Brueckner rearrangement effects in ${}^{5}_{\Lambda}$He and ${}^{6}_{\Lambda\Lambda}$He

M. Kohno,[1] Y. Fujiwara,[2] and Y. Akaishi[3]
[1]*Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan*
[2]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[3]*Institute of Particle and Nuclear Studies, KEK, Tsukuba 305-0801, Japan*
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Rearrangement effects in light hypernuclei are investigated in the framework of the Brueckner theory. We can estimate without detailed numerical calculations that the energy of the α core is reduced by more than 2.5 MeV when the Λ adheres to ^{4}He to form ${}^{5}_{\Lambda}$He. Similar assessment of rearrangement contributions is essential to deduce the strength of $\Lambda\Lambda$ interaction from experimentally observed $\Delta B_{\Lambda\Lambda}$. The recently observed experimental value of ~ 1 MeV for the $\Delta B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He suggests that the matrix element of $\langle \Lambda\Lambda | v | \Lambda\Lambda \rangle$ in ${}^{6}_{\Lambda\Lambda}$He is around -2 MeV.

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I. INTRODUCTION

The hypernuclei serve as an invaluable source of information for hyperon-nucleon and hyperon-hyperon interactions. New discovery of the double hypernucleus ${}^{6}_{\Lambda\Lambda}$He [1] provided more reliable data which would revise previous understanding based on the old data [2–5] that the $\Lambda\Lambda$ interaction was fairly strong. A quantity $\Delta B_{\Lambda\Lambda}$ (${}^{6}_{\Lambda\Lambda}$He), which is defined as $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) - 2B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$ or equivalently $2E({}^{5}_{\Lambda}\mathrm{He}) - E({}^{6}_{\Lambda\Lambda}\mathrm{He}) - E({}^{4}\mathrm{He})$, is meant to deduce the strength of the $\Lambda\Lambda$ interaction. The new data indicated that $\Delta B_{\Lambda\Lambda}$ (${}^{6}_{\Lambda\Lambda}$He) ~ 1 MeV. However, it contains many-body effects. Since exact few-body calculations are not practical yet apart from very light ordinary nuclei, the understanding of these effects from the viewpoint of standard nuclear many-body theory is useful. It has also been pointed out that hypernuclear systems work as an interesting testing ground of nuclear many-body theories. In this paper we estimate rearrangement effects [6] in a framework of the Brueckner theory [7,8] for ${}^{5}_{\Lambda}$He and ${}^{6}_{\Lambda\Lambda}$He. When matrix elements between nucleons and Λ's are needed, we use the SU(6) quark model potential [9,10], which provides a successful unified description for octet baryon-baryon interactions.

Rearrangement effects in theoretical consideration of hypernuclei have been discussed by many authors. Variational calculations [11,12] with a Jastrow trial function addressed the problem of the core polarization effect. The repulsive energy change due to the nuclear core polarization was also discussed in mean field calculations with a relativistic parametrization [13] and Skyrme type of density dependent effective forces [14,15]. Our discussion in this paper is focused on Brueckner rearrangement energies which arise through the energy dependence of the reaction matrix and the Pauli principle. The correspondence of these effects to higher order correlations in Fermi hypernetted chain approach is not straightforward.

We first present the treatment of the energy change of the ^{4}He core in ${}^{5}_{\Lambda}$He from ^{4}He. This problem was discussed by Bando and Shimodaya [16] in relation with the overbinding problem of ${}^{5}_{\Lambda}$He. The estimation of the rearrangement energy for the Λ in nuclear matter was presented by Dabrowski and Köhler [17] as early as in 1964. In the present consideration we show that numerical evaluation of relevant matrix elements can be avoided and that the potential energy change is shown to be connected to the Λ separation energy of ${}^{5}_{\Lambda}$He and the wound integral of nucleon pairs. Thus the estimation is more solid than that of Bando and Shimodaya. Recently, Nemura, Akaishi, and Suzuki [18] showed by their variational calculations that the addition of the Λ decreases the ^{4}He core energy in ${}^{5}_{\Lambda}$He by 4.7 MeV. The variational calculation in Refs. [11,12], on the other hand, suggested that the rearrangement energy is small. The discussion of the corresponding effects in the standard nuclear many-body theory is helpful.

Next we consider the rearrangement energy in $\Delta B_{\Lambda\Lambda}$ (${}^{6}_{\Lambda\Lambda}$He). Again it is shown that the essential ingredient can be written by the Λ separation energy of ${}^{5}_{\Lambda}$He, wound integrals of nucleon pairs, and nucleon-Λ pairs. In this case the Pauli rearrangement contribution of the Λ particle also appears. The recognition of these possible rearrangement effects in hypernuclei is important, because sophisticated calculations in which the structure dependence of effective interactions is ignored have been often presented, particularly, in cluster models.

II. REARRANGEMENT EFFECTS IN ${}^{5}_{\Lambda}$He

When one or two lambda particles are added to ^{4}He, the interaction between the Λ and nucleons causes a change of correlations among nucleons and thus the energy expectation value in the nucleon sector, Fig. 1(a), would change. A part of this change is through the change of wave functions. Another important effect comes from the change of three-body correlations. In the framework of the lowest order Brueckner theory [7,8], they are represented through the modification of Pauli effects and starting energy dependence. Since the change of nucleon Pauli effects is absent for the addition of hyperons, a relevant correlation is the potential insertion for hole states, Fig. 1(b). A requirement of self-consistency [6] for the hole energies means the inclusion of some second-order hole-line insertions, Figs. 1(c) and 1(d).

In the lowest order Brueckner theory, a ground state po-

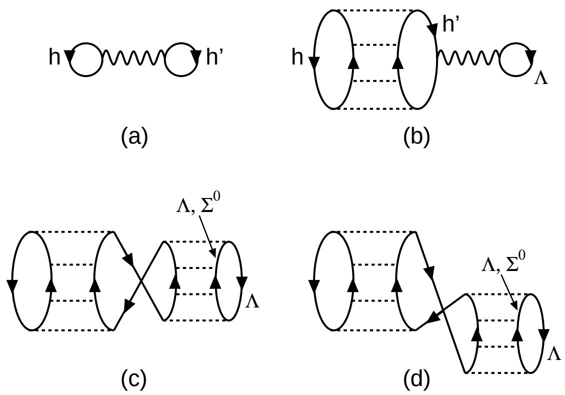

FIG. 1. Diagrams in the hole-line expansion in the Brueckner theory.

tential energy of ^{4}He, starting from the realistic NN interaction v_{NN}, is given by introducing G-matrix elements:

$$PE_\alpha(4)=\frac{1}{2}\sum_{hh'}\langle hh'|G_{NN}(4,\omega=e_h(4)+e_{h'}(4))|hh'\rangle_{as}, \tag{1}$$

$$e_h(4)=\langle h|t_N|h\rangle+\sum_{h'}\langle hh'|G_{NN}(4,\omega=e_h(4)+e_{h'}(4))|hh'\rangle_{as}, \tag{2}$$

$$G_{NN}(4,\omega)=v_{NN}+v_{NN}\frac{Q}{\omega-H_0}G_{NN}(4,\omega), \tag{3}$$

where h and h' correspond to a sole occupied $0s$ state besides implicit spin and isospin summations. The standard choice of the intermediate spectra H_0 is a QTQ prescription. The self-consistency of e_h and G means that the hole-line potential insertion, a part of three-body correlations, is taken into account. In finite nuclei Hartree-Fock condition is also required. However, in ^{4}He nucleus a single harmonic oscillator function provides a very good approximation. It is also known that the ^{4}He core in $^5_\Lambda$He differs little from ^{4}He. In the following discussion we assume that the single-particle wave function is given by the same harmonic oscillator $(0s)$ function both for ^{4}He and $^5_\Lambda$He. Then, the potential energy of the ^{4}He core in $^5_\Lambda$He is also given by an expression similar to Eq. (1):

$$PE_\alpha(5)=\frac{1}{2}\sum_{hh'}\langle hh'|G_{NN}(5,\omega=e_h(5)+e_{h'}(5))|hh'\rangle_{as}, \tag{4}$$

$$e_h(5)=\langle h|t_N|h\rangle+\sum_{h'}\langle hh'|G_{NN}(5,\omega=e_h(5)+e_{h'}(5))|hh'\rangle_{as}+\langle h\Lambda|G_{N\Lambda}(5,\omega=e_h(5)+e_\Lambda(5))|h\Lambda\rangle, \tag{5}$$

$$G_{NN}(5,\omega)=v_{NN}+v_{NN}\frac{Q}{\omega-H_0}G_{NN}(5,\omega). \tag{6}$$

In this case the single-particle potential includes a contribution of the Λ particle. The added Λ particle does not modify Pauli exclusion operators for nucleons in Eqs. (3) and (6). In the above framework the difference between $PE_\alpha(4)$ and $PE_\alpha(5)$ comes from the difference of e_h.

To estimate the difference between $PE_\alpha(5)$ and $PE_\alpha(4)$, we rewrite the expression as follows:

$$\Delta PE\equiv PE_\alpha(5)-PE_\alpha(4)=\frac{1}{2}\sum_{hh'}\langle hh'|G_{NN}(5)-G_{NN}(4)|hh'\rangle_{as}. \tag{7}$$

Using the well-known relation

$$\begin{aligned}\langle hh'|G_{NN}(5)-G_{NN}(4)|hh'\rangle_{as}\\ &=\langle hh'|G_{NN}(4)\left\{\frac{Q}{e_h(5)+e_{h'}(5)-QTQ}-\frac{Q}{e_h(4)+e_{h'}(4)-QTQ}\right\}G_{NN}(5)|hh'\rangle_{as}\\ &=\langle hh'|G_{NN}(4)\frac{Q}{e_h(4)+e_{h'}(4)-QTQ}[e_h(4)+e_{h'}(4)-e_h(5)-e_{h'}(5)]\frac{Q}{e_h(5)+e_{h'}(5)-QTQ}G_{NN}(5)|hh'\rangle_{as},\end{aligned} \tag{8}$$

ΔPE is expressed as

$$\Delta PE=-\frac{1}{2}\sum_{hh'}\Delta e_h\langle hh'|G_{NN}(4)\frac{Q}{e_h(4)+e_{h'}(4)-QTQ}\frac{Q}{e_h(5)+e_{h'}(5)-QTQ}G_{NN}(5)|hh'\rangle_{as}, \tag{9}$$

where Δe_h is

$$\Delta e_h \equiv e_h(5)+e_{h'}(5)-e_h(4)-e_{h'}(4)=2[e_h(5)-e_h(4)]. \tag{10}$$

At this stage we may introduce an approximation of $G_{NN}(5)\sim G_{NN}(4)$ and $e_h(5)\sim e_h(4)$ in the denominator. Using the relation

$$\frac{\partial G_{NN}}{\partial \omega}=-G_{NN}\frac{Q}{\omega-QTQ}\frac{Q}{\omega-QTQ}G_{NN}, \tag{11}$$

the difference of the potential energy expectation value becomes

$$\Delta PE \simeq \frac{1}{2}\sum_{hh'} \Delta e_h \langle hh'|\frac{\partial G_{NN}}{\partial \omega}|hh'\rangle_{as} = -\frac{1}{2}\sum_{h} \Delta e_h \kappa_N, \tag{12}$$

where we introduced $\kappa_N \equiv -\Sigma_{h'}\langle hh'|\partial G_{NN}/\partial\omega|hh'\rangle_{as}$ which has been known as a wound integral.

There being only one single-particle state, Δe_h is also expressed in terms of $\partial G_{NN}/\partial\omega$:

$$\begin{aligned}\Delta e_h &= 2[e_h(5)-e_h(4)] = 2\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle \\ &\quad + 2\sum_{h'} \langle hh'|G_{NN}(5)-G_{NN}(4)|hh'\rangle_{as} \\ &\simeq 2\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle - 2\Delta e_h \kappa_N. \end{aligned} \tag{13}$$

Thus we find

$$\Delta e_h \simeq \frac{2}{1+2\kappa_N}\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle. \tag{14}$$

Substituting this result for Δe_h in Eq. (12), we finally obtain the following expression:

$$\Delta PE \simeq -\sum_{h} \frac{\kappa_N}{1+2\kappa_N}\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle. \tag{15}$$

Estimation of the matrix element $\langle h\Lambda|G_{N\Lambda}|h\Lambda\rangle$ requires a knowledge of the Λ and nucleon wave functions. However, this matrix element can be related to the Λ separation energy ϵ_Λ which is known experimentally to be -3.12 MeV;

$$\begin{aligned}\epsilon_\Lambda &= E({}^{5}_{\Lambda}\mathrm{He})-E({}^{4}\mathrm{He}) \\ &= \langle \Lambda|t_\Lambda|\Lambda\rangle + \sum_{h} \langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle + \Delta PE + \Delta T_{\mathrm{c.m.}}, \end{aligned} \tag{16}$$

where we write the difference of the center of mass kinetic energy as $\Delta T_{\mathrm{c.m.}}$. In order to simplify expressions we utilize the fact that there is only one nucleon single-particle state. Inserting ΔPE of Eq. (15) into the right hand side, we obtain

$$\epsilon_\Lambda(5)-\langle \Lambda|t_\Lambda|\Lambda\rangle - \Delta T_{\mathrm{c.m.}} \simeq \sum_{h} \frac{1+\kappa_N}{1+2\kappa_N}\langle h\Lambda|G_{\Lambda N}(5)|h\Lambda\rangle. \tag{17}$$

Hence, eliminating the ΛN matrix element in Eq. (14), we end up with the following estimation:

$$\Delta PE \simeq -\frac{\kappa_N}{1+\kappa_N}[\epsilon_\Lambda(5)-\langle \Lambda|t|\Lambda\rangle - \Delta T_{\mathrm{c.m.}}]. \tag{18}$$

It is worthwhile to note that this difference of the potential energy contributions is due to the effect through the starting energy dependence of NN G matrices. The addition of the Λ to ^{4}He makes the single-particle energy of the nucleon deeper, which induces less attractive NN reaction matrices. Thus, the expression of Eq. (18) does not explicitly include the quantity such as ΛN correlations.

Supposing a single harmonic oscillator wave function with the oscillator constant ν_Λ, the kinetic energy expectation value $\langle \Lambda|t_\Lambda|\Lambda\rangle$ is expressed as $\frac{3}{4}(\hbar^2/m_\Lambda)\nu_\Lambda$, while $\Delta T_{\mathrm{c.m.}}$ is given by

$$\frac{3}{4}\frac{\hbar^2}{m}\left(\frac{m_\Lambda}{4m+m_\Lambda}\nu - \frac{m}{4m+m_\Lambda}\nu_\Lambda\right)$$

with m and m_Λ being the nucleon and Λ masses, respectively. The wound integral κ_N is estimated by employing nuclear matter G-matrix calculations in ^{4}He in the following scheme. The matrix element $\langle 00(\nu/2)|V_{NN}|00(\nu/2)\rangle$ between $0s$ nucleon states with the oscillator constant ν is evaluated by

$$\begin{aligned}&\left\langle 00\left(\frac{\nu}{2}\right)\middle| V_{NN} \middle| 00\left(\frac{\nu}{2}\right)\right\rangle \\ &\quad = \left(\frac{8\pi}{\nu}\right)^{3/2}\frac{1}{(2\pi^2)^2}\int_0^\infty dq'\int_0^\infty dq\, q'^2 q^2 e^{-(1/\nu)(q^2+q'^2)} \\ &\qquad \times \langle q'|G_{NN}^{\ell=0}(k_F)|q\rangle, \end{aligned} \tag{19}$$

where $\langle q'|G_{NN}^{\ell=0}(k_F)|q\rangle$ is obtained by the equation

$$\begin{aligned}&\langle q'|G_{NN}(k_F)|q\rangle \\ &\quad = \langle q'|v_{NN}|q\rangle + \langle q'|v_{NN}\frac{Q(k_F)}{\omega-QTQ}G_{NN}(k_F)|q\rangle. \end{aligned} \tag{20}$$

In numerical calculations, a standard oscillator constant of $\nu=0.56$ fm^{-2} was taken for ^{4}He, and the Fermi momentum k_F was set to be 1.2 fm^{-1} since the average density $\overline{\rho} \equiv \int\{\rho(r)\}^2 r^2 dr/\int \rho(r) r^2 dr$ is 0.106 fm^{-3} which corresponds to $k_F\sim 1.2$ fm^{-1}. The energy dependence of $\langle q'|G^{\ell=0}(k_F)|q\rangle$ tells us that κ_N is about 0.2, which is reasonable. Expecting ν_Λ to be 0.4–0.5 fm^{-2}, we obtain ΔPE as 2.5–2.9 MeV. Although further contributions from orbital rearrangement and other higher order correlations are expected, it is important to settle the order of magnitude of rearrangement effects in ${}^{5}_{\Lambda}$He by simple and transparent arguments.

III. REARRANGEMENT EFFECTS IN $\Delta B_{\Lambda\Lambda}$ OF ${}^{6}_{\Lambda\Lambda}$He

In this section, we consider the energy $\Delta B_{\Lambda\Lambda} \equiv 2E({}^{5}_{\Lambda}\mathrm{He}) - E({}^{6}_{\Lambda\Lambda}\mathrm{He}) - E({}^{4}\mathrm{He})$. It is straightforward to decompose it to each matrix element:

$$\Delta B_{\Lambda\Lambda} = \frac{1}{2}\sum_{hh'} \langle hh'|2G_{NN}(5) - G_{NN}(4) - G_{NN}(6)|hh'\rangle_{as} + 2\sum_{h} \langle h\Lambda|G_{N\Lambda}(5) - G_{N\Lambda}(6)|h\Lambda\rangle - \langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle_{as} + \Delta T_{\Lambda\Lambda}, \tag{21}$$

where the single-particle wave functions are assumed to be common in ^{4}He, ${}^{5}_{\Lambda}$He, and ${}^{6}_{\Lambda\Lambda}$He, and $\Delta T_{\Lambda\Lambda}$ is the contribution of kinetic energy terms which is discussed later. In order to obtain an information for the strength of the $\Lambda\Lambda$ interaction $\langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle_{as}$ from experimental data of $E({}^{5}_{\Lambda}\mathrm{He})$, $E({}^{6}_{\Lambda\Lambda}\mathrm{He})$, and $E({}^{4}\mathrm{He})$, it is necessary to estimate rearrangement contributions which correspond to the first and second terms of the above expression. Since the changes from $G_{NN}(4)$ to $G_{NN}(5)$ and from $G_{NN}(5)$ to $G_{NN}(6)$ are the same in the leading order, the first term of Eq. (21) should be small because of the cancellation $[G_{NN}(5) - G_{NN}(4)] - [G_{NN}(6) - G_{NN}(5)] \sim 0$. The second term represents the main source of the rearrangement effect. As in Eq. (8), it is straightforward to obtain

$$\begin{aligned}\langle h\Lambda|G_{N\Lambda}(6) - G_{N\Lambda}(5)|h\Lambda\rangle &= \langle h\Lambda|G_{N\Lambda}(5)\left\{\frac{Q(6)}{e_h(6)+e_\Lambda(6)-Q(6)TQ(6)} - \frac{Q(5)}{e_h(5)+e_\Lambda(5)-Q(5)TQ(5)}\right\}G_{N\Lambda}(6)|h\Lambda\rangle \\ &= \langle h\Lambda|G_{N\Lambda}(5)\frac{Q(6)-Q(5)}{e_h(5)+e_\Lambda(5)-Q(5)TQ(5)}G_{N\Lambda}(6)|h\Lambda\rangle \\ &\quad - \langle h\Lambda|G_{N\Lambda}(5)\frac{\Delta e_{h\Lambda}Q(6) - [Q(6)-Q(5)]TQ(6)}{[e_h(5)+e_\Lambda(5)-Q(5)TQ(5)][e_h(6)+e_\Lambda(6)-Q(6)TQ(6)]}G_{N\Lambda}(6)|h\Lambda\rangle,\end{aligned} \tag{22}$$

where $\Delta e_{h\Lambda} \equiv e_h(6) + e_\Lambda(6) - e_h(5) - e_\Lambda(5)$ is a single-particle energy difference. In this case the change of the Pauli blocking for Λ also contributes. In the following presentation, we neglect the contribution from the term including $[Q(6) - Q(5)]TQ(6)$ because of its restricted summation compared with $\Delta e_{h\Lambda}Q(6)$.

Writing $Q(5) - Q(6)$ as $\Sigma_p |p\Lambda_0\rangle\langle p\Lambda_0|$ where Λ_0 stands for a Λ $0s$ state and $|p\rangle$ for a nucleon unoccupied state,

$$\langle h\Lambda|G_{N\Lambda}(6) - G_{N\Lambda}(5)|h\Lambda\rangle = -\sum_{p} \frac{\langle h\Lambda|G_{N\Lambda}(5)|p\Lambda_0\rangle\langle p\Lambda_0|G_{N\Lambda}(6)|h\Lambda\rangle}{e_h(5)+e_\Lambda(5)-Q(5)TQ(5)} + \Delta e_{h\Lambda}\langle h\Lambda|\frac{\partial G_{N\Lambda}(5)}{\partial\omega}|h\Lambda\rangle. \tag{23}$$

The first term is positive because the numerator is positive and the denominator is negative. The second term is also positive if $\Delta e_{h\Lambda} < 0$, as the ΛN interaction $\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle$ becomes more attractive when the starting energy ω becomes shallower and thus the derivative of the matrix element with respect to ω is negative. To estimate $\Delta e_{h\Lambda}$, we calculate the energy differences $e_h(6) - e_h(5)$ and $e_\Lambda(6) - e_\Lambda(5)$ as follows:

$$\begin{aligned} e_h(6) - e_h(5) &= \sum_{h'} \langle hh'|\frac{\partial G_{NN}}{\partial\omega}|hh'\rangle\Delta e_h + \langle h\Lambda|\frac{\partial G_{N\Lambda}}{\partial\omega}|h\Lambda\rangle\Delta e_{h\Lambda} \\ &\quad - \sum_{p} \frac{\langle h\Lambda|G_{N\Lambda}(5)|p\Lambda_0\rangle\langle p\Lambda_0|G_{N\Lambda}(6)|h\Lambda\rangle}{e_h(5)+e_\Lambda(5)-Q(5)TQ(5)} \\ &\quad + \langle h\Lambda|G_{N\Lambda}(6)|h\Lambda\rangle, \end{aligned} \tag{24}$$

$$\begin{aligned} e_\Lambda(6) - e_\Lambda(5) &= \sum_{h'} \langle\Lambda h'|\frac{\partial G_{\Lambda N}}{\partial\omega}|\Lambda h'\rangle\Delta e_{h'\Lambda} \\ &\quad - \sum_{h'p} \frac{\langle\Lambda h'|G_{\Lambda N}(5)|\Lambda_0 p\rangle\langle\Lambda_0 p|G_{\Lambda N}(6)|\Lambda h'\rangle}{e_{h'}(5)+e_\Lambda(5)-Q(5)TQ(5)} \\ &\quad + \langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle. \end{aligned} \tag{25}$$

In order to simplify these expressions, we introduce the following notations:

$$W = \sum_{h'} \langle\Lambda h'|\frac{\partial G_{\Lambda N}}{\partial\omega}|\Lambda h'\rangle\Delta e_{h'\Lambda} = -\Delta e_{h\Lambda}\kappa_\Lambda, \tag{26}$$

$$P = -\sum_{h'p} \frac{\langle\Lambda h'|G_{\Lambda N}(5)|\Lambda_0 p\rangle\langle\Lambda_0 p|G_{\Lambda N}(6)|\Lambda h'\rangle}{e_{h'}(5)+e_\Lambda(5)-Q(5)TQ(5)}, \tag{27}$$

$$D=\sum_{h}\langle h\Lambda|G_{N\Lambda}(6)-G_{N\Lambda}(5)|h\Lambda\rangle=W+P. \quad (28)$$

Using the wound integral $\kappa_\Lambda=-\Sigma_{h'}\langle\Lambda h'|\partial G_{\Lambda N}/\partial\omega|\Lambda h'\rangle$ for the ΛN pair, the energy differences are written as

$$\begin{aligned} e_h(6)-e_h(5)&=-\Delta e_h\kappa_N+\tfrac{1}{4}D+\langle h\Lambda|G_{N\Lambda}(6)|h\Lambda\rangle\\ &=-\Delta e_h\kappa_N+\tfrac{1}{2}D+\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle, \end{aligned} \quad (29)$$

$$e_\Lambda(6)-e_\Lambda(5)=D+\langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle. \quad (30)$$

Then, noticing the relation $e_h(6)-e_h(5)=\frac{1}{2}\Delta e_h$, $\Delta e_{h\Lambda}=e_h(6)-e_h(5)+e_\Lambda(6)-e_\Lambda(5)$ becomes

$$\begin{aligned} \Delta e_{h\Lambda}=&\frac{3+4\kappa_N}{2(1+2\kappa_N)}D+\frac{1}{1+2\kappa_N}\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle\\ &+\langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle. \end{aligned} \quad (31)$$

Inserting this into $D=W+P=-\kappa_\Lambda\Delta e_{h\Lambda}+P$, D can be expressed as

$$\begin{aligned} D=&\frac{1}{1+\kappa_\Lambda\dfrac{3+4\kappa_N}{2(1+2\kappa_N)}}\left(\frac{-\kappa_\Lambda}{1+2\kappa_N}\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle\right.\\ &\left.-\kappa_\Lambda\langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle+P\right). \end{aligned} \quad (32)$$

The estimation of $\Delta B_{\Lambda\Lambda}$ finally reads

$$\Delta B_{\Lambda\Lambda}\sim-\langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle-2D+\Delta T_{\Lambda\Lambda}. \quad (33)$$

Since D is positive as remarked above, the $\Delta B_{\Lambda\Lambda}$ becomes smaller than $-\langle\Lambda\Lambda|G_{\Lambda\Lambda}(6)|\Lambda\Lambda\rangle$. Some comments are necessary for the contribution of $\Delta T_{\Lambda\Lambda}$. The assumption of the same single-particle wave functions implies

$$\Delta T_{\Lambda\Lambda}=\frac{3\hbar^2}{4m}\nu\frac{2m_\Lambda^2}{(4m+m_\Lambda)(4m+2m_\Lambda)}\left(1-\frac{m}{m_\Lambda}\frac{\nu_\Lambda}{\nu}\right), \quad (34)$$

which amounts to 0.37 MeV with $\nu=0.56\ \text{fm}^{-2}$ and $\nu_\Lambda=0.5\ \text{fm}^{-2}$, and thus partly cancels the negative contribution of $-2D$. However, $\Delta T_{\Lambda\Lambda}$ is sensitive to the change of single-particle wave functions. It has been known that the addition of the Λ to ${}^{5}_{\Lambda}$He makes the Λ single-particle wave function compact [19]. In that case, $\Delta T_{\Lambda\Lambda}$ becomes even negative. In the estimation below we leave out this contribution.

The matrix element $\langle h\Lambda|G_{N\Lambda}(5)|h\Lambda\rangle$ in Eq. (32) is estimated by Eq. (17). New ingredients here are the wound integral κ_Λ and the Pauli blocking effect P. The energy dependence of the ΛN reaction matrix element is much weaker than that of the NN interaction, since the ΛN tensor force, which is an important source of the ω dependence, is weak because of the absence of the lowest order pion exchange. We find $\kappa_\Lambda\sim0.05$ from the ω dependence of the calculated ΛN reaction matrices. The Pauli blocking effect is estimated by evaluating the matrix element $\langle 00(\nu)|V_{\Lambda N}|00(\nu)\rangle$ from the hypernuclear matter G matrix $\langle q'|G_{\Lambda N}(k_F^N,k_F^\Lambda)|q\rangle$ by solving the equation similar to Eq. (20). Half of the difference between matrix elements with $(k_F^N,k_F^\Lambda)=(1.2\ \text{fm}^{-1},0\ \text{fm}^{-1})$ and $(k_F^N,k_F^\Lambda)=(1.2\ \text{fm}^{-1},1.2\ \text{fm}^{-1})$ gives $P\sim0.23$ MeV. The calculation of $\langle\Lambda\Lambda|G_{\Lambda\Lambda}|\Lambda\Lambda\rangle$ is carried out, as in Eq. (19), by the momentum space folding of nuclear matter $\Lambda\Lambda$ G matrix, in which the coupling with the $\Sigma\Sigma$ and ΞN channels is included. The quark model potential fss2 [9] indicates that $\langle\Lambda\Lambda|G_{\Lambda\Lambda}|\Lambda\Lambda\rangle$ is -2.17 MeV for $\nu=0.40\ \text{fm}^{-2}$, and -2.65 MeV for $\nu=0.50\ \text{fm}^{-2}$. In these calculations, the starting energy $\omega=2e_\Lambda(6)$ in the energy denominator of the G-matrix equation is set to be -12 MeV. The κ_Λ is 0.05 for $\nu=0.40\ \text{fm}^{-2}$, and 0.06 MeV for $\nu=0.50\ \text{fm}^{-2}$. Then we obtain $2D=0.93$ MeV, namely $\Delta B_{\Lambda\Lambda}=1.24$ MeV, for $\nu=0.40\ \text{fm}^{-2}$ and $2D=1.12$ MeV, namely $\Delta B_{\Lambda\Lambda}=1.53$ MeV, for $\nu=0.50\ \text{fm}^{-2}$. Since we should expect other effects not considered here such as the change of wave functions and the contribution of the first term of Eq. (21), more quantitative evaluation of the actual rearrangement contribution would be desirable.

It is instructive to present the matrix element $\langle\Lambda\Lambda|G_{\Lambda\Lambda}|\Lambda\Lambda\rangle$ calculated with other available $\Lambda\Lambda$ interactions. Lanskoy and Yamamoto [20] parametrized k_F^N-dependent $\Lambda\Lambda$ G matrices [21] in a three-range Gaussian form obtained from the $\Lambda\Lambda$-ΞN sectors of the Nijmegen hard core model D [22] and the Nijmegen soft core model [23]. Nishizaki, Yamamoto, and Takatsuka [24] also gave parameters of the k_F^N-dependent effective $\Lambda\Lambda$ interactions in a four-range Gaussian form, based on G matrices of the Nijmegen model D and model F potentials [22]. The obtained matrix elements with these effective forces at $k_F^N=1.2\ \text{fm}^{-1}$ are shown and compared with our results of the quark model potential fss2 [9] in Table I. Referring to the newly determined value of $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He})\sim1$ MeV [1], the Nijmegen model D gives stronger attraction, though there is ambiguity in the choice of the hard core radius. On the other hand, the Nijmegen soft core model has an insufficient $\Lambda\Lambda$

TABLE I. Matrix elements $\langle\Lambda\Lambda|G_{\Lambda\Lambda}|\Lambda\Lambda\rangle$ calculated by k_F^N-dependent effective $\Lambda\Lambda$ interactions parametrized by Lanskoy and Yamamoto (LY) [20], and Nishizaki, Yamamoto, and Takatsuka (NYT) [24], which are based on G matrices obtained from three models by the Nijmegen group [22,23]: model D (ND), model F (NF), and the soft core model (NS). Our results of the quark model potential fss2 [9] are also included. Calculations are done for two harmonic oscillator constants of the Λ single-particle wave function. k_F^N is set to be 1.2 fm^{-1}. Entries are in MeV.

ν (fm^{-2})	LY-ND	LY-NS	NYT-ND	NYT-NF	fss2
0.4	-4.03	-1.27	-3.68	$+2.68$	-2.17
0.5	-4.90	-1.57	-4.29	$+3.90$	-2.65

attraction, when we take into account the Brueckner rearrangement effects. Vidaña *et al.* [25] calculated bond energies $\Delta B_{\Lambda\Lambda}$ for heavier double-Λ hypernuclei by a new set of the Nijmegen soft core potentials [26]. Their results suggested that the new soft core potentials have a weaker $\Lambda\Lambda$ attraction.

IV. CONCLUSION

We have estimated rearrangement contributions in the energy expectation values of the ${}^{5}_{\Lambda}$He and ${}^{6}_{\Lambda\Lambda}$He systems. The knowledge of this quantity is important to deduce the information of strengths of hyperon-nucleon and hyperon-hyperon interactions from experimental binding energies. In the standard framework of the lowest order Brueckner theory [7,8], from which invaluable understanding of nuclear systems has been accumulated for more than 40 years, the principal contribution to rearrangement energies is due to the starting energy dependence of the effective interaction as well as the change of Pauli blocking effect. Since we have avoided specific numerical calculations as much as possible, our results may not be very quantitative. On the other hand, the magnitude of the estimated values is rather robust. The energy of the α-core part is reduced by about 2 MeV when the single-particle energy becomes deeper by the addition of Λ to ^{4}He. This effect is connected with the overbinding problem of the *s*-shell hypernuclei, as Bando and Shimodaya [16] discussed in 1980. The new aspect of our presentation is the treatment of the estimation of the matrix element $\langle \Lambda h|G_{\Lambda N}|\Lambda h\rangle$. Instead of calculating it directly, we relate it to the Λ separation energy, which is by itself influenced by the rearrangement energy. As it should be, our result confirmed their estimation.

The rearrangement energy discussed in this paper is certainly not a sole answer for the overbinding problem. Core polarization effects should be considered, though the ^{4}He core is rather rigid. Several estimations [11,12,15] suggested that the energy change due to the core polarization is about 0.5 MeV or less. Three-body correlations and ΛNN three-body forces should also be taken into account. In the former case the coupling to the Σ intermediate state plays a characteristic role. It is important to treat three-body correlation effects together with three-body forces, which is an interesting subject for future investigation.

We next considered rearrangement effects to obtain the matrix element of the $\Lambda\Lambda$ interaction from the observed masses of double Λ hypernuclei. The experimental determination of the $\Delta B_{\Lambda\Lambda}$ which is defined as $2E({}^{5}_{\Lambda}\mathrm{He}) - M({}^{4}\mathrm{He}) - M({}^{6}_{\Lambda\Lambda}\mathrm{He})$ does not directly tell the strength of the $\Lambda\Lambda$ interaction, since it contains the rearrangement energies. This investigation is important in view of the recent experimental finding, which would renew previous understanding that the $\Lambda\Lambda$ interaction is rather strong. We derived a compact expression for the $\Delta B_{\Lambda\Lambda}$, assuming common wave functions for ^{4}He, ${}^{5}_{\Lambda}$He, and ${}^{6}_{\Lambda\Lambda}$He. The present estimation of the rearrangement contribution is about 1 MeV. Thus the *s*-wave matrix element of the $\Lambda\Lambda$ interaction in ${}^{6}_{\Lambda\Lambda}$He should be around -2 MeV.

Our calculation is based on the argument of the many-body theory in the model space. The extension of few-body-type calculations using bare interactions for hypernuclei such as ${}^{5}_{\Lambda}$He and ${}^{6}_{\Lambda\Lambda}$He would clarify the same subject directly from the basic microscopic viewpoint. As for ${}^{5}_{\Lambda}$He, Nemura, Akaishi, and Suzuki [18] recently showed by variational calculations that ^{4}He core energy in ${}^{5}_{\Lambda}$He decreases by 4.7 MeV. The difference of about 2 MeV may come from actual change of wave functions and higher order effects. It is interesting and gratifying to observe the correspondence between these studies and our treatments in the standard many-body theory.

Finally we comment that similar considerations should be applied to ${}^{9}_{\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be, where *ad hoc* effective interactions tend to have been employed.

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