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α decay of odd-A nuclei with an extra nucleon outside a closed shell

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The newly discovered α decay of ^{209}Bi [Marcillac *et al.*, Nature (London) **422**, 876 (2003)] is investigated in the cluster model of α decay. It is found that the cluster model can reproduce the data of this longest-lived α emitter in all known α-decay nuclei. This decay belongs to a special class of α decays occurring in odd-A nuclei with an extra nucleon outside a closed shell. By combining the cluster model of α decay with a microscopic model of preformation α cluster, we can successfully describe the half-lives of odd-A $N=127$ isotones. The cluster model of the favored α decays is interestingly generalized to the hindered α decays of odd-A nuclei.

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I. INTRODUCTION

It was believed for a long period that the naturally occurring nuclide ^{209}Bi is the heaviest stable nucleus in nature and this is written in many textbooks of modern physics and nuclear physics. However, the recent observation of α decay of ^{209}Bi by Marcillac *et al.* [1] has changed this view. It has been measured [1] that ^{209}Bi has an α-decay energy of $Q_\alpha = 3.137 \pm 0.002$ MeV and a half-life of $T_{1/2} = (1.9 \pm 0.2) \times 10^{19}$ yr. This half-life is the longest in all known α decays. Therefore it is interesting to see whether current models of α decay can be generalized to explain such an extraordinary observation result. Testing of the decay theory to an extremely long half-life is also useful for further development of α-decay models.

There are many calculations [2–10] on favored α decays occurring between the ground states of even-even nuclei. Mang [2] made significant contributions to the theory of α decay. An earlier review paper of α decay can be found in Ref. [3]. Varga, Lovas, and Liotta calculated the half-life of α decay from the ground state of ^{212}Po in a microscopic model which incorporates both shell-model and cluster-model configurations. In this paper we concentrate on the cluster model proposed by Buck *et al.* [8–10]. The cluster model [8–10] is widely used for calculations of favored α decay between the ground states of medium and heavy nuclei. By a few parameters the model can reproduce experimental half-lives within a factor of $2\sim3$. This indicates clear agreement between the model and the data because half-lives vary in a very wide range from nanoseconds (10^{-9} s) to 10^{19} yr. Buck *et al.* [8–10] calculated the favored α decays between the ground states of parent and daughter nuclei where they have the same spin and parity. Although a great amount of research has been carried out on favored decays, studies on hindered transitions are relatively few. Importantly there are rare investigations on a special class of α decays, i.e., the α decays occurring in the ground states of odd-A nuclei with an extra nucleon outside a closed shell. The α particle carries an odd angular momentum in this case due to the different parity of parent and daughter nuclei. The newly discovered α decay of ^{209}Bi [1] is one of this class. The α decays of odd-A $N=127$ isotones also belong to this class. The decays involve valence nucleons in different major shells. It is expected that the preformation probability of the α cluster in these nuclei is lower than other nuclei due to both the necessity of nucleon excitations across the shell and the blocking effect of odd nucleon on pairing correlations or α correlations [11–13]. In this paper we generalize the cluster model of the favored α decay to this kind of decay. The main conclusions on the preformation probability from a simple microscopic model [11–13] will be absorbed into the cluster model of α decay [8–10].

This paper is organized in the following way. In Sec. II, we will briefly discuss the formalism of the cluster model of α decay. In Sec. III, the numerical results for half-lives of ^{209}Bi and of $N=127$ isotones are presented and discussed. This includes the results for both a constant preformation factor of α cluster and a Z-dependent preformation factor based on a microscopic model. A summary is given in Sec. IV.

II. THE FORMALISM OF THE CLUSTER MODEL OF α DECAY

The cluster model of α decay [8–10] invokes an extreme cluster picture where the ground state of the parent nucleus is assumed to be an α particle orbiting the daughter nucleus. The orbit is denoted by a large value of the global quantum number $G=2n+L$, where n is the node number of radial motion and L is the angular momentum [8–10]. The Bohr-Sommerfeld quantization is used to describe the motion of the α particle in a given potential. It is assumed that the α particle is preformed in the parent nucleus and therefore a preformation factor of the α particle is introduced. Usually the preformation factor is assumed to be a constant in Buck *et al.* calculations [8–10]. We will improve their assumption on the preformation factor later.

In the cluster model the α-core potential [8–10] is written as

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TABLE I. The variation of theoretical half-life of α decay with proton number without nuclear structure effect where $G=23$ for $N=127$ isotones and $G=21$ for $N=126$ (^{209}Bi) are chosen in calculations (choice 1).

^{A}Z	^{A}Z	I_i	I_f	L_α	R(fm)	Q_α(MeV)	T_α(expt.)	T_α(Calc.)
^{209}Bi	^{205}Tl	$9/2^-$	$1/2^+$	5	7.234	3.137	1.9×10^{19} yr	1.8×10^{19} yr
^{211}Po	^{207}Pb	$9/2^+$	$1/2^-$	5	7.642	7.599	516 ms	156 ms
^{213}Rn	^{209}Po	$9/2^+$	$1/2^-$	5	7.650	8.248	25 ms	10 ms
^{215}Ra	^{211}Rn	$9/2^+$	$1/2^-$	5	7.660	8.869	1.59 ms	1.14 ms
^{217}Th	^{213}Ra	$9/2^+$	$1/2^-$	5	7.672	9.428	252 μs	222 μs
^{219}U	^{215}Th	$9/2^+$	$1/2^-$	5	7.687	9.870	55 μs	92 μs

$$V(r)=V_N(r)+V_C(r)+\frac{\hbar^2}{2\mu}\frac{\left(L+\frac{1}{2}\right)^2}{r^2}, \tag{1}$$

where the nuclear potential is given by a "cosh" geometry of depth V_0, diffuseness a, and radius R.

$$V_N(r)=-V_0\frac{1+\cosh(R/a)}{\cosh(r/a)+\cosh(R/a)} \tag{2}$$

and the Coulomb potential is taken to be

$$\begin{aligned}V_C(r)&=\frac{Z_1Z_2e^2}{r}\quad (r\geqslant R)\\ &=\frac{Z_1Z_2e^2}{2R}\left[3-\left(\frac{r}{R}\right)^2\right]\quad (r\leqslant R).\end{aligned} \tag{3}$$

In these equations Z_1 and Z_2 are the charges of the α particle and the core, respectively. We have written the centrifugal barrier in its Langer modified form with $L(L+1)$ replaced by $(L+\frac{1}{2})^2$ to ensure that the subsequently required integrals are well defined for all values of L [8–10].

The classical turning points (r_1, r_2, and r_3 in order of increasing distance from the origin) are found by numerical solutions of the equation $V(r)=Q$. The radius parameter R can be determined separately for each decay by applying the Bohr-Sommerfeld quantization condition

$$\begin{aligned}&\int_{r_1}^{r_2}dr\sqrt{\frac{2\mu}{\hbar^2}[Q-V_N(r)-V_C(r)]-\frac{\left(L+\frac{1}{2}\right)^2}{r^2}}\\ &\quad=(2n+1)\frac{\pi}{2}=(G-L+1)\frac{\pi}{2}.\end{aligned} \tag{4}$$

In semiclassical approximation, the α-decay width Γ_α [8–10] is given by

$$\Gamma=PF\frac{\hbar^2}{4\mu}\exp\left[-2\int_{r_2}^{r_3}dr\,k(r)\right]. \tag{5}$$

The normalization factor F is

$$F\int_{r_1}^{r_2}dr\frac{1}{k(r)}\cos^2\left[\int_{r_1}^{r}dr'k(r')-\frac{\pi}{4}\right]=1, \tag{6}$$

with the wave number $k(r)$ given by

$$k(r)=\sqrt{\frac{2\mu}{\hbar^2}|Q-V(r)|}. \tag{7}$$

The α-decay half-life is then related to the width by

$$T_{1/2}=\hbar\ln 2/\Gamma. \tag{8}$$

The parameters of the α-core potential are chosen to be $V_0=162.3$ MeV and $a=0.40$ fm by Buck *et al.* [8–10]. We still use these values as inputs of our calculations for consistence. For the favored α decays between the ground states of nuclei usually there is no change of the angular momentum and parity between parent and daughter nuclei. The global quantum number $G=2n+L$ is an even number. Buck *et al.* [8–10] choose $G=20$ for $82<N\leqslant 126$ and $G=22$ for $126<N$. They have also pointed out that the choice of G is not unique in some cases. The global quantum number G ranges from $G=18$ to $G=24$ in their systematic calculations of the favored α decays with various potentials. The existence of the decays with a change of parity leads to a natural choice of G as an odd number. Because the cluster model of the favored α decays is extended to hindered decays, it is difficult to pin down a unique value of G. For the hindered decays studied in this paper there are two reasonable choices. One is that $G=23$ for $N=127$ and $G=21$ for $N=126$

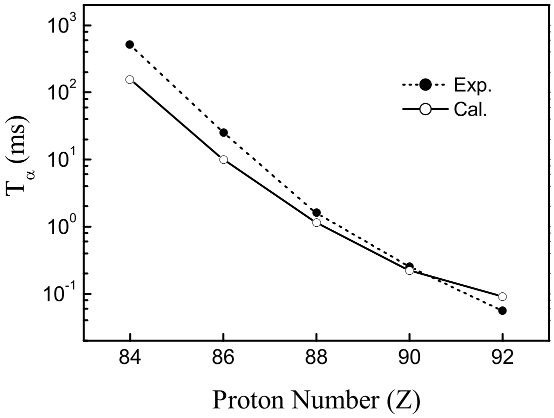

FIG. 1. The variation of theoretical half-life of α decay with proton number without nuclear structure effect where $G=23$ is chosen for $N=127$ isotones. The black circles are experimental half-lives. The hollow circles are theoretical half-lives.

TABLE II. The α-decay half-life of odd-A nuclei with an extra nucleon outside a closed shell where the nuclear structure effect of the preformation α cluster is included. $G=23$ is chosen for $N=127$ isotones (choice 1).

${}^{A}Z$	${}^{A}Z$	I_i	I_f	L_α	R(fm)	Q_α(MeV)	T_α(expt.)	T_α(calc.)
^{211}Po	^{207}Pb	$9/2^+$	$1/2^-$	5	7.642	7.599	516 ms	583 ms
^{213}Rn	^{209}Po	$9/2^+$	$1/2^-$	5	7.650	8.248	25 ms	20 ms
^{215}Ra	^{211}Rn	$9/2^+$	$1/2^-$	5	7.660	8.869	1.59 ms	1.42 ms
^{217}Th	^{213}Ra	$9/2^+$	$1/2^-$	5	7.672	9.428	252 μs	208 μs
^{219}U	^{215}Th	$9/2^+$	$1/2^-$	5	7.687	9.870	55 μs	69 μs

(choice 1). Another is that $G=21$ for $N=127$ and $G=19$ for $N=126$ (choice 2). In this paper the calculations will be carried out for both of them.

After the global quantum numbers G are fixed, we have to choose the preformation factor of the α cluster. Usually the experimental preformation factor of α cluster in α-transfer reactions and α decays ranges from 0.005 to 1.0 [6]. It is known experimentally that this factor is lower near magic nuclei and higher in open-shell nuclei [6]. A theoretical study with a microscopic model [11–13] clearly shows that there exist shell and blocking effects in α preformation probability. This agrees with the experimental facts. Because we investigate in this paper the α decays with an extra nucleon outside a closed shell, the overlap of the wave functions of parent nuclei and daughter nuclei is poor. The α cluster can be formed with some particle-hole excitations where the variation of parity may happen. Therefore the preformation factor of the α cluster in this kind of nucleus should be smaller than other nucleus. At first we use the same approximation such as Buck *et al.* [8–10] and take the preformation factor of α cluster P_α as a constant. Then we will use a Z-dependent preformation factor for $N=127$ isotones based on a microscopic model [11–13]. For the first choice of G we use $P_\alpha=0.03$. For the second choice of G we use $P_\alpha=0.3$. These values of P_α lie in the experimental range of $P_\alpha=0.005-1.0$ [6].

III. NUMERICAL RESULTS AND DISCUSSIONS

At first we calculate by choice 1 the half-lives of α decays of ^{209}Bi and of some odd-A $N=127$ isotones and list the numerical results and experimental data in Table I.

In Table I, the first column marks the parent nuclei. The second column marks the daughter nuclei. The third and fourth columns are the spin and parity of the ground states of parent and daughter nuclei, respectively. The angular momentum of the α particle is given in column 5 and the calculated radius parameter R in column 6. The decay energy is listed in column 7. Experimental half-life and theoretical one are given in the last two columns. The experimental values are taken from Ref. [1] and the nuclear mass table [14].

It is seen from the last two columns of Table I that theoretical half-lives are in reasonable agreement with experimental ones. The ratios between experimental half-lives and theoretical ones are just a few times. It clearly demonstrates that the cluster model of the favored α decay can be generalized to explain the decays of the ground states of nuclei with different spin and parity. Specially the longest half-life of ^{209}Bi in all known α decays can be well reproduced. Therefore the cluster model is valid for the extreme case of the half-life in α decays.

In order to see the systematic behavior of the agreement between the model and the data, we draw the variation of the theoretical half-life and experimental one of $N=127$ isotones in Fig. 1. In Fig. 1 the X axis is proton number and the Y axis is the experimental half-life and theoretical one. Although the theoretical curve in Fig. 1 agrees qualitatively with experimental one, there is an apparent discrepancy in quantity. The experimental value is underestimated in low-Z nuclei such as ^{211}Po and ^{213}Rn and overestimated in high-Z nuclei such as ^{219}U. This discrepancy is systematic and also interesting. The cause of this discrepancy lies in the choice of a constant preformation factor of α cluster for different nuclei on an isotonic chain. This choice means the omission of the nuclear structure effect such as the nuclear shell effect in the cluster model. This shell effect of the preformation factor of α cluster exists in both the (n,α) reactions and the α decays [6]. This effect also manifests itself in calculations of a microscopic model [11,12].

The simple microscopic model of a two level with interacting protons and neutrons [11,12] is the mock-up of heavy nuclei with valence protons and neutrons in two major shells. The formation of the α cluster must involve the nucleon excitations across the shell. In addition to the proton-proton and neutron-neutron pairing correlations the proton-neutron pairing correlation is also included in the two-level model.

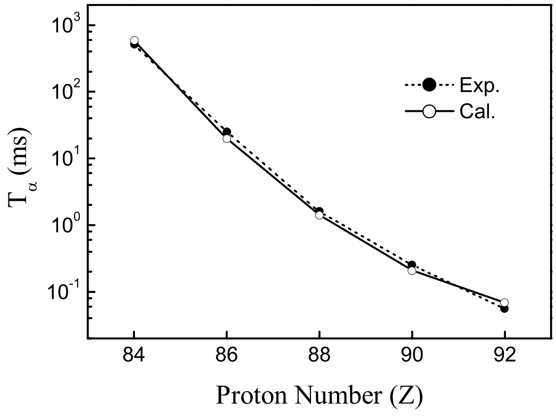

FIG. 2. The variation of theoretical half-life of α decay with proton number where the nuclear structure effect of the preformation α cluster is included. $G=23$ is chosen for $N=127$ isotones. The black circles are experimental half-lives. The hollow circles are theoretical half-lives.

TABLE III. The variation of theoretical half-life of α decay with proton number without nuclear structure effect where $G=21$ for $N=127$ isotones and $G=19$ for $N=126$ (^{209}Bi) are chosen in calculations (choice 2).

^{A}Z	^{A}Z	I_i	I_f	L_α	R(fm)	Q_α(MeV)	T_α(expt.)	T_α(calc.)
^{209}Bi	^{205}Tl	$9/2^-$	$1/2^+$	5	6.692	3.137	1.9×10^{19} yr	2.6×10^{19} yr
^{211}Po	^{207}Pb	$9/2^+$	$1/2^-$	5	7.107	7.599	516 ms	163 ms
^{213}Rn	^{209}Po	$9/2^+$	$1/2^-$	5	7.117	8.248	25 ms	11 ms
^{215}Ra	^{211}Rn	$9/2^+$	$1/2^-$	5	7.128	8.869	1.59 ms	1.19 ms
^{217}Th	^{213}Ra	$9/2^+$	$1/2^-$	5	7.141	9.428	252 μs	231 μs
^{219}U	^{215}Th	$9/2^+$	$1/2^-$	5	7.158	9.870	55 μs	96 μs

The cluster can be clearly defined and the preformation factor can be calculated microscopically with the model [11,12]. The preformation factor of α cluster in the model is Z dependent for an isotonic chain and N dependent for an isotopic chain [11,12]. The final expression of the α-cluster probability can be simplified for an isotonic chain as $P_\alpha = \text{const.}\times Z_v(1-Z_v/\Omega)$, where Z_v is the valence proton number and the quantity in the bracket is from the effect of the Pauli principle [11,12]. In this expression Ω is a large number which corresponds to the maximum number of protons in a major shell. For $N=127$ isotones from $Z=84$ to $Z=92$ this corresponds to the beginning of a new shell beyond $Z=82$. The number of valence protons varies from $Z_v=84-82=2$ to $Z_v=92-82=10$. Therefore the quantity in the bracket of above expression can be approximated as 1.0 and the expression of the preformation factor is $P_\alpha \approx \text{const.}\times(Z-82)$. Although this expression is very simple, the nuclear structure effect is included as it is based on a microscopic model. We choose this constant as 0.004. $P_\alpha \approx 0.004(Z-82)$ is input into the α-cluster model of decays and we recalculate the half-lives of the $N=127$ isotones. The results are given in Table II and Fig. 2 where we have used the same symbols as those in Table I and Fig. 1.

It is seen clearly from Table II and Fig. 2 that very good agreement between the model and the data is achieved. Importantly the systematic discrepancy between the experimental curve and the theoretical one disappears. Therefore to include the nuclear structure effect of the preformation factor in decays is interesting and it is also an important extension of Buck *et al.* model to the decays with different spin and parity.

After we present the numerical results for the first choice of G, we carry out the calculations with another choice in order to see whether our conclusions are general. The numerical results of $G=21$ for $N=127$ isotones and of $G=19$ for $N=126$ (^{209}Bi) are given in Tables III and IV. Table III is similar to Table I where a constant G is used. Table IV is similar to Table II where the Z-dependent preformation factor $P_\alpha=0.04(Z-82)$ is used. From Tables III and IV it is seen again that experimental data are reproduced well. Good agreement between theory and experiment is obtained by employing a Z-dependent preformation factor suggested by a simple microscopical model. Because the theoretical half-lives of Table III are very close to those of Table I and the theoretical half-lives of Table IV are very close to those of Table II, it is not necessary for us to draw the numerical results of Tables III and IV in figures. All previous discussions on Tables I and II hold true for Tables III and IV. Our conclusions on Tables I and II and on Figs. 1 and 2 are still valid for Tables III and IV. Here we do not repeat them. Finally it is interesting to mention that the radius parameters R in above tables are reasonable as compared with Buck *et al.* values [8–10]. It is known that the values of R are directly related to the values of G. Therefore the values of R of $N=127$ isotones for the first choice ($G=23$) are slightly larger than those of neighboring even-even nuclei ($G=22$). The values of R for the second choice ($G=21$) are slightly smaller than those of neighboring even-even nuclei ($G=22$).

IV. SUMMARY

The newly observed extraordinary α decay of ^{209}Bi [1] has inspired us to study the decay problem more carefully. We find that the decays of ^{209}Bi and of $N=127$ isotones belong to a new class of α decays occurring between the ground states of odd-A nuclei with an extra nucleon outside a closed shell where both spin and parity are different for parent and daughter nuclei. Usually this kind of decay is strongly suppressed as compared with the favored α decays.

TABLE IV. The α-decay half-life of odd-A nuclei with an extra nucleon outside a closed shell where the nuclear structure effect of the preformation α cluster is included. $G=21$ is chosen for $N=127$ isotones.

^{A}Z	^{A}Z	I_i	I_f	L_α	R(fm)	Q_α(MeV)	T_α(expt.)	T_α(calc.)
^{211}Po	^{207}Pb	$9/2^+$	$1/2^-$	5	7.107	7.599	516 ms	610 ms
^{213}Rn	^{209}Po	$9/2^+$	$1/2^-$	5	7.117	8.248	25 ms	20 ms
^{215}Ra	^{211}Rn	$9/2^+$	$1/2^-$	5	7.128	8.869	1.59 ms	1.48 ms
^{217}Th	^{213}Ra	$9/2^+$	$1/2^-$	5	7.141	9.428	252 μs	217 μs
^{219}U	^{215}Th	$9/2^+$	$1/2^-$	5	7.158	9.870	55 μs	72 μs

In the past it was difficult to detect this hindered decay of nuclei such as the case of ^{209}Bi. However the use of new experimental instruments makes it possible to observe this extraordinary decay process. Therefore it is necessary to generalized the original cluster model of the favored α decays [8–10] to study the hindered α decays. We develop the cluster model of α decay to calculate the new class of α decays from the ground states of ^{209}Bi and of $N=127$ isotones. The model reproduces the half-life of the newly discovered α emitter of ^{209}Bi which has the longest half-life in known α decays. By combining the cluster model with a microscopic model of α formation [11,12], we include the nuclear structure effect of the preformation factor of α cluster. The half-lives of odd-A $N=127$ isotones can be very well reproduced. This nuclear structure effect is well grounded from the systematical trend of both available experimental data of α decay and theoretical results of a microscopic model. This is an interesting generalization of the cluster model of the favored α decays to the hindered α decays where the ground states of parent and daughter nuclei have different spin and parity. It is expected that this extension of the cluster model may be useful for future calculations of α-decay half-lives in recently discovered superheavy nuclei [15,16].

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