Fluctuations of rapidity gaps in nucleus-nucleus collisions: Evidence of erraticity

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The fluctuations of the spatial pattern from event to event have been investigated by analyzing the data of multipion production in ^{24}Mg-AgBr interactions and ^{12}C-AgBr interactions at 4.5*A* GeV and ^{16}O-AgBr interactions at 2.1*A* GeV and 60*A* GeV. Two entropylike quantities S_q and Σ_q have been estimated from two different moments of rapidity gaps of produced pions. The investigation provides evidence of the erraticity of rapidity gaps of produced pions in relativistic nuclear collisions.

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In the beginning of the era of multiparticle production, most of the theoretical and experimental studies on multiparticle production in high-energy collisions were mainly based on analyzing average properties, such as the average multiplicity, the first few moments of the multiplicity distribution, and the rapidity distribution. Then with the increase of accelerator energy, the total rapidity range became large enough to permit meaningful partitioning of the rapidity interval into smaller bins of various sizes and the study of multiplicity fluctuation as a function of the bin size attracting the attention of physicists. In this way the concepts of intermittency was introduced by Bialas and Peschanski [1], which refer to the power-law dependence of the normalized factorial moments F_q on bin size. The method of factorial moments F_q does not completely reveal all the fluctuations that the system exhibits. In a multipion production process, the density of emitted pion spectra fluctuates from bin to bin for each interaction (namely, spatial fluctuations) and these fluctuations also fluctuate from one interaction to another (namely, event space fluctuations). In the case of vertically averaged horizontal moments, only the spatial fluctuation is taken into account neglecting the event space fluctuation. On the other hand, horizontally averaged vertical moments lose information about spatial fluctuation and only measure the fluctuations from event to event. Thus, the common methods lead to the loss of information on the erratic nature of the multiparticle production processes.

One of the important characteristics of multipion production is the fluctuations of spatial patterns from event to event. It can reveal greater details about the underlying dynamics of particle production. Up to now two methods have been proposed to describe these fluctuations: one is the moment of distribution of factorial moments F_q from event to event, which is based on bin multiplicities [2–4] and the other is a gap moment based on rapidity gaps [5].

Fu *et al.* [6] pointed out that the first is not suitable to measure the dynamical fluctuations for erraticity analysis in the case of low multiplicity events. The origin of the problem is that if the event multiplicity n is low and the number of bins M is high, then the average bin multiplicity in an event n/M is much less than 1. If there is any bin (ith bin) having multiplicity $n_i \geqslant q$ then only it will contribute to F_q and in the case of low multiplicity events, most of the events will be non contributing. Again F_q is not able to locate the position of the bins which contribute to it. Thus F_q does not describe the spatial pattern of an event very well. To overcome that deficiency, it is preferred to emphasize rapidity gaps not on bin multiplicities. It is intuitively obvious that the two quantities are complementary: the former measures how far apart neighboring particles are while the latter measures how many particles fall into the same bin.

In this investigation we deal with the fluctuations of spatial patterns from event to event for the nuclear emulsion data of multipion production in ^{24}Mg-AgBr and ^{12}C-AgBr interactions at 4.5*A* GeV and ^{16}O-AgBr interactions at 2.1*A* and 60*A* GeV. The nuclear emulsion covers 4π geometry and provides very good accuracy in pseudorapidity of the order of 0.1 pseudorapidity units. It is worthwhile to mention that the emulsion technique possesses very high spatial resolution, which makes it a very effective detector for studying the erratic behavior of rapidity gaps in multipion production. The details of the data sets were given in our earlier papers [7–10]. Since the values of the average multiplicities of the data sets are not too high, we have adopted the rapidity gap analysis proposed by Hwa *et al.* [5] as a measure of the fluctuations of spatial patterns from event to event.

Two moments of the rapidity gap distribution, namely, Γ_q and H_q, are suggested for characterizing an event. Two entropylike quantities S_q and Σ_q quantify the event-to-event fluctuations of gap moments Γ_q and H_q respectively. The q dependence of S_q and Σ_q have also been investigated through our study.

The single-particle density distribution in pseudorapidity space is nonflat. As the shape of this distribution may influence the scaling behavior of the moments, we shall use the "cumulative" variable $X(\eta)$ instead of η [11]. The cumulative variable $X(\eta)$ is given by

$$X(\eta)=\int_{\eta_1}^{\eta}\rho(\eta')\partial\eta' \Big/ \int_{\eta_1}^{\eta_2}\rho(\eta')\partial\eta', \tag{1}$$

where η_1 and η_2 are two extreme points in the distribution $\rho(\eta)$. Thus the accessible range of η is mapped to $X(\eta)$ between 0 and 1 and the density of particles in $X(\eta)$ space is uniform.

One can consider an event with n particles labeled by $i=1,2,\ldots,n$, located in $X(\eta)$ space at X_i, ordered from left to right. The distances between neighboring particles are given by

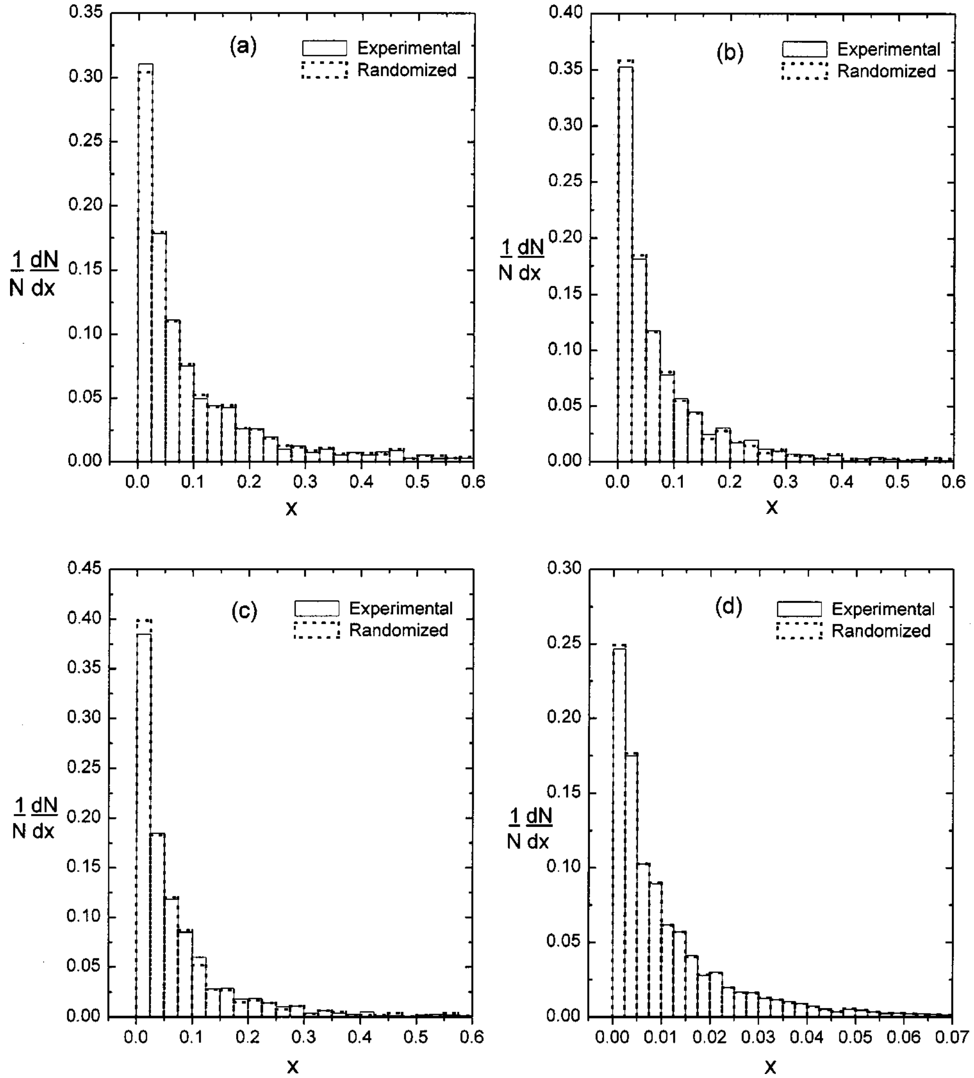

FIG. 1. Rapidity gap ($x = X_{i+1} - X_i$) distribution for (a) ^{12}C-AgBr interactions at 4.5*A* GeV, (b) ^{24}Mg-AgBr interactions at 4.5*A* GeV, (c) ^{16}O-AgBr interactions at 2.1*A* GeV and (d) ^{16}O-AgBr interactions 60*A* GeV.

$$x_i = X_{i+1} - X_i, \quad i = 0,1,\ldots,n, \tag{2}$$

where $X_0 = 0$ and $X_{n+1} = 1$ are the boundaries of $X(\eta)$ space. Every event e is thus characterized by a set S_e of $n+1$ numbers $S_e = \{x_i | i = 0,\ldots,n\}$, which clearly satisfy

$$\sum_{i=0}^{n} x_i = 1. \tag{3}$$

These numbers are referred to as "rapidity gaps."

To study the fluctuation of S_e from event to event, the moment of x_i for each event is defined as [5]

$$\Gamma_q = \frac{1}{n+1} \sum_{i=0}^{n} x_i^q, \tag{4}$$

where q is the order for spatial fluctuation. Since $x_i < 1$, the Γ_q's are usually $\ll 1$. It is obvious from Eqs. (3) and (4) that

$$\Gamma_0 = 1 \text{ and } \Gamma_1 = \frac{1}{n+1}.$$

At higher q, Γ_q are progressively smaller but are increasingly more dominated by the large x_i components in S_e, which in turn emphasize the large rapidity gaps. The moment Γ_q fluctuates from event to event. This event-to-event fluctuation of Γ_q can be quantified by the erraticity measure

$$s_q = -\langle \Gamma_q \ln \Gamma_q \rangle, \tag{5}$$

where angular brackets stand for the average over all events.

The moment Γ_q does not filter out statistical fluctuations. However, one can estimate how much S_q stands out above the statistical fluctuation by first calculating

$$s_q^{\mathrm{st}} = -\langle \Gamma_q^{\mathrm{st}} \ln \Gamma_q^{\mathrm{st}} \rangle, \tag{6}$$

where Γ_q^{st} is determined by constructing a reference sample using only the statistical distribution of the gaps, i.e., when

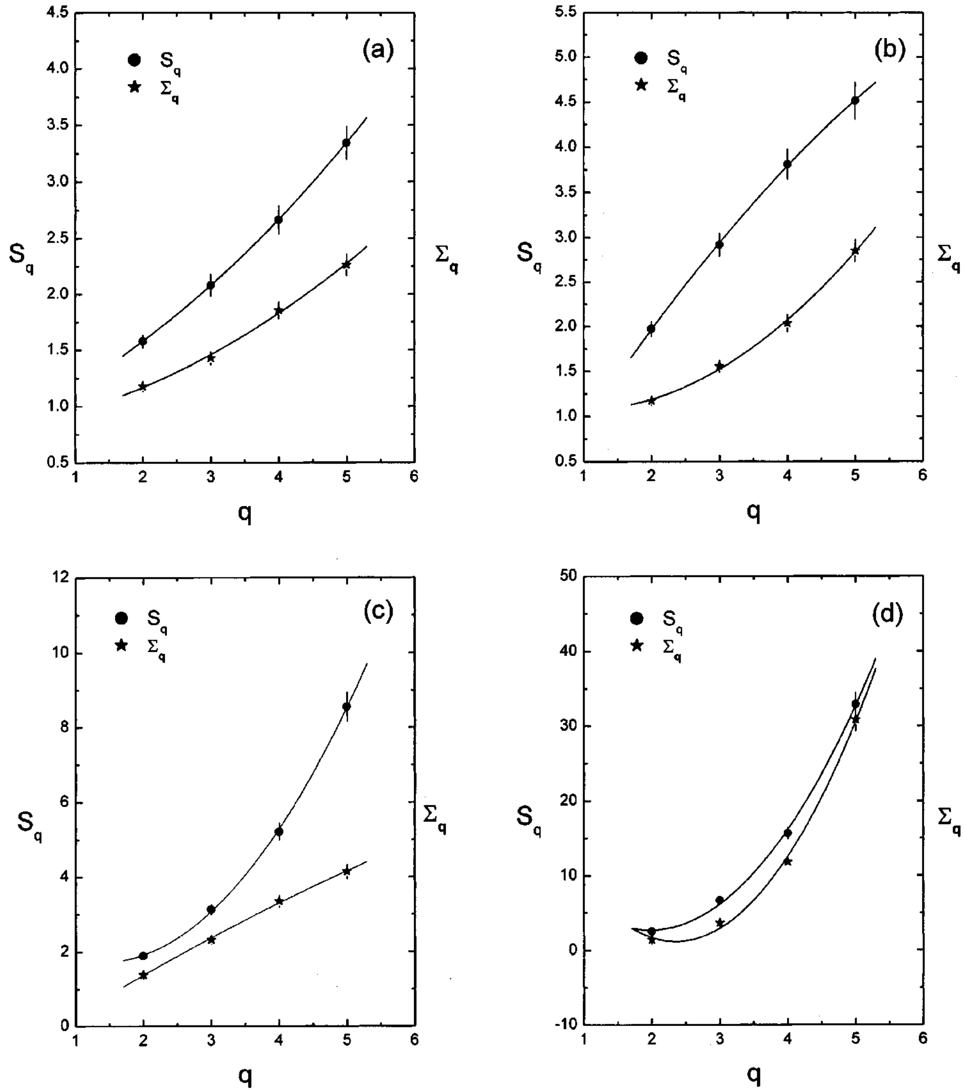

FIG. 2. S_q and Σ_q versus q plots for (a) ^{12}C-AgBr interactions at 4.5*A* GeV, (b) ^{24}Mg-AgBr interactions at 4.5*A* GeV, (c) ^{16}O-AgBr interactions at 2.1*A* GeV and (d) ^{16}O-AgBr interactions 60*A* GeV.

all n particles in an event are distributed randomly in $X(\eta)$ space and at the same time describes the inclusive distribution of experimental rapidity gaps and then taking the ratio

$$S_q = s_q / s_q^{\mathrm{st}}. \tag{7}$$

The deviation of S_q from 1 will indicate the dynamical erraticity behavior of multipion production. The q dependence of S_q will be determined from the analysis. However, the specific way in which S_q depends on q has no physical significance [5].

We have constructed a randomly distributed reference sample corresponding to experimental data sets of ^{24}Mg-AgBr and ^{12}C-AgBr interactions at 4.5*A* GeV and ^{16}O-AgBr interactions at 2.1*A* and 60*A* GeV as described earlier. Figure 1 shows the rapidity gap ($x = X_{i+1} - X_i$) distributions of experimental and random data sets. We have

TABLE I. The second-order polynomial fit parameters ($y = A + B1x + B2x^2$) of S_q versus q plots with the corresponding χ^2 values.

Interactions	Energy	A	B1	B2	χ^2
^{12}C-AgBr	4.5*A* GeV	0.85±0.02	0.28±0.01	0.044±0.001	0.00001
^{24}Mg-AgBr	4.5A GeV	−0.34±0.17	1.27±0.11	−0.06±0.01	0.00091
^{16}O-AgBr	2.1*A* GeV	2.7±0.5	−1.4±0.3	0.52±0.04	0.00798
^{16}O-AgBr	60*A* GeV	15.4±4.3	−12.9±2.7	3.3±0.4	0.5731

TABLE II. The second-order polynomial fit parameters ($y=A+B1x+B2x^2$) of Σ_q versus q plots and their relative parameters.

Interactions	Energy	A	B1	B2	χ^2
^{12}C-AgBr	4.5*A* GeV	0.8±0.3	0.10±0.16	0.04±0.02	0.00212
^{24}Mg-AgBr	4.5*A* GeV	1.2±0.3	−0.20±0.18	0.11±0.03	0.00272
^{16}O-AgBr	2.1*A* GeV	−0.8±0.4	1.2±0.3	−0.03±0.04	0.00557
^{16}O-AgBr	60*A* GeV	24.4±6.4	−19.7±3.9	4.2±0.5	1.23169

calculated the Γ_q moment for each event of all four interactions using Eq. (4). Here the order for spatial fluctuation q is varied from 2 to 5. For each q, Γ_q moments for all four data sets fluctuate from event to event. The erraticity measure s_q has been calculated using Eq. (5) to probe this event-to-event fluctuation. To eliminate the statistical part of this measure we have calculated $s_q^{\rm st}$ for the samples statistical produced following the same procedure and have taken the ratio $S_q = s_q / s_q^{\rm st}$. The S_q values are plotted against q in Figs. 2(a)–2(d) respectively for the ^{12}C-AgBr interactions at 4.5*A* GeV, ^{24}Mg-AgBr interactions at 4.5*A* GeV, and ^{16}O-AgBr interactions at 2.1*A* and 60*A* GeV. It is evident from the Fig. 2 that the entropylike quantities S_q deviate significantly from 1 for all the interactions implying that it is a statistically significant measure of the erraticity of rapidity gaps in multipion production. The S_q values increase with the increase of q putting more weight on large gaps. Evidently, the result indicates a polynomial behavior in q. The values of the second-order polynomial fit parameters for all four interactions are tabulated in Table I.

Another moment of x_i for each event has also been defined [5] to study the fluctuation of S_e from event to event as

$$H_q = \frac{1}{n+1} \sum_{i=0}^{n} (1-x_i)^{-q}. \tag{8}$$

As with the Γ_q moments, these moments also receive a dominant contribution from large x_i, but H_q can become $\gg 1$ unlike Γ_q.

The fluctuation of event patterns implies fluctuation of H_q, which can be quantified by the erraticity measure defined as

$$\sigma_q = \langle H_q \ln H_q \rangle \tag{9}$$

and

$$\Sigma_q = \sigma_q / \sigma_q^{\rm st}, \tag{10}$$

where the denominator is the statistical contribution to σ_q which can be calculated by constructing a randomly distributed reference sample which can describe the inclusive distribution of experimental rapidity gaps of the produced particles in $X(\eta)$ space.

We have calculated the H_q moment with $q=2$, 3, 4, and 5 for each event of ^{24}Mg-AgBr and ^{12}C-AgBr interactions at 4.5*A* GeV and ^{16}O-AgBr interactions at 2.1*A* and 60*A* GeV using Eq. (8). For each interaction, the fluctuations of H_q moments from event to event have been captured by calculating the erraticity measure σ_q using Eq. (9). For each case we have calculated $\sigma_q^{\rm st}$, the statistical part of this measure, and have taken the ratio $\Sigma_q = \sigma_q / \sigma_q^{\rm st}$. The Σ_q values are also plotted against q in Figs. 2(a)–2(d) with corresponding S_q values. The entropylike quantities Σ_q deviate significantly from 1, as is evident from Fig. 2, indicating the erratic behavior of rapidity gaps of produced pions. In general, Σ_q also shows polynomial behavior in q. The values of the second-order polynomial fit parameters for all four interactions are tabulated in Table II.

In this article, an erraticity analysis has been performed for three different types of projectile beams and for three distinct energies. In all cases, the entropylike quantities S_q and Σ_q deviate significantly from 1, which in turn provides the first strong evidence in favor of erratic fluctuations of rapidity gaps from event to event in the case of relativistic nuclear collisions.

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