Searching for X(5) behavior in nuclei

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We have searched even-even nuclei with $Z \geqslant 20$, $N \geqslant 20$ to find examples displaying the predicted characteristics of X(5) critical point behavior. On the basis of the yrast state energies and yrast intraband transition strengths, the best candidates are ^{126}Ba, ^{130}Ce, and the previously suggested examples of the $N=90$ isotones of Nd, Sm, Gd, and Dy.

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Notable benchmarks of collective nuclear behavior are the harmonic vibrator [1], the symmetrically deformed rotor [2], and the triaxially soft rotor [3]. While nuclei may display behavior near these idealized limits, many lie in transitional regions between them. Recently, it has been suggested that a useful approach is to apply the ideas of a phase transition of the nuclear shape and to try to define critical points of the shape change as new benchmarks against which nuclear properties can be compared [4,5]. In particular, the transition from a spherical harmonic vibrator to an axially deformed rotor has been described analytically [5] by introducing a dynamic symmetry, denoted as X(5), which arises when the potential in the Bohr Hamiltonian [2] is decoupled into two components—an infinite square well potential for the quadrupole deformation parameter β and a harmonic potential well for the triaxiality deformation parameter γ.

Several empirical examples of nuclei that may be close to an X(5) critical point have been suggested. These include ^{150}Nd ($Z=60$, $N=90$) [6], ^{152}Sm ($Z=62$, $N=90$) [7], and ^{104}Mo ($Z=42$, $N=62$) [8]. For the $N=90$ isotones a recent paper [9] has shown that some of the properties of these nuclei, specifically the energy spacings of the nonyrast states and the intersequence transition strengths, are not accurately reproduced by the X(5) description. In the case of ^{104}Mo, the reduced transition strengths, derived from recent lifetime measurements of states in the yrast sequence [10,11], were used to demonstrate that this nucleus does not display X(5) behavior [11].

If the X(5) description is to be taken as a benchmark for describing shape transitional behavior, then it is important to find nuclei that follow the predicted behavior more closely than the examples discussed above. Motivated by such considerations we have searched the ENSDF data file[1] [12] for examples of even-even nuclei, with $Z \geqslant 20$, $N \geqslant 20$, which display the predicted characteristics of the X(5) critical point description.

The experimental signatures for X(5) behavior are the following. (a) The energies of the yrast states, $E(I_1^+)$, should show characteristic ratios lying between those of a vibrator and a rotor; (b) The strength of transitions between yrast states as reflected in the $B(E2;I\rightarrow I-2)$ values should increase with angular momentum I at a rate intermediate between the values for a vibrator and rotor; (c) the position of the first excited collective 0_2^+ state is 5.67 times the energy of 2_1^+ level; (d) the nonyrast states based on the 0_2^+ level have larger energy spacings than the yrast sequence; (e) the $B(E2;I\rightarrow I-2)$ values for intrasequence transitions should be lower for the nonyrast sequence relative to those of the yrast sequence (these latter two points reflect the fact that the nonyrast states have a lower expectation value of β deformation than the states in the yrast sequence); (f) intersequence $B(E2)$ values should show a characteristic pattern. We shall use all of the above points in our search for nuclei displaying behavior similar to the X(5) predictions.

As a first step we used the energy ratio $E(4_1^+)/E(2_1^+)$. As pointed out by Mallmann [13] this ratio (and other similar ratios) is characteristic of different collective motions of the nucleus. An axially symmetric rotor should have $E(4_1^+)/E(2_1^+)=3.33$, an harmonic vibrator has $E(4_1^+)/E(2_1^+)=2.00$, while X(5) behavior should have $E(4_1^+)/E(2_1^+)=2.91$. We searched for even-even nuclei with $Z \geqslant 20$, $N \geqslant 20$ with $2.71 < E(4_1^+)/E(2_1^+) < 3.11$. This yielded 35 candidates as listed in Table I. The nuclei found in this way belong to several identifiable groups: (a) a group near ^{104}Mo, (b) a group near ^{128}Ce, (c) the $N=90$ isotonic chain from ^{148}Ba to ^{158}Er, (d) a group near ^{166}Hf, (e) $^{188-192}$Os, (f) ^{224}Ra, ^{224}Th. All these nuclei occupy transitional regions in the sense that they are known to exhibit shape softness.

We examined the energies of the yrast sequences in these nuclei. A figure of merit was defined as $F^2=1/(N-1)\Sigma(E_{expt}-E_{X(5)})^2$, where N is the number of data points (typically $N=5$ since we did not use states above $I^\pi=10^+$). E_{expt} ($E_{X(5)}$) are the yrast state energies normalized to the energy of the first 2^+ state energy from experiment [X(5) prediction]. The resultant figures of merit are given in Table I. Of the top 30 candidates, as ranked by their F^2 values, reliable lifetime measurements up to $I^\pi=8_1^+$ (in most of the cases up to $I^\pi=10_1^+$) were known for fifteen nuclei. In Fig. 1 we present the energies of the yrast sequences (normalized to the energy of their respective 2_1^+ levels) in these nuclei and compare them with the expected behavior of an harmonic vibrator, an axially deformed rotor, and the X(5)

[1]The ENSDF data file used in our search was last updated in December 2002. It does not necessarily include all published information up to that date since certain mass chains may not have been evaluated for several years.

TABLE I. The nuclei found in a search with the requirement that the ratio of the energy of the first 4^+ state to the energy of the first 2^+ state was $2.71 < E(4^+)/E(2^+) < 3.11$. In each case the Z and N of the nucleus is indicated along with the values of the ratio $E(4^+)/E(2^+)$ and the figure of merit, F^2. The latter quantity was defined as $F^2 = 1/(N-1)\Sigma(E_{expt} - E_{X(5)})^2$, where N is the number of data points (typically $N=5$). E_{expt} ($E_{X(5)}$) are the yrast state energies normalized to the energy of the first 2^+ state energy from experiment (from X(5) prediction).

Nucleus	Z	N	$E(4^+)/E(2^+)$	F^2	Nucleus	Z	N	$E(4^+)/E(2^+)$	F^2
^{98}Sr	38	60	3.01	2.38	^{154}Gd	64	90	3.02	0.58
^{104}Mo	42	62	2.92	0.19	^{156}Dy	66	90	2.93	0.07
^{106}Mo	42	64	3.03	1.84	^{158}Er	68	90	2.74	0.58
^{108}Mo	42	66	2.92	0.36	^{160}Er	68	92	3.10	1.40
^{108}Ru	44	64	2.75	0.23	^{162}Yb	70	92	2.92	0.008
^{110}Ru	44	66	2.75	0.17	^{164}Hf	72	92	2.78	0.45
^{112}Ru	44	68	2.72	0.60	^{166}Hf	72	94	2.96	0.06
^{124}Ba	56	68	2.83	0.05	^{168}W	74	94	2.82	0.32
^{126}Ba	56	70	2.78	0.13	^{170}W	74	96	2.94	0.01
^{126}Ce	58	68	3.05	0.94	^{172}W	74	98	3.07	0.50
^{128}Ce	58	70	2.93	0.03	^{180}Os	76	104	3.10	0.79
^{130}Ce	58	72	2.80	0.32	^{188}Os	76	112	3.08	1.43
^{130}Nd	60	70	3.06	0.56	^{190}Os	76	114	2.93	0.12
^{132}Nd	60	72	2.86	0.44	^{192}Os	76	116	2.82	0.05
^{134}Sm	62	72	2.94	0.003	^{224}Ra	88	136	2.99	0.17
^{146}Ba	56	90	2.84	0.17	^{224}Th	90	134	2.90	0.002
^{148}Ce	58	90	2.87	0.17					
^{150}Nd	60	90	2.93	0.02					
^{152}Sm	62	90	3.00	0.47					

prediction (see caption of Fig. 1). In Fig. 2 we present the $B(E2;I\rightarrow I-2)$ reduced transition strength [normalized to their respective $B(E2;2_1^+\rightarrow 0_1^+)$ values] and again compare them with the expected behavior for an harmonic vibrator, an axially deformed rotor, and the X(5) prediction (see caption of Fig. 2). In most cases the $B(E2;I\rightarrow I-2)$ values used were the accepted values from the latest Nuclear Data Sheet. However, in a few cases we used more accurate data from recent measurements (^{104}Mo [10,11], ^{126}Ba [14], ^{150}Nd [6], ^{158}Er [15]).

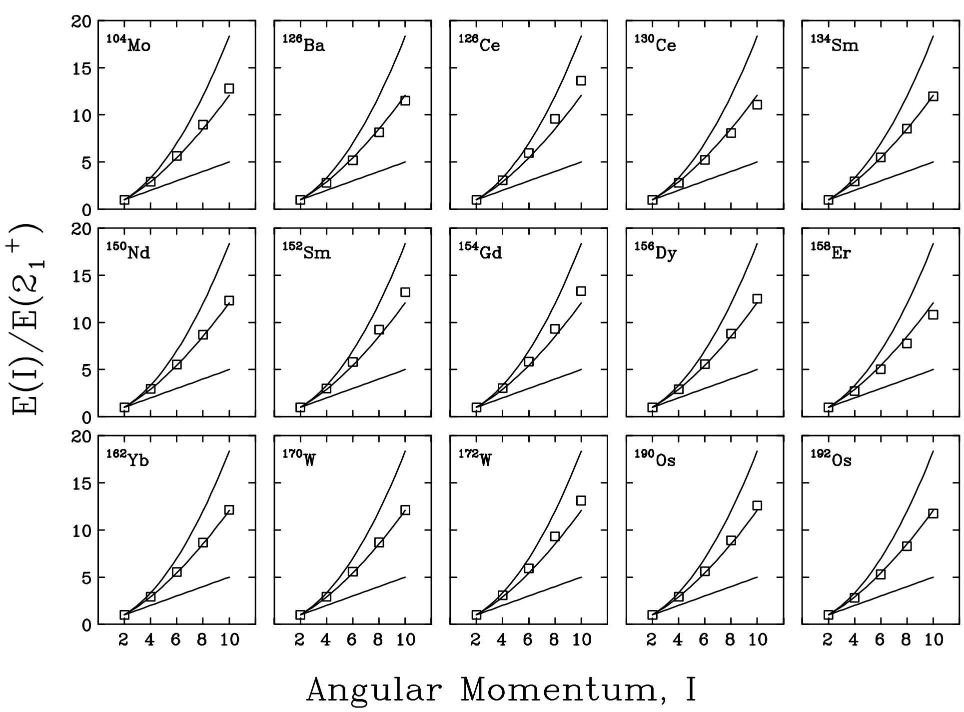

FIG. 1. Plots of the normalized energies for the yrast sequences of 15 candidate nuclei. The relevant nucleus is indicated in the top left of each panel and the experimental data is plotted with open squares. For comparison the expected energies for a harmonic vibrator (lowest solid line), an axially deformed rotor (highest solid line), and an X(5) critical point nucleus (intermediate solid line) are also shown.

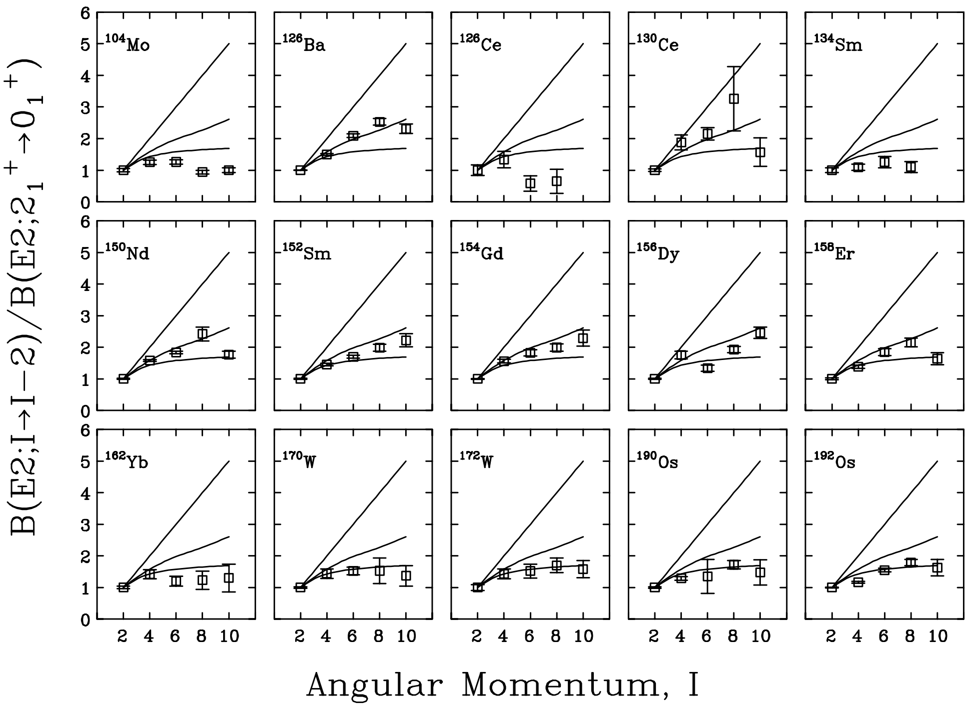

FIG. 2. Plots of the normalized $B(E2;I\rightarrow I-2)$ values for transitions in the yrast sequences for 15 candidate nuclei. The relevant nucleus is indicated in the top left of each panel and the experimental data is plotted with open squares. For comparison the expected values for a harmonic vibrator (highest solid line), an axially deformed rotor (lowest solid line), and an X(5) critical point nucleus (intermediate solid line) are also shown.

It is clear from Table I and Fig. 1 that there are many examples of nuclei with yrast energies that closely follow the X(5) prediction. However, as can be seen from Fig. 2, in most of these cases X(5) behavior can be excluded on the basis of the deduced yrast $B(E2;I\rightarrow I-2)$ values. From the available data, the only nuclei that remain candidates are ^{126}Ba, ^{130}Ce, and the $N=90$ isotones from Nd ($Z=60$) to Er ($Z=68$).

For this subset of nuclei, we can examine the properties of excited states and the transitions from them. Figure 3 shows the behavior of the energies of the yrast and excited sequences in ^{126}Ba, ^{130}Ce, ^{150}Nd, ^{152}Sm, ^{154}Gd (we do not show the plots for ^{156}Dy and ^{158}Er since they are very similar to those of the other $N=90$ isotones and there are no accurate lifetime data for states in the excited sequences). In each nucleus, the excited sequence to be compared with the X(5) prediction should be that based on the first excited collective 0^+ state (the 0_2^+ state in each of the five cases shown in Fig. 3).

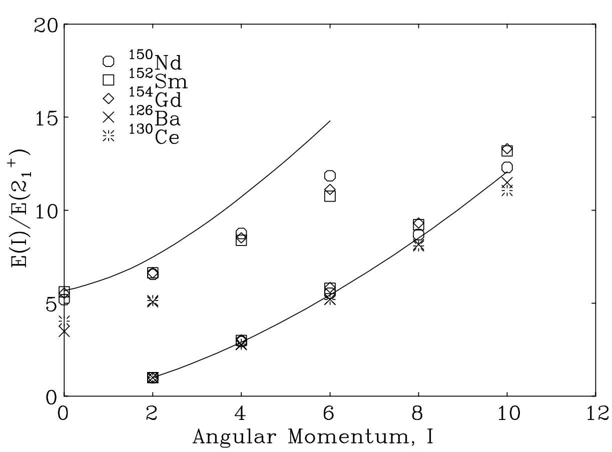

FIG. 3. Plots of the normalized energies of the yrast and excited sequences in ^{150}Nd (open circles), ^{152}Sm (open squares), ^{154}Gd (open diamonds), ^{126}Ba (crosses), and ^{130}Ce (stars). The solid curves are the predictions from the X(5) description.

For ^{126}Ba [16] only the positions of the excited 0^+ and 2^+ states are known. For ^{130}Ce [17] the excited 0^+ and 2^+ states are only tentatively assigned. However, as seen in Fig. 3 the position of the excited 0^+ and 2^+ states in these two nuclei deviate from the X(5) prediction.

For the $N=90$ isotones, the positions of the excited 0^+ states are close to the X(5) prediction but we note that the spacings in the excited sequence do not follow the expected behavior. Indeed, the known sequences look like well developed rotational bands with properties similar to the yrast sequences. To investigate further we looked at the available data on the strengths of transitions from the excited states. The intrasequence $B(E2;I\rightarrow I-2)$ values for the excited sequence are expected to be lower than the corresponding values in the yrast sequence in the X(5) picture. We present the available experimental data in Table II where we have normalized the values to the $B(E2;2_1^+\rightarrow 0_1^+)$ value. The available data are consistent with a possible drop of $B(E2;2_2^+\rightarrow 0_2^+)$ relative to $B(E2;2_1^+\rightarrow 0_1^+)$. ^{154}Gd seems to show the most statistically significant lowering of $B(E2;2_2^+\rightarrow 0_2^+)$ with respect to $B(E2;2_1^+\rightarrow 0_1^+)$.

TABLE II. The normalized experimental intrasequence $B(E2;I\rightarrow I-2)$ values for ^{150}Nd, ^{152}Sm, and ^{154}Gd. The predicted X(5) values are given in the second column.

Transition	X(5)	^{150}Nd	^{152}Sm	^{154}Gd
$2_1^+\rightarrow 0_1^+$	100	100(2)	100(2)	100(1)
$4_1^+\rightarrow 2_1^+$	158	158(3)	145(4)	156(6)
$6_1^+\rightarrow 4_1^+$	198	183(4)	170(5)	182(10)
$8_1^+\rightarrow 6_1^+$	227	242(22)	198(11)	199(11)
$10_1^+\rightarrow 8_1^+$	261	177(11)	222(21)	229(25)
$2_2^+\rightarrow 0_2^+$	79	99(20)	77(19)	62(6)
$4_2^+\rightarrow 2_2^+$	120	148(44)	142(27)	
$6_2^+\rightarrow 4_2^+$	146			

The intersequence $B(E2)$ values are another test of X(5) behavior since they should show a characteristic pattern. For the cases of ^{150}Nd and ^{152}Sm, where there have been detailed measurements of the lifetimes and branching ratios of the intersequence transitions, a recent paper [9] showed that the intersequence $B(E2)$ values are not well reproduced by the X(5) description. Lifetimes are also known [18] for the excited 0^+ and 2^+ states in ^{154}Gd and the deduced $B(E2)$ values are similar to those in ^{152}Sm and ^{150}Nd.

To conclude, we have searched the available data on even-even nuclei with $Z\geqslant 20$, $N\geqslant 20$ in an effort to find examples that display the predicted characteristics of X(5) critical point behavior. On the basis of the yrast state energies and yrast intraband transition strengths, the best candidates were found to be ^{126}Ba, ^{130}Ce, and the $N=90$ isotones of Nd, Sm, Gd, and Dy. While the X(5) picture reproduces the position of the first excited 0_2^+ in the $N=90$ isotones, none of these nuclei display the predicted behavior of the energy spacings of the excited states or the intersequence transition strengths. It should be noted that other treatments, notably the model of Davydov and Chaban [19], can also reproduce accurately the energies of the 0_2^+ states [20]. This model is based on a similar collective Hamiltonian but uses a different assumption for the shape of the potential in quadrupole deformation. It would be worth reexamining its predictions for the energy spacings and transition strengths in more detail.

Our investigations suggest that future experiments should focus on more detailed measurements of the excited states in ^{154}Gd and ^{156}Dy (a recent paper has been published on ^{156}Dy [21] which substantially revised the low-lying decay scheme) and to get detailed information on states above the collective 0_2^+ levels in ^{126}Ba and ^{130}Ce. These studies would be important for understanding the collective excitations in transitional nuclei regardless of the applicability of the X(5) description.

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