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Further observations on midrapidity E_T distributions with aperture corrected scale

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In a previous publication [T. Abbott *et al.*, E802 Collaboration, Phys. Rev. C **63**, 064602 (2001); **64**, 029901(E) (2001)], measurements of the A dependence and pseudorapidity interval ($\delta\eta$) dependence of midrapidity E_T distributions in a half-azimuth ($\Delta\phi=\pi$) electromagnetic calorimeter were presented for p+Be, p+Au, O+Cu, Si+Au, and Au+Au collisions at the BNL-AGS. The validity of the "nuclear geometry" characterization versus $\delta\eta$ was illustrated by plots of the $E_T(\delta\eta)$ distribution in each $\delta\eta$ interval in units of the measured $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$ in the same $\delta\eta$ interval for p+Au collisions. These plots, with aperture corrected scale in the physically meaningful units of number of average observed p+Au collisions, were nearly universal as a function of $\delta\eta$, confirming that the reaction dynamics for E_T production at midrapidity at AGS energies is governed by the number of projectile participants and can be well characterized by measurements in apertures as small as $\Delta\phi=\pi, \delta\eta=0.3$. A key ingredient in these analyses is the probability p_0 for no signal to be detected in a given aperture $\delta\eta$ for the fundamental p+Au collision. In fact the measured $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$ is biased and the true $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}_{p+\mathrm{Au}}$ for the detector aperture is the measured value times $1-p_0$. The issues and merits of measuring the $E_T(\delta\eta)$ distribution in units of $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$ or $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}_{p+\mathrm{Au}}$ in the same $\delta\eta$ interval are presented and discussed. This method has application at RHIC, where p-p data could be used as the reference distribution for two participants. The E_T distributions for $B+A$ collisions, with $E_T(\delta\eta)$ scale normalized by $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}_{p\text{-}p}$ in the same aperture for p-p collisions, would then be given directly in the popular unit "per participant-pair" [K. Adcox *et al.*, PHENIX Collaboration, Phys. Rev. Lett. **86**, 3500 (2001); I. G. Bearden *et al.*, BRAHMS Collaboration, Phys. Lett. B**523**, 227 (2001); B. B. Back *et al.*, PHOBOS Collaboration, Phys. Rev. C **65**, 031901(R) (2002); C. Adler *et al.*, STAR Collaboration, Phys. Rev. Lett. **89**, 202301 (2002)].

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I. INTRODUCTION

In a previous publication [1], a systematic study of transverse energy (E_T) distributions at midrapidity in a half-azimuth ($\Delta\phi=\pi$) electromagnetic (EM) calorimeter was presented for 14.6*A* GeV/*c* $p+$Be, $p+$Au, O+Cu, Si+Au, and Au+Au collisions[1] at the BNL-AGS. E_T is an event-by-event variable defined as

$$E_T=\sum_i E_i \sin\theta_i \quad \text{and} \qquad dE_T(\eta)/d\eta=\sin\theta(\eta)\, dE(\eta)/d\eta, \tag{1}$$

$\eta=-\ln\tan\theta/2$ is the pseudorapidity and the sum is taken over all particles emitted in an event into a fixed but large solid angle. The importance of E_T distributions in relativistic heavy ion (RHI) collisions is that they are largely dominated by the *nuclear geometry* of the reaction and so provide a measure of the overall character or *centrality* of individual RHI interactions: as the impact parameter is reduced from grazing impact, more nucleons participate (there are fewer spectators) so more energy is transferred from the projectile and target rapidity regions to the transverse direction and toward midrapidity. Extensive references to the literature and details of the measurement are given in the previous publication [1].

Two issues addressed in the previous study [1] were whether the limited acceptance and the limited calorimeter response (electromagnetic instead of hadronic) affected the centrality characterization and whether the success [3,4,1] in reconstructing the measured O+A, Si+A, Au+Au ($B+A$) midrapidity E_T spectra as the sum of the one to B-fold convolutions of the measured $p+$Au spectrum, weighted according to the "geometric"probability for 1, 2, ..., B of the projectile nucleons to interact in the target (wounded projectile nucleon model, or WPNM), could be an artifact of the limited coverage.

One obvious problem with E_T distributions from a limited aperture EM calorimeter in comparison to measurements using 4π hadronic calorimeters is the difficulty in relating the end points or the overall E_T scales of the spectra to the total available energy for the reaction. In the previous study, the E_T scale for each aperture was normalized by the measured $\langle E_T\rangle$ in the same aperture for $p+$Au collisions [1], so that the distributions became transformed to the physically meaningful units of "number of average $p+$Au collisions," equivalent to projectile participants at AGS energies [1]. The issue of whether or how precisely the E_T distributions in limited apertures are proportional to the number of projectile participants could then be read directly from the data without any recourse to external centrality definition or other correction of the E_T spectra for limited aperture and calorimeter response. It was found that the upper percentiles of these distributions, in units of $E_T(\delta\eta)/\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$, varied only slightly ($\leqslant 5\%$) with $\delta\eta$ over the range 7 percentile to 0.5 percentile typically used to define *central collisions*. Notably, this small-observed variation was significantly less than would be expected if the data were perfectly described by the WPNM. Also the measured percentiles in units of projectile participants (i.e., wounded projectile nucleons) were smaller than the percentiles of a pure WPNM or wounded nucleon model (WNM) calculation, for instance as given recently by the four experiments at RHIC [2].[2]

II. THE IMPORTANCE OF p_0 FOR LIMITED APERTURES

A key quantity for E_T distributions in limited apertures is the probability p_0 for a $p+$Au collision (or other reference reaction) to produce no signal on the aperture, the $\delta\eta$ inter-

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[1]The Au+Au measurements were at 11.6*A* GeV/*c* and scaled up in E_T by a factor of 1.155 to correspond to the higher beam energy.

[2]The WNM and WPNM calculations are degenerate for Au+Au, since by symmetry the total number of wounded nucleons is simply twice the number of wounded projectile nucleons at any impact parameter.

TABLE I. Measured quantities from $p+\mathrm{Au}$ data from Ref. [1]. Errors quoted are statistical only. The $\langle E_T\rangle$ observed on each interval is computed from a Γ distribution fit to the measured E_T spectrum. The probability p_0 for a $p+\mathrm{Au}$ reaction to produce zero signal on the interval $\delta\eta$ is computed by taking $1-p_0$ as the ratio of σ, the observed cross section on the interval, to the inelastic $p+\mathrm{Au}$ cross section of 1.662 b (0.176 b) from a nuclear geometry calculation [5]. Also note that on each interval $\langle E_T\rangle^{\mathrm{true}} = (1-p_0)\langle E_T\rangle$.

Measured quantities from $p+\mathrm{Au}$ collisions in Ref. [1]					
$\delta\eta$	σ (b)	$\langle E_T\rangle$ (GeV)	$1-p_0$	$\langle E_T\rangle^{\mathrm{true}}$ (GeV)	$\langle E_T\rangle^{\mathrm{true}}/\delta\eta$ (GeV)
1.30	1.52 ± 0.02	0.353 ± 0.007	0.917 ± 0.011	0.324 ± 0.008	0.249 ± 0.006
0.966	1.40 ± 0.02	0.286 ± 0.006	0.844 ± 0.011	0.241 ± 0.006	0.250 ± 0.006
0.624	1.29 ± 0.02	0.197 ± 0.006	0.774 ± 0.014	0.152 ± 0.005	0.244 ± 0.009
0.378	1.29 ± 0.06	0.119 ± 0.007	0.774 ± 0.034	0.092 ± 0.007	0.244 ± 0.018

val in the present discussion. Since the measured $\langle E_T\rangle$ corresponds to the case where a signal is detected on the aperture, it only corresponds to a fraction $1-p_0$ of all $p+\mathrm{Au}$ collisions and is thus biased. The true $\langle E_T\rangle$ for all $p+\mathrm{Au}$ collisions is

$$\langle E_T\rangle^{\mathrm{true}}_{p+\mathrm{Au}} = (1-p_0)\langle E_T\rangle_{p+\mathrm{Au}}, \tag{2}$$

where we keep the notation from Ref. [1] that $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$ without the superscript "true" means the measured or observed value.

For experiments where the cross section $d\sigma/dE_T$ is measured, the probability p_0 for the reference reaction to produce zero signal on an interval can be computed by taking $1-p_0$ as the ratio of σ, observed cross section on the interval (from the integral of the $d\sigma/dE_T$), to the known cross section for the reference reaction [4,5]. In Ref. [1] this was done for $p+\mathrm{Au}$ collisions as a function of $\delta\eta$ (see Table I). From Table I, the bias of the measured $\langle E_T\rangle$ is evident: for instance, when the $\delta\eta$ is reduced from 1.30 to 0.624, a factor of 2.08, the observed $\langle E_T\rangle$ falls by only a factor of 1.79, from 0.353 to 0.197 GeV. It is easy to realize that the only case for which splitting an interval in half will result in exactly half the measured $\langle E_T\rangle$ on each half interval is when p_0 is the same on both the full and each of the half intervals. In general, this can only happen when $p_0=0$ on both half intervals. Alternatively, if the measured $\langle E_T\rangle$ is corrected to $\langle E_T\rangle^{\mathrm{true}}$ on all intervals, then a true split occurs. This can be seen in Table I where $\langle E_T\rangle^{\mathrm{true}}$ drops by a factor of 2.1 from 0.353 to 0.197 GeV and $\langle E_T\rangle^{\mathrm{true}}/\delta\eta$ is constant from $\delta\eta=1.30$ to 0.378 as expected for a constant $dE_T/d\eta$ [6].

In Ref. [1], the E_T scales for each aperture were normalized by the measured $\langle E_T\rangle$ in the same aperture for $p+\mathrm{Au}$ collisions [1], and we noted above that this normalization transformed the distributions to physically meaningful units which we called "number of average $p+\mathrm{Au}$ collisions." It is clear from the present discussion that we should have called these units number of average *observed* $p+\mathrm{Au}$ collisions. Distributions in the true number of average $p+\mathrm{Au}$ collisions are obtained by normalizing the E_T scales for each aperture by $\langle E_T\rangle^{\mathrm{true}} = (1-p_0)\langle E_T\rangle$ in the same aperture for $p+\mathrm{Au}$ collisions. The difference in results for the two cases is investigated in the following sections.

In Sec. III we show how normalization of the E_T scale by $\langle E_T\rangle^{\mathrm{true}}$ gives an excellent representation of the WPNM in all apertures, while using the observed $\langle E_T\rangle$ gives results far from the WPNM values. In Sec. IV we compare the data for the two cases, with surprising results.

III. WOUNDED NUCLEON MODELS

In the extreme-independent-collision models of $B+A$ nuclear scattering, such as the wounded nucleon model [7,8] and wounded projectile nucleon model, the effect of the nuclear geometry of the interaction can be calculated independently of the dynamics of particle production, which can be taken directly from experimental measurements. In these models, the nuclear geometry is represented as the relative probability per interaction for a given number of total participants (WNM), projectile participants (WPNM), or other basic elements of particle production such as wounded projectile quarks (additive quark model) [9] or binary nucleon-nucleon collisions, integrated over the impact parameter of the $B+A$ reaction. Typically, Woods-Saxon densities are used for both the projectile and target nuclei, and the nucleon-nucleon inelastic cross section appropriate to the center of mass energy of the collision is taken. For the present discussion, 30 mb is used, corresponding to a nucleon-nucleon mean free path of $\sim$2.2 fm at nuclear density. Once the nuclear geometry is specified in this manner, experimental measurements can be used to derive the distribution (in the actual detector) of E_T or multiplicity (or other additive quantity) for the elementary collision process, i.e., a wounded nucleon or a wounded projectile nucleon, which is then used as the basis of the analysis of a nuclear scattering as the result of multiple independent elementary collision processes. The key experimental issue then becomes the linearity of the detector response to multiple collisions (better than 1% in the present case), instead of detailed instrumental corrections to obtain, e.g., the total hadronic E_T impinging on the detector by correcting the measured E_T using the response function or hadronic simulation.

The WPNM calculation for a B+A reaction is given by the sum

$$\left(\frac{d\sigma}{dE_T}\right)_{\mathrm{WPNM}} = \sigma_{BA}\sum_{n=1}^{B} w_n\, P_n(E_T), \tag{3}$$

where σ_{BA} is the measured $B+A$ cross section in the interval $\delta\eta$, w_n is the relative probability for n projectile participants in the $B+A$ reaction, and $P_n(E_T)$ is the calculated E_T distribution on the $\delta\eta$ interval for n independently interacting projectile nucleons. If $f_1(E_T)$ is the measured E_T spectrum on the $\delta\eta$ interval for one projectile nucleon, in the present case the $p+$Au spectrum, and p_0 is the probability for a p $+$Au collision to produce no signal in the $\delta\eta$ interval, then, the correctly normalized E_T distribution for one projectile participant is

$$P_1(E_T)=(1-p_0)f_1(E_T)+p_0\delta(E_T), \qquad (4)$$

where $\delta(E_T)$ is the Dirac delta function and $\int f_1(E_T)dE_T$ $=1$. $P_n(E_T)$ (including the p_0 effect) is obtained by convoluting $P_1(E_T)$ with itself $n-1$ times,

$$P_n(E_T)=\sum_{i=0}^{n}\frac{n!}{(n-i)!\,i!}p_0^{n-i}(1-p_0)^i f_i(E_T), \qquad (5)$$

where $f_0(E_T)\equiv\delta(E_T)$ and $f_i(E_T)$ is the ith convolution of $f_1(E_T)$:

$$f_i(x)=\int_0^x dy\, f_1(y)f_{i-1}(x-y). \qquad (6)$$

Substituting Eq. (5) into Eq. (3) and reversing the indices gives a form that is less physically transparent, but considerably easier to compute:

$$\left(\frac{d\sigma}{dE_T}\right)_{\rm WPNM}=\sigma_{BA}\sum_{i=1}^{B} w_i'(p_0)f_i(E_T), \qquad (7)$$

where

$$w_i'(p_0)=(1-p_0)^i\sum_{n=i}^{B}\frac{n!}{(n-i)!\;\;i!}p_0^{n-i}w_n\,. \qquad (8)$$

It is interesting to note that the mean of the nth convolution of the observed reference distribution, Eq. (6),

$$\int\;E_Tf_n(E_T)dE_T=n\langle E_T\rangle, \qquad (9)$$

gives n times the observed $\langle E_T\rangle$, as it should, while the mean of the distribution $P_n(E_T)$ for n independently interacting projectile nucleons, Eq. (5), gives the correct value for

$$\int\;E_TP_n(E_T)dE_T=n\langle E_T\rangle(1-p_0)=n\langle E_T\rangle^{\rm true}. \qquad (10)$$

The WPNM calculations from Ref. [1] for Au+Au (14.6A GeV/c) with the $E_T(\delta\eta)$ scale normalized by the measured $\langle E_T(\delta\eta)\rangle_{p+{\rm Au}}$ are shown in Fig. 1(a) while the same calculations with the $E_T(\delta\eta)$ scale normalized by $\langle E_T(\delta\eta)\rangle^{\rm true}_{p+{\rm Au}}$ are shown in Fig. 1(b). As noted in Ref. [1] the positions of the knees of the curves in Fig. 1(a) decrease proportionally to $1-p_0$ and the steep falloffs above the knees reflect the shapes of the underlying $p+$Au distributions. This is especially clear for the curves for $\delta\eta=0.624$ and 0.378 in Fig. 1(a) which have the same p_0 but different shape $p+$Au reference spectra. In Fig. 1(b), the curves with $E_T(\delta\eta)$ scales normalized by $\langle E_T(\delta\eta)\rangle^{\rm true}_{p+{\rm Au}}$ nicely illustrate the expected behavior of WPNM calculations with different shape reference distributions. The WPNM calculations closely follow the relative probability for n projectile participants w_n until just below the knee $\sim$160 projectile participants, roughly the upper 4–5 percentile. Above the knee, the shapes of the distributions no longer reflect the simple nuclear geometry (w_n), which is exhausted at $\sim$193 projectile participants, but are sensitive to the underlying dynamics, in this case represented by the difference in shapes of the reference distributions, which fluctuate more (are flatter) the smaller the aperture. It is clear from Fig. 1 that normalizing the scale of $E_T(\delta\eta)$ by $\langle E_T(\delta\eta)\rangle^{\rm true}_{p+{\rm Au}}$ really does give distributions which can be directly read in projectile participants up to the top 5 percentile and is better from the theoretical standpoint than normalizing by the measured $\langle E_T(\delta\eta)\rangle_{p+{\rm Au}}$ which does not correctly reflect the true num-

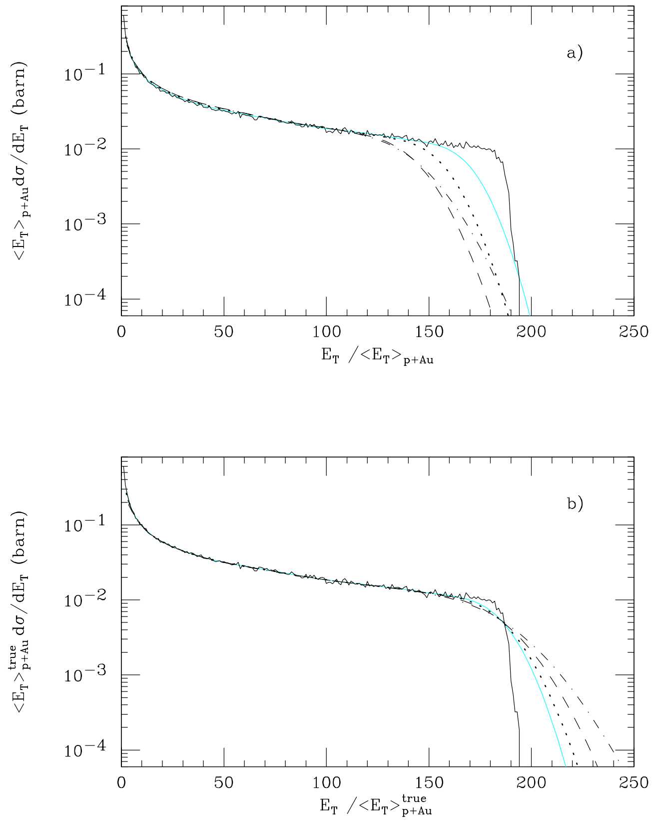

FIG. 1. (a) WPNM distributions for Au+Au at 14.6A GeV/c on the four $\delta\eta$ intervals, $\delta\eta=1.30$ (light solid), 0.966 (dots), 0.624 (dashes), 0.378 (dot dash), with $E_T(\delta\eta)$ scales normalized by the measured $\langle E_T(\delta\eta)\rangle_{p+{\rm Au}}$ on the same interval. (b) WPNM distributions for Au+Au at 14.6A GeV/c on the four $\delta\eta$ intervals, with $E_T(\delta\eta)$ scales normalized by the true $\langle E_T(\delta\eta)\rangle^{\rm true}_{p+{\rm Au}}$ on the same interval. The solid-jagged curve is the relative probability w_n for n projectile participants. The integrals of all curves are normalized to 6.47 b, the measured Au+Au cross section on all four intervals (to within negligible statistical error).

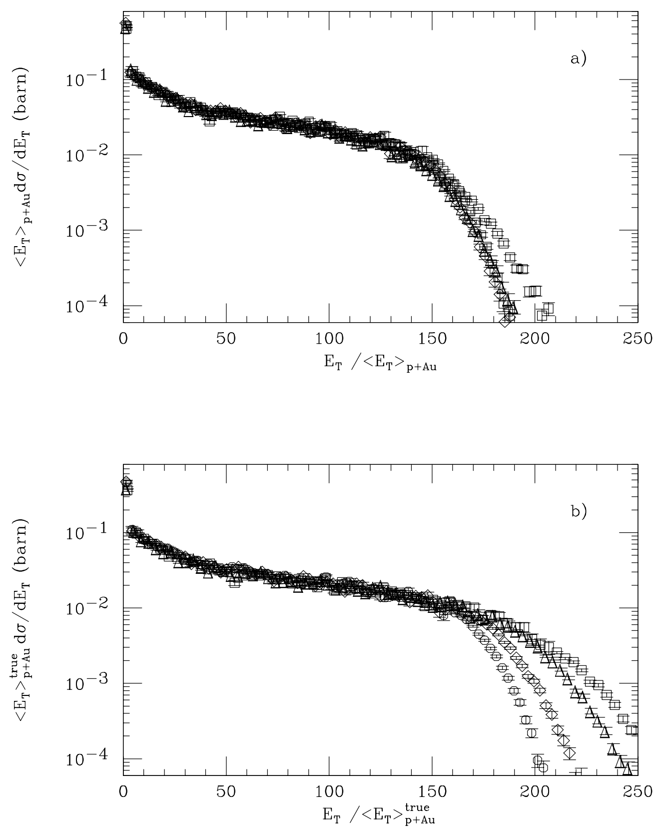

FIG. 2. (a) Measured distributions for Au+Au [1] at 14.6*A* GeV/*c* on the four $\delta\eta$ intervals, $\delta\eta = 1.30$ (circles), 0.966 (diamonds), 0.624 (triangles), 0.378 (squares), with $E_T(\delta\eta)$ scales normalized by the measured $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$ on the same interval. (b) Measured distributions for Au+Au on the four $\delta\eta$ intervals, with $E_T(\delta\eta)$ scales normalized by the true $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}_{p+\mathrm{Au}}$ on the same interval.

ber of projectile participants because of bias in the measured spectrum. Perhaps this should have been obvious from Eqs. (3) and (10).

IV. THE MEASUREMENTS

The measured E_T distributions from Ref. [1] for Au+Au, corrected[1] to 14.6*A* GeV/*c*, are shown in Fig. 2(a) with the $E_T(\delta\eta)$ scale normalized by the measured $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$ and in Fig. 2(b) with the $E_T(\delta\eta)$ scale normalized by $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}_{p+\mathrm{Au}}$. By comparing Figs. 2(b) and 1(b), it is easy to see from the distributions of the data and WPNM in units of $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}_{p+\mathrm{Au}}$ that the data largely follow the WPNM, but, as noted in Ref. [1], systematically vary from the WPNM predictions as a function of $\delta\eta$. The data in the $\delta\eta = 0.966$ aperture are closest to the WPNM, while the larger $\delta\eta$ spectrum is below the WPNM and the smaller $\delta\eta$ spectra are increasingly above the WPNM. Nevertheless, the data in Fig. 2(b) all closely follow the w_n distribution to ~ 160 units and above the knee exhibit a larger fluctuation, the smaller the aperture, just like the model. On the other hand, comparison of Figs. 2(a) and 1(a), the distributions of the data and WPNM with the $E_T(\delta\eta)$ scale in units of the measured $\langle E_T(\delta\eta)\rangle_{p+\mathrm{Au}}$, reveals that the small systematic variations of the data from the WPNM produce data distributions which overlap entirely over the whole measured range for the largest three $\delta\eta$ intervals, with the smallest $\delta\eta$ interval deviating slightly only in the upper tail. This spectacular empirical scaling law was perhaps understated in Ref. [1] with the description that the upper percentiles of the data distributions showed "small-observed variation" as a function of $\delta\eta$, "significantly less than would be expected" in the WPNM. It is worth remarking that the empirical scaling illustrated in Fig. 2(a) would likely have been missed if we had followed in Ref. [1] the correct procedure for normalizing the $E_T(\delta\eta)$ scales outlined in the present work. It is also worth noting that empirical scaling behavior of E_T distributions in disagreement with the WNM was seen in α-α collisions at $\sqrt{s_{NN}} = 31$ GeV at the CERN ISR [10].

V. CONCLUSIONS

The procedure for obtaining E_T distributions with aperture corrected scale outlined in Ref. [1] is amended in the present work by using the true $\langle E_T\rangle^{\mathrm{true}}$ for the reference distribution in the aperture, rather than the observed $\langle E_T\rangle$, to normalize the E_T scale in the same aperture for $B+A$ collisions. The measured $\langle E_T\rangle$ is biased because only a fraction $1-p_0$ of the reference collisions produce a signal on the aperture, so that $\langle E_T\rangle^{\mathrm{true}}$ for the reference distribution is related to the measured $\langle E_T\rangle$ by $\langle E_T\rangle^{\mathrm{true}} = (1-p_0)\langle E_T\rangle$. As demonstrated in Figs. 1(b) and 2(b), normalizing the scale of the measured $E_T(\delta\eta)$ distribution for Au+Au collisions by $\langle E_T(\delta\eta)\rangle^{\mathrm{true}}$ in the same aperture for the reference distribution really does give results which can be read directly in physically meaningful units (projectile participants for the present discussion) up to the top 5 percentile without recourse to external centrality definition or correction of the E_T spectra for limited aperture and calorimeter response. For the data at AGS energies, the reference distribution used was p+Au, which at midrapidity was shown [1] to represent the E_T distribution of a projectile participant. At higher energies, such as at RHIC, *p-p* data could be used as the reference distribution for two participants. The E_T distributions for $B+A$ collisions with E_T scale normalized by $\langle E_T\rangle^{\mathrm{true}}$ in the same aperture for *p-p* collisions would then be given directly[2] in the popular unit, "per participant-pair." Of course, one should also keep alert for possible additional unexpected empirical scaling laws for E_T distributions.

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