Decay out of superdeformed bands

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A consistent theory of the decay out of superdeformed rotational levels is presented. It is based on exact equations for Green's functions, taking into account simultaneously both the residual and the electromagnetic interactions. In the weak-coupling limit, we generalized the two-level model of Stafford and Barrett to the case where the superdeformed level is coupled with an infinite equidistant spectrum of normal compound states. For nonoverlapping levels, the general equations of Vigezzi *et al.* are rederived. The possibility of the nonexponential decay law is discussed.

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I. INTRODUCTION

In recent years, the decay mechanism of superdeformed (SD) rotational bands has been intensively studied both theoretically [1–13] and experimentally [14–26]. There are numerous observations of γ spectra, produced by deexcitation transitions within SD bands. At some small spins *I*, the intensity of such spectral lines is found to drop suddenly, so that the γ spectrum quenches. This observation has been explained by a statistical model [1–3], based on the fact that the SD level $|s\rangle$ with small spin lies high above the normal yrast line. As a result, the collective SD level is surrounded by a dense spectrum of excited compound states of a normally (*N*) deformed nucleus $|\alpha\rangle$, which decay, emitting mainly $E1$ photons. The residual interaction $\hat{V}'$ causes mixing of the SD and *N* states. Then, $E1$ transitions begin competing with $E2$ transitions within the SD band, which leads to a sudden reduction of their intensity. The statistical model describes the normal states in terms of the Gaussian orthogonal ensemble of random matrices. It has been developed further by Weidenmüller and co-workers [9,11], who represented the relative intensity of transitions within the SD band $\overline{\mathcal{I}_{in}}$ as a sum,

$$\overline{\mathcal{I}_{in}}=\mathcal{I}_{av}+\overline{\mathcal{I}_{fluc}}, \tag{1}$$

where $\mathcal{I}_{av}$ and $\overline{\mathcal{I}_{fluc}}$ stand for the averaged and fluctuating parts of the intensity, respectively. Specifically, $\mathcal{I}_{av}$ is given by

$$\mathcal{I}_{av}=\frac{\Gamma_s}{\Gamma_s+\Gamma}, \tag{2}$$

where Γ_s is the radiative width of the SD level and Γ is the spreading width of the SD level. The latter is determined by the average value of Fermi's golden rule:

$$\Gamma=2\pi v'^2/D_N, \tag{3}$$

where D_N is the average spacing of the states $|\alpha\rangle$ and $v'=\langle\alpha|\hat{V}'|s\rangle$. But Fermi's golden rule holds for continuous spectra. Therefore, the question remains open: at what densities of normal levels their spectrum can be considered as quasicontinuum and Eq. (3) can be used?

The mixing of the SD level with a single close-lying *N* one has been accurately studied by Stafford and Barrett [8], who analyzed both mixing and electromagnetic decays on an equal footing. Special attention has been paid to the case, typical for the nuclei in mass-190 region, where

$$\Gamma_N \gg \Gamma_s, \Gamma^{\downarrow}. \tag{4}$$

Here, Γ_N is the radiative width of the normal state and

$$\Gamma^{\downarrow}=\frac{v'^2(\Gamma_s+\Gamma_N)}{[(\Gamma_s+\Gamma_N)/2]^2+\Delta^2} \tag{5}$$

is the decay-out width, depending on the difference $\Delta=E_0-E_s$ of the unperturbed energies of the normal (E_0) and superdeformed (E_s) states. Combining Eqs. (4) and (5), one can rewrite inequality (4) as

$$\Gamma_N \gg \Gamma_s, \quad v' \ll \sqrt{\Delta^2+(\Gamma_N/2)^2}. \tag{6}$$

The Hamiltonian of the nucleus can be represented as $\hat{H}_N=\hat{H}_N^{(0)}+\hat{V}'$, where the unperturbed Hamiltonian $\hat{H}_N^{(0)}$ is a sum of the terms $\hat{H}_{rot}, \hat{H}_{vib}(\beta,\gamma)$, and $\hat{H}_{intr}(\beta,\gamma;\xi)$, which describe the rotation, vibrations of the shape, and intrinsic motion of the nucleons (treated in the framework of the shell model), respectively. In the adiabatic approximation with respect to slow β and γ vibrations, one can first omit $\hat{H}_{vib}(\beta,\gamma)$ [27]. Then, we should solve the Schrödinger equation with the reduced Hamiltonian $\hat{H}_{rot}+\hat{H}_{intr}$, depending on the parameters β and γ. An eigenvalue of such reduced operator, as a function of the deformation parameter β, at a fixed value of γ plays the role of the potential energy $V_I(\beta)$ in the one-dimensional Schrödinger equation for the deformed nuclear shape motion. We are interested in nuclei, for which function $V_I(\beta)$ has two minima, associated with the normal and superdeformed shapes. The Schrödinger equation with such an asymmetric potential has been solved quasiclassically in a preceding paper [13] in full analogy with the familiar symmetric case [28]. Its solution $\varphi_I(\beta)$ is spread over both wells simultaneously. In other words, it is

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represented by a superposition of the wave functions $\varphi^{(N)}(\beta)$ and $\varphi^{(S)}(\beta)$, which describe vibrations in N and SD wells with frequencies ω_N and ω_S, respectively. In the absence of the barrier tunneling, there are separate harmonic vibrations in the wells N and SD with energies $\epsilon_1 = \hbar\omega_N(n_1+1/2)$ and $\epsilon_2=\hbar\omega_S(n_2+1/2)$, where n_1 and n_2 are integers. The tunneling ensures shifting of these levels and the mixing of the wave functions $\varphi^{(N)}(\beta)$ and $\varphi^{(S)}(\beta)$ with the amplitudes c_N and c_S, respectively. Recall that for nuclei with stable octupole deformations, the tunneling leads to mixing of the wave functions for the mirror octupole shapes with equal weights (see, e.g., Ref. [29]).

A typical spacing $|\epsilon_1-\epsilon_2|$ of the vibrational levels greatly exceeds the tunneling strength

$$v=(\hbar\omega_0/2\pi)\exp(-A), \tag{7}$$

where

$$\omega_0^2=\omega_N\omega_S, \quad A=\pi W_I/\hbar\omega_B, \tag{8}$$

while W_I stands for the height of the barrier, approximated by an inverse parabola with frequency ω_B. In such a case, one of the amplitudes c_N or c_S is much less than unity, i.e., the wave function is concentrated mainly to one of the wells. In particular, wave function

$$\varphi_s(\beta)=c_N^s\varphi_s^{(N)}(\beta)+c_S^s\varphi_s^{(S)}(\beta), \tag{9}$$

which describes the deformed motion in the SD state, is located almost completely in the SD well; it has only a weak tail in the normal well, allowing the coupling of the superdeformed state with the compound states $|\alpha\rangle$ [see also Fig. 1(b) of the paper by Vigezzi *et al.* [3]]. The amplitudes in Eq. (9) are [13]

$$c_S^s\approx 1, \quad |c_N^s|\approx\frac{\exp(-A)}{2|\sin\alpha_N|}\ll 1, \tag{10}$$

where α_N denotes the angle

$$\alpha_N=\pi(\epsilon_1-\epsilon_2)/\hbar\omega_N. \tag{11}$$

When $|\epsilon_1-\epsilon_2|\gg v$, the normal component $\varphi_s^{(N)}(\beta)$ of the wave function $\varphi_s(\beta)$ is represented by a decomposition in terms of the harmonic oscillator wave functions, which describe vibrations in the N well with different phonon numbers n_1.

The eigenfunctions of the unperturbed nuclear Hamiltonian $\hat{H}_N^{(0)}$ are superpositions, characterized by the definite signature, of the products of the rotational wave function $\sqrt{(2I+1)/8\pi^2}D_{KM}^I(\theta)$, depending on the Euler angles θ as well as functions describing intrinsic motion $\Phi(\beta,\gamma;\xi)$ and motion of the deformed shape $\varphi(\beta)$. Specifically, the wave function for the SD level ψ_s contains as a factor the function $\varphi_s(\beta)$. Using expression (9), we rewrite this wave function as $\psi_s=c_N^s|N\rangle+c_S^s|S\rangle$, where the components $|N\rangle$ and $|S\rangle$ describe pure normal and superdeformed shapes, respectively. From the smallness of c_N^s it follows that $\Gamma_s\approx\Gamma_S$, where Γ_S is the radiative width of the pure superdeformed state $|S\rangle$. Since wave functions $|\alpha\rangle$ of the compound states overlap only with the normal component of the wave function ψ_s, the interaction strength v' factorizes:

$$v'=c_N^s\langle\alpha|\hat{V}'|N\rangle. \tag{12}$$

In this paper, we shall generalize the approach of Stafford and Barrett [8] to the case where the SD level is coupled with an arbitrary number of N states $|\alpha\rangle$. We use the model of equidistant compound states $|\alpha\rangle$, which allows an analytical treatment.

II. GREEN'S FUNCTIONS

The total Hamiltonian of the system (nucleus plus electromagnetic field) may be written as

$$\hat{H}=\hat{H}_0+\hat{V}, \quad \hat{H}_0=\hat{H}_N+\hat{H}_{rad}, \tag{13}$$

where $\hat{H}_N$ and $\hat{H}_{rad}$ denote the Hamiltonians of the nucleus and electromagnetic field, respectively. The perturbation operator is

$$\hat{V}=\hat{V}_r+\hat{V}', \tag{14}$$

where $\hat{V}_r$ is the interaction operator of the nucleus with electromagnetic field. The eigenfunctions of the unperturbed Hamiltonian $\hat{H}_0$ will be products of the functions for the nucleus and the electromagnetic field. In particular, the initial state of the system at $t=0$ is described by wave function

$$\Psi_s(0)=|s\rangle=\psi_s|0\rangle, \tag{15}$$

where $|0\rangle$ stands for the vacuum function of the field. The corresponding energy of the system equals energy E_s of the SD level. State $|s\rangle$ is coupled to intermediate states of the system $|\alpha\rangle=\psi_\alpha|0\rangle$ with energies E_α. Below, the set of states $|s\rangle$ and $|\alpha\rangle$, which may have in principle close-lying energies, will be also labeled by $|a\rangle$ or $|a'\rangle$. The wave functions of final states $|b\rangle$ are products of the functions for the nucleus in the N or SD state and the field with one photon.

Time evolution of the wave function at $t\geqslant 0$ is determined by equation [30]

$$\Psi_s(t)=-\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\varepsilon e^{-i\varepsilon t/\hbar}\hat{G}^+(\varepsilon)\Psi_s(0), \tag{16}$$

where Green's operator

$$\hat{G}^+(\varepsilon)=(\varepsilon+i\eta-\hat{H})^{-1}, \quad \eta\to+0. \tag{17}$$

The probability of finding the system at moment t in state $|b\rangle$ or $|a'\rangle$ is given by

$$P_{b(a')}(t)=|\mathcal{G}_{b(a')}(t)|^2, \tag{18}$$

where

$$\mathcal{G}_{b(a')}(t)=-\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\varepsilon e^{-i\varepsilon t/\hbar}\langle b(a')|\hat{G}^{+}(\varepsilon)|s\rangle. \tag{19}$$

Following Ref. [30] we easily find that Green's matrices G^{+}_{ba} and $G^{+}_{a'a}$ are related by (see also Ref. [31])

$$G^{+}_{ba}(\varepsilon)=(\varepsilon+i\eta-E_b)^{-1}\sum_{a'}V_{ba'}G^{+}_{a'a}(\varepsilon), \tag{20}$$

while $G^{+}_{a'a}$ is determined by the system of algebraic equations [31]

$$\sum_{a''}\{(\varepsilon-E_a)\delta_{aa''}-R^{+}_{aa''}(\varepsilon)\}G^{+}_{a''a'}(\varepsilon)=\delta_{aa'}, \tag{21}$$

where the R matrix is given by the expansion

$$R^{+}_{aa'}(\varepsilon)=V_{aa'}+\sum_{b\neq a,a'}\frac{V_{ab}V_{ba'}}{\varepsilon+i\eta-E_b}+\cdots. \tag{22}$$

We shall assume that all states $|\alpha\rangle$ have common radiative width Γ_N and they are coupled to the SD state with equal strength v', i.e.,

$$R^{+}_{ss}=-i\Gamma_s/2,\ \ R^{+}_{\alpha\alpha}=-i\Gamma_N/2,\ \ R^{+}_{s\alpha}=v'. \tag{23}$$

Inserting Eq. (23) into Eqs. (21), one has their solution

$$G^{+}_{ss}(\varepsilon)=\left\{\varepsilon-E_s+i\frac{\Gamma_s}{2}-\sum_{\alpha}\frac{v'^2}{\varepsilon-E_\alpha+i\Gamma_N/2}\right\}^{-1} \tag{24}$$

and

$$G^{+}_{\alpha s}(\varepsilon)=\frac{v'}{\varepsilon-E_\alpha+i\Gamma_N/2}G^{+}_{ss}(\varepsilon). \tag{25}$$

Summing $P_b(\infty)$ over all possible states of emitted photons and final SD nuclear states, one gets the probability that the nucleus will conserve superdeformed shape after decay (see also Ref. [8]):

$$F_S=\frac{\Gamma_s}{2\pi}\int_{-\infty}^{\infty}d\varepsilon|G^{+}_{ss}(\varepsilon)|^2. \tag{26}$$

Here, F_S has the same meaning as $\mathcal{I}_{in}$ in Refs. [9,11]. It is evident that the decay-out probability will be $F_N=1-F_S$.

Following Ref. [32], we take an infinite equidistant spectrum of normal states $E_\alpha=E_0+\alpha D_N$, where $\alpha=0,\pm1,\pm2,\ldots$, and E_0 stands for the energy nearest to E_s. Then, the sum over α in Eq. (24) is easily calculated [32], yielding

$$G^{+}_{ss}(\varepsilon)=\frac{1}{\varepsilon-E_s+i\Gamma_s/2+i(\Gamma/2)\Phi(\varepsilon)}, \tag{27}$$

where the spreading width Γ is defined by Eq. (3), and

$$\Phi(\varepsilon)=\frac{e^{-iz}+e^{iz}}{e^{-iz}-e^{iz}},\quad z=\pi(\varepsilon-E_0+i\Gamma_N/2)/D_N. \tag{28}$$

Function $\Phi(\varepsilon)$ tends to unity when $e^{-iz}\to\infty$, i.e., when $\Gamma_N/2D_N\to\infty$. In this case, Green's function (27) reduces to

$$G^{+}_{ss}(\varepsilon)=\frac{1}{\varepsilon-E_s+i(\Gamma_s+\Gamma)/2}. \tag{29}$$

On the other hand, at $\Gamma_N/2\gg D_N$ we can replace the summation over α in Eq. (24) by an integral that provides the same result (29). For such a dense spectra of overlapping normal levels, the decay becomes purely exponential with attenuation $\Gamma_s+\Gamma$, being independent of Γ_N. Then, the branching ratio F_S is determined by the same equation (2) as $\mathcal{I}_{av}$. Thus, for nuclei with equidistant normal levels we can introduce the spreading width (3) when these levels greatly overlap. Such a statement is consistent with the conclusion of the papers [9,11] that, in nuclei with random spectra, the term $\mathcal{I}_{av}$ dominates over $\overline{\mathcal{I}_{fluc}}$ at $\Gamma_N\gg D_N$. However, typical experimental data are $D_N\sim1$–10^3 eV and $\Gamma_N\sim1$–10 meV (see, e.g., Ref. [8]), i.e., in most cases $\Gamma_N/2D_N\ll1$.

From Eqs. (21) one sees that $G_{aa'}(\varepsilon)$ has an inverse matrix

$$G^{-1}_{aa'}(\varepsilon)=\varepsilon\delta_{aa'}-\mathcal{H}_{aa'}, \tag{30}$$

where $\mathcal{H}_{aa'}$ is an effective non-Hermitian Hamiltonian matrix (see also Ref. [33]),

$$\mathcal{H}_{aa'}=E_a\delta_{aa'}+R^{+}_{aa'}(\varepsilon). \tag{31}$$

When $\Gamma_N\neq\Gamma_s$, matrix $\mathcal{H}_{aa'}$ is not normal, since it does not commute with its conjugate [34]. Therefore, its eigenvectors $|i\rangle$, satisfying equation

$$\hat{\mathcal{H}}|i\rangle=\mu_i|i\rangle, \tag{32}$$

are not orthogonal to each other, i.e., $\langle i|i'\rangle\neq0$ when $i\neq i'$. Moreover, its eigenvalues are complex numbers, $\mu_i=\mathcal{E}_i-i\Gamma_i/2$. They determine the poles of function $G^{+}_{ss}(\varepsilon)$ on the complex plane $\varepsilon=\varepsilon'+i\varepsilon''$. If the values μ_i are not degenerate, these poles are simple, and $G^{+}_{ss}(\varepsilon)$ is represented by the following sum of resonant terms:

$$G^{+}_{ss}(\varepsilon)=\sum_i\frac{\mathcal{A}_i}{\varepsilon-\mu_i}, \tag{33}$$

where $\mathcal{A}_i$ are the residues of $G^{+}_{ss}(\varepsilon)$ at the poles μ_i. Substituting this expression into Eq. (19), we obtain the nonexponential decay law for the SD level:

$$P_s(t)=\left|\sum_i\mathcal{A}_ie^{-i\mu_it/\hbar}\right|^2. \tag{34}$$

Thus, in the general case, the probabilities of finding the excited nucleus in one of the potential wells undergo attenuating Rabi beats with several frequencies $|\mathcal{E}_i-\mathcal{E}_{i'}|/\hbar$ (see also Ref. [8]).

III. NONOVERLAPPING LEVELS

Calculations of $\mathcal{A}_i$ and μ_i simplify in the case of nonoverlapping resonant levels, whose spacings greatly exceed their attenuation, so that $|\mathcal{E}_i-\mathcal{E}_{i'}|\gg\Gamma_i,\Gamma_{i'}$ if $i\neq i'$. Then, it is useful to rewrite the effective Hamiltonian matrix (31) as a sum

$$\hat{\mathcal{H}}=\hat{\mathcal{H}}^{(0)}+\hat{\mathcal{H}}', \tag{35}$$

containing the unperturbed matrix

$$\mathcal{H}^{(0)}_{aa'}=E_a\delta_{aa'}+V'_{aa'}, \tag{36}$$

and the perturbation

$$\mathcal{H}'_{aa'}=-i(\Gamma_a/2)\delta_{aa'} \tag{37}$$

responsible for the radiative decays. The symmetric matrix $\hat{\mathcal{H}}^{(0)}$ has an orthonormal set of eigenvectors

$$|i\rangle=\sum_a c_a(i)|a\rangle, \tag{38}$$

where $c_a(i)$ stands for an orthogonal matrix. The corresponding eigenvalue equation reads

$$\hat{\mathcal{H}}^{(0)}|i\rangle=\mathcal{E}^{(0)}_i|i\rangle. \tag{39}$$

The corrections to energies $\mathcal{E}^{(0)}_i$ in the first order of the perturbation theory are

$$\langle i|\hat{\mathcal{H}}'|i\rangle=-i\Gamma_i/2, \tag{40}$$

where the radiative widths of states $|i\rangle$, which are mixtures of N and SD functions, are equal to

$$\Gamma_i=c_s^2(i)\Gamma_s+[1-c_s^2(i)]\Gamma_N. \tag{41}$$

Ignoring small perturbation corrections to the wave functions, one gets

$$G^+_{ss}(\varepsilon)=\sum_i\frac{c_s^2(i)}{\varepsilon-\mathcal{E}^{(0)}_i+i\Gamma_i/2}. \tag{42}$$

Substituting Eq. (42) into Eq. (26) gives branching ratios, derived previously in Ref. [2]:

$$F_S=\sum_i c_s^4(i)(\Gamma_s/\Gamma_i),$$
$$F_N=\sum_i c_s^2(i)[1-c_s^2(i)](\Gamma_N/\Gamma_i), \tag{43}$$

where Γ_i are determined by Eq. (41). In the case considered, periods $2\pi\hbar/|\mathcal{E}^{(0)}_i-\mathcal{E}^{(0)}_{i'}|$ of temporary beats of $P_{N(s)}(t)$ are much less than the attenuation times $\hbar/\Gamma_i$. Such swift Rabi oscillations should be yet averaged within the time of observation. Then, the averaged probabilities, to be measured experimentally, will not represent oscillations:

$$\overline{P}_s(t)=\sum_i c_s^4(i)e^{-\Gamma_i t/\hbar}. \tag{44}$$

IV. WEAK-COUPLING LIMIT

Another simplification of the Green's function is achieved when conditions (6) are fulfilled. Let us first analyze the analytical behavior of function $G^+_{ss}(\varepsilon)$ on the complex plane $\varepsilon=\varepsilon'+i\varepsilon''$ in such a weak-coupling limit. Its poles are determined by equation

$$\varepsilon=E_s-i(\Gamma_s/2)+(\pi v'^2/D_N)ctgz, \tag{45}$$

where z is specified by Eq. (28). The pole $\varepsilon_0(s)$ near the point $\varepsilon_0^{(0)}(s)=E_s-i\Gamma_s/2$ may be found by iterating Eq. (45). In the first order, one has

$$\mathrm{Im}\varepsilon_0(s)\approx-(\Gamma_s+\Gamma^{\downarrow})/2. \tag{46}$$

where the decay-out width $\Gamma^{\downarrow}$ is already determined by

$$\Gamma^{\downarrow}=\frac{gv'^2}{\xi_-^2+\sin^2(\pi\Delta/D_N)}, \tag{47}$$

and the following notations are used:

$$g=2\pi\xi_+\xi_-/D_N, \tag{48}$$

$$\xi_\pm=\frac{1}{2}[\exp(\pi\Gamma_N/2D_N)\pm\exp(-\pi\Gamma_N/2D_N)].$$

Usually, $\Gamma_N\ll D_N$. In this approximation, Eq. (47) becomes

$$\Gamma^{\downarrow}=\frac{v'^2\Gamma_N}{(\Gamma_N/2)^2+(D_N/\pi)^2\sin^2(\pi\Delta/D_N)}. \tag{49}$$

Note that the decay-out width (47) coincides with the spreading one only at $\Gamma_N\gg D_N$.

Other poles $\varepsilon_0(\alpha)$ of $G^+_{ss}(\varepsilon)$ lie in the vicinity of points $\varepsilon_0^{(0)}(\alpha)=E_\alpha-i\Gamma_N/2$. Substituting their shifts $\Delta\varepsilon_0(\alpha)=\varepsilon_0(\alpha)-\varepsilon_0^{(0)}(\alpha)$ into Eq. (45), we obtain the quadratic equation

$$[\Delta\varepsilon_0(\alpha)]^2+(\Delta+\alpha D_N-i\Gamma_N/2)\Delta\varepsilon_0(\alpha)-v'^2=0. \tag{50}$$

Its small root is

$$\Delta\varepsilon_0(\alpha)\approx\frac{v'^2}{\Delta+\alpha D_N-i\Gamma_N/2}. \tag{51}$$

Keeping in mind condition (6), we see that

$$\mathrm{Im}\varepsilon_0(\alpha)\approx-\Gamma_N/2. \tag{52}$$

Hence, poles $\varepsilon_0(\alpha)$ lie much farther from the real axis on the ε plane than pole $\varepsilon_0(s)$. Moreover, the residues at these poles $\mathcal{A}_\alpha=\mathrm{res}\,G^+_{ss}(\varepsilon_0(\alpha))$ are small:

$$\mathcal{A}_\alpha=\left(\frac{v'}{\Delta+\alpha D_N-i\Gamma_N/2}\right)^2. \tag{53}$$

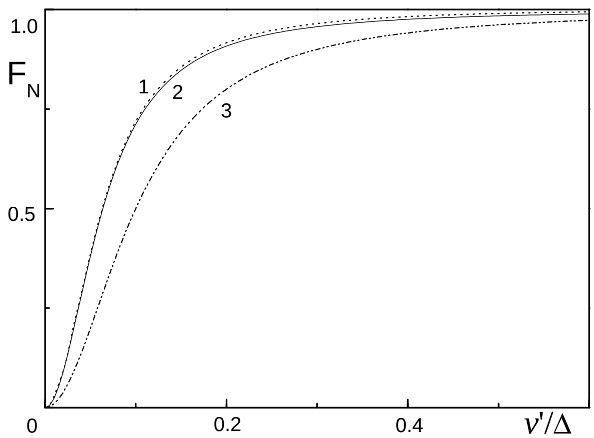

FIG. 1. The branching ratio F_N versus v'/Δ, calculated with the aid of exact Eq. (25) (curve 1) as well as approximate Eqs. (49) and (5) (curves 2 and 3, respectively) for $\Delta = D_N/2$.

Therefore, contribution from poles $\varepsilon_0(\alpha)$ to integrals (19) or (26), calculated by means of the contour integration, can be omitted compared to the contribution from $\varepsilon_0(s)$. Similar arguments are presented in Ref. [32] in connection with the discussion of the exponential law of decay. So expansion (33) for the Green's function $G^+_{ss}(\varepsilon)$ can be replaced by the single term

$$G^+_{ss}(\varepsilon) = \frac{1}{\varepsilon - E_s + i(\Gamma_s + \Gamma^\downarrow)/2}. \tag{54}$$

Then, the branching ratios become

$$F_S = \frac{\Gamma_s}{\Gamma_s + \Gamma^\downarrow}, \quad F_N = \frac{\Gamma^\downarrow}{\Gamma_s + \Gamma^\downarrow}. \tag{55}$$

The usefulness of our formulas is illustrated in Figs. 1 and 2, showing the dependence of F_N on v'/Δ. Curve 1 represents direct numerical calculations, using exact Eqs. (24) and (26). Curves 2 and 3 exhibit model calculations based on Eq. (54), which uses for $\Gamma^\downarrow$, respectively, our Eqs. (49) and (5), derived in Ref. [8]. The parameters are chosen as follows:

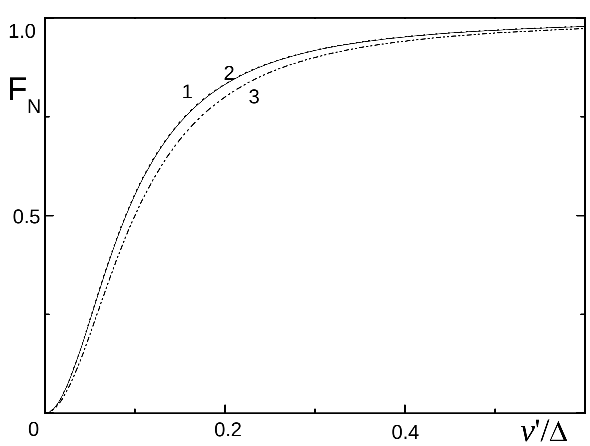

FIG. 2. As in Fig. 1, but for $\Delta = D_N/4$.

$D_N = 100$ eV, $\Gamma_s = 0.1$ meV, and $\Gamma_N = 10$ meV, while $\Delta = D_N/2$ in Fig. 1 and $\Delta = D_N/4$ in Fig. 2. Our expression (49) is seen to provide a better accuracy than Eq. (5) at large Δ.

Usually, probabilities F_N and F_S are averaged over some distribution of the energy difference Δ (see, e.g., Ref. [2]). For uniform distribution of Δ, the mean value of $F_N(\Delta)$ is

$$\overline{F_N} = \frac{1}{D_N} \int_{-D_N/2}^{D_N/2} F_N(\Delta)\, d\Delta. \tag{56}$$

Substituting Eq. (47) into Eqs. (55) and (56), one finds

$$\overline{F_N} = \frac{v'^2}{\sqrt{[\xi_-^2 \Gamma_S/g + v'^2][(1+\xi_-^2)\Gamma_S/g + v'^2]}}. \tag{57}$$

For $\Gamma_N \ll D_N/\pi$, Eq. (57) simplifies to

$$\overline{F_N} \approx \frac{v'^2}{\sqrt{[\Gamma_N \Gamma_s/4 + v'^2][(D_N/\pi)^2 (\Gamma_s/\Gamma_N) + v'^2]}}. \tag{58}$$

These expressions allow us to find the coupling strength v', which hereafter will be referred to as $\overline{v'}$. From Eq. (58) one finds

$$\overline{v'} \approx \frac{D_N}{\pi} \sqrt{\frac{\Gamma_s}{\Gamma_N} \left(\frac{\overline{F}_N^2}{1 - \overline{F}_N^2} \right)}. \tag{59}$$

Then using this evaluation together with designation (3), we can rewrite expression (58) as

$$\overline{F_N} \approx \sqrt{\frac{\Gamma \Gamma_N}{D_N \Gamma_s}} \left(\frac{2}{\pi} + \frac{\Gamma \Gamma_N}{D_N \Gamma_s} \right)^{-1/2}. \tag{60}$$

In reality, the energy difference Δ takes some definite value between $-D_N/2$ and $D_N/2$ (see also Ref. [8]). According to Eqs. (47) and (55), v' may vary in the interval confined by the values

$$v'_{min} = \tfrac{1}{2} \sqrt{F_N/(1-F_N)} \sqrt{\Gamma_s \Gamma_N},$$
$$v'_{max} = (2D_N/\pi \Gamma_N) v'_{min}, \tag{61}$$

where v'_{min} and v'_{max} correspond to $\Delta = 0$ and $|\Delta| = D_N/2$, respectively.

Using experimental data for F_s and the parameters Γ_N, Γ_S, and D_N, listed in Refs. [8,11,26], we found with the aid of Eq. (59) the coupling constants $\overline{v'}$ for some SD levels of Hg and Pb isotopes. All these parameters and constants are presented in the table together with the limiting quantities v'_{min} and v'_{max}. Knowing the magnitude of the nuclear matrix element $\langle \alpha | \hat{V} | N \rangle$, we would be able to find by means of Eqs. (10) and (12) action A and the barrier height W_I, respectively. Following Ref. [35], we accept that $|\langle \alpha | \hat{V} | N \rangle| \sim 1$ MeV. In addition, taking $\alpha_N = \pi/2$ and $\hbar \omega_0 = \hbar \omega_B = 0.6$ MeV, we estimated the barrier heights W_I, shown also in the Table I.

TABLE I. The parameters characterizing decay out of nuclei with mass ~ 190.

	I ($\hbar$)	F_S	Γ_S (meV)	Γ_N (meV)	D_N (eV)	$\overline{v'}$ (eV)	$v'_{\rm min}$ (meV)	$v'_{\rm max}$ (eV)	$\overline{W}$ (MeV)	$W_{\rm min}$ (MeV)	$W_{\rm max}$ (MeV)
^{192}Hg-1 [8]	12^+	0.87	0.116	10.3	34	0.16	0.21	0.45	2.86	2.65	4.12
^{192}Hg-1 [8]	10^+	0.09	0.054	10.3	30	1.51	1.18	2.19	2.43	2.36	3.79
^{194}Hg-1 [26]	12^+	0.60	0.108	21	344	3.41	0.62	6.41	2.27	2.15	3.92
^{194}Hg-1 [26]	10^+	0.03	0.046	20	493	30.04	2.73	42.83	1.85	1.79	3.64
^{194}Hg-3 [11]	15^+	0.90	0.230	4.0	26.5	0.20	0.16	0.68	2.81	2.57	4.17
^{194}Hg-3 [11]	13^+	0.84	0.110	4.5	19.9	0.32	0.15	0.43	2.74	2.66	4.18
^{194}Hg-3 [11]	11^+	<0.07	0.048	6.4	7.2	0.41	1.02	0.72	2.78	2.57	3.83
^{194}Pb-1 [11]	8^+	0.62	0.014	0.50	2200	64.12	0.01	95.79	1.73	1.62	4.46
^{194}Pb-1 [11]	6^+	<0.09	0.003	0.65	1400	66.45	0.04	106.21	1.74	1.63	4.33

V. DISCUSSION

The radiative decay of the SD and N levels as well as their mixing are simultaneously taken into account in exact equations (21) and (22) for Green's functions. In the general case, their solution is rather cumbersome and the decay of the SD level, described by Eq. (34), is nonexponential due to Rabi oscillations between the SD level and the normal deformed states. The Green's function reduces to one resonant term (29), if $\Gamma_N/2 \gg D_N$. Here, width Γ_N characterizes uncertainty of the energy of states $|\alpha\rangle$. When such uncertainty largely exceeds distance D_N between levels $|\alpha\rangle$, they behave like a continuous spectrum and the rate of the decay out of the SD level into such a quasicontinuous spectrum is provided by Fermi's golden rule. The corresponding decay-out width of the SD level equals its spreading width Γ [32], while the complete width of the SD level $\Gamma + \Gamma_s$ does not depend on Γ_N. Time dependence of the SD level decay is described now by the exponent in correspondence with the general theory [30], which describes decay of any level to a continuous spectrum (for a discrete spectra of final states, there will be temporary beats superimposed on the attenuating exponent).

As indicated in Ref. [32], the spreading width Γ can be introduced also in the strong-coupling limit, when $v' \gg D_N$ and, respectively, the SD level is spread over a large number of normal levels. But, for nuclei with mass ~ 190, the opposite limit of weak coupling, $v' \ll D_N$, is realized. Then, one cannot introduce the strength function [32], characterized by the width Γ, so that Γ loses its physical sense, remaining only a helpful designation. Nevertheless, one can employ the decay-out width $\Gamma^{\downarrow}$ instead of Γ, if $\Gamma_N \gg \Gamma_s$, $\Gamma^{\downarrow}$. Then, $G^{+}_{ss}(\varepsilon)$ is again reduced to a simple resonant term (54), as in the case with Γ. It is worth noting that now $\Gamma^{\downarrow}$ depends on Γ_N. Expression (49) for $\Gamma^{\downarrow}$ transforms to Eq. (5), derived previously in Ref. [8], when $|\Delta| \ll D_N$ and only a single close-lying normal level is significant.

Although the energy difference Δ has a definite value between $-D_N/2$ and $D_N/2$ for all the nuclei, we derived the averaged decay-out probability $\overline{F_N}$. Expression (60) contains an extra factor $(2/\pi + \Gamma\Gamma_N/D_N\Gamma_s)^{-1/2}$ compared to Eq. (13) of Ref. [3]. Recall, that Vigezzi *et al.* [3] considered mixing of the SD level only with a single normal compound state, ignoring their overlapping at small $|\Delta|$ and the role of other N states, whereas we took into account all these effects.

Formula (59) allows us to find the averaged values of v' or Γ from experimental data for F_N, using estimations of Γ_N, Γ_s, and D_N, made by other authors. Usually, the spreading width Γ is identified with the tunneling width $\Gamma_{tunn} = (\hbar\omega_0/2\pi)\exp(-2A)$ in order to calculate action A and the barrier height W_I, respectively (see, e.g. Ref. [12,26]). This expression for the tunneling width may be applied when the SD state is coupled to the continuous spectrum of the deformed motion. This would be the case if we considered the tunneling through an external potential barrier. But we deal with the deformed motion only within two asymmetric potential wells separated by a barrier, disregarding the nuclear fission. The corresponding solutions (9) of the Schrödinger equation describe bound states having discrete energies. The quasicontinuous spectrum arises when we include into consideration normal deformed states $|\alpha\rangle$, associated with the excited intrinsic motion of the nucleons, i.e., when we begin dealing with other degrees of freedom. Then, mixing of the superdeformed collective state ψ_s with such normal states is due to the residual interaction but not to the quantum-mechanical tunneling. To do this more formally, we substitute Eqs. (10) and (12) into Eq. (3). Then, the spreading width Γ transforms to the form

$$\Gamma = (\pi |\langle \alpha | \hat{V}' | N\rangle|^2 / 2D_N \sin^2\alpha_N) e^{-2A}, \tag{62}$$

explicitly showing its difference from Γ_{tunn}. Therefore, when calculating the barrier heights W_I, we were forced to adopt the above estimations for angle α_N and for the matrix element $\langle \alpha | \hat{V}' | N\rangle$.

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