Progress in the determination of the J/ψ-π cross section

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Improving previous calculations, we compute the $J/\psi\pi\rightarrow$charmed meson cross section using QCD sum rules. Our sum rules for the $J/\psi\pi\rightarrow\bar{D}D^*$, $D\bar{D}^*$, $\bar{D}^*D^*$, and $\bar{D}D$ hadronic matrix elements are constructed by using vacuum-pion correlation functions, and we work up to twist-4 in the soft-pion limit. Our results suggest that using meson exchange models is perfectly acceptable, provided that they include form factors and that they respect chiral symmetry. After evaluating a thermal average we get $\langle\sigma^{\pi J/\psi}v\rangle\sim 0.3$ mb at $T=150$ MeV.

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I. INTRODUCTION

For a long time charmonium suppression has been considered as one of the best signatures of quark gluon plasma (QGP) formation [1]. Recently this belief was questioned by some works. Detailed simulations [2,3] of a population of c-$\bar{c}$ pairs traversing the plasma suggested that, given the large number of such pairs, the recombination effect of the pairs into charmonium Coulomb bound states is nonnegligible and can even lead to an enhancement of J/ψ production. This conclusion received support from the calculations of Refs. [4,5]. Taking the existing calculations seriously, it is no longer clear that an overall suppression of the number of J/ψ's will be a signature of QGP. A more complex pattern can emerge, with suppression in some regions of the phase space and enhancement in others [6,7]. Whatever the new QGP signature (involving charm) turns out to be, it is necessary to understand better the J/ψ dissociation mechanism by collisions with comoving hadrons.

Since there is no direct experimental information on J/ψ absorption cross sections by hadrons, several theoretical approaches have been proposed to estimate their values. In order to elaborate a theoretical description of the phenomenon, first we have to choose the relevant degrees of freedom. Already at this point no consensus has been found. Some approaches were based on charm quark-antiquark dipoles interacting with the gluons of a larger (hadron target) dipole [8–10] or quark exchange between two (hadronic) bags [11,12], whereas other works used the meson exchange mechanism [13–18]. In this case it is not easy to decide in favor of quarks or hadrons because we are dealing with charm quark bound states, which are small and massive enough to make perturbation theory meaningful, but not small enough to make nonperturbative effects negligible. Charmonium is "the border-guard of the mysterious border of perturbative world of quarks and gluons and the nonperturbative world of hadrons" [19].

In principle, different approaches apply to different energy regimes, and we might think that at lower energies we can use quark-interchange models [11,12] or meson exchange models [13–18] and at higher energies we can use perturbative QCD [8–10]. However, even at low energies, the short distance aspects may become dominant and spoil a nonperturbative description. In a similar way, nonperturbative effects may be important even at very high energies [20].

In spite of the difficulties, some progress has been achieved. This can be best realized if we compare our knowledge on the subject today with what we knew a few years ago, described by Mueller (in 1999) [21] as "... the state of the theory of interactions between J/ψ and light hadrons is embarrassing. Only three serious calculations exist (after more than 10 years of intense discussion about this issue!) and their results differ by at least two orders of magnitude in the relevant energy range. There is a lot to do for those who would like to make a serious contribution to an important topic." In the subsequent three years about 30 papers on this subject appeared and now the situation is much better, at least in what concerns the determination of the order of magnitude, which, as it will be discussed below, in the case of the J/ψ pion interaction, is determined to be $1<\sigma_{J/\psi\text{-}\pi}<10$ mb in the energy region close to the open charm production threshold.

One of the main things that we have learned is the near-threshold behavior of $\sigma_{J/\psi\text{-}\pi}$. This is quite relevant because in a hadron gas at temperatures of 100–300 MeV most of the J/ψ-π interactions occur at relatively low energies, barely sufficient to dissociate the charmonium state. In some calculations a rapid growth of the cross sections with the energy was found [15,16,14]. This behavior was criticized and considered to be incompatible with empirical information extracted from J/ψ photoproduction [22]. This criticism, however, made use of the vector meson dominance hypothesis, which, in the case of charm, is rather questionable [23]. The introduction of form factors in the effective Lagrangian approach, while reducing the order of magnitude of the cross section, did not change this fast growing trend around the threshold. Later, again in the context of meson exchange models, it was established [18] that the correct implementation of chiral symmetry prevents the cross section from rising steeply around the threshold. In a different approach, with QCD sum rules (QCDSR) [24], the behavior found in Ref. [18] was confirmed and in the present work, with improved QCDSR, we confirm again the smooth threshold behavior. We, thus, believe that this question has been answered.

Another phenomenologically less important but conceptually interesting issue is the energy dependence in the region far from threshold. Results obtained with the nonrelativistic quark model [11] indicated a rapidly falling cross section. This behavior is due to the Gaussian tail of the quark wave functions used in the quark exchange model. This result of the quark model approach could be mimicked within chiral meson Lagrangian approaches with the introduction of $\sqrt{s}$ dependent form factors [25,17]. For J/ψ-N interactions, it was found in Ref. [10] that this behavior depends ultimately on the gluon distribution in the proton at low x. In the case of J/ψ-N, for certain parametrizations of the gluon density one could find a falling trend for the cross section [10], but no definite conclusion could yet be drawn.

If J/ψ is treated as an ordinary hadron, its cross section for interaction with any other ordinary hadron must increase smoothly at higher energies, in much the same way as the proton-proton or pion-proton cross sections. The underlying reason is the increasing role played by perturbative QCD dynamics and the manifestation of the partonic nature of all hadrons. Among the existing calculations, no one is strictly valid at $\sqrt{s}\simeq 20$ GeV, except the one of Ref. [20], which is designed to work at very high energies and which gives, for the J/ψ-nucleon cross section the value $\sigma_{J/\psi\text{-}N}\simeq 5$ mb. This number can be considered as a guide for J/ψ-π cross section in the high energy regime. It should, however, be pointed out, that the calculation of Ref. [20] is based on a purely nonperturbative QCD approach. The inclusion of a perturbative contribution will add to the quoted value and will have a larger weight at higher energies. A similar conclusion was reached in Ref. [26]. In the traditional short distance QCD approach the cross section grows monotonically [8–10].

As a side product, the theoretical effort to estimate the charmonium-hadron cross section motivated a series of calculations [27–29], within the framework of QCD sum rules, of form factors and coupling constants involving charmed hadrons, which may be relevant also to other problems in hadron physics.

In this work we improve the calculation done in Ref. [24] by considering sum rules based on a three-point function with a pion. We work up to twist-4, which allows us to study the convergence of the one-pion exchange (OPE) expansion. Since the method of the QCDSR uses QCD explicitly, we believe that our work brings a significant progress to this important topic.

The paper is organized as follows: in the following section we review the method of QCD sum rules, giving special emphasis to the QCD side. In Sec. III we present some formulas for the computation of open charm production amplitudes and in Sec. IV we give our numerical results. Finally some concluding remarks are given in Sec. V.

II. THE METHOD

In the QCDSR approach [30,31], the short range perturbative QCD is extended by an OPE expansion of the correlator, giving a series in inverse powers of the squared momentum with Wilson coefficients. The convergence at low momentum is improved by using a Borel transform. The coefficients involve universal quark and gluon condensates. The quark-based calculation of a given correlator is equated to the same correlator, calculated using hadronic degrees of freedom via a dispersion relation, giving sum rules from which a hadronic quantity can be estimated.

Let us start with the general vacuum-pion correlation function

$$\Pi_{\mu 34}=\int d^4x d^4y e^{-ip_2 y} e^{ip_3 x} \times\langle 0|T\{j_3(x)j_4(0)j_\mu^{\psi}(y)\}|\pi(p_1)\rangle, \qquad (1)$$

with the currents given by $j_\mu^{\psi}=\bar{c}\gamma_\mu c$, $j_3=\bar{u}\Gamma_3 c$, and $j_4=\bar{c}\Gamma_4 d$. p_1, p_2, p_3, and p_4 are the four-momenta of the mesons π, J/ψ, M_3, and M_4 respectively, and Γ_3 and Γ_4 denote specific γ matrices corresponding to the process involving the mesons M_3 and M_4. For instance, for the process $J/\psi\ \pi\rightarrow \bar{D}D^*$ we will have $\Gamma_3=\gamma_\nu$ and $\Gamma_4=i\gamma_5$. The advantage of this approach as compared with the four-point calculation in Ref. [24] is that we can consider more terms in the OPE expansion of the correlation function in Eq. (1) and, therefore, check the "convergence" of the OPE expansion.

Following Ref. [32], we can rewrite Eq. (1) as

$$\Pi_{\mu 34}=-\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}(S_{ac}(p_3-k)\gamma_\mu \times S_{cb}(p_3-p_2-k)\Gamma_4 D_{ab}(k,p_1)\Gamma_3), \qquad (2)$$

where

$$S_{ab}(p)=i\,\frac{\not{p}+m_c}{p^2-m_c^2}\,\delta_{ab} \qquad (3)$$

is the free c-quark propagator and $D_{ab}(k,p)$ denotes the quark-antiquark component with a pion, which can be separated into three pieces depending on the Dirac matrices involved [32,33]:

$$D_{ab}(k,p)=\delta_{ab}[i\gamma_5 A+\gamma_\alpha\gamma_5 B^\alpha+\gamma_5\sigma_{\alpha\beta}C^{\alpha\beta}], \qquad (4)$$

with a, b, and c being color indices. The three invariant functions of k,p are defined by

$$A(k,p)=\frac{1}{12}\int d^4x e^{ikx}\langle 0|\bar{d}(x)i\gamma_5 u(0)|\pi(p)\rangle,$$

$$B^\alpha(k,p)=\frac{1}{12}\int d^4x e^{ikx}\langle 0|\bar{d}(x)\gamma^\alpha\gamma_5 u(0)|\pi(p)\rangle,$$

$$C^{\alpha\beta}(k,p)=-\frac{1}{24}\int d^4x e^{ikx}\langle 0|\bar{d}(x)\sigma^{\alpha\beta}\gamma_5 u(0)|\pi(p)\rangle. \qquad (5)$$

Using the soft-pion theorem, partially conserved axial-vector current and working at $O(p_\mu p_\nu)$ we get up to twist-4 [32,34],

$$A(k,p)=\frac{(2\pi)^4}{12}\frac{\langle\bar{q}q\rangle}{f_\pi}\left[-2+ip_{\alpha_1}\frac{\partial}{i\partial k_{\alpha_1}}+\frac{1}{3}p_{\alpha_1}p_{\alpha_2}\frac{\partial}{i\partial k_{\alpha_1}}\frac{\partial}{i\partial k_{\alpha_2}}\right]\delta^{(4)}(k),$$

$$B_\alpha(k,p)=\frac{(2\pi)^4}{12}f_\pi\left[ip_\alpha+\frac{1}{2}p_\alpha p_{\alpha_1}\frac{\partial}{i\partial k_{\alpha_1}}+\frac{i\delta^2}{36}(5p_\alpha g_{\alpha_1\alpha_2}-2p_{\alpha_2}g_{\alpha\alpha_1})\frac{\partial}{i\partial k_{\alpha_1}}\frac{\partial}{i\partial k_{\alpha_2}}\right]\delta^{(4)}(k),$$

$$C_{\alpha\beta}(k,p)=-\frac{(2\pi)^4}{24}\frac{\langle\bar{q}q\rangle}{3f_\pi}(p_\alpha g_{\beta\alpha_1}-p_\beta g_{\alpha\alpha_1})\times\left[i\frac{\partial}{i\partial k_{\alpha_1}}-\frac{p_{\alpha_2}}{2}\frac{\partial}{i\partial k_{\alpha_1}}\frac{\partial}{i\partial k_{\alpha_2}}\right]\delta^{(4)}(k), \quad (6)$$

where δ^2 is defined by the matrix element $\langle 0|\bar{d}g_s\tilde{\mathcal{G}}^{\alpha\beta}\gamma_\beta u|\pi(p)\rangle=i\delta^2 f_\pi p^\alpha$, where $\tilde{\mathcal{G}}_{\alpha\beta}=\epsilon_{\alpha\beta\sigma\tau}\mathcal{G}^{\sigma\tau}/2$ and $\mathcal{G}_{\alpha\beta}=t^A G_{\alpha\beta}$.

The additional contributions to the OPE come from the diagrams where one gluon, emitted from the c-quark propagator, is combined with the quark-antiquark component. Specifically, the c-quark propagator with one gluon being attached is given by [31]

$$-\frac{g_s\mathcal{G}_{\alpha\beta}}{2(k^2-m_c^2)^2}[k_\alpha\gamma_\beta-k_\beta\gamma_\alpha+(\not{k}+m_c)i\sigma_{\alpha\beta}]. \quad (7)$$

Taking the gluon stress tensor into the quark-antiquark component, one can write down the correlation function in the form

$$\Pi_{\mu 34}=-4\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}([S_{\alpha\beta}(p_3-k)\gamma_\mu S(p_3-p_2-k)+S(p_3-k)\gamma_\mu S_{\alpha\beta}(p_3-p_2-k)]\Gamma_4 D^{\alpha\beta}(k,p_1)\Gamma_3), \quad (8)$$

where we have already contracted the color indices, and where we have defined

$$S_{\alpha\beta}(k)=-\frac{1}{2(k^2-m_c^2)^2}[k_\alpha\gamma_\beta-k_\beta\gamma_\alpha+(\not{k}+m_c)i\sigma_{\alpha\beta}] \quad (9)$$

and

$$D^{\alpha\beta}(k,p)=\gamma_5\sigma_{\rho\lambda}E^{\rho\lambda\alpha\beta}(k,p)+\gamma^\tau\epsilon^{\alpha\beta\theta\delta}F_{\tau\theta\delta}(k,p), \quad (10)$$

with

$$E^{\rho\lambda\alpha\beta}(k,p)=-\frac{1}{32}\int d^4x e^{ikx}\langle 0|\bar{d}(x)\gamma_5\sigma^{\rho\lambda}g_s G^{\alpha\beta}u|\pi(p)\rangle,$$

$$F_{\tau\theta\delta}(k,p)=\frac{1}{32}\int d^4x e^{ikx}\langle 0|\bar{d}(x)\gamma_\tau g_s\tilde{G}_{\theta\delta}u|\pi(p)\rangle. \quad (11)$$

Up to twist-4 and at $O(p_\mu p_\nu)$, the two functions appearing above are given by [32,34]

$$E^{\rho\lambda\alpha\beta}=\frac{i}{32}f_{3\pi}(p^\alpha p^\rho g^{\lambda\beta}-p^\beta p^\rho g^{\lambda\alpha}-p^\alpha p^\lambda g^{\rho\beta}+p^\beta p^\lambda g^{\rho\alpha})\times(2\pi)^4\delta^{(4)}(k),$$

$$F_{\tau\theta\delta}=-\frac{i\delta^2 f_\pi}{3(32)}(p_\theta g_{\tau\delta}-p_\delta g_{\tau\theta})(2\pi)^4\delta^{(4)}(k), \quad (12)$$

where $f_{3\pi}$ is defined by the vacuum-pion matrix element $\langle 0|\bar{d}g_s\sigma_{\alpha\beta}\gamma_5\tilde{\mathcal{G}}^{\alpha\beta}u|\pi(p)\rangle$ [34].

The phenomenological side of the correlation function $\Pi_{\mu 34}$ is obtained by the consideration of J/ψ, M_3, and M_4 state contribution to the matrix element in Eq. (1). The hadronic amplitudes are defined by the matrix element

$$i\mathcal{M}=\langle\psi(p_2,\mu)|M_3(-p_3,\nu)M_4(-p_4,\rho)\pi(p_1)\rangle=i\mathcal{M}_{\mu 34}(p_1,p_2,p_3,p_4)\epsilon_2^\mu f_3^{*\nu}f_4^{*\rho}, \quad (13)$$

where $f_i^{*\alpha}=\epsilon_i^{*\alpha}$ for the D^* meson and $f_i^{*\alpha}=1$ for the D meson.

The phenomenological side of the sum rule can be written as (for the part of the hadronic amplitude that will contribute to the cross section) [24]

$$\Pi_{\mu 34}^{phen}=-\frac{m_\psi f_\psi\lambda_3\lambda_4\mathcal{M}_{\mu 34}}{(p_2^2-m_\psi^2)(p_3^2-m_3^2)(p_4^2-m_4^2)}+\mathrm{hr}, \quad (14)$$

where hr means higher resonances and where λ_i is related with the corresponding meson decay constant: $\langle D|j_D|0\rangle=-\lambda_D=m_D^2 f_D/m_c$ and $\langle 0|j_\alpha|D^*\rangle=\lambda_{D^*}\epsilon_\alpha=m_{D^*}f_{D^*}\epsilon_\alpha$.

III. HADRONIC AMPLITUDES FOR $J/\psi\pi\to$ OPEN CHARM

The OPE of Eq. (1) and the corresponding hadronic amplitude vanishes when contracted with the four-momentum of J/ψ. This is because the charm quark number has to be conserved locally during the process. Among the possible tensor structures of the hadronic amplitudes, we will only consider terms that contribute to the physical cross sections. Furthermore, since we are working in the soft-pion limit, we expect that nonchiral terms should not be present [18]. Hence, leaving out terms that vanish after contracting with the physical polarization tensors of external vector particles and chiral broken terms, we can write as follows.

(1) For the process $J/\psi\pi\to\bar{D}D^*$,

$$\mathcal{M}_{\mu\nu}=\Lambda_1^{DD^*}p_{1\mu}p_{1\nu}+\Lambda_2^{DD^*}p_{1\mu}p_{2\nu}+\Lambda_3^{DD^*}p_{1\nu}p_{3\mu}. \tag{15}$$

(2) For the process $J/\psi\pi\to\bar{D}D$,

$$\mathcal{M}_{\mu}=\Lambda_{DD}\epsilon_{\mu\alpha\beta\sigma}p_1^\alpha p_3^\beta p_4^\sigma. \tag{16}$$

(3) For the process $J/\psi\pi\to\bar{D}^*D^*$,

$$\begin{aligned}\mathcal{M}_{\mu\nu\rho}=&\Lambda_1^{D^*D^*}H_{\mu\nu\rho}+\Lambda_2^{D^*D^*}J_{\mu\nu\rho}\\&+\Lambda_3^{D^*D^*}g_{\nu\rho}\epsilon_{\mu\alpha\beta\gamma}p_1^\alpha p_2^\beta p_3^\gamma+\Lambda_4^{D^*D^*}\epsilon_{\nu\rho\alpha\beta}p_{3\mu}p_1^\alpha p_3^\beta\\&+\Lambda_5^{D^*D^*}\epsilon_{\nu\rho\alpha\beta}p_{3\mu}p_1^\alpha p_2^\beta+\Lambda_6^{D^*D^*}\epsilon_{\mu\nu\alpha\beta}p_{3\rho}p_1^\alpha p_2^\beta\\&+\Lambda_7^{D^*D^*}\epsilon_{\mu\nu\alpha\beta}p_{1\rho}p_1^\alpha p_2^\beta+\Lambda_8^{D^*D^*}\epsilon_{\nu\rho\alpha\beta}p_{1\mu}p_1^\alpha p_4^\beta\\&+\Lambda_9^{D^*D^*}\epsilon_{\mu\nu\rho\alpha}p_1^\alpha+\Lambda_{10}^{D^*D^*}\epsilon_{\nu\rho\alpha\beta}p_{1\mu}p_1^\alpha p_3^\beta\\&+\Lambda_{11}^{D^*D^*}\epsilon_{\mu\nu\alpha\beta}p_{1\rho}p_1^\alpha p_3^\beta+\Lambda_{12}^{D^*D^*}\epsilon_{\mu\nu\alpha\beta}p_{3\rho}p_1^\alpha p_3^\beta,\end{aligned} \tag{17}$$

with $H_{\mu\nu\rho}=(\epsilon_{\nu\alpha\beta\gamma}g_{\mu\rho}-\epsilon_{\rho\alpha\beta\gamma}g_{\mu\nu})p_1^\alpha p_2^\beta p_3^\gamma+\epsilon_{\mu\rho\alpha\beta}p_{2\nu}p_1^\alpha p_2^\beta$ and $J_{\mu\nu\rho}=(\epsilon_{\nu\rho\alpha\beta}p_{1\mu}+\epsilon_{\mu\rho\alpha\beta}p_{1\nu}+\epsilon_{\mu\nu\alpha\beta}p_{1\rho})p_2^\alpha p_3^\beta+\epsilon_{\mu\nu\alpha\beta}p_{2\rho}p_1^\alpha p_3^\beta$.

In Eqs. (15)–(17), Λ_i are the parameters that we will evaluate from the sum rules. In principle, all the independent structures appearing in $H_{\mu\nu\rho}$ and $J_{\mu\nu\rho}$ would have independent parameters Λ_i. However, since in our approach we get exactly the same sum rules for all of them, we decided to group them with the same parameters.

Inserting the results in Eqs. (6) and (12) into Eqs. (2) and (8) we can write a sum rule for each of the structures appearing in Eqs. (15)–(17). To improve the matching between the phenomenological and theoretical sides we follow the usual procedure and make a single Borel transformation to all the external momenta taken to be equal: $-p_2^2=-p_3^2=-p_4^2=P^2\to M^2$. The problem of doing a single Borel transformation is the fact that terms associated with the pole continuum transitions are not suppressed [35]. In the present case we have two kinds of these transitions: double pole continuum and single pole-continuum. In the limit of similar meson masses it is easy to show that the Borel behavior of the three-pole, double-pole-continuum, and single-pole-continuum contributions are $e^{-m_M^2/M^2}/M^4$, $e^{-m_M^2/M^2}/M^2$, and $e^{-m_M^2/M^2}$, respectively. Therefore, we can single out the three-pole contribution from the others by introducing two parameters in the phenomenological side of the sum rule, which will account for the double-pole-continuum and single-pole-continuum contributions. The expressions for all 19 sum rules are given in Appendixes A–C.

IV. RESULTS AND DISCUSSION

The parameter values used in all calculations are $m_c=1.37$ GeV, $m_\pi=140$ MeV, $m_D=1.87$ GeV, $m_{D^*}=2.01$ GeV, $m_\psi=3.097$ GeV, $f_\pi=131.5$ MeV, $\langle\bar{q}q\rangle=-(0.23)^3$ GeV^3, $m_0^2=0.8$ GeV^2, $\delta^2=0.2$ GeV^2, and $f_{3\pi}=0.0035$ GeV^2 [34]. For the charmed mesons decay constants we use the values from Ref. [34] for f_D and f_{D^*} and the experimental value for f_ψ:

$$f_\psi=270\ \text{MeV},\quad f_D=170\ \text{MeV},\quad f_{D^*}=240\ \text{MeV}. \tag{18}$$

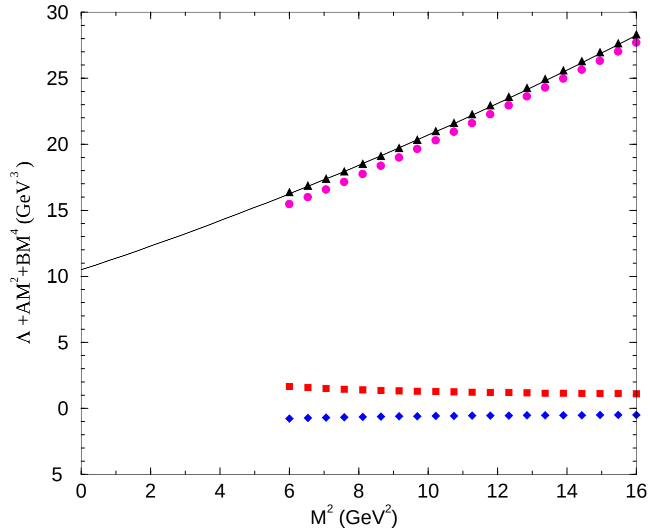

FIG. 1. Sum rule for $\Lambda_1^{D^*D^*}$ related to the process $J/\psi\pi\to\bar{D}^*D^*$ as a function of the Borel mass. The dots, squares, and diamonds give the twist-2, -3, and -4 contributions to the sum rule. The triangles give the result from Eq. (C1). The solid line gives the fit to the QCDSR results.

In Ref. [36] we have analyzed the sum rule for the process $J/\psi\pi\to\bar{D}D$. Here we choose to show the sum rule for $\Lambda_1^{D^*D^*}$ in Eq. (C1), as an example of the sum rules for the process $J/\psi\pi\to\bar{D}^*D^*$.

In Fig. 1 we show the QCD sum rule results for $\Lambda_1^{D^*D^*}+A_1^{D^*D^*}M^2+B_1^{D^*D^*}M^4$ as a function of M^2. The dots, squares, and diamonds give the twist-2, -3, and -4 contributions, respectively. The triangles give the final QCDSR results. We see that the twist-3 and -4 contributions are small as compared with the twist-2 contribution, following the same behavior as the sum rule for the process $J/\psi\pi\to\bar{D}D$ given in Ref. [36]. In general all the other sum rules are similar and contain twist-2, twist-3, and twist-4 contributions corresponding to the first, second, and third terms inside the brackets in the right hand side of Eq. (C1). Only the sum rules for $\Lambda_{10}^{D^*D^*}$ up to $\Lambda_{12}^{D^*D^*}$ do not get the leading twist contribution, and will be neglected in the evaluation of the cross section. It is also interesting to notice that if we consider only the leading twist contributions we recover the sum rules obtained in Ref. [24]. The triangles in Fig. 1 follow almost a straight line in the Borel region $6\leq M^2\leq16$ GeV^2. This show that the single-pole-continuum transitions contribution is small. The value of the amplitude $\Lambda_1^{D^*D^*}$ is obtained by the extrapolation of the fit to $M^2=0$ [29,28,35]. Fitting the QCD sum rule results to a quadratic form we get

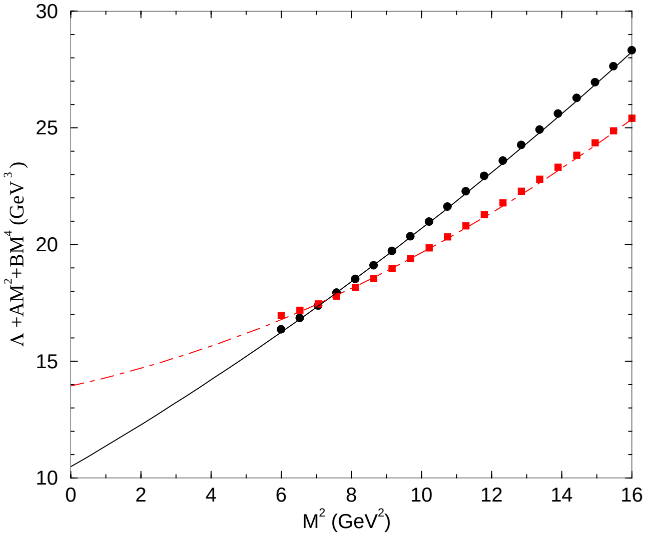

FIG. 2. Sum rule for $\Lambda_1^{D^*D^*}$ related to the process $J/\psi\pi \to \bar{D}^*D^*$ as a function of the Borel mass. The dots and squares give the results from Eq. (C1) when using respectively numerical values or the two-point sum rules for the meson decay constants. The solid and dot-dashed lines give the fits to the QCDSR results.

$$\Lambda_1^{D^*D^*} \simeq 10.5 \ \ \mathrm{GeV}^{-3}. \tag{19}$$

Since we worked in the soft-pion limit, $\Lambda_1^{D^*D^*}$, as well as all other Λ, is just a number. All particle momenta dependence of the amplitudes is contained in the Dirac structure.

In obtaining the results shown in Fig. 1 we have used the numerical values for the meson decay constants given in Eq. (18). However, it is also possible to use the respective sum rules, as done in Ref. [24]. The two-point sum rules for the meson decay constants are given in Appendix D. The behavior of the results for the hadronic amplitudes does not change significantly if we use the two-point sum rules for the meson decay constants instead of the numerical values, leading only to a small change in the value of the amplitudes. In Fig. 2 we show, for a comparison, both results in the case of $\Lambda_1^{D^*D^*}$.

Using the respective sum rules for the meson decay constants we get

$$\Lambda_1^{D^*D^*} \simeq 13.9 \ \ \mathrm{GeV}^{-3}. \tag{20}$$

We will use these two procedures to estimate the errors in our calculation. It is important to mention that our results agree completely with the value obtained in Ref. [24].

The results for all other sum rules show a similar behavior and the amplitudes can be extracted by the extrapolation of the fit to $M^2=0$. The QCDSR results, evaluated using the numerical values for the meson decay constants, as well as the quadratic fits for the amplitudes associated with the process $J/\psi\pi \to \bar{D}D^*$ are shown in Fig. 3.

The values for the parameters associated with the process $J/\psi\pi \to \bar{D}\, D^*$ are given in Table I.

For the process $J/\psi\pi \to \bar{D}\, D$ we have only one parameter which is given by [36]

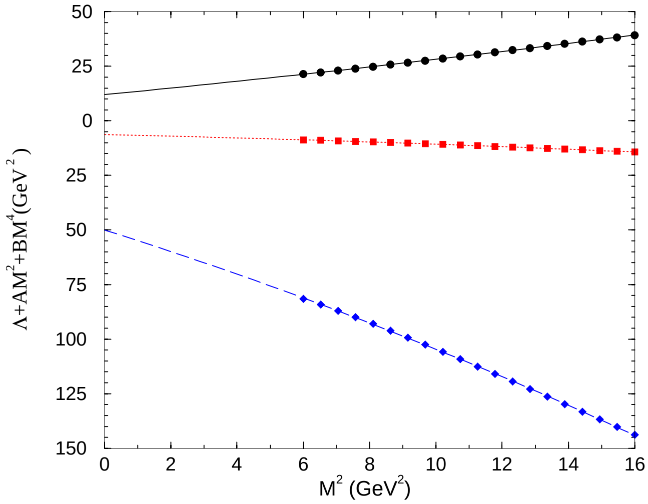

FIG. 3. Sum rules for $\Lambda_i^{DD^*}$ related to the process $J/\psi\pi \to \bar{D}D^*$ as a function of the Borel mass. The dots, squares, and diamonds give the results from the QCDSR for $\Lambda_1^{DD^*}$ up to $\Lambda_3^{DD^*}$, respectively. The solid, dotted, and dashed lines give the fits to the QCDSR results.

$$\Lambda_{DD} = 13.2 \pm 1.8 \ \ \mathrm{GeV}^{-3}, \tag{21}$$

and the 12 parameters associated with the process $J/\psi\pi \to \bar{D}^*D^*$ are given in Table II.

The errors in all parameters were estimated by the evaluation of the sum rules using the numerical values and the two-point QCDSR for the meson decay constants.

Having the QCD sum rule results for the amplitudes of the three processes $J/\psi\pi \to \bar{D}D^*$, $\bar{D}D$, $\bar{D}^*D^*$, given in Eqs. (15)–(17) we can evaluate the cross sections. After including isospin factors, the differential cross section for the J/ψ-π dissociation is given by

$$\frac{d\sigma}{dt} = \frac{1}{96\pi s p_{i,cm}^2} \sum_{spin} |\mathcal{M}|^2, \tag{22}$$

where $\mathbf{p}_{i,cm}$ is the three-momentum of p_1 (or p_2) in the center of mass frame [with $p_1(p_2)$ being the four-momentum of the $\pi(J/\psi)$]:

$$p_{i,cm}^2 = \frac{\lambda(s, m_\pi^2, m_\psi^2)}{4s}, \tag{23}$$

with $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$, $s=(p_1+p_2)^2$, and $t=(p_1-p_3)^2$.

TABLE I. The best fitted values for the parameters associated with the process $J/\psi\pi \to \bar{D}D^*$.

$\Lambda_1^{DD^*}$	$\Lambda_2^{DD^*}$	$\Lambda_3^{DD^*}$
14 ± 2 GeV^{-2}	-7.2 ± 0.9 GeV^{-2}	-58 ± 8 GeV^{-2}

TABLE II. The best fitted values for the parameters associated with the process $J/\psi\pi\rightarrow\bar{D}^*D^*$.

$\Lambda_1^{D^*D^*}$	$\Lambda_2^{D^*D^*}$	$\Lambda_3^{D^*D^*}$	$\Lambda_4^{D^*D^*}$
12.2 ± 1.7 GeV^{-3}	-12.8 ± 1.8 GeV^{-3}	12.5 ± 1.7 GeV^{-3}	-24.6 ± 3.4 GeV^{-3}
$\Lambda_5^{D^*D^*}$	$\Lambda_6^{D^*D^*}$	$\Lambda_7^{D^*D^*}$	$\Lambda_8^{D^*D^*}$
9.8 ± 1.6 GeV^{-3}	9.7 ± 1.6 GeV^{-3}	-13.0 ± 1.8 GeV^{-3}	-13.8 ± 1.8 GeV^{-3}
$\Lambda_9^{D^*D^*}$(GeV^{-1})	$\Lambda_{10}^{D^*D^*}$	$\Lambda_{11}^{D^*D^*}$	$\Lambda_{12}^{D^*D^*}$
-5.4 ± 0.9 GeV^{-1}	2.5 ± 0.2 GeV^{-3}	-0.022 ± 0.002 GeV^{-3}	0.53 ± 0.03 GeV^{-3}

In Eq. (22), the sum over the spins of the amplitude squared is given by

$$\sum_{spin}|\mathcal{M}|^2=\mathcal{M}_{\mu\nu}\mathcal{M}^*_{\mu'\nu'}\left(\frac{g^{\mu\mu'}-p_2^{\mu}p_2^{\mu'}}{m_\psi^2}\right)\times\left(\frac{g^{\nu\nu'}-p_3^{\nu}p_3^{\nu'}}{m_{D^*}^2}\right) \tag{24}$$

for $J/\psi\pi\rightarrow\bar{D}D^*$, with $p_3(p_4)$ being the four-momentum of D^* (D),

$$\sum_{spin}|\mathcal{M}|^2=\mathcal{M}_{\mu}\mathcal{M}^*_{\mu'}\left(g^{\mu\mu'}-\frac{p_2^{\mu}p_2^{\mu'}}{m_\psi^2}\right) \tag{25}$$

for $J/\psi\pi\rightarrow\bar{D}\ D$, and

$$\sum_{spin}|\mathcal{M}|^2=\mathcal{M}_{\mu\nu\alpha}\mathcal{M}^*_{\mu'\nu'\alpha'}\left(g^{\mu\mu'}-\frac{p_2^{\mu}p_2^{\mu'}}{m_\psi^2}\right)\times\left(g^{\nu\nu'}-\frac{p_3^{\nu}p_3^{\nu'}}{m_{D^*}^2}\right)\left(g^{\alpha\alpha'}-\frac{p_4^{\alpha}p_4^{\alpha'}}{m_{D^*}^2}\right) \tag{26}$$

for $J/\psi\pi\rightarrow\bar{D}^*D^*$.

As mentioned before, the sum rules for $\Lambda_{10}^{D^*D^*}$ up to $\Lambda_{12}^{D^*D^*}$ do not get the leading order contribution and will be neglected when evaluating the cross sections. It is important to keep in mind that since our sum rules were derived in the limit $p_1\rightarrow0$, we cannot extend our results to large values of $\sqrt{s}$. For this reason we will limit our calculation to $\sqrt{s}\leqslant4.5$ GeV.

In Fig. 4 we show separately the contributions for each one of the process. Our first conclusion is that our results show that, for values of $\sqrt{s}$ far from the $J/\psi\pi\rightarrow\bar{D}^*D^*$ threshold, $\sigma_{J/\psi\pi\rightarrow\bar{D}^*D^*}\geqslant\sigma_{J/\psi\pi\rightarrow\bar{D}D^*+D\bar{D}^*}\geqslant\sigma_{J/\psi\pi\rightarrow\bar{D}D}$, in agreement with the model calculations presented in Ref. [14] but in disagreement with the results obtained with the nonrelativistic quark model of Ref. [11], which show that the state $\bar{D}^*D$ has a larger production cross section than $\bar{D}^*D^*$. Furthermore, our curves indicate that the cross section grows monotonically with the center of mass system (c.m.s.) energy but not as fast, near the thresholds, as it does in the calculations in Refs. [14–18]. Again, this behavior is in opposition to Ref. [11], where a peak just after the threshold followed by continuous decrease in the cross section was found.

In Fig. 5 we show, for comparison, our result for the total cross section for the J/ψ-π dissociation (solid lines) and the results from meson exchange model [14] obtained with a cutoff $\Lambda=1$ GeV (dot-dashed line), quark exchange model [11] (dashed line), and short distance QCD [9,10] (dotted line). The shaded area in our results gives an evaluation of the uncertainties in our calculation obtained with the two procedures described above. It is very interesting to note that below the DD^* threshold, our result and the results from meson exchange and quark exchange models are in a very good agreement. However, as soon as the DD^* channel is open the cross section obtained with the meson exchange and quark exchange models show a very fast growth, as a function of $\sqrt{s}$, as compared with our result. This is due to the fact that chiral symmetry is broken in these two model calculations. Since this is the energy region where this process is more likely to happen, it is very important to use models that respect chiral symmetry when evaluating the J/ψ-π cross section.

The momentum distribution of thermal pions in a hadron gas depends on the effective temperature T with an approxi-

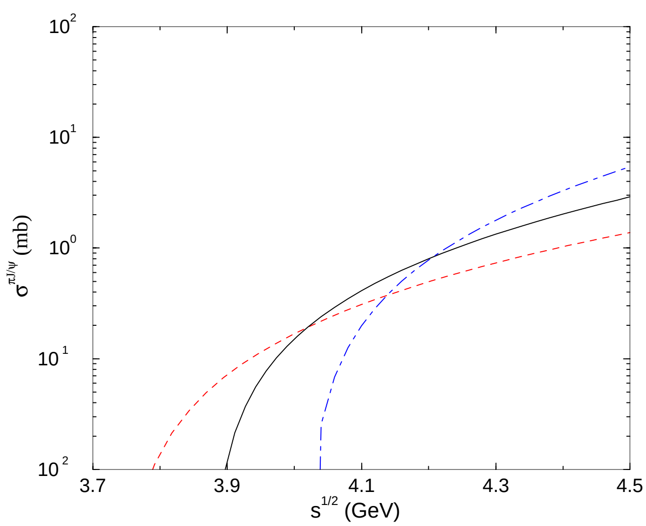

FIG. 4. J/ψ-π dissociation cross sections for the processes $J/\psi\pi\rightarrow\bar{D}D^*+D\ \bar{D}^*$ (solid line), $J/\psi\pi\rightarrow\bar{D}D$ (dashed line), and $J/\psi\pi\rightarrow\bar{D}^*D^*$ (dot-dashed line).

mate Bose-Einstein distribution. Therefore, in a hadron gas, pions collide with J/ψ at different energies, and the relevant quantity is not the value of the cross section at a given energy, but the thermal average of the cross section. The thermal average of the cross section is defined by the product of the dissociation cross section and the relative velocity of initial state particles averaged over the energies of the pions, $\langle \sigma^{\pi J/\psi} v \rangle$, and is given by [16]

$$\langle \sigma^{\pi J/\psi} v \rangle = \frac{\int_{z_0}^{\infty} dz [z^2-(\alpha_1+\alpha_2)^2][z^2-(\alpha_1-\alpha_2)^2] K_1(z) \sigma^{\pi J/\psi}(s=z^2T^2)}{4\alpha_1^2 K_2(\alpha_1)\alpha_2^2 K_2(\alpha_2)}, \tag{27}$$

where $\alpha_i = m_i/T$ ($i=1$ to 4), $z_0=\max(\alpha_1+\alpha_2, \alpha_3+\alpha_4)$, and K_i is the modified Bessel function.

As shown in Fig. 6, $\langle \sigma^{\pi J/\psi} v \rangle$ increases with the temperature. Since the J/ψ dissociation by a pion requires energetic pions to overcome the energy threshold, it has a small thermal average at low temperatures. The magnitude of our thermal average cross section is of the same order as the meson exchange model calculation in Ref. [16] with a cutoff $\Lambda = 1$ GeV. The shaded area in Fig. 6 gives an evaluation of the uncertainties in our calculation due to the two procedures used to extract the hadronic amplitudes.

V. SUMMARY AND CONCLUSIONS

We have evaluated the hadronic amplitudes for the J/ψ dissociation by pions using the QCD sum rules based on a three-point function using vacuum-pion correlation functions. We have considered the OPE expansion up to twist-4 and we have worked in the soft-pion limit. Our work improves the former QCDSR calculation, done with a four-point function at the pion pole [24], since we have included more terms in the OPE expansion. We have shown that the twist-3 and twist-4 contributions to the sum rules are small when compared with the leading order contribution, showing a good convergence of the OPE expansion. We have checked that, taking the appropriate limit, we recover the previous result of Ref. [24].

From the theoretical point of view, the use of QCDSR in this problem was responsible for real progress, being a step beyond models and beyond the previous leading twist calculations [8–10,37,24]. This is specially true in the low energy region, close to the open charm production threshold. At higher energies our treatment is less reliable due to the approximations employed.

In the nonrelativistic quark exchange model of Wong, Swanson, and Barnes [11], the cross section increases with the energy in the region close to the threshold and then decreases rapidly with $\sqrt{s}$. All other approaches to the problem, namely, the short distance QCD [8–10], the effective Lagrangian [13–18], and the QCDSR treatment lead to monotonically increasing (with the energy) cross section. The monotonic growth of the cross section comes simply from the growth of the final state phase space. All these ap-

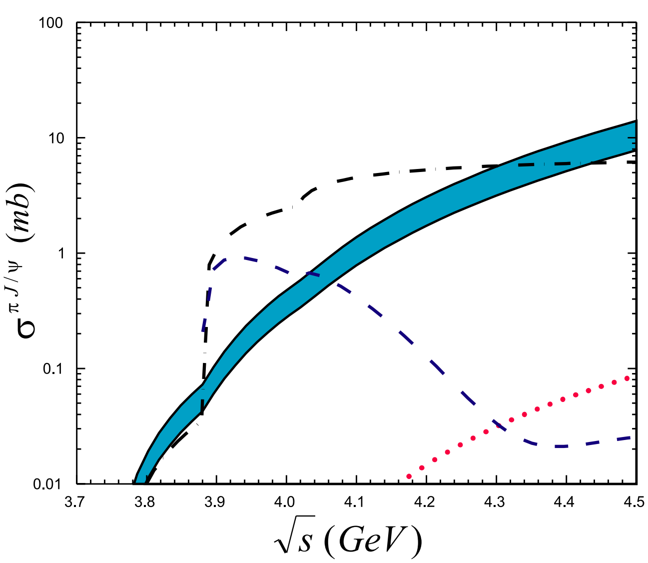

FIG. 5. J/ψ-π dissociation cross sections from meson exchange model [14] (dot-dashed line), quark exchange model [11] (dashed line), short distance QCD [9,10] (dotted line), and QCD sum rules (solid lines). The shaded area gives an evaluation of the uncertainties in our calculation.

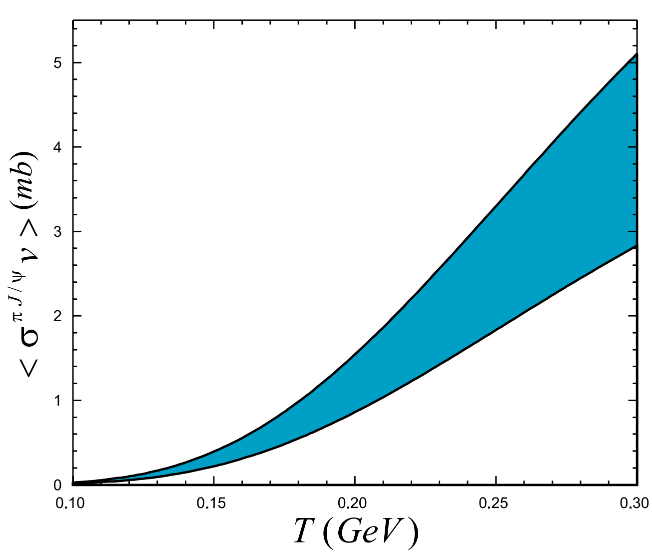

FIG. 6. Thermal average of J/ψ dissociation cross section by pions as a function of temperature T. The shaded area give an evaluation of the uncertainties in our calculation.

proaches, except for short distance QCD, gradually lose their validity at higher energies, but it is impossible to give a precise number up to which we should believe the results.

In the effective Lagrangian approach, in spite of the fact that there are some mesonic masses, the size of the cross sections is ultimately given by nonperturbative ingredients such as the form factors (with both their shapes and cutoff parameters), and the coupling constants. In other words, these cross sections are determined by the spatial extension of the charm charges, which should be put in by hand or fitted to experimental data. In the short distance QCD treatment, the quarkonium is considered to be compact enough to resolve the gluon/partonic content of the target (typically a pion or a nucleon), and therefore, the LO cross section is proportional to the gluon-quarkonium cross section convoluted with the gluon distribution of these targets. As the collision energy increases, gluons with smaller momentum fraction x can contribute to the breakup of the quarkonium. Hence, the cross section grows with increasing energy as the available number of gluons rises at small x. Finally, the physical picture emerging from our QCD sum rule calculation is the following: the dominant contribution comes from the twist-2 operators, or equivalently from the quark condensate. As we know that the quark condensate is stronger in the vacuum and weaker in the interior of hadrons, we can conclude that the charmonium ''sees'' and interacts with the surface of the pions where there is a ''halo'' of condensates.

Although a more sophisticated analysis of our uncertainties is still to be done, the shaded area in Fig. 5 shows that we can make some unambiguous statement concerning the behavior of $\sigma_{\pi J/\psi}$ with the energy $\sqrt{s}$. Our cross section grows monotonically with the c.m.s. energy but not as fast, near the thresholds, as it does in the calculations using meson exchange models [14–18]. We have also shown the importance of respecting chiral symmetry, since the increase of the cross section near the threshold is strongly intensified when chiral symmetry is broken. In other words, our results suggest that *using meson exchange models is perfectly acceptable, provided that they include form factors and that they respect chiral symmetry*. With these precautions, they can be a good tool to make predictions at somewhat higher energies.

We have also evaluated the thermal average of the J/ψ-π dissociation cross section. It increases with the temperature and at $T=150$ MeV we get $\langle\sigma^{\pi J/\psi}v\rangle\sim 0.2$–$0.4$ mb which is compatible with the values presented in Fig. 5 of Ref. [16], i.e., in a meson exchange model with monopole form factor with cutoff $\Lambda=1$ GeV. The use of this information will reduce the uncertainties in the calculations of the hadronic lifetimes of J/ψ, which are needed in simulations such as those of Ref. [5].

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APPENDIX A: SUM RULES FOR THE PROCESS $J/\psi\pi\to\bar{D}D^*$

Using $\Gamma_3=\gamma_\nu$ and $\Gamma_4=i\gamma_5$ in Eqs. (2) and (8), we obtain the following sum rules for the structures in Eq. (15):

$$\Lambda_1^{DD^*}+A_1^{DD^*}M^2+B_1^{DD^*}M^4 =\frac{1}{C_{DD^*}\ f_{DD^*}(M^2)}\left[-f_\pi m_c+f_{3\pi}\left(3+\frac{m_c^2}{2M^2}\right)\right]\times\frac{e^{-m_c^2/M^2}}{M^2}, \tag{A1}$$

$$\Lambda_2^{DD^*}+A_2^{DD^*}M^2+B_2^{DD^*}M^4 =\frac{1}{C_{DD^*}f_{DD^*}(M^2)}\left\{f_\pi m_c+\frac{4\langle\bar{q}q\rangle}{3f_\pi}-\frac{m_c}{6M^2}\left[\frac{4\langle\bar{q}q\rangle m_c}{f_\pi}+\frac{\delta^2 f_\pi}{3}\left(19+\frac{5m_c^2}{M^2}\right)\right]\right\}\frac{e^{-m_c^2/M^2}}{M^2}, \tag{A2}$$

$$\Lambda_3^{DD^*}+A_3^{DD^*}M^2+B_3^{DD^*}M^4 =\frac{1}{C_{DD^*}f_{DD^*}(M^2)}\left\{2f_\pi m_c-\frac{8\langle\bar{q}q\rangle}{3f_\pi}-\frac{m_c}{6M^2}\left[\frac{8\langle\bar{q}q\rangle m_c}{f_\pi}+\frac{\delta^2 f_\pi}{3}\left(-2+\frac{10m_c^2}{M^2}\right)\right]\right\}\frac{e^{-m_c^2/M^2}}{M^2}, \tag{A3}$$

where

$$f_{DD^*}(M^2)=\frac{1}{m_\psi^2-m_{D^*}^2}\left[\frac{e^{-m_D^2/M^2}-e^{-m_\psi^2/M^2}}{m_\psi^2-m_D^2}-\frac{e^{-m_D^2/M^2}-e^{-m_{D^*}^2/M^2}}{m_{D^*}^2-m_D^2}\right] \tag{A4}$$

and

$$C_{DD^*}=\frac{m_c}{m_D^2 m_{D^*}m_\psi f_D f_{D^*}f_\psi}. \tag{A5}$$

$A_i^{DD^*}$ and $B_i^{DD^*}$ are the parameters introduced to account for double-pole-continuum and single-pole-continuum transitions, respectively.

APPENDIX B: SUM RULES FOR THE PROCESS $J/\psi\pi\to\bar{D}D$

Using $\Gamma_3=i\gamma_5$ and $\Gamma_4=i\gamma_5$ in Eqs. (2) and (8), we obtain the following sum rule for the structure in Eq. (16) [36]:

$$\frac{\Lambda_{DD}+A_{DD}M^2+B_{DD}M^4}{m_\psi^2-m_D^2}\left[\frac{e^{-m_D^2/M^2}}{M^2}-\frac{e^{-m_D^2/M^2}-e^{-m_\psi^2/M^2}}{m_\psi^2-m_D^2}\right]$$
$$=\frac{m_c^2}{m_D^4 m_\psi f_D^2 f_\psi}\frac{e^{-m_c^2/M^2}}{M^2}\left[f_\pi-\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi M^2}-\frac{f_\pi\delta^2}{18M^2}\left(17+\frac{5m_c^2}{M^2}\right)\right]. \tag{B1}$$

APPENDIX C: SUM RULES FOR THE PROCESS $J/\psi\pi\to\bar{D}^*D^*$

Using $\Gamma_3=\gamma_\nu$ and $\Gamma_4=\gamma_\rho$ in Eqs. (2) and (8), we obtain the following sum rules for the structures in Eq. (17):

$$\Lambda_1^{D^*D^*}+A_1^{D^*D^*}M^2+B_1^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[-f_\pi+\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi M^2}+\frac{f_\pi\delta^2}{6M^2}\left(\frac{25}{3}+\frac{5m_c^2}{3M^2}\right)\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C1}$$

$$\Lambda_2^{D^*D^*}+A_2^{D^*D^*}M^2+B_2^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[f_\pi-\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi M^2}-\frac{f_\pi\delta^2}{6M^2}\left(\frac{13}{3}+\frac{5m_c^2}{3M^2}\right)\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C2}$$

$$\Lambda_3^{D^*D^*}+A_3^{D^*D^*}M^2+B_3^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[-f_\pi+\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi M^2}+\frac{f_\pi\delta^2}{6M^2}\left(\frac{19}{3}+\frac{5m_c^2}{3M^2}\right)\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C3}$$

$$\Lambda_4^{D^*D^*}+A_4^{D^*D^*}M^2+B_4^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[2f_\pi-\frac{4m_c\langle\bar{q}q\rangle}{3f_\pi M^2}-\frac{f_\pi\delta^2}{3M^2}\left(\frac{23}{3}+\frac{5m_c^2}{3M^2}\right)\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C4}$$

$$\Lambda_5^{D^*D^*}+A_5^{D^*D^*}M^2+B_5^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[-f_\pi+\frac{23f_\pi\delta^2}{18M^2}\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C5}$$

$$\Lambda_6^{D^*D^*}+A_6^{D^*D^*}M^2+B_6^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[-f_\pi+\frac{25f_\pi\delta^2}{18M^2}\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C6}$$

$$\Lambda_7^{D^*D^*}+A_7^{D^*D^*}M^2+B_7^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[f_\pi-\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi M^2}-\frac{3m_c f_{3\pi}}{M^2}\right]\times\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C7}$$

$$\Lambda_8^{D^*D^*}+A_8^{D^*D^*}M^2+B_8^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[f_\pi-\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi M^2}+\frac{2m_c f_{3\pi}}{M^2}\right]\times\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C8}$$

$$\Lambda_9^{D^*D^*}+A_9^{D^*D^*}M^2+B_9^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[-\frac{m_c^2 f_\pi}{2}+\frac{m_c\langle\bar{q}q\rangle}{2f_\pi}+\frac{f_\pi\delta^2}{3}\left(\frac{53}{6}-\frac{13m_c^2}{6M^2}+\frac{5m_c^4}{12M^4}\right)\right]\frac{e^{-m_c^2/M^2}}{M^2}, \tag{C9}$$

$$\Lambda_{10}^{D^*D^*}+A_{10}^{D^*D^*}M^2+B_{10}^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[\frac{2m_c\langle\bar{q}q\rangle}{3f_\pi}+m_c f_{3\pi}\right]\frac{e^{-m_c^2/M^2}}{M^4}, \tag{C10}$$

$$\Lambda_{11}^{D^*D^*}+A_{11}^{D^*D^*}M^2+B_{11}^{D^*D^*}M^4$$
$$=\frac{1}{C_{D^*D^*}f_{D^*D^*}(M^2)}\left[2m_c f_{3\pi}-\frac{f_\pi\delta^2}{3}\right]\frac{e^{-m_c^2/M^2}}{M^4}, \tag{C11}$$

$$\Lambda_{12}^{D^*D^*}+A_{12}^{D^*D^*}M^2+B_{12}^{D^*D^*}M^4$$
$$=-\frac{2}{C_{D^*D^*}f_{D^*D^*}(M^2)}\frac{f_\pi\delta^2}{3}\frac{e^{-m_c^2/M^2}}{M^4}, \tag{C12}$$

where

$$f_{D^*D^*}(M^2)=\frac{1}{m_{D^*}^2-m_\psi^2}\left[\frac{e^{-m_{D^*}^2/M^2}}{M^2}-\frac{e^{-m_{D^*}^2/M^2}-e^{-m_\psi^2/M^2}}{m_\psi^2-m_{D^*}^2}\right] \tag{C13}$$

and

$$C_{D^*D^*}=\frac{1}{m_{D^*}^2 m_\psi f_{D^*}^2 f_\psi}. \tag{C14}$$

APPENDIX D: SUM RULES FOR THE MESON DECAY CONSTANTS

For consistency we use in our analysis the QCDSR expressions for the decay constants of the J/ψ, D^*, and D mesons up to dimension four in lowest order of α_s:

$$f_D^2=\frac{3m_c^2}{8\pi^2 m_D^4}\int_{m_c^2}^{s_D} ds\frac{(s-m_c^2)^2}{s}e^{(m_D^2-s)/M^2} - \frac{m_c^3}{m_D^4}\langle\overline{q}q\rangle e^{(m_D^2-m_c^2)/M^2}, \tag{D1}$$

$$f_{D^*}^2=\frac{1}{8\pi^2 m_{D^*}^2}\int_{m_c^2}^{s_{D^*}} ds\left[\frac{(s-m_c^2)^2}{s}\left(2+\frac{m_c^2}{s}\right)e^{(m_{D^*}^2-s)/M^2}\right] - \frac{m_c}{m_{D^*}^2}\langle\overline{q}q\rangle e^{(m_{D^*}^2-m_c^2)/M^2}, \tag{D2}$$

$$f_\psi^2=\frac{1}{4\pi^2}\int_{4m_c^2}^{s_\psi} ds(s+2m_c^2)\sqrt{\frac{s-4m_c^2}{s^{3/2}}}e^{(m_\psi^2-s)/M^2}, \tag{D3}$$

where s_M stands for the continuum threshold of the meson M, which we parametrize as $s_M=(m_M+\Delta_s)^2$. The values of s_M are, in general, extracted from the two-point function sum rules for f_D and f_{D^*} and f_ψ in Eqs. (D1)–(D3). Using the Borel region $3\leqslant M_M^2\leqslant 6$ GeV^2 for the D^* and D mesons and $6\leqslant M_M^2\leqslant 12$ GeV^2 for the J/ψ, we found good stability for f_D, f_{D*}, and f_ψ with $\Delta_s\sim 0.6$ GeV. We obtained $f_D=160\pm 5$ MeV, $f_{D*}=220\pm 10$ MeV, and $f_\psi=280\pm 10$ MeV, which are compatible with the numerical values in Eq. (18).

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