$ by a forward integration, starting with the unity matrix inside the repulsive core. We also find by a backward integration the matrix of external outgoing fundamental solutions $\mathcal{H}^{(+)}(R)$, starting at large distances with Hankel-Coulomb functions $H^{(+)}(R)$ on the diagonal. The radial vector function $f(R)$ is built as a linear combination of these fundamental solutions for the internal and external regions separately. We match them at some radius R_1 inside the barrier, i.e.,

TABLE I. The rotational basis (l, I_1, I_2).

No.	l	I_1	I_2
1	0	0	0
2	0	2	2
3	0	4	4
4	2	2	0
5	2	0	2
6	2	2	2
7	2	4	4
8	2	4	2
9	2	2	4
10	4	4	0
11	4	0	4
12	4	2	2
13	4	4	4
14	4	4	2
15	4	2	4

$$f(R_1) = \mathcal{R}(R_1) C = \mathcal{H}^{(+)}(R_1) S. \tag{8}$$

By using a similar condition for derivatives one obtains the following secular equation

$$\det \begin{bmatrix} \mathcal{R}(R_1) & \mathcal{H}^{(+)}(R_1) \\ d\mathcal{R}(R_1)/dR & d\mathcal{H}^{(+)}(R_1)/dR \end{bmatrix} = 0. \tag{9}$$

The roots of system (9) correspond to the poles of the S matrix and do not depend upon the matching radius R_1.

The two vectors of coefficients C and S are fully determined from the normalization of the wave function in the internal region. By using the continuity equation one obtains the total decay width as a sum over partial channel widths, i.e.,

$$\Gamma = \sum_{l I_1 I_2} \Gamma_{l I_1 I_2} = \sum_{l I_1 I_2} \hbar v_{I_1 I_2} \lim_{R \to \infty} |f_{l I_1 I_2}(R)|^2 = \sum_{l I_1 I_2} \hbar v_{I_1 I_2} |S_{l I_1 I_2}|^2, \tag{10}$$

where $v_{I_1 I_2}$ is the center of mass velocity at infinity in the channel (l, I_1, I_2), i.e.,

$$v_{I_1 I_2} = \sqrt{\frac{2(E - E_{I_1} - E_{I_2})}{\mu}}. \tag{11}$$

We investigated the following splitting:

$$^{252}\mathrm{Cf} \to {}^{104}\mathrm{Mo} + {}^{148}\mathrm{Ba}, \tag{12}$$

where only 2^+ and 4^+ states of the rotational ground band were detected [7]. Therefore in both expansions (5) we used the same rotational basis (l, I_1, I_2) given in Table I, containing angular momenta $0,2,4$.

We analyzed the cold fission (neutronless) process, i.e., by supposing ground-state deformations of final fragments. The fragments are neutron rich unstable nuclei. Their deformations can be determined from electromagnetic transitions, but these values are not available at this moment. Therefore we choose ground-state deformations from the systematics given in Ref. [22]. The corresponding values are $\beta_2^{(1)}=0.349, \beta_4^{(1)}=0.030$ for ^{104}Mo and $\beta_2^{(2)}=0.236, \beta_4^{(2)}=0.131$ for ^{148}Ba. We mention that in Ref. [17] the important role played by hexadecapole deformations on the penetration process was clearly evidenced.

The most favorable fissioning configuration, where the Coulomb barrier has the lowest possible value, is the pole-to-pole one, with $\Omega_1=\Omega_2=\Omega$, i.e.,

$$V_{p\text{-}p}(R) = V(R, \Omega, \Omega, \Omega). \tag{13}$$

As we pointed out the above expression does not depend upon Ω. Before the scission point this potential rapidly increases around this configuration in the direction of angular variables [23]. By approaching the scission point the potential becomes gradually flatter, and for large distances, where the fragments are separated, the interaction is given only by the Coulomb term. In this region the two nuclei are left in excited rotational states. Therefore in our calculation we considered as a spherical component the pole-to-pole interaction $V_{p\text{-}p}(R)$ until the intersection with the true spherical part $V_0(R)$, as in Fig. 1 of Ref. [19]. Beyond this point, which is close to the touching configuration, $V_0(R)$ becomes energetically more favorable and the two fragments start to rotate separately. The energy of the resonant state is given by the Q value, i.e., $E=214.67$ MeV. We adjust it by using the strength of a repulsive core, such as in Ref. [19]. This is a standard procedure used to describe α or heavy cluster decays. It is important to point out that $\Gamma/Q<10^{-4}$, and therefore the roots of the system, given by Eq. (9), are practically real numbers.

For proton densities we considered a standard diffusivity $a_p^{(k)}=0.5$ fm and radial parameter $w_p^{(k)}=0$. The only free parameters, according to Eq. (3) remain $a_n^{(k)}$ and $w_n^{(k)}$. First of all we studied the influence of the neutron diffusivity on the decay widths by considering $w_n^{(k)}=0$. Our calculation showed that partial decay widths remain practically unchanged for a constant value of the sum $a_n^{(1)}+a_n^{(2)}$. We will present our results for a common value $a_n \equiv a_n^{(1)}=a_n^{(2)}$.

It turns out that a small increase of this parameter from $a_n=0.50$ fm up to $a_n=0.53$ fm changes the behavior of radial components beyond the external turning point. The radial probabilities in this region, according to Eq. (10), are proportional to the partial decay widths. As a criterium selecting the resonant state we used the angular distribution of fragments around the pole-to-pole configuration. The first resonance inside the pocketlike potential has a much narrower distribution than higher states and therefore it is the best candidate describing the fission process. The 15 radial components of the wave function $f_l(R), l \to (l, I_1, I_2)$ are plotted in Fig. 1(a) for $a_n=0.50$ fm and in Fig. 1(b) for $a_n=0.53$ fm. They are mainly concentrated in the internal part of the potential and have zero nodes in this region. One can see an increasing of the oscillatory tail in the case (b) with respect to (a).

Therefore we expect an important dependence of partial rotational widths upon the neutron diffusivity. To this purpose we investigated partial yields for each fragment, defined as follows

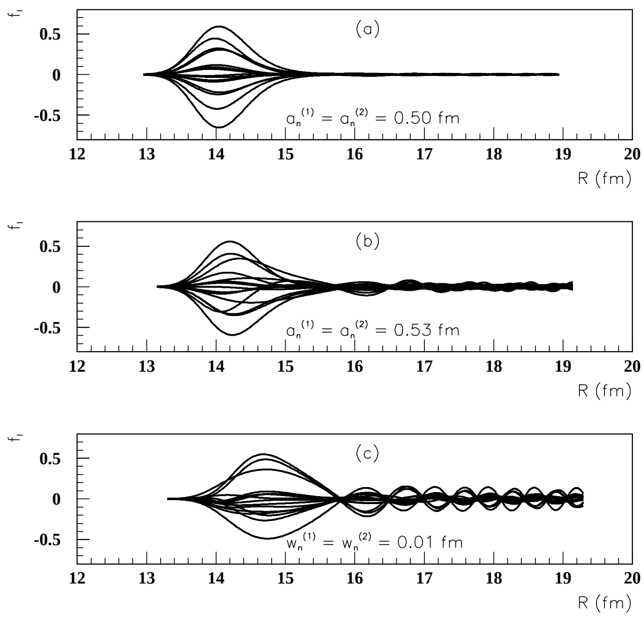

FIG. 1. The radial wave function components $f_l(R)$ for neutron density parameters in Eq. (3) $a_n=0.50$ fm, $w_n=0$ (a), $a_n=0.53$ fm, $w_n=0$ (b), and $w_n=0.01$ fm, $a_n=0.5$ fm (c). The basis states are given in Table I. The fission process is $^{252}\mathrm{Cf} \to {}^{104}\mathrm{Mo} + {}^{148}\mathrm{Ba}$.

$$y_0^{(k)} = 100[\Gamma_0^{(k)} + \Gamma_2^{(k)} + \Gamma_4^{(k)}]/\Gamma, \quad k=1,2, \tag{14}$$

$$y_2^{(k)} = 100[\Gamma_2^{(k)} + \Gamma_4^{(k)}]/\Gamma,$$

$$y_4^{(k)} = 100\ \Gamma_4^{(k)}/\Gamma,$$

where the decay widths for each fragment are, respectively, given by

$$\Gamma_I^{(1)} = \sum_{lI'} \Gamma_{lII'}, \quad \Gamma_I^{(2)} = \sum_{lI'} \Gamma_{lI'I}. \tag{15}$$

Here the upper index (1) denotes ^{104}Mo, while (2) ^{148}Ba. The summed yields (14) can be measured by using triple γ-rays coincidence, except $y_0^{(k)}$ measured by charge detectors. In Fig. 2(a) the summed yields/fragment (14) as a function of a_n are given. By solid lines we give the values of ^{104}Mo and by dashes of ^{148}Ba for different angular momenta.

At this moment only the following relative yields are available:

$$\gamma_4^{(k)} = 100\frac{y_4^{(k)}}{y_2^{(k)}}, \quad k=1,2. \tag{16}$$

The dependence of relative hexadecapole yields versus a_n is given in Fig. 2(b). From comparable values their ratio increases by three times over the investigated interval. It is important to point out that by changing neutron diffusivity the relative hexadecapole yields approach the experimental values, namely, $\gamma_4^{(1)}=80\pm20, \gamma_4^{(2)}\leqslant 15$ [7].

Then we investigated the influence of the radial parameter $w_n=w_n^{(1)}=w_n^{(2)}$ by keeping a constant diffusivity $a_n=0.5$ fm.

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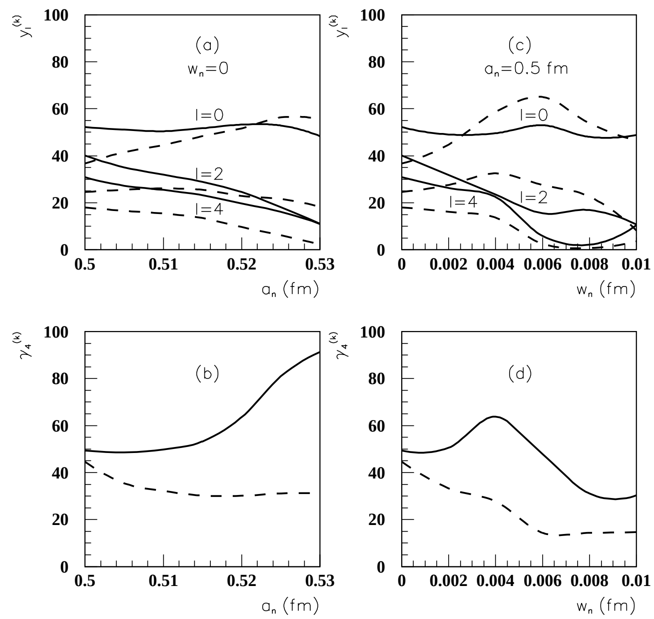

FIG. 2. (a) The yields/fragment $y_I^{(k)}$ defined by Eq. (14) vs the neutron diffusivity parameter $a_n \equiv a_n^{(k)}$. The radial parameters are $w_p^{(k)} = w_n^{(k)} = 0$. By solid lines are given the yields of the ^{106}Mo ($k=1$) and by dashes those of ^{146}Ba ($k=2$). (b) The relative hexadecapole yields $\gamma_4^{(k)}$ defined by Eq. (16) vs the diffusivity parameter a_n. The radial parameters are $w_p^{(k)} = w_n^{(k)} = 0$. The solid line corresponds to ^{106}Mo ($k=1$) and dashed line to ^{146}Ba ($k=2$). (c) The same as in (a) vs the radial parameter $w_n \equiv w_n^{(k)}$ for a_n=0.5 fm. (d) The same as in (b) vs the radial parameter w_n for a_n=0.5 fm.

The results remain again qualitatively unchanged for a constant sum. We changed this parameter from 0 up to 0.01 fm. This corresponds to an increase of the neutron radius by 0.05 fm$\approx 0.01 A^{1/3}$. In Fig. 1(c) we give the radial components for w_n=0.01 fm. The oscillatory tails are even larger in this case. In Fig. 2(c) the yields (14) as functions of w_n are given. From Fig. 2(d) one can see a strong dependence of the relative yields(16).

In Ref. [4] relativistic Hartree-Bogoliubov calculations are performed for Sn isotopes, i.e., between the emitted fragments. By using these values we found for a similar proton-neutron asymmetry larger diffusivity and radial parameter of the neutron density, namely, $a_n \approx 0.55$ fm and $w_n \approx 0.04$ fm, respectively. For the considered deformations the resonant state vanishes at these values, but by a small decrease of quadrupole deformations it is possible to obtain these values of the neutron diffusivity and radial parameter. Therefore in order to measure the density profile it is necessary to have a careful analysis of not only the absolute yields (14) of the cold fission, but also of electromagnetic transitions in each rotational band, giving a realistic information about the deformation parameters.

It is known that proton emission to excited states is very sensitive to the mean field details of exotic proton rich nuclei. This is the "lightest" member of the cluster decay family. We analyzed here the mean field of neutron rich nuclei by using the "heaviest" extreme of the same family, namely, cold fission. The effects in this case are very much enhanced, due to the following three factors: (1) the double fine structure of the measured widths, (2) large fragment charges, and (3) a relative low Coulomb barrier, about 3 MeV.

In conclusion in this paper we proposed to use the double fine structure of the cold fission process as a tool to investigate the mean field of neutron rich nuclei. We showed that relative yields of rotational states are very sensitive to the neutron density.

We generalized our previous coupled channels formalism to estimate decay probabilities to rotational states from a fissioning nucleus. We estimated the fission barrier by using the double folding procedure with M3Y two-body plus Coulomb forces. The energy was fixed by an internal repulsive core.

It turns out that the results remain unchanged for a constant sum of neutron diffusivities. We showed that the relative hexadecapole yields are very sensitive to this parameter. They increase three times if one changes the diffusivity by 0.03 fm.

We also investigated the influence of the neutron radius and found out a significant change over an interval of 0.05 fm.

Thus, the cold fission process involving transitions to low-lying rotational states is a useful tool to investigate nuclear mean field of neutron rich nuclei. This is a very important feature because the radius and diffusivity cannot be directly measured by other experiments for such unstable nuclei. We hope that the conclusions of this paper will encourage experimentalists to perform an extensive analysis concerning the double fine structure of neutron rich products.

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