Coulomb corrections in the calculation of ultrarelativistic heavy ion production of continuum e^+e^- pairs

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Coulomb corrections to perturbation theory for producing electron-positron pairs in ultrarelativistic heavy ion collisions are considered in a part-analytical, part-numerical approach. Production probabilities are reduced from perturbation theory with increasing charge of the colliding heavy ions, as has been previously argued in the literature. It is shown here that the reduction from perturbation theory comes from the appropriate physical spatial cutoff of the electromagnetic potentials arising from the colliding ultrarelativistic heavy ions.

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I. INTRODUCTION

The problem of calculating heavy ion induced continuum e^+e^- pair production to all orders in $Z\alpha$ has received some renewed interest in the past several years. Realization that in an appropriate gauge [1], the electromagnetic field of a relativistic heavy ion is to a very good approximation a δ function in the direction of motion of the heavy ion times the two dimensional solution of Maxwell's equations in the transverse direction [2], led to an exact solution of the appropriate Dirac equation for excitation of bound-electron positron pairs [3]. Given this solution, it was perhaps not surprising that the solution of the Dirac equation was obtained independently and practically simultaneously by two different collaborations [4–6] for the analogous case of continuum e^+e^- pair production induced by the corresponding countermoving δ function potentials produced by ultrarelativistic heavy ions in a collider such as RHIC. An extended discussion and reanalysis of this solution, with comments on early parallel work in the literature, shortly followed [7]. One apparent physical consequence of this solution was that the rates for pair production in the exact solution agreed with the corresponding perturbation theory result [5–7].

Several authors subsequently argued [8–10] that a correct regularization of the exact Dirac equation amplitude should lead to deviations from perturbation theory, the so called Coulomb corrections. Although, as has been pointed out [11], the derived exact semiclassical Dirac amplitude is not simply the exact amplitude for the excitation of a particular (correlated) electron-positron pair, there are observables, such as the total pair production cross section, that can be constructed from this derived amplitude. The exact amplitude for a correlated electron-positron pair will not be treated here. It is the Coulomb corrections to the observables that *can* be constructed from this exact Dirac equation amplitude that are the topic of this paper.

In what follows it will be shown from a somewhat different approach from what has been done before that Coulomb corrections must exist, that they arise from the physical cutoff of the transverse Coulomb potential, and the accuracy of their evaluation has been up to now limited by an effective two-peak approximation to the exact retarded Dirac amplitude.

II. THE DIRAC EQUATION SOLUTION

One begins the semiclassical Dirac solution by representing the electromagnetic effect of one heavy ion on the other as the Liénhard-Wiechart potential produced by a point charge on a straight-line trajectory,

$$V(\boldsymbol{\rho},z,t)=\frac{\alpha Z(1-v\,\alpha_z)}{\sqrt{[(\mathbf{b}-\boldsymbol{\rho})/\gamma]^2+(z-vt)^2}}, \tag{1}$$

$\mathbf{b}$ is the impact parameter, perpendicular to the z axis along which the ions travel, $\boldsymbol{\rho}$, z, and t are the coordinates of the potential relative to a fixed target (or ion), α_z is the Dirac matrix, and Z, v, and γ are the charge, velocity, and relativistic γ factor of the moving ion. If one makes a gauge transformation on the wave function [1]

$$\psi=e^{-i\chi(\mathbf{r},t)}\psi', \tag{2}$$

where

$$\chi(\mathbf{r},t)=\frac{\alpha Z}{v}\ln[\,\gamma(z-vt)+\sqrt{b^2+\gamma^2(z-vt)^2}\,] \tag{3}$$

the interaction potential $V(\boldsymbol{\rho},z,t)$ is gauge transformed to

$$V(\boldsymbol{\rho},z,t)=\frac{\alpha Z(1-v\,\alpha_z)}{\sqrt{[(b-\boldsymbol{\rho})/\gamma]^2+(z-vt)^2}}-\frac{\alpha Z[1-(1/v)\,\alpha_z]}{\sqrt{b^2/\gamma^2+(z-vt)^2}}. \tag{4}$$

The second term is pure gauge and serves to reduce the range of the potential in $(z-vt)$ to more closely map the $(z-vt)$ range of the $\mathbf{B}$ and $\mathbf{E}$ fields, which have the denominator to the $\frac{3}{2}$ power rather than the $\frac{1}{2}$ power of the untransformed Lorentz gauge potential Eq. (1).

In the ultrarelativistic limit (ignoring correction terms in $[(b-\boldsymbol{\rho})/\gamma]^2$) [2]

$$V(\boldsymbol{\rho},z,t)=-\,\delta(z-t)(1-\alpha_z)\,\alpha Z_P\ln(\mathbf{b}-\boldsymbol{\rho})^2. \tag{5}$$

This is the potential that allowed the closed form solution of the Dirac equation for the bound-electron–positron problem. The full solution of the problem is in perturbation theory form, but with an eikonalized interaction in the transverse direction,

$$V(\boldsymbol{\rho},z,t) = -i\,\delta(z-t)(1-\alpha_z)\{\exp[-i\alpha Z_P \ln(\mathbf{b}-\boldsymbol{\rho})^2]-1\}, \tag{6}$$

in place of the perturbation interaction, Eq. (5), producing the higher order effect in $Z\alpha$. Recall that this exact semiclassical solution produced a reduction of a little less than 10% in the predicted cross section for Au + Au at RHIC [3]; one can identify this reduction as a Coulomb correction to bound-electron–positron pair production.

In the bound-electron–positron problem one conveniently takes the electromagnetic field of one moving heavy ion seen in the rest frame of the heavy ion that receives the created electron. For production of continuum pairs in an ultrarelativistic heavy ion reaction one may work in in the center of mass frame and the electromagnetic interaction goes to the limit of two countermoving δ function potentials

$$V(\boldsymbol{\rho},z,t) = \delta(z-t)(1-\alpha_z)\Lambda^-(\boldsymbol{\rho}) + \delta(z+t)(1+\alpha_z)\Lambda^+(\boldsymbol{\rho}), \tag{7}$$

where

$$\Lambda^{\pm}(\boldsymbol{\rho}) = -Z\alpha \ln\frac{(\boldsymbol{\rho}\pm\mathbf{b}/2)^2}{(b/2)^2}. \tag{8}$$

The semiclassical Dirac equation with this potential has been solved in closed form [4–7]. Baltz and McLerran [5] noted the apparent agreement of the obtained amplitude with that of perturbation theory even for large Z. Segev and Wells [6] also noted the agreement with perturbation theory and noted the scaling with $Z_1^2 Z_2^2$ seen in CERN SPS data [12]. These data were obtained from reactions of 160 GeV/c Pb ions on C, Al, Pa, and Au targets as well as 200 Gev/c S ions on the same C, Al, Pa, and Au targets. The group presenting the CERN data, Vane *et al.*, stated their findings in summary: ‘‘Cross sections scale as the product of the squares of the projectile and target nuclear charges.’’ On the other hand, it is well known that photoproduction of e^+e^- pairs on a heavy target shows a negative (Coulomb) correction proportional to Z^2 that is well described by the Bethe-Maximon theory [13].

III. COULOMB CORRECTIONS

As noted in the Introduction, several authors have argued that a correct regularization of the exact Dirac equation amplitude must lead to Coulomb corrections. The first analysis was done in a Weizsacker-Williams approximation [8]. Subsequently, Lee and Milstein argued [9,10] the existence of Coulomb corrections by an approximate analysis of the closed form solution of the Dirac equation. We will take as our starting point a somewhat extended consideration of the results of Lee and Milstein.

To begin let us write the previously derived semiclassical amplitude for electron-positron pair production [4–7] in the notation of Lee and Milstein [9],

$$M(p,q) = \int \frac{d^2k}{(2\pi)^2}\exp[i\,\mathbf{k}\cdot\mathbf{b}]\mathcal{M}(\mathbf{k})F_B(\mathbf{k})F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k}). \tag{9}$$

p and q are the four-momenta of the produced electron and positron, respectively, $\mathbf{k}$ is an intermediate transverse photon momentum to be integrated over,

$$\mathcal{M}(\mathbf{k}) = \overline{u}(p)\frac{\boldsymbol{\alpha}(\mathbf{k}-\mathbf{p}_\perp)+\gamma_0 m}{-p_+q_- - (\mathbf{k}-\mathbf{p}_\perp)^2 - m^2 + i\epsilon}\gamma_- u(-q) + \overline{u}(p)\frac{-\boldsymbol{\alpha}(\mathbf{k}-\mathbf{q}_\perp)+\gamma_0 m}{-p_-q_+ - (\mathbf{k}-\mathbf{q}_\perp)^2 - m^2 + i\epsilon}\gamma_+ u(-q), \tag{10}$$

and the effect of the potential Eqs. (7) and (8) is contained in integrals, F_B and F_A, over the transverse spatial coordinates taking the form

$$F(\mathbf{k}) = \int d^2\rho\,\exp[-i\mathbf{k}\cdot\boldsymbol{\rho}]\{\exp[-i2Z\alpha\ln\rho]-1\} = 2\pi\int_0^\infty \rho\,d\rho J_0(k\rho)\{\exp[-i2Z\alpha\ln\rho]-1\}. \tag{11}$$

$F(\mathbf{k})$ has to be regularized or cut off at large ρ. How it is regularized is the key to understanding Coulomb corrections. If one merely regularizes the integral itself at large ρ one obtains [5–7] apart from a trivial phase

$$F(\mathbf{k}) = \frac{4\pi\alpha Z}{k^{2-2i\alpha Z}}. \tag{12}$$

All the higher order $Z\alpha$ effects in $M(p,q)$ are contained only in the phase of the denominator of Eq. (12). As we will see, it directly follows that calculable observables are equal to perturbative results.

A. Observables

Before considering the Lee and Milstein analysis, we will discuss the observables that can be calculated [14–17] from the solution of a Dirac equation such as Eqs. (9)–(12). We have pointed out that the derived semiclassical Dirac amplitude $M(p,q)$ is not simply the exact amplitude for the excitation of an electron-positron pair [11]. The point is that exact solution of the semiclassical Dirac equation may be used to compute the inclusive average number of pairs—not an exclusive amplitude for a particular pair. Calculating the exact exclusive amplitude to all orders in $Z\alpha$ is not easily tractable due to need for Feynman propagators [11]. The possibility of solutions of the semiclassical Dirac equation is connected to the retarded propagators involved. In this paper we do not consider the exclusive (Feynman propagator) amplitude at all. We concentrate on observables that *can* be constructed from the above amplitude and investigate the Coulomb corrections contained in them.

The occupation number or inclusive number of electrons created in state p (at impact parameter b) is

$$N(p) = \int \frac{m\,d^3q}{(2\pi)^3\epsilon_q}|M(p,q)|^2. \tag{13}$$

Likewise the inclusive number of positrons created in state q is

$$N(q)=\int \frac{m\,d^3p}{(2\pi)^3\epsilon_p}|M(p,q)|^2. \quad (14)$$

These inclusive expressions say nothing about correlations between electrons in state p and and positrons in state q.

The mean number of electron-positron pairs is of course equal to either the mean number of positrons or the mean number of electrons and may be obtained by integrating over either of the previous expressions,

$$N=\int \frac{m\,d^3p}{(2\pi)^3\epsilon_p}N(p)=\int \frac{m\,d^3q}{(2\pi)^3\epsilon_q}N(q) \quad (15)$$

$$=\int \frac{m^2d^3p\,d^3q}{(2\pi)^6\epsilon_p\epsilon_q}|M(p,q)|^2. \quad (16)$$

It is possible to calculate well-defined observables from the occupation numbers by integrating over the impact parameter b,

$$d\sigma(p)=\int d^2bN(p)=\int d^2b\frac{m\,d^3q}{(2\pi)^3\epsilon_q}|M(p,q)|^2, \quad (17)$$

$$d\sigma(q)=\int d^2bN(q)=\int d^2b\frac{m\,d^3p}{(2\pi)^3\epsilon_p}|M(p,q)|^2 \quad (18)$$

and

$$\sigma_T=\int d^2bN=\int d^2b\frac{m^2d^3p\,d^3q}{(2\pi)^6\epsilon_p\epsilon_q}|M(p,q)|^2. \quad (19)$$

$d\sigma(p)$ is the cross section for an electron of momentum (p) where the state of the positron is unspecified. Likewise, $d\sigma(q)$ is the cross section for a positron of momentum (q) with the state of the electron unspecified. Note that σ_T corresponds to a peculiar type of inclusive cross section which we should call the "number weighted total cross section,"

$$\sigma_T=\int d^2bN=\int d^2b\sum_{n=1}^{\infty} nP_n(b), \quad (20)$$

in contrast to the usual definition of an inclusive total cross section σ_I for pair production,

$$\sigma_I=\int d^2b\sum_{n=1}^{\infty} P_n(b). \quad (21)$$

Now we can write for the factor common to all the cross sections,

$$\int d^2b|M(p,q)|^2=\int d^2b\int \frac{d^2k}{(2\pi)^2}\int \frac{d^2k'}{(2\pi)^2} \times\exp[i\,(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}]\mathcal{M}(\mathbf{k})\mathcal{M}(\mathbf{k}')^* \times F_B(\mathbf{k})F_B(\mathbf{k}')^*F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k}) \times F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k}')^*. \quad (22)$$

Integrating $\exp[i\,(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}]$ over the impact parameter b in the usual way gives $(2\pi)^2\delta(\mathbf{k}-\mathbf{k}')$ and so

$$\int d^2b|M(p,q)|^2=\int \frac{d^2k}{(2\pi)^2}|\mathcal{M}(\mathbf{k})|^2 \times|F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k})|^2|F_B(\mathbf{k})|^2. \quad (23)$$

One now obtains expressions for $d\sigma(p)$, $d\sigma(q)$, and σ_T that appear identical to the result of perturbation theory (scaling as $Z_A^2Z_B^2$) when our previous expression for $F(\mathbf{k})$, Eq. (12), is employed,

$$d\sigma(p)=\int \frac{m\,d^3q}{(2\pi)^3\epsilon_q}\int \frac{d^2k}{(2\pi)^2}|\mathcal{M}(\mathbf{k})|^2 \times|F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k})|^2|F_B(\mathbf{k})|^2, \quad (24)$$

$$d\sigma(q)=\int \frac{m\,d^3p}{(2\pi)^3\epsilon_p}\int \frac{d^2k}{(2\pi)^2}|\mathcal{M}(\mathbf{k})|^2 \times|F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k})|^2|F_B(\mathbf{k})|^2, \quad (25)$$

$$\sigma_T=\int \frac{m^2d^3p\,d^3q}{(2\pi)^6\epsilon_p\epsilon_q}\int \frac{d^2k}{(2\pi)^2}|\mathcal{M}(\mathbf{k})|^2 \times|F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k})|^2|F_B(\mathbf{k})|^2. \quad (26)$$

Obviously F_B and F_A still have to be regularized or cut off at small $|\mathbf{k}|$ and $|\mathbf{q}_\perp+\mathbf{p}_\perp-k|$.

B. The regularization of Lee and Milstein

The strategy of the first paper of Lee and Milstein [9] was to evaluate Coulomb corrections by Taylor expanding $\mathcal{M}$ around $\mathbf{k}=0$, i.e., $\mathcal{M}(\mathbf{k})\simeq\mathbf{k}\cdot\mathbf{L}$. The derivative $\mathbf{L}$ is evaluated at $\mathbf{k}=0$, and also in the evaluation of, e.g., Eq. (26) $\mathbf{k}$ is ignored in $F_A(\mathbf{q}_\perp+\mathbf{p}_\perp-\mathbf{k})$. All the $\mathbf{k}$ dependence of the integral is then contained in $d^2k\,k^2|F_B(\mathbf{k})|^2$. Lee and Milstein then invite us to consider the integral representing the difference between the exact solution and the perturbative solution,

$$G=\int \frac{d^2k}{(2\pi)^2}k^2[|F(\mathbf{k})|^2-|F^0(\mathbf{k})|^2], \quad (27)$$

where

$$F(\mathbf{k})=\int d^2\rho\,\exp[-i\,\mathbf{k}\cdot\boldsymbol{\rho}]\{\exp[-i\chi(\boldsymbol{\rho})]-1\}, \quad (28)$$

with the transverse form of the potential not yet specified,

$$\chi(\boldsymbol{\rho})=\int_{-\infty}^{\infty}dzV(z,\boldsymbol{\rho}) \quad (29)$$

and

$$F^0(\mathbf{k}) = -i \int d^2\rho \exp[-i\,\mathbf{k}\cdot\boldsymbol{\rho}]\chi(\boldsymbol{\rho}) \quad (30)$$

is the perturbative expression limit of $F(\mathbf{k})$.

Lee and Milstein keep the $2Z\alpha\ln(\rho)$ form for $\chi(\boldsymbol{\rho})$ but switch the order of integration between ρ and k. They integrate k to some finite upper limit Q and then claim to set Q to infinity in the resulting expression. Actually Q simply falls out of the problem by a rescaling of ρ to ρ/Q. Next, after integrating over the rescaled ρ, the expression they obtain is a universal function of $Z\alpha$,

$$G = -8\pi(Z\alpha)^2[\mathrm{Re}\psi(1+iZ\alpha)+\gamma_{Euler}], \quad (31)$$

where $\psi(1+iZ\alpha)$ is the digamma function and γ_{Euler} is Euler's constant. This expression may be alternatively expressed as

$$G = -8\pi(Z\alpha)^2 f(Z\alpha), \quad (32)$$

where $f(Z\alpha)$ is the same function that was derived by Bethe and Maximon for Coulomb corrections to e^+e^- photoproduction on heavy nuclei and takes the form

$$f(Z\alpha) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n[n^2+(Z\alpha)^2]}. \quad (33)$$

The derivation and result may seem a little mystifying. Lee and Milstein state, "Thus, we come to a remarkable conclusion: although the main contribution to the integral in Eq. (4) comes from the region of small k, where $|F(\mathbf{k})|$ differs from ($|F^0(\mathbf{k}|) = 4\pi Z\alpha/k^2$ and depends on the regularization parameters (the radius of screening), nevertheless the integral G itself is a universal function of $Z\alpha$." As we will see later, the only part of this quoted statement that is completely true is that G is a universal function of $Z\alpha$.

G is then used by Lee and Milstein to calculate the Coulomb correction arising from ion B by taking ion A to lowest order in $Z\alpha$. Generalizing this approach, the corresponding Coulomb correction arising from ion A is also evaluated [10]. The sum of these two contributions then agrees with the Coulomb corrections as evaluated by Ivanov, Schiller, and Serbo [8] using the Weizsacker-Williams method.

C. A physical regularization

Let us try to understand Lee and Milstein's result by putting in a physical cutoff to the transverse potential $\chi(\boldsymbol{\rho})$ (which has been up to now set to $2Z\alpha\ln\rho$). Instead of regularizing the integral itself and letting the cutoff radius go to infinity as was originally done [4–7], we will apply an appropriate physical cutoff to the interaction potential. In the Weizsacker-Williams or equivalent photon treatment of electromagnetic interactions the potential is cut off at impact parameter $b \simeq \gamma/\omega$, where γ is the relativistic boost of the ion producing the photon and ω is the energy of the photon. As Lee and Milstein subsequently recall (but do not utilize) if

$$\chi(\boldsymbol{\rho}) = \int_{-\infty}^{\infty} dz\, V(\sqrt{z^2+\rho^2}) \quad (34)$$

and $V(r)$ is cut off in a physically motivated way, such as an equivalent photon cutoff, then

$$V(r) = \frac{-Z\alpha \exp[-r\omega_{A,B}/\gamma]}{r}, \quad (35)$$

where

$$\omega_A = \frac{p_+ + q_+}{2}, \quad \omega_B = \frac{p_- + q_-}{2} \quad (36)$$

with ω_A the energy of the photon from ion A moving in the positive z direction and ω_B the energy of the photon from ion B moving in the negative z direction. For simplicity we will suppress the subscripts on ω, remembering, however, for possible use in future that $\omega_{A,B}$ are well defined in terms of $p_\pm$ and $q_\pm$. Integral (34) can be carried out to obtain

$$\chi(\rho) = -2Z\alpha K_0(\rho\omega/\gamma), \quad (37)$$

and

$$F_{A,B}(\mathbf{k}) = 2\pi \int d\rho\, \rho J_0(k\rho)\{\exp[2iZ_{A,B}\alpha K_0(\rho\omega/\gamma)]-1\}. \quad (38)$$

The modified Bessel function $K_0(\rho\omega/\gamma) = -\ln(\rho\omega/2\gamma)$ for small ρ and cuts off exponentially at $\rho \sim \gamma/\omega$. This is the physical cutoff to the transverse potential.

One may define $\xi = k\rho$ and rewrite Eq. (38),

$$F_{A,B}(\mathbf{k}) = \frac{2\pi}{k^2} \int d\xi\, \xi J_0(\xi)\{\exp[2iZ_{A,B}\alpha K_0(\xi\omega/\gamma k)]-1\}. \quad (39)$$

It is now clear that $F_{A,B}$ is a function of $1/k^2$ times some function of $(\gamma k/\omega)$. The perturbative limit $F^0_{A,B}(\mathbf{k})$ is analytically solvable and takes the form

$$F^0_{A,B}(\mathbf{k}) = \frac{4\pi Z_{A,B}\alpha}{k^2+\omega^2/\gamma^2} = \frac{4\pi Z_{A,B}\alpha}{k^2(1+\omega^2/k^2\gamma^2)}. \quad (40)$$

Figure 1 displays the results of numerical calculation of the scaled magnitude of $F(\mathbf{k})$ as a function of $k\gamma/\omega$ for $Z=1$ [essentially the perturbative form Eq. (40)] and for $Z=82$. Note that the upper cutoff of ρ at γ/ω has the effect of regularizing $F(\mathbf{k})$ at small k. $F(\mathbf{k})$ goes to the constant $4\pi\gamma^2/\omega^2$ as k goes to zero in the $Z=1$ perturbative case; it goes to a reduced constant value as k goes to zero for $Z=82$. The form of the original solution, Eq. (12),

$$F(\mathbf{k}) = \frac{4\pi\alpha Z}{k^{2-2i\alpha Z}} \quad (41)$$

is simply wrong because it is unphysical. Since it lacks a proper physical cutoff in ρ, it not only blows up at $k=0$, but

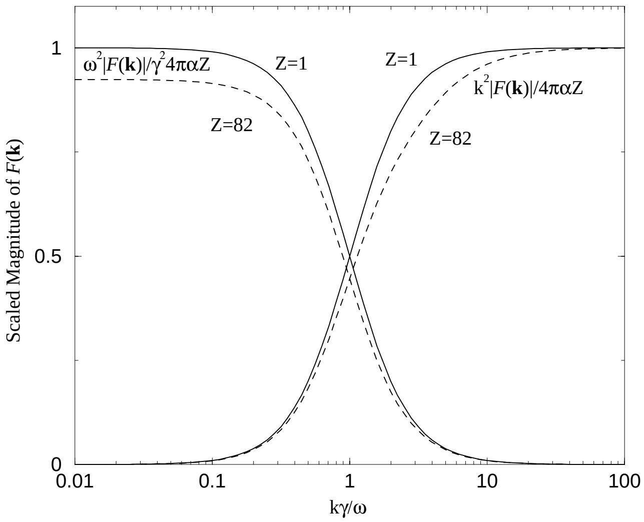

FIG. 1. The decrease in the magnitude of the transverse integral F with Z. The two sets of curves have been normalized to display that the finite Coulomb correction only rescales down the $|F(\mathbf{k})| \sim 1/\omega^2$ behavior at $k\omega/\gamma = 0$ and that the negative Coulomb corrections do not vanish until well above the onset of $|F(\mathbf{k})| \sim 1/k^2$ dominant behavior.

it also fails to exhibit the correct reduction in magnitude that occurs when $k\gamma/\omega$ is not too large.

Figure 2 is an alternate display of results of the numerical calculations showing the fractional decrease in the ratio $|F(\mathbf{k})|/|F^0(\mathbf{k})|$ for various values of Z as a function of $k\gamma/\omega$. It is clear from the two figures that for increasing Z Coulomb corrections reduce $F(\mathbf{k})$ from the perturbative result for $k\gamma/\omega \ll 100$. Only for $k > \sim 100\ \omega/\gamma$ does the magnitude of $F(\mathbf{k})$ go over into the original form of Eq. (41).

Now let us consider G again with the specific forms of $F(\mathbf{k})$ displayed in Figs. 1 and 2,

$$G = \int \frac{d^2k}{(2\pi)^2} k^2 [|F(\mathbf{k})|^2 - |F^0(\mathbf{k})|^2]. \tag{42}$$

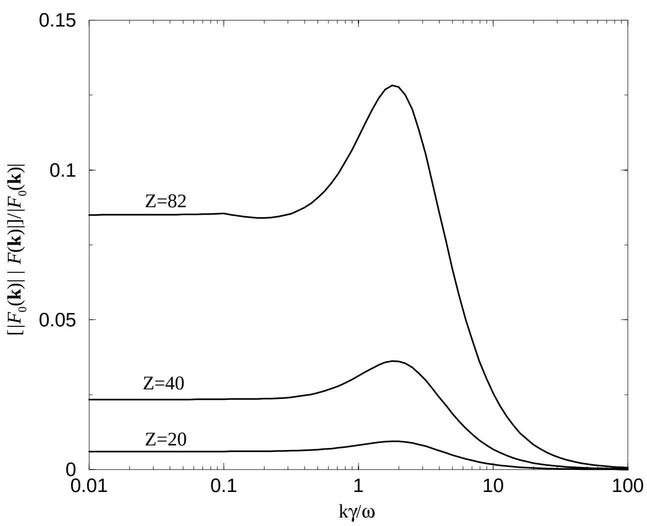

FIG. 2. The curves display the ratio $[|F_0(\mathbf{k})| - |F(\mathbf{k})|]/|F_0(\mathbf{k})|$ as a function of Z.

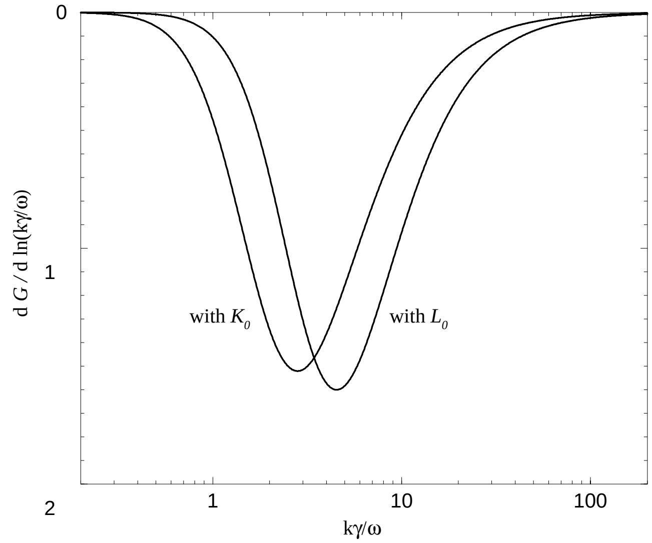

FIG. 3. Region of $k\gamma/\omega$ contributing to the Coulomb correction integral G for $Z=82$.

Note that, given the $1/k^2$ dependence of $F(\mathbf{k})$ and of Eqs. (39) and (40), this is a logarithmic integral of k (i.e., dk/k) times a function of $k\gamma/\omega$. Therefore the integration is really over the combination variable $k\gamma/\omega$. Thus γ/ω falls out of the integral, and the Coulomb correction function G does not depend on γ or ω.

I have evaluated G numerically and found it exactly converging to Lee and Milstein's result according to the expected improved precision with decreasing mesh size. I attained agreement to one part in 10^6.

Conjecturing that the detail of the cutoff should not matter, I replaced the function $K_0(\rho\omega/\gamma)$ with a different function that also goes as $-\ln(\rho\omega/2\gamma)$ (plus an irrelevant constant, $1/2 + \gamma_{Euler}$) for small ρ and also cuts off exponentially at $\rho \sim \gamma/\omega$:

$$L_0(\rho\omega/\gamma) = \frac{(\rho\omega/\gamma)^2}{2} [K_1^2(\rho\omega/\gamma) - K_0(\rho\omega/\gamma) K_2(\rho\omega/\gamma)]. \tag{43}$$

Calculations of G with L_0 in place of K_0 similarly converge numerically to the result of Lee and Milstein with agreement to one part in 10^6. Note, however, the nonidentical shapes of the contribution to G as a function of $k\gamma/\omega$ for the K_0 and L_0 transverse potential forms exhibited in Fig. 3, even though the area above the two curves (the value of G) is identical.

Now we can begin to understand the result of Lee and Milstein. The reason that "the integral G itself is a universal function of $Z\alpha$" is that the first order k^2 factor from the expansion makes the integral G logarithmic and so, contrary to what Lee and Milstein state, G does not "depend . . . on the regularization parameters (the radius of screening)." The radius of screening, i.e., γ/ω, is finite, but it has fallen out of the problem. Furthermore "the main contribution to the integral" does not "come from the region of small k" but, as is seen from the plot of the physically motivated K_0 curve in

Fig. 3, the main contribution is peaked at $\gamma k/\omega=2.8$ and spreads out between half maxima at 1.3 and 7.5.

Note that the decoupling of the Coulomb corrections from γ/ω seen in G is only valid to first order in k. Including higher order terms in k or, alternatively, carrying out a full numerical evaluation of, e.g., Eq. (26), would necessarily restore some dependence on γ/ω to the Coulomb corrections. A previous Monte Carlo perturbation theory calculation of Bottcher and Strayer [18] displays the pair production cross section as a function of $P_T=p_\perp+q_\perp$, and shows a significant deviation between an exact Monte Carlo evaluation of the cross section and evaluation using a two peak approximation (in particular, see Fig. 9 of Ref. [18]). Since in carrying out their calculation, Lee and Milstein made a variety of a two peak approximation (assuming $P_T=p_\perp+q_\perp$ small), one has to assume that the precision of their results is limited.

IV. GENERAL OBSERVATIONS

To lowest order in transverse momentum (small k and small $P_T=p_\perp+q_\perp$), Coulomb corrections do exist as a universal function of $f(\alpha Z)$, where $f(\alpha Z)$ is the same function of Bethe and Maximon derived for Coulomb corrections to electron-positron pair photoproduction. These Coulomb corrections reduce the uncorrelated electron or positron production cross sections and the number weighted total pair cross section.

In general, and not limited to lowest order in transverse momentum, Coulomb corrections are a function of only Z and the combination variable $k\gamma/\omega$. Coulomb corrections arise from the finite cutoff of the transverse spatial integral at γ/ω and vanish for large $k\gamma/\omega$.

Since the CERN data cover a large part of the momentum range of produced positrons and scale perturbatively, they still seem to present a puzzle. It would be useful to carry out full calculations of the total number weighted cross section σ_T as well as of the uncorrelated momentum dependent electron and positron cross sections $d\sigma(p)$ and $d\sigma(q)$, utilizing the transverse integrals with a correct physical cutoff. Since the CERN data only detect positrons, comparison with a full calculation of $d\sigma(q)$ is appropriate.

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