Scalar susceptibility and chiral symmetry restoration in nuclei

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We study the nuclear modification of the scalar QCD susceptibility, calculated as the derivative of the quark condensate with respect to the quark mass. We show that it has two origins. One is the low lying nuclear excitations. At normal nuclear density this part is constrained by the nuclear incompressibility. The other part arises from the individual nucleon response and it is dominated by the pion cloud contribution. Numerically the first contribution dominates. The resulting increase in magnitude of the scalar susceptibility at normal density is such that it becomes close to the pseudoscalar susceptibility, while it is quite different in the vacuum. We interpret it as a consequence of chiral symmetry restoration in nuclei.

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I. INTRODUCTION

The chiral phase transition is a hot topic of QCD. The attention has been focused in particular on a baryonic rich environment, the simplest one being ordinary nuclei. In view of the difficulties of the lattice methods to treat baryonic matter, studies based on models have revealed their interest. In the first stage the order parameter, which is the quark condensate, was investigated [1,2]. With the realization that the amount of restoration is large, as the order parameter has decreased by about 35% at ordinary nuclear density, the interest has also been focused on precursor effects linked to the partial restoration of chiral symmetry in the form of dropping hadron masses [3–5] or axial-vector correlator mixing [6,7]. The problem has also been studied in the framework of effective field theories and chiral perturbation theory [8,9]. The last topic to attract attention is the question of the susceptibilities in QCD [10,11]. In the broken symmetry phase, where the order parameter introduces a privileged direction, the susceptibility is splitted as in magnetism in the parallel susceptibility, along the magnetization axis, and the perpendicular one. In QCD these are the scalar (χ_S) and pseudoscalar (χ_{PS}) susceptibilities related to the fluctuations of the scalar and pseudoscalar quark density, respectively.

The in-medium QCD susceptibilities have been discussed by Chanfray and Ericson [11]. For the scalar one they used the linear sigma model which provides the coupling of the quark scalar density fluctuations to the nucleonic ones through sigma exchange. They ignored the coupling to the pion density fluctuations, expected to have a smaller influence. Their approach is basically a dispersive one, with the introduction of the in-medium scalar spectral function. In terms of graphs their effect corresponds to the one of Fig. 1, and the dressing by the pion lines to that of Fig. 2.

The present work uses a totally different approach which relies on the very definition of the longitudinal susceptibility as the derivative of the order parameter with respect to the perturbation responsible for the explicit symmetry breaking. In magnetism it is the external magnetic field. In QCD it is the quark mass and we have the generic definition of the scalar susceptibility:

$$\chi_S = \frac{\partial}{\partial m_q} \langle \overline{q} q \rangle, \tag{1}$$

where q is the quark field. The two methods are known to be equivalent in their principle. However in practice, in the dispersive approach, truncations are made in the intermediate accessible states. In fact, our new derivation incorporates terms as well as interference effects which were previously absent. The present study is also free from the specific features of the linear sigma model.

In Sec. II, we show how Eq. (1) leads to a natural decomposition of χ_S into a vacuum contribution $\chi_S(\text{vac.})$, a contribution noted χ_S^N which is related to the *nucleonic* excitations, and a contribution noted $\chi_S^{nuclear}$ which is related to the *nuclear* excitations. In Sec. III, we use the Fermi gas model to estimate $\chi_S^{nuclear}$ and we compare the result with the one obtained previously with the linear sigma model [11]. We conclude that $\chi_S^{nuclear}$ depends essentially on the zero momentum particle-hole propagator which we relate to the nuclear incompressibility. In Sec. IV, we study the nucleonic contribution χ_S^N and we find that it is dominated by the pion cloud, the quark core giving only about 6% of χ_S^N. The model dependence of the pion cloud contribution is moderated by the fact that, within a factor of order two, its value is fixed by the leading nonanalytic piece of the sigma term. In Sec. V, we present some numerical estimates based on the results of Secs. II, III, and we discuss their implications concerning the restoration of chiral symmetry in nuclei.

II. NUCLEAR SUSCEPTIBILITY

Since our aim is to evaluate the modification of the susceptibility with respect to its vacuum value, we note $\langle \overline{q} q(\rho) \rangle$

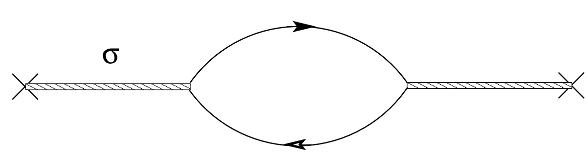

FIG. 1. Modification of the σ propagator by the particle-hole polarization propagator. The cross represents the condensate.

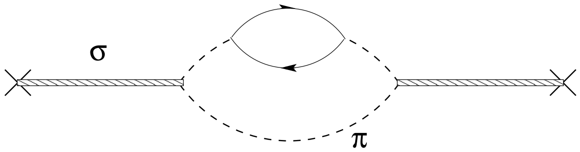

FIG. 2. Modification of the σ propagator by the in-medium 2π propagator. The cross represents the condensate.

the in-medium value of the quark condensate and we introduce

$$\chi_S(\rho)=\frac{\partial}{\partial m_q}\langle\overline{q}q(\rho)\rangle, \tag{2}$$

where ρ is the nuclear density.

For a collection of independent nucleons, the in-medium condensate $\langle\overline{q}q(\rho)\rangle$ writes

$$\langle\overline{q}q(\rho)\rangle=\langle\overline{q}q(\text{vac.})\rangle\left[1-\frac{\Sigma_N\rho_S}{f_\pi^2 m_\pi^2}\right]. \tag{3}$$

Here ρ_S is the nucleon scalar density, $\langle\overline{q}q(\text{vac.})\rangle$ denotes the vacuum expectation value of the condensate, and Σ_N is the nucleon sigma term:

$$\Sigma_N=\langle N|[Q_5,\dot{Q}_5]|N\rangle=2m_q\int d\vec{x}\langle N|\overline{q}q(\vec{x})-\overline{q}q(\text{vac.})|N\rangle. \tag{4}$$

Using the Gellmann, Oakes, and Renner relation:

$$f_\pi^2 m_\pi^2=-2m_q\langle\overline{q}q(\text{vac.})\rangle, \tag{5}$$

the in-medium condensate expression (3) can also be written in the form

$$\langle\overline{q}q(\rho)\rangle=\langle\overline{q}q(\text{vac.})\rangle+\frac{\Sigma_N\rho_S}{2m_q}. \tag{6}$$

The result above follows from the Feynman-Hellman theorem which relates the condensate to the thermodynamical grand potential per unit volume $\omega=\epsilon-\mu\rho$ through

$$\langle\overline{q}q(\rho)\rangle=\frac{1}{2}\left(\frac{\partial\omega}{\partial m_q}\right)_\mu, \tag{7}$$

where the derivative has to be taken at constant baryonic chemical potential μ (which controls the density ρ). As an illustration in a free Fermi gas, one has after substracting the vacuum energy:

$$\omega=4\int\frac{d\vec{p}}{(2\pi)^3}(E_p-\mu)\Theta(\mu-E_p), \tag{8}$$

with $E_p=\sqrt{p^2+M^2}$. The medium contribution to the condensate is obtained as

$$\langle\overline{q}q(\rho)\rangle-\langle\overline{q}q(\text{vac.})\rangle=\frac{1}{2}\left(\frac{\partial\omega}{\partial M}\right)_\mu\frac{\partial M}{\partial m_q}=\rho_S\frac{\Sigma_N}{2m_q}, \tag{9}$$

where the scalar density is defined by

$$\rho_S=4\int\frac{d\vec{p}}{(2\pi)^3}\frac{M}{E_p}\Theta(\mu-E_p), \tag{10}$$

while the ordinary density is

$$\rho=4\int\frac{d\vec{p}}{(2\pi)^3}\Theta(\mu-E_p)=\frac{2}{3\pi^2}p_F^3, \tag{11}$$

with $p_F^2=\mu^2-M^2$.

Note that in Eq. (9) the contribution of the δ function from the derivative of the Heaviside function vanishes. It will not be the case for the susceptibility. Starting from Eq. (6), one gets

$$\chi_S(\rho)=\frac{\partial}{\partial m_q}\langle\overline{q}q(\text{vac.})\rangle+\rho_S\frac{\partial}{\partial m_q}\left(\frac{\Sigma_N}{2m_q}\right)+\frac{\Sigma_N}{2m_q}\frac{\partial\rho_S}{\partial m_q}, \tag{12}$$

which contains the following three contributions.

(1) The derivative of $\langle\overline{q}q(\text{vac.}\rangle$, which is the vacuum susceptibility $\chi_S(\text{vac.})$. Its evaluation would require a nonperturbative QCD model, which is outside the scope of this work. So we focus on the difference $\chi_S(\rho)-\chi_S(\text{vac.})$.

(2) The derivative of $\Sigma_N/2m_q$, which is in fact the nucleon scalar susceptibility χ_S^N.[1] This follows from the relation between the sigma term and the condensate:

$$\Sigma_N=2m_q\int d\vec{x}\langle N|\overline{q}q(\vec{x})-\overline{q}q(\text{vac.})|N\rangle. \tag{13}$$

Thus,

$$\chi_S^N=\frac{\partial}{\partial m_q}\int d\vec{x}\langle N|\overline{q}q(\vec{x})-\overline{q}q(\text{vac.})|N\rangle=\frac{\partial}{\partial m_q}\left(\frac{\Sigma_N}{2m_q}\right). \tag{14}$$

Therefore this second term, which writes $\rho_S\chi_S^N$, can be interpreted as the individual nucleon contribution to the nuclear susceptibility.

(3) The derivative of ρ_S which gives the third term, noted $\chi_S^{nuclear}$:

$$\chi_S^{nuclear}=\frac{\Sigma_N}{2m_q}\frac{\partial\rho_S}{\partial m_q}. \tag{15}$$

We shall see that it represents the effect of the nuclear excitations by contrast to the second term which is due to the nucleon excitations.

[1]This quantity has not the same dimension as χ_S due to the normalization volume of infinite nuclear matter.

In summary, we are going to study separately the two terms of the quantity

$$\chi_S(\rho)-\chi_S(\text{vac.})=\rho_S\chi_S^N+\chi_S^{nuclear}. \tag{16}$$

III. NUCLEAR EXCITATION CONTRIBUTION

We first examine the term $\chi_S^{nuclear}$ in Eq. (16). We use the free Fermi gas, for which the scalar nucleon density is given in Eq. (10). Taking its derivative with respect to the quark mass at constant chemical potential, we get two terms, the second one arising from the derivative of the Heaviside function:

$$\frac{\partial\rho_S}{\partial m_q}=\frac{\Sigma_N}{m_q}4\int\frac{d\vec{p}}{(2\pi)^3}\left[\frac{p^2}{E_p^3}\Theta(E_p-\mu)-\frac{M^2}{E_p^2}\delta(E_p-\mu)\right]. \tag{17}$$

The first term in Eq. (17) represents the polarization through nucleon-antinucleon ($\bar{N}N$) excitations of a relativistic Fermi gas submitted to a scalar perturbation (see the Appendix). As it vanishes in the nonrelativistic limit we shall neglect this $\bar{N}N$ contribution.

From the second term in Eq. (17), we obtain

$$\chi_S^{nuclear}=\frac{\Sigma_N^2}{2m_q^2}\left(-\frac{2}{\pi^2}\frac{p_FM^2}{\mu}\right). \tag{18}$$

In the nonrelativistic limit ($\mu\simeq M$) the parenthesis in the above equation reduces to $-2Mp_F/\pi^2$ which is actually the particle-hole polarization propagator $\Pi_{ph}(q)$ of the nonrelativistic free Fermi gas taken in the static situation, i.e., for $q_0=0$, and taking the limit of vanishing three momentum ($\vec{q}\to 0$). We have then

$$\chi_S^{nuclear}\equiv\frac{\Sigma_N}{2m_q}\frac{\partial\rho_S}{\partial m_q}\simeq\frac{\Sigma_N^2}{2m_q^2}\Pi_{ph}(0,\vec{0})\simeq-\frac{2Mp_F}{\pi^2}\frac{\Sigma_N^2}{2m_q^2}. \tag{19}$$

The presence of Π_{ph} indicates that the origin of this term lies in the nuclear excitations.

A. Comparison with $\chi_S^{nuclear}$ obtained in the sigma model

In the sigma model the scalar susceptibility is related to the σ propagator [11]. In the medium this propagator is modified by the particle-hole insertions, which gave

$$\chi_S^{nuclear}=2\frac{\langle\bar{q}q(\text{vac.})\rangle^2}{f_\pi^2}\frac{g_\sigma^2}{m_\sigma^4}\Pi_S(0,\vec{0}), \tag{20}$$

where Π_S is the full scalar particle-hole polarization propagator and g_σ is the sigma nucleon coupling constant. In order to link the two expressions of $\chi_S^{nuclear}$, we first need to evaluate the nucleon sigma term Σ_N within the linear sigma model. It is built up of two pieces. The first one corresponds to the exchange of a sigma between the condensate and the nucleon as illustrated in Fig. 3(a). The second one is the contribution of the pion cloud of the nucleon which comes into play as two pions exchange, as shown in Fig. 3(b). The result is

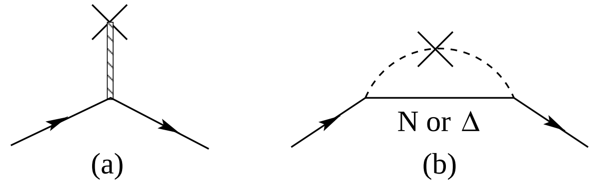

FIG. 3. Nucleon sigma term in the σ model.

$$\Sigma_N=\Sigma_N^\sigma+\Sigma_N^\pi=f_\pi m_\pi^2\frac{g_S}{m_\sigma^2}+\frac{m_\pi^2}{2}\int d\vec{x}\langle N|\phi^2(\vec{x})|N\rangle. \tag{21}$$

In the context of our previous work [11], the pionic contribution Σ_N^π did not appear naturally and thus was ignored. To make the comparison meaningful, we should then retain only the sigma exchange part Σ_N^σ and insert it in our expression (19) which leads to

$$\begin{aligned}\chi_S^{nuclear}(\sigma)&=\frac{m_\pi^4f_\pi^2}{2m_q^2}\frac{g_\sigma^2}{m_\sigma^4}\Pi_{ph}(0,\vec{0})\\&=2\frac{\langle\bar{q}q(\text{vac.})\rangle^2}{f_\pi^2}\frac{g_\sigma^2}{m_\sigma^4}\Pi_{ph}(0,\vec{0}).\end{aligned} \tag{22}$$

This is essentially the result of the sigma model calculation [see Eq. (20)], provided we replace the full Π_S by the free Fermi gas expression Π_{ph}. The present derivation is more satisfactory in the sense that it does not rely on the sigma model to derive the coupling between the quark density fluctuations and the nucleon ones. In particular, it incorporates other couplings than through sigma exchange, such as the two pion exchange term Σ_N^π of Σ_N. Moreover the interferences between the various components of Σ_N are automatically incorporated in the crossed terms of Σ_N^2. One of these interferences, which is the $\Sigma_N^\sigma\Sigma_N^\pi$ term, is illustrated in Fig. 4.

On the other hand talking also about the limitations of the present work, it applies to a free Fermi gas (its generalization is in progress), while our previous approach did not restrict to this situation. In the dressing of the sigma line by ph states, the full ph propagator entered. The latter is to some extent constrained by nuclear phenomenology [11] and the

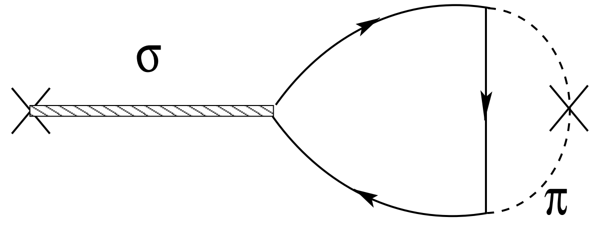

FIG. 4. Interference between σ and 2π exchange.

argument goes as follows. At normal nuclear density there is no distinction between the scalar and ordinary density operators. Now, at ordinary density, the ph propagator Π_ρ is the response of the system to a perturbation which couples to the nucleon density. In other words it is the nuclear compressibility, with the relation:

$$\Pi_\rho = -\frac{9\rho}{K}, \tag{23}$$

where the nuclear incompressibility K is related to the energy per particle E/A by

$$K = \frac{d}{d\rho}\left(\rho^2 \frac{d}{d\rho}\frac{E}{A}\right). \tag{24}$$

Its experimental value at the saturation density ρ_0 is in the range 200–300 MeV [12]. This value is compatible with the free Fermi gas value computed at the same density. Even though this agreement may result from a cancellation between several many-body influences, such as the effective mass and residual interaction effects, it justifies to some extent the use of the free Fermi gas model.

IV. NUCLEON SCALAR SUSCEPTIBILITY χ_S^N

For a structureless nucleon, one has of course $\chi_S^N = 0$ but we do not make such a restriction. In general, $\chi_S^N \neq 0$ because the true nucleon responds to a scalar perturbation by adjusting its internal structure. One can estimate this response using models, for instance the MIT bag model, but it could also be extracted from lattice calculations when the latter are done at realistic quark mass values.

A. Valence quark contribution to χ_S^N

The scalar susceptibility of a free nucleon has been introduced by Guichon *et al.* [13] in another context, concerning a pure nuclear physics problem, that is, the question of saturation of nuclear matter. In his quark-meson coupling model the saturation follows from the response of the nucleon to a scalar field. In the bag model the total scalar charge of the bag, defined as

$$Q_S = \int d\vec{x} \langle N | \bar{q}q(\vec{x}) | N \rangle, \tag{25}$$

depends on the quark mass. The linear term in the quark mass expansion of Q_S:

$$Q_S(m_q) = Q_S(0) + \chi_S^N(\mathrm{Bag}) m_q + \cdots, \tag{26}$$

defines the susceptibility of the bag. It is found to be $\chi_S^N(\mathrm{Bag}) \simeq 0.5R \simeq 2.5\times 10^{-3}$ MeV^{-1}, where the numerical value corresponds to a bag radius of $R=0.8$ fm. It turns out that this susceptibility due to the quark structure can stabilize the scalar nuclear field and provide a mechanism for saturation [14]. In our case, however, its contribution to the nuclear susceptibility is negligible (see Sec. V) with respect to the one due to the nuclear excitations. This is not a surprise since the nucleonic excitations, which control $\chi_S^N(\mathrm{Bag})$, are much higher than the nuclear ones. What is interesting however is the positive sign of this term. Its origin is rather obvious: when its mass increases the quark becomes less relativistic. This tends to increase the scalar charge of the nucleon as the quark scalar density $\bar{q}q$ involves the difference of the large and small components of the quark spinor.

Another important point is that $\chi_S^N(\mathrm{Bag})$ only includes the valence quark contribution and not those from the pion or sigma clouds. The argument about the energy of the corresponding excitations may not apply to the pion cloud contribution in view of the small pion mass. It is therefore interesting to evaluate the corresponding susceptibility.

B. Pionic contribution to χ_S^N

The nucleon sigma commutator is largely influenced by the presence of the pion cloud. If the nucleon remains unexcited after pion emission and in the heavy baryon limit, the corresponding contribution Σ_N^π is equal to [15]

$$\Sigma_N^\pi = \frac{m_\pi^2}{2}\int d\vec{x} \langle N | \phi^2(\vec{x}) | N \rangle$$
$$= \frac{3m_\pi^2}{16\pi^2}\left(\frac{g_A}{f_\pi}\right)^2 \int_0^\infty dq \frac{q^4}{(q^2+m_\pi^2)^2} F^2(q), \tag{27}$$

where ϕ is the pion field and $F(q)$ the πNN form factor. With this explicit expression of Σ_N^π, it is straightforward to calculate the derivatives with respect to m_q (or m_π^2) involved in the corresponding susceptibility:

$$\chi_S^N(\pi) = \frac{d}{dm_q}\frac{\Sigma_N^\pi}{2m_q} = \frac{2\langle \bar{q}q(\mathrm{vac.})\rangle^2}{f_\pi^4}\frac{d}{dm_\pi^2}\frac{\Sigma_N^\pi}{m_\pi^2}$$
$$= -\frac{3\langle \bar{q}q(\mathrm{vac.})\rangle^2}{4\pi^2 f_\pi^4}\left(\frac{g_A}{f_\pi}\right)^2 \int_0^\infty dq \frac{q^4}{(q^2+m_\pi^2)^3} F^2(q). \tag{28}$$

For a monopole form factor $F(q) = \Lambda^2/(\Lambda^2+q^2)$, we find

$$\chi_S^N(\pi) = -\frac{9m_\pi^3}{64\pi}\frac{\langle \bar{q}q(\mathrm{vac.})\rangle^2}{f_\pi^4}\left(\frac{g_A}{f_\pi}\right)^2\left(\frac{\Lambda}{\Lambda+m_\pi}\right)^4$$
$$= -4\times 10^{-2}\ \mathrm{MeV}^{-1}, \tag{29}$$

where the numerical value corresponds to $\Lambda = 5m_\pi$. This value is about 15 times larger than the susceptibility due to the quark bag structure estimated in the preceding section.

A rough estimate of the pionic contribution can be obtained if the form factor $F(q)$ is omitted in Eq. (28), which corresponds to the limit $\Lambda \to \infty$. In this case, $\chi_S^N(\pi)$ can be written in terms of the leading nonanalytical term of the sigma term Σ_N^{LNA} according to

$$\chi_S^N(\pi)=\frac{2\langle\overline{q}q(\mathrm{vac.})\rangle^2}{f_\pi^4}\frac{d}{dm_\pi^2}\frac{\Sigma_N^{LNA}}{m_\pi^2}=\frac{\langle\overline{q}q(\mathrm{vac.})\rangle^2}{f_\pi^4 m_\pi^4}\Sigma_N^{LNA}, \tag{30}$$

where

$$\Sigma_N^{LNA}=-\frac{9}{64\pi}\left(\frac{g_A}{f_\pi}\right)^2 m_\pi^3\simeq-23\ \ \mathrm{MeV}. \tag{31}$$

This approximation leads to $\chi_S^N(\pi)\simeq-8.6\times10^{-2}\ \mathrm{MeV}^{-1}$, which is about double the value obtained with the form factor.

There is also a contribution to the nucleon sigma term due to intermediate $\pi\Delta$ states [see Fig. 3(b)]. In the narrow width approximation, it reads

$$\Sigma_N^{\pi\Delta}=\frac{3m_\pi^2}{16\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{4}{9}\left(\frac{g_{\pi N\Delta}}{g_{\pi NN}}\right)^2\int_0^\infty dq$$
$$\times\left(\frac{q^4}{2\omega_q^2(\omega_q+\Delta_q)^2}+\frac{q^4}{2\omega_q^3(\omega_q+\Delta_q)}\right)F^2(q), \tag{32}$$

with $\omega_q=\sqrt{q^2+m_\pi^2}$ and $\Delta_q=M_\Delta-M_N+q^2/2M_\Delta$. The corresponding contribution to the susceptibility, $\chi_S^N(\pi\Delta)$, is obtained from the derivative of the above expression with respect to m_π^2. The presence of the large energy denominator $M_\Delta-M_N$ makes it less sensitive to the pion mass. Numerically with the ratio $g_{\pi N\Delta}/g_{\pi NN}=\sqrt{72}/5$, we find $\chi_S^N(\pi\Delta)\simeq-1.4\times10^{-2}\ \mathrm{MeV}^{-1}$. The overall susceptibility of a free nucleon due to the pion cloud is thus about $-5.4\times10^{-2}\ \mathrm{MeV}^{-1}$.

Our conclusion on the nucleon scalar polarizability is that, as the electric one, it is dominated by the pion cloud. Within a factor of 2, it can be expressed in a model independent way in terms of the leading nonanalytic part of the nucleon sigma term. In this context, it is legitimate to wonder why the pionic susceptibility $\chi_S^N(\pi)$ which is so dominant in this problem does not also dominate the nuclear saturation problem where instead it is totally ignored. The answer lies in the chiral properties of the scalar field responsible for the nuclear attraction that we have studied in Ref. [16]. We have stressed that this field has to be chiral invariant rather than σ, the chiral partner of the pion. It couples derivatively to the pion. Therefore, in the long wavelength and static limit, the pion cloud is weakly coupled to this nuclear scalar field.

C. Interpretation of the pion cloud contribution in the sigma model

It is interesting to look at the nuclear susceptibility $\rho_S\chi_S^N(\pi)$ in the framework of the sigma model. The sigma, which transmits the quark fluctuations, is dressed not only by the particle-hole excitations but also by the two pions excitations, as shown in Fig. 5. Since we are interested in the modification of the susceptibility with respect to its vacuum value, at least one of the pions in this graph has to be a nuclear one. So it is dressed by particle-hole insertions. To lowest order in the density, the contribution of this graph to the nuclear susceptibility is

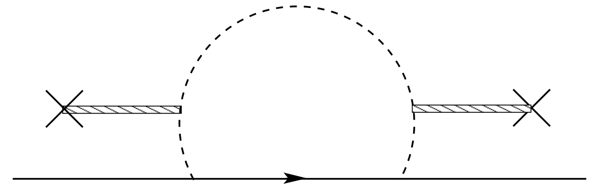

FIG. 5. Pionic contribution to the nucleon susceptibility.

$$\chi_S^{nuclear,2\pi}=3\frac{2\langle\overline{q}q(\mathrm{vac.})\rangle^2}{f_\pi^2 m_\sigma^4}\frac{m_\sigma^4}{4f_\pi^2}\left(\frac{g_A}{f_\pi}\right)^2$$
$$\times\int\frac{dq^0 d\vec{q}}{(2\pi)^4}\frac{\vec{q}^2}{(q^2-m_\pi^2)^3}\Pi_L(q^0,\vec{q})F^2(q). \tag{33}$$

Here the subscript L in Π_L indicates the spin longitudinal character. In the static limit the domain where $\Pi(q^0,\vec{q})$ has a nonvanishing value is pushed to zero energy. In this case the pions do not carry energy so the integral over q^0 which then involves only $\Pi(q^0,\vec{q})$ reduces to

$$\int\frac{dq_0}{2\pi}\Pi_L(q_0,\vec{q})=\int\frac{dq_0}{2\pi}\int d\omega\left(-\frac{2\omega}{\pi}\right)\frac{\mathrm{Im}\Pi_L(\omega\vec{q})}{q_0^2-\omega^2+i\eta}$$
$$=\int d\omega\left(-\frac{1}{\pi}\mathrm{Im}\Pi_L(\omega\vec{q})\right)$$
$$=\rho\int d\omega R_L(\omega\vec{q})=\rho S_L(q), \tag{34}$$

where $R_L(\omega\vec{q})=-\mathrm{Im}\Pi_L(\omega\vec{q})/\pi\rho$ is the nuclear longitudinal spin-isospin response and S_L its integral over energy. For a free Fermi gas, one has

$$S_L(q)=\Theta(q-2p_F)+\Theta(2p_F-q)\left[\frac{3}{2}\frac{q}{2p_F}-\frac{1}{2}\left(\frac{q}{2p_F}\right)^3\right]. \tag{35}$$

Ignoring the Pauli blocking effect, the quantity $S_L(q)$ reduces to unity, which leads to

$$\chi_S^{nuclear,2\pi}=\frac{\langle\overline{q}q(\mathrm{vac.})\rangle^2}{2f_\pi^4}\rho\frac{d}{dm_\pi^2}\left(\frac{\Sigma_N^\pi}{m_\pi^2}\right)=\rho\chi_S^N(\pi). \tag{36}$$

This is exactly the nucleonic polarizability arising from the pion cloud multiplied by the density. In fact, a mere comparison of the many-body graph of Fig. 2 with the one of Fig. 5 which represents the free nucleon susceptibility leads to this conclusion. To leading order the evaluation of the nuclear QCD scalar susceptibility arising from the 2π intermediate states does not need any calculation as it is simply related to the nucleon one, if Pauli blocking is ignored.

V. NUMERICAL ESTIMATES

We have now all ingredients to proceed to the numerical evaluation of the in-medium modification of the scalar susceptibility. To fix the idea, we shall use $\Sigma_N \simeq 45$ MeV and $K \simeq 230$ MeV^{-1}. At normal density the contribution of the nuclear excitations to the susceptibility is then

$$\chi_S^{Nuclear}(\rho_0) = -8.2\times 10^5\ \mathrm{MeV}^2.$$

Turning now to the nucleonic participation to the scalar susceptibility, we have to take into account the Pauli blocking which reduces the pionic cloud contribution from πN intermediate states by about 25% at normal density, bringing the in-medium nucleon susceptibility to the effective value $\tilde{\chi}_S^N \simeq -4.9\times 10^{-2}$ MeV^{-1}. We multiply it by the nuclear density which gives $\rho_0 \tilde{\chi}_S^N \simeq -6.8\times 10^4$ MeV^2. This number is smaller than the nuclear excitation contribution. Altogether the scalar susceptibility of nuclear matter at normal density is

$$\chi_S(\rho_0) = \chi_S(\mathrm{vac.}) - 8.9\times 10^5 \mathrm{MeV}^2. \tag{37}$$

It is interesting to give a scale to compare this nuclear modification of the scalar susceptibility. The susceptibility of the vacuum χ_S(vac.) is not known but due to the large mass of the scalar meson, it is certainly much smaller than the pseudoscalar susceptibility χ_{PS}(vac). The latter is actually dominated by the Goldstone boson, i.e., the pion, which allows its evaluation. Chanfray and Ericson gave the following expression [11]:

$$\chi_{PS}(vac) = -\frac{2\langle \overline{q}q(\mathrm{vac.})\rangle^2}{f_\pi^2 m_\pi^2} \simeq -1.3\times 10^6\ \mathrm{MeV}^2. \tag{38}$$

From this we infer that (i) $\chi_{S(\mathrm{vac.})}$ can reasonably be neglected on the right hand side (RHS) of Eq. (37); (ii) at $\rho = \rho_0$ the nuclear scalar susceptibility is comparable to the vacuum pseudoscalar susceptibility χ_{PS}(vac). Moreover Chanfray and Ericson have shown [11] that $\chi_{PS}(\rho)$ follows the density evolution of the quark condensate, i.e., at normal density it has decreased by 35%, which brings it to $\chi_{PS}(\rho_0) \simeq -9\times 10^5$ MeV^2. This is nearly equal to the value $\chi_S(\rho_0) \simeq -8.9\times 10^5$MeV^2 which we get when we neglect $\chi_{S(\mathrm{vac.})}$ in Eq. (37). This implies that the scalar and pseudoscalar susceptibilities, which are so different in the vacuum, become nearly equal at normal nuclear density, a feature normally expected only near the chiral phase transition. As our evaluation uses the value of the free nucleon sigma term, this convergence of $\chi_S(\rho_0)$ and $\chi_{PS}(\rho_0)$ toward a common value must be taken with a grain of salt due to the possible medium renormalization: $\Sigma_N \rightarrow \tilde{\Sigma}_N(\rho_0)$, which we have not taken into account in this work. However, this manifestation of the restoration of chiral symmetry is so spectacular that we do not expect it to be totally destroyed by these renormalization effects. A systematic investigation of this problem, as well as the extension of this study at larger densities, deserves further work.

In summary, we have found that the two QCD susceptibilities, namely the scalar and pseudoscalar ones, undergo a strong modification in the nuclear medium in such a way they become close to each other already at normal nuclear matter density. It is a spectacular consequence of partial chiral restoration which may have consequences in processes involving higher densities.

APPENDIX

Here we want to interpret the first term in Eq. (17) in terms of nucleon antinucleon excitations. For this we can consider a relativistic free Fermi gas described by a Hamiltonian H_0 and add a perturbation,[2]

$$\lambda W = \lambda \int d\vec{r}\, \overline{\psi}\psi = \lambda \int d\vec{r}\, \rho_S(\vec{r}), \tag{A1}$$

which changes the nucleon mass by the amount $\delta M = \lambda$. If we note $|\lambda\rangle$ the ground state of the system in presence of the perturbation then, by the Feynman Hellman theorem, we have

$$\frac{\partial}{\partial\lambda}\langle\lambda| \int d\vec{r}\, \rho_s(\vec{r})|\lambda\rangle_{\lambda=0} = V\frac{\partial}{\partial\lambda}\langle\lambda|\rho_S(0)|\lambda\rangle|_{\lambda=0}$$
$$= 2\langle\lambda=0|W\frac{1}{E_0 - H_0}W|\lambda=0\rangle, \tag{A2}$$

where we assume $\langle\lambda|\lambda\rangle = 1$ for simplicity and V is the volume of the gas. To simplify we note

$$\frac{\partial}{\partial\lambda}\langle\lambda|\rho_S(0)|\lambda\rangle|_{\lambda=0} = \frac{\partial\rho_S}{\partial\lambda}. \tag{A3}$$

From the canonical field expansion, the part of W which produces the $N\overline{N}$ intermediate states is

$$W_{N\overline{N}} = \int d\vec{p}\frac{1}{2E_p}[\overline{u}(\vec{p})v(-\vec{p})b^\dagger(\vec{p})d^\dagger(-\vec{p})] + \mathrm{h.c.}$$
$$= -\int d\vec{p}\frac{\vec{\sigma}\cdot\vec{p}}{E_p}[b^\dagger(\vec{p})d^\dagger(-\vec{p})] + \mathrm{h.c.}, \tag{A4}$$

where b,d are, respectively, the destruction operators of the nucleon and antinucleon. So the N-$\overline{N}$ contribution to the RHS of Eq. (A2) is

[2] In the general case, one should start with a space dependent perturbation and let it tend to a constant at the end. For the $N\overline{N}$ contribution this step is not necessary.

$$\frac{\partial\rho_S}{\partial\lambda}|_{N\bar{N}}=\frac{2}{V}\langle\lambda=0|W_{N\bar{N}}\frac{1}{E_0-H_0}W_{N\bar{N}}|\lambda=0\rangle$$
$$=\frac{2}{V}\langle\lambda=0|\int d\vec{p}\frac{\vec{\sigma}\cdot\vec{p}}{E_p}b(\vec{p})d(-\vec{p})\frac{1}{E_0-H_0}$$
$$\times\int d\vec{p}\,'\frac{\vec{\sigma}.\vec{p}\,'}{E_p'}d^\dagger(-\vec{p}\,')b^\dagger(\vec{p}\,')|\lambda=0\rangle$$
$$=\frac{2}{V}\int d\vec{p}\frac{p^2}{E_p^2}\left(\frac{1}{-2E_p}\right)\langle\rho|b(\vec{p})b^\dagger(\vec{p})|\rho\rangle$$
$$=\frac{1}{V}\int d\vec{p}\frac{p^2}{E_p^3}\langle\rho|b^\dagger(\vec{p})b(\vec{p})|\rho\rangle-C_\infty$$
$$=\frac{4}{(2\pi)^3}\int_0^{p_F}d\vec{p}\frac{p^2}{E_p^3}-C_\infty, \tag{A5}$$

where we have used

$$\langle\lambda=0|b^\dagger(\vec{p})b(\vec{p})|\lambda=0\rangle=4\,\delta(\vec{0})\,\theta(p_F-p)$$
$$=\frac{4V}{(2\pi)^3}\,\theta(p_F-p). \tag{A6}$$

The infinite term C_∞ is independent of the density so it drops out when we substract the vacuum contribution. Since perturbation (A1) is equivalent to a change $\delta M=\lambda$ of the nucleon mass, we can write

$$\left.\frac{\partial\rho_S}{\partial m_q}\right|_{N\bar{N}}=\frac{\partial M}{\partial m_q}\left.\frac{\partial\rho_S}{\partial\lambda}\right|_{N\bar{N}}=\frac{\Sigma_N}{m_q}\frac{4}{(2\pi)^3}\int_0^{p_F}d\vec{p}\frac{p^2}{E_p^3}, \tag{A7}$$

which is the first term in Eq. (17). In other terms the derivative of the scalar density at fixed density is entirely due to the $N\bar{N}$ excitations.

Note that this does not allow us to say that the term

$$\frac{\Sigma_N}{2m_q}\left.\frac{\partial\rho_S}{\partial m_q}\right|_{N\bar{N}} \tag{A8}$$

in Eq. (15) is the nuclear susceptibility due to $N\bar{N}$ excitations. To reach such a conclusion, we should start with a perturbation of the form

$$m_q\int d\vec{r}[\bar{u}u(\vec{r})+\bar{d}d(\vec{r})]=m_qW, \tag{A9}$$

which is the true mass term of QCD. Defining the nuclear susceptibility (per unit volume) as

$$\chi_S^{nuclear}=\frac{1}{2V}\frac{\partial}{\partial m_q}\langle m_q|\int d\vec{r}[\bar{u}u(\vec{r})+\bar{d}d(\vec{r})]|m_q\rangle|_{m_q=0}, \tag{A10}$$

where $|m_q\rangle$ denotes the nuclear ground state in presence of the quark mass term (A9), then the Feynman-Hellman theorem gives

$$\chi_S^{nuclear}=\left(\frac{1}{2V}\right)2\langle m_q=0|W\frac{1}{E-H_0}W|m_q=0\rangle. \tag{A11}$$

To compute this quantity using the Fermi gas approximation we need to know the matrix element $\langle N|W|N\rangle$ for the ph excitations and $\langle N\bar{N}|W|\mathrm{vac.}\rangle$ for the $N\bar{N}$ excitations. There is no problem with the first one since we know it from the nucleon sigma term Σ_N. On the other hand, the second one is essentially unknown. We can quantify this problem by introducing the NN and $N\bar{N}$ scalar form factors in the standard way:

$$m_q\langle N(p')|\bar{u}u(0)+\bar{d}d(0)|N(p)\rangle$$
$$=S^{NN}[(p-p')^2]\bar{u}(p')u(p),$$
$$m_q\langle N(p')\bar{N}(p)|\bar{u}u(0)+\bar{d}d(0)|\mathrm{vac.}\rangle$$
$$=S^{N\bar{N}}[(p+p')^2]\bar{u}(p')v(p). \tag{A12}$$

From the definition of the nucleon sigma term

$$\Sigma_N=\frac{1}{\langle N(0)|N(0)\rangle}\langle N(0)|\int d\vec{r}m_q[\bar{u}u(\vec{r})+\bar{d}d(\vec{r})]|N(0)\rangle, \tag{A13}$$

we get $S^{NN}(0)=\Sigma_N$ and by the crossing rule $S^{N\bar{N}}$ and S^{NN} are the same function. So we can write

$$\langle N(p')\bar{N}(p)|\bar{u}u(0)+\bar{d}d(0)|\mathrm{vac.}\rangle$$
$$=\frac{\Sigma_N}{m_q}g[(p+p')^2]\bar{u}(p')u(p), \tag{A14}$$

where we have defined $g(x)=S^{NN}(x)/S^{NN}(0)$. A straightforward calculation then leads to the following expression[3] for the $N\bar{N}$ contribution to the nuclear susceptibility:

$$\chi_S^{nuclear}(N\bar{N})=\frac{\Sigma_N^2}{2m_q^2}\frac{4}{(2\pi)^3}\int_0^{p_F}d\vec{p}|g(4E_p^2)|^2\frac{p^2}{E_p}$$
$$\approx|g(4M^2)|^2\frac{\Sigma_N}{2m_q}\left.\frac{\partial\rho_S}{\partial m_q}\right|_{N\bar{N}}, \tag{A15}$$

which differs from Eq. (A8) by the factor $|g(4M^2)|^2$. This factor is probably very small because the transition *vacuum* $\rightarrow N\bar{N}$ through the one-body quark operator $\bar{u}u+\bar{d}d$ is suppressed as compared to the elastic transition $N\rightarrow N$.

[3]After substraction of the vacuum contribution.

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