Quark model analysis of the charge symmetry breaking in nuclear force

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In order to investigate the charge symmetry breaking (CSB) in the short-range part of the nuclear force, we calculate the difference of the masses of the neutron and the proton, ΔM, the difference of the scattering lengths of the p-p and n-n scatterings, Δa, and the difference of the analyzing power of the proton and the neutron in the n-p scattering, $\Delta A(\theta)$, by a quark model. In the present model the sources of CSB are the mass difference of the up and down quarks and the electromagnetic interaction. We investigate how much each of them contributes to ΔM, Δa, and $\Delta A(\theta)$. It is found that the contribution of CSB of the short-range part in the nuclear force is large enough to explain the observed $\Delta A(\theta)$, while Δa is rather underestimated.

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I. INTRODUCTION

The charge symmetry is the invariance under the charge-reflection, i.e., the reflection about the 1-2 plane in the isospin space. If this were an exact symmetry, the masses of the proton and the neutron would be the same, as well as the binding energies of the mirror nuclei or the scattering lengths of the p-p and n-n scatterings. The charge symmetry holds only approximately in the real world. There are small but nonzero differences such as

$$\Delta M = M_n - M_p = 1.29 \ \ (\text{MeV}),$$
$$\Delta a = a_{pp} - a_{nn} = 1.5(\text{fm}). \qquad (1)$$

These differences are a manifestation of the charge symmetry breaking (CSB).

CSB appears also in spin-dependent observables. For example, the $\vec{p}$-n system is the mirror of $\vec{n}$-p, where $\vec{p}(\vec{n})$ is a polarized nucleon. There was found small difference in the analyzing powers of $\vec{p}$ and $\vec{n}$ in the medium energy scattering [1,2],

$$\Delta A(\theta) = A_n(\theta) - A_p(\theta). \qquad (2)$$

The study of $\Delta A(\theta)$ is important because there is no Coulomb interaction between n and p.

It is important to understand CSB from the quantum chromodynamics (QCD) viewpoint [3]. From QCD we find that CSB has two origins: (i) the difference of the masses of the up and down quarks and (ii) the electromagnetic interaction. Thus the study of CSB phenomena can be a good probe to examine the behavior of the quarks and gluons in the low-energy region. The ultimate goal of the CSB study may be understanding their effects on hadron spectra and hadronic interactions directly from QCD, by, e.g., lattice QCD simulation. As the direct approach is not available up to now indirect approaches have been taken for the CSB study.

An often used approach to CSB is based on the meson exchange picture of the nuclear force. It was suggested that CSB of the nuclear force is generated by mixings of $I=0$ and $I=1$ mesons such as the ρ-ω mixing [4]. A model based on such a picture was reported to explain Δa well [5]. But it was also pointed out that the effect of the ρ-ω mixing to CSB may be suppressed by the off-shell effect of the ρ-ω mixing [6]. Thus, this problem is still open [7]. A class IV interaction [8] is also generated by the neutron-proton mass difference in the one-pion-exchange interaction [9]. It was pointed out that the effects of OPE and ρ-ω mixing explain $\Delta A(\theta)$ fairly well.

On the other hand, CSB appearing in the short-range part should be investigated by introducing subnucleonic degrees of freedom. One of the pioneering works to apply a quark model to CSB is found in Ref. [10], where the isovector mass shifts of isospin multiplets and the isospin-mixing matrix elements in $1s0d$-shell nuclei are investigated by using the quark cluster model (QCM) [11–15]. It was concluded that the u-d quark constituent mass difference produces significant effects, which may explain the observed Okamoto-Nolen-Schiffer anomaly [16] well.

In the present work, we investigate CSB in ΔM, Δa, and $\Delta A(\theta)$ by employing essentially the same model for all these three observables: a quark potential model for ΔM and QCM for Δa, and $\Delta A(\theta)$. The CSB sources are taken to be (a) the difference of the masses of the up and down constituent quarks and (b) the electromagnetic interaction between the constituent quarks. Our aim is to estimate the effect of CSB sources (a) and (b) on nuclear force by investigating the above three observables simultaneously.

Chemtob and Yang [17] (CY) calculated Δa using QCM, suggesting that the quark mass difference contributes to Δa significantly. Later, Bräuer *et al.* [18,19] studied Δa and $\Delta A(\theta)$ using QCM and concluded that the effects of CSB sources (a) and (b) are too small to explain the observed value. However, their calculation of $\Delta A(\theta)$ suffers from a wrongly chosen factor, from omitting the symmetric spin-orbit term and from inconsistent use of the operators and wave functions (See Sec. IV).

In the present paper, we extend CY's and Bräuer's works in order to obtain more integrated knowledge on CSB. We investigate CSB in ΔM, Δa, and $\Delta A(\theta)$ simultaneously. Also, we introduce the instanton induced interaction (III) [20–26], which comes from the nonperturbative effects of QCD and explains the η-η' mass splitting. Since III does not break the charge symmetry, its role in this study is mainly to make the effective strength of the one-gluon-exchange interaction smaller. The strength becomes reasonably small, which fits to the picture that this term represents the perturbative effect of the gluons (See Sec. IV). Moreover, we include the symmetric spin-orbit term in the analysis of $\Delta A(\theta)$, whose effect is as large as the antisymmetric one. Furthermore, we solve QCM to obtain the relative wave function and use it to evaluate the matrix elements of Δa and $\Delta A(\theta)$.

In Sec. II, we show the Hamiltonian for quarks and the CSB sources. In Sec. III, we explain the detail of the calculations of ΔM, Δa, and $\Delta A(\theta)$. Results are discussed in Sec. IV. Summary is given in Sec. V.

II. HAMILTONIAN

We employ the constituent quark model with quark masses of order $m \simeq 300$ (MeV) in this study. The Hamiltonian is given by

$$H = K + V. \tag{3}$$

K is the quark kinetic energy and considered as semirelativistic in calculation of ΔM (see Sec. III A) and as nonrelativistic in calculations of Δa and $\Delta A(\theta)$ (see Sec. III B) in this study. The quark-quark interactions are represented by a static potential, which consists of the confinement (CF), the one-gluon-exchange (OGE) [27], the electromagnetic (EM), and the III's:

$$V = V_{\mathrm{conf}} + V_{\mathrm{OGE}} + V_{\mathrm{EM}} + V_{\mathrm{III}}, \tag{4}$$

$$V_{\mathrm{CF}} = \sum_{i<j} -a(\vec{\lambda}_i \cdot \vec{\lambda}_j) r_{ij}, \tag{5}$$

$$\begin{aligned} V_{\mathrm{OGE}} = \sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) \frac{\alpha_s}{4} & \times \left\{ \frac{1}{r_{ij}} - \left(\frac{\pi}{2m_i^2} + \frac{\pi}{2m_j^2} + \frac{2\pi}{3m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \delta(\vec{r}_{ij}) \right. \\ & - \left[\frac{1}{2r_{ij}^3} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j} \right) \right] \vec{L}_{ij} \cdot \frac{\vec{\sigma}_i + \vec{\sigma}_j}{2} \\ & \left. - \left[\frac{1}{4r_{ij}^3} \left(\frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \right] \vec{L}_{ij} \cdot \frac{\vec{\sigma}_i - \vec{\sigma}_j}{2} \right\}, \end{aligned} \tag{6}$$

$$\begin{aligned} V_{\mathrm{EM}} = \sum_{i<j} e_i e_j \alpha_{em} & \left\{ \frac{1}{r_{ij}} - \left(\frac{\pi}{2m_i^2} + \frac{\pi}{2m_j^2} + \frac{2\pi}{3m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \delta(\vec{r}_{ij}) \right. \\ & - \left[\frac{1}{2r_{ij}^3} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j} \right) \right] \vec{L}_{ij} \cdot \frac{\vec{\sigma}_i + \vec{\sigma}_j}{2} \\ & \left. - \left[\frac{1}{4r_{ij}^3} \left(\frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \right] \vec{L}_{ij} \cdot \frac{\vec{\sigma}_i - \vec{\sigma}_j}{2} \right\}, \end{aligned} \tag{7}$$

$$\begin{aligned} V_{\mathrm{III}} = V_0^{(2)} \sum_{i<j} & \left(1 + \frac{3}{32} \vec{\lambda}_i \cdot \vec{\lambda}_j + \frac{9}{32} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \delta(\vec{r}_{ij}) \\ & - \frac{1}{8} \left\{ \left(-1 + \frac{3}{16} \lambda_i \cdot \lambda_j \right) \frac{2}{\overline{m}^2} + \frac{9}{8\overline{m}^2} \lambda_i \cdot \lambda_j \right\} \\ & \times \frac{\delta(\vec{r}_{ij})}{r^2} \vec{L}_{ij} \frac{\vec{\sigma}_i + \vec{\sigma}_j}{2}. \end{aligned} \tag{8}$$

$\vec{\lambda}_i$ is the color SU(3) Gell-Mann matrix and e_i is the quark electric charge in units of the proton charge e. In this study it is assumed that the confinement potential does not break the charge symmetry. This is a natural assumption based on the confining potential obtained, for instance, from lattice QCD calculation. Yet there may exist velocity dependent terms associated with confinement which break the charge symmetry. We do not consider such terms in this study. Taking the Breit-Fermi interaction naively, non-Galilei invariant terms appear in the LS terms. But we consider only the Galilei invariant terms such as the spin-orbit term in Eqs. (6) and (7). It should be noted that the III is effective only on the flavor singlet (isosinglet) quark-quark state. In other words, it works only on a pair of up and down quarks. Thus III does not break the charge symmetry.

In this Hamiltonian the terms including the quark mass and the electric charge may break the charge symmetry. In order to show the CSB terms explicitly, we rewrite the quark mass and the electric charge in terms of the isospin operator,

$$\begin{aligned} m_i &= \frac{m_d + m_u}{2} - \frac{m_d - m_u}{2} \tau_3^{(i)} \\ &= \overline{m} \left(1 - \frac{\Delta m}{2\overline{m}} \tau_3^{(i)} \right) \\ &= \overline{m} (1 - \epsilon \tau_3^{(i)}), \end{aligned} \tag{9}$$

$$e_i = \frac{\tau_3^{(i)}}{2} + \frac{1}{6}, \tag{10}$$

where

$$\overline{m}=\frac{m_d+m_u}{2}, \quad \Delta m=m_d-m_u,$$

$$\epsilon=\frac{\Delta m}{2\overline{m}}. \tag{11}$$

Using the typical constituent quark mass $\overline{m}\simeq 300$ MeV and the up and down quark mass difference $\Delta m\simeq 6$ MeV, $\epsilon \simeq 6/2\times 300=1/100$ is as small as the electromagnetic coupling constant, $\alpha_{e.m.}\simeq 1/137$. So we divide the Hamiltonian into the charge symmetric part $\overline{H}$ and the charge symmetry breaking part ΔH_{CSB}, and treat ΔH_{CSB} perturbatively.

The CSB part of the Hamiltonian is given, to the leading order in ϵ and $\alpha_{e.m.}$, by

$$\Delta V_{\mathrm{CSB}}=\Delta V_{\mathrm{CSB}}^{\mathrm{OGE}}+\Delta V_{\mathrm{CSB}}^{\mathrm{EM}}, \tag{12}$$

$$\Delta V_{\mathrm{CSB}}^{\mathrm{OGE}}=\sum_{i<j}(\vec{\lambda}_i\cdot\vec{\lambda}_j)\frac{\alpha_s}{4}\epsilon\Bigg\{-\frac{\pi}{\overline{m}^2}(\tau_3^{(i)}+\tau_3^{(j)})\times\left(1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\delta(\vec{r}_{ij})-\frac{3\alpha_s}{4\overline{m}^2 r_{ij}^3}\vec{L}_{\mathrm{ij}}\cdot(\vec{\sigma}_i+\vec{\sigma}_j)\times(\tau_3^{(i)}+\tau_3^{(j)})-\frac{\alpha_s}{4\overline{m}^2 r_{ij}^3}\vec{L}_{\mathrm{ij}}\cdot(\vec{\sigma}_i-\vec{\sigma}_j)(\tau_3^{(i)}-\tau_3^{(j)})\Bigg\}, \tag{13}$$

$$\Delta V_{\mathrm{CSB}}^{\mathrm{EM}}=\sum_{i<j}\frac{\tau_3^{(i)}+\tau_3^{(j)}}{12}\alpha_{e.m.}\left\{\frac{1}{r_{ij}}-\frac{\pi}{\overline{m}^2}\left(1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\delta(\vec{r}_{ij})-\frac{3}{4\overline{m}^2 r_{ij}^3}\vec{L}_{\mathrm{ij}}\cdot(\vec{\sigma}_i+\vec{\sigma}_j)\right\}. \tag{14}$$

We ignore the second order terms $\mathcal{O}(\epsilon^2,\alpha_{e.m.}^2,\epsilon\alpha_{e.m.})$. The CSB terms from the tensor interaction are excluded because the tensor interactions between quarks are small. But we consider them when solving the charge symmetric equation for the unperturbated wave function.

The Hamiltonian has five parameters, α_s, $\overline{m}$, a, $V_0^{(2)}$, and Δm. The parameters are determined so as to reproduce the single baryon properties and the results are shown in Sec. IV.

III. CALCULATIONS

In this section we present the formulas of the neutron-proton mass difference, ΔM, the difference of the scattering lengths of the p-p and n-n scattering, Δa, and the difference of the analyzing power of the neutron and the proton of the n-p scattering, $\Delta A(\theta)$.

A. The proton-neutron mass difference ΔM

The differences of the mass of the isodoublet hadrons were evaluated in the constituent quark model by Isgur [28]. We also evaluate the neutron-proton mass difference in order to determine the mass difference of the up and down constituent quarks. Our approach is different in the following two points. First, we consider the semirelativistic kinetic energy term,

$$K=\sum_i^3\sqrt{m_i^2+\mathrm{P}_i^2}. \tag{15}$$

Equation (15) can be divided into the charge symmetric part and the charge symmetry breaking part,

$$K=\overline{K}+\Delta K_{\mathrm{CSB}}, \tag{16}$$

$$\overline{K}=\sum_i^3\sqrt{\overline{m}^2+p_i^2}, \tag{17}$$

$$\Delta K_{\mathrm{CSB}}=-\sum_i^3\frac{\overline{m}^2}{\sqrt{\overline{m}^2+p_i^2}}\epsilon\tau_3^{(i)}. \tag{18}$$

Equation (15) contains the kinetic energy of the center-of-mass coordinate, which must be subtracted in order to calculate the baryon mass. For the semirelativistic kinematics, the center-of-mass energy cannot be treated exactly. Therefore we use the following approximation:

$$M_{\mathrm{N}}=\langle\sqrt{H^2-P_{\mathrm{G}}^2}\rangle\simeq\langle H\rangle-\frac{\langle P_{\mathrm{G}}^2\rangle}{2\langle H\rangle}. \tag{19}$$

The relativistic effect is partially included as the convergence of the expansion in $\langle P_G\rangle/\langle H\rangle$ is better than that in $\langle p_i/m_i\rangle$. Then the nucleon mass can be written in terms of $\overline{H}$ and ΔH_{CSB} as

$$M_{\mathrm{N}}=\langle\overline{H}\rangle-\frac{\langle P_{\mathrm{G}}^2\rangle}{2\langle\overline{H}\rangle}+\langle\Delta H_{\mathrm{CSB}}\rangle\left(1+\frac{\langle P_{\mathrm{G}}^2\rangle}{2\langle\overline{H}\rangle^2}\right), \tag{20}$$

where

$$H=\overline{H}+\Delta H_{\mathrm{CSB}}, \tag{21}$$

$$\overline{H}=\overline{K}+\overline{V}, \tag{22}$$

$$\Delta H_{\mathrm{CSB}}=\Delta K_{\mathrm{CSB}}+\Delta V_{\mathrm{CSB}}. \tag{23}$$

$\overline{H}$ is the charge symmetric part of the Hamiltonian and ΔH_{CSB} contains Eqs. (18) and (12). The first two terms of Eq. (20) give the average mass of the nucleon and the third term contributes to ΔM. The up-down quark mass difference Δm is determined so as to reproduce ΔM by using Eq. (20).

The second difference from the Isgur's work is that the III is considered in this study. III has the contact spin-spin interaction and contributes to the difference of the masses of the nucleon and $\Delta(1232)$ just like the color magnetic interaction. We choose the coupling constant of the OGE, α_s, and the III, $V_0^{(2)}$, so as to reproduce the nucleon-Δ mass difference in total. So α_s becomes smaller effectively by considering III.

B. CSB in the *N*-*N* scattering

In the calculation of the scattering lengths and analyzing powers, we employ the QCM [11–15], which describes two-nucleon systems in terms of their quark coordinates. The scattering wave functions, which are used as the unperturbated states, are calculated by solving the resonating group method (RGM) equation. By mainly technical reasons the kinetic energy term is treated purely in the nonrelativistic way, i.e., the semirelativistic kinematics is not taken into account contrary to the case of single baryon mass. This approximation can be justified because the relativistic effect on the kinetic energy term is smaller for the motion of the two baryons. Then the kinetic energy is given as

$$K=\sum_{i}^{6} K_{\rm i}-K_{\rm G}, \tag{24}$$

$$K_{\rm i}=\left(m_i+\frac{p_i^2}{2m_i}\right), \tag{25}$$

$$K_{\rm G}=\frac{P_{\rm G}^2}{2M_{\rm G}}, \tag{26}$$

where

$$M_{\rm G}=\sum_{i}^{6} m_i, \quad P_{\rm G}=\sum_{i}^{6} p_i. \tag{27}$$

The RGM equation for the baryon A and baryon B is as follows:

$$\int \phi_A(\xi_A)\phi_B(\xi_B)(H-E) \times \mathcal{A}[\phi_A(\xi_A)\phi_B(\xi_B)\chi(R_{AB})]d\xi_A d\xi_B=0, \tag{28}$$

$$\phi_A(\xi_A)=\left(\frac{1}{2\pi b^2}\right)^{3/4}\left(\frac{2}{3\pi b^2}\right)^{3/4}\exp\left(-\frac{\xi_{A1}^2}{4b^2}-\frac{\xi_{A2}^2}{3b^2}\right). \tag{29}$$

$\phi_{A(B)}$ and $\xi_{A(B)}$ are the internal wave function and coordinates of the baryon $A(B)$. R_{AB} are the relative coordinates of the baryons A and B. The parameter b is the Gaussian size parameter, which represents a nucleon size. $\mathcal{A}$ is the antisymmetrization operator for six quarks and is written as follows:

$$\mathcal{A}=1-\mathcal{A}'=1-\sum_{i\in A, j\in B} P_{\rm ij}. \tag{30}$$

In the end, the following equation is obtained:

$$\left[\frac{P_{AB}^2}{2\mu_{AB}}+V_{\rm rel}^{(D)}(R)-\frac{k^2}{2\tilde{\mu}_{AB}}\right]\chi(R)-\int dR'[K^{(EX)}(R,R') + V^{(EX)}(R,R')-EN^{(EX)}(R,R')]\chi(R')=0, \tag{31}$$

where P_{AB} is the momentum operator of the relative motion of the baryons A and B, and

$$E=\tilde{M}_A+\tilde{M}_B+\frac{k^2}{2\tilde{\mu}_{AB}}, \tag{32}$$

$$\frac{1}{\mu_{AB}}=\frac{1}{M_A}+\frac{1}{M_B}, \tag{33}$$

$$M_{A(B)}=\sum_{i\in A(B)}^{3} m_i, \tag{34}$$

$$\frac{1}{\tilde{\mu}_{AB}}=\frac{1}{\tilde{M}_A}+\frac{1}{\tilde{M}_B}, \tag{35}$$

$$\tilde{M}_{A(B)}: \text{ observed mass of the baryon } A(B). \tag{36}$$

It should be noted here that $M_{A(B)}$ and $\tilde{M}_{A(B)}$ may not agree with each other completely. We take $m_i=313$ (MeV) in our calculation so that the difference is small, but for the charge symmetry breaking we assume that $\mu_{AB}=\tilde{\mu}_{AB}$. The observed masses of the proton and neutron are given by

$$\tilde{M}_A=\tilde{M}(1-\epsilon_N\tau_3^{(A)}), \tag{37}$$

$$\epsilon_N=\frac{\Delta\tilde{M}}{2\tilde{M}}, \tag{38}$$

$$\tilde{M}=\frac{\tilde{M}_p+\tilde{M}_n}{2}=939 \text{ MeV}, \tag{39}$$

$$\Delta\tilde{M}=\tilde{M}_n-\tilde{M}_p=1.29 \text{ MeV}. \tag{40}$$

Therefore we may rewrite the kinetic energy terms as

$$\frac{P_{AB}^2}{2\tilde{\mu}_{AB}}-\frac{k^2}{2\tilde{\mu}_{AB}}=\frac{P_{AB}^2-k^2}{2\tilde{\mu}}\left(1+\frac{\tau_3^{(A)}+\tau_3^{(B)}}{2}\epsilon_N\right) \tag{41}$$

and the energy in Eq. (31) as

$$\begin{aligned} E&=2\tilde{M}\left(1-\frac{\tau_3^{(A)}+\tau_3^{(B)}}{2}\epsilon_{\rm N}\right)+\frac{k^2}{2\tilde{\mu}}\left(1+\frac{\tau_3^{(A)}+\tau_3^{(B)}}{2}\epsilon_{\rm N}\right)\\ &=2\tilde{M}+\frac{k^2}{2\tilde{\mu}}+\left(-2\tilde{M}+\frac{k^2}{2\tilde{\mu}}\right)\left(\frac{\tau_3^{(A)}+\tau_3^{(B)}}{2}\epsilon_{\rm N}\right)\\ &=\overline{E}+\Delta E_{\rm CSB}, \end{aligned} \tag{42}$$

because

$$\frac{1}{2\tilde{\mu}_{AB}}=\frac{1}{2\tilde{\mu}}\left(1+\frac{\tau_3^{(A)}+\tau_3^{(B)}}{2}\epsilon_{\rm N}\right), \tag{43}$$

$$\tilde{\mu}=\frac{\tilde{M}}{2}. \tag{44}$$

The RGM kernels $V_{\rm rel}^{(D)}, N^{(EX)}, K^{(EX)}, V^{(EX)}$ are defined by

$$V_{\rm rel}^{(D)}(R)=\int d\xi_A d\xi_B dR_{AB}\phi_A(\xi_A)\phi_B(\xi_B)\times\sum_{i\in A j\in B}V_{ij}\delta(R-R_{AB})\phi_A(\xi_A)\phi_B(\xi_B), \tag{45}$$

$$\begin{pmatrix} N^{(EX)}(R',R)\\ K^{(EX)}(R',R)\\ V^{(EX)}(R',R)\end{pmatrix} =\int d\xi_A d\xi_B dR_{AB}\phi_A(\xi_A)\phi_B(\xi_B)\delta(R'-R_{AB})\times\left(\begin{pmatrix}1\\K\\V\end{pmatrix},\mathcal{A}'[\delta(R-R_{AB})\phi_A(\xi_A)\phi_B(\xi_B)]\right) =\begin{pmatrix}\overline{N}^{(EX)}(R',R)\\ \overline{K}^{(EX)}(R',R)+\Delta K_{\rm CSB}(R',R)\\ \overline{V}^{(EX)}(R',R)+\Delta V_{\rm CSB}(R',R)\end{pmatrix}. \tag{46}$$

K and V are given by Eqs. (24) and (4) and can be divided into the charge symmetric part $\overline{K},\overline{V}$ and the charge symmetry breaking part $\Delta K_{\rm CSB},\Delta V_{\rm CSB}$. Therefore RGM kernels are divided into the charge symmetric part $\overline{K}^{(EX)},\overline{V}^{(EX)}$ and the charge symmetry breaking part $\Delta K_{\rm CSB}^{(EX)},\Delta V_{\rm CSB}^{(EX)}$.

In order to treat the CSB part perturbatively, we employ the distorted wave Born approximation (DWBA) in this study. We solve the following equation to obtain the distorted wave:

$$\left[\frac{P_{AB}^2}{2\tilde{\mu}}-\frac{k^2}{2\tilde{\mu}}\right]\chi_{\rm dist}(R)-\int dR'[\overline{K}^{(EX)}(R,R')+\overline{V}^{(EX)}(R,R')-E\overline{N}^{(EX)}(R,R')]\chi_{\rm dist}(R')=0. \tag{47}$$

The direct kernel $V_{\rm rel}^{(D)}(R)$ comes from the electromagnetic interaction of quarks and corresponds to the electromagnetic interaction of baryons. We are interested in effects of CSB at the quark level, not at the hadron level. So we ignore the direct kernel. But we consider the exchange kernel of the electromagnetic interaction of quarks. Using the distorted wave $\chi_{\rm dis}(R)$, we estimate the following CSB parts:

TABLE I. Parameters.

	$P_{\rm III}$	Δm	$\overline{m}$ (MeV)	b (fm)	α_s	a (MeV/fm)	$V_0^{(2)}$ (MeV fm^3)
A	0.4	7.3	313	0.6	0.91	44.29	−177.2
B	0.5	5.2	313	0.6	0.76	40.34	−221.5

$$(\text{CSB part})=\frac{P_{AB}^2-k^2}{2\tilde{\mu}}\left(\frac{\tau_3^{(A)}+\tau_3^{(B)}}{2}\epsilon_N\right)\chi_{\rm dist}(R)-\int dR'[\Delta K_{\rm CSB}^{(EX)}(R,R')+\Delta V_{\rm CSB}^{(EX)}(R,R')-\Delta E_{\rm CSB}\overline{N}^{(EX)}(R,R')]\chi_{\rm dist}(R'). \tag{48}$$

C. CSB in the analyzing power

There is a special CSB interaction in the neutron-proton system, which is called the class IV interaction, according to the classification by Henley and Miller [8],

$$V_{\rm IV}\propto(\tau_3^A-\tau_3^B)(\vec{\sigma}_A-\vec{\sigma}_B) \tag{49}$$

or

$$(\vec{\tau}_A\times\vec{\tau}_B)_z(\vec{\sigma}_A\times\vec{\sigma}_B). \tag{50}$$

One sees that the class IV interaction mixes spin-singlet states and spin-triplet states. The spin singlet-triplet mixing induces asymmetries of spin polarization observables such as the analyzing power. At the level of the quark-quark interaction, CSB in the spin-orbit interactions is given as [see Eqs. (6) and (7)]

$$V_{\rm CSB}^{LS}=V_{\rm qSLS}^{\rm OGE}+V_{\rm qALS}^{\rm OGE}+V_{\rm qSLS}^{\rm EM}, \tag{51}$$

$$V_{\rm qSLS}^{\rm OGE}=-\sum_{i<j}(\vec{\lambda}_i\cdot\vec{\lambda}_j)\frac{3\alpha_s\epsilon}{16\overline{m}^2}\frac{\vec{L}_{ij}}{r_{ij}^3}[(\vec{\sigma}_i+\vec{\sigma}_j)(\tau_3^{(i)}+\tau_3^{(j)})], \tag{52}$$

$$V_{\rm qALS}^{\rm OGE}=-\sum_{i<j}(\vec{\lambda}_i\cdot\vec{\lambda}_j)\frac{\alpha_s\epsilon}{16\overline{m}^2}\frac{\vec{L}_{ij}}{r_{ij}^3}[(\vec{\sigma}_i-\vec{\sigma}_j)(\tau_3^{(i)}-\tau_3^{(j)})], \tag{53}$$

$$V_{\rm qSLS}^{\rm EM}=-\sum_{i<j}\frac{\alpha_{em}}{16\overline{m}^2}\frac{\vec{L}_{ij}}{r_{ij}^3}[(\vec{\sigma}_i+\vec{\sigma}_j)(\tau_3^{(i)}+\tau_3^{(j)})]. \tag{54}$$

The first two terms of Eq. (51) come from the one-gluon-exchange interaction and the third term from the electromagnetic interaction of quarks. It should be noted that the symmetric spin-orbit interaction of quarks (qSLS) induces the class IV interaction of baryons as well as the antisymmetric one (qALS). Bräuer *et al.* calculated $\Delta A(\theta)$ using a similar model without including the qSLS terms [19]. They con-

TABLE II. Contributions to ΔM for $\Delta m=6$ MeV.

$P_{\rm III}$	Kin	OGE	EM	M_n-M_p
0	4.72	−5.54	−0.41	−1.23
0.1	4.72	−5.54	−0.41	−0.67
0.2	4.72	−4.99	−0.41	−0.12
0.3	4.72	−4.43	−0.41	0.44
0.4	4.72	−3.88	−0.41	0.99
0.5	4.72	−2.77	−0.41	1.54

TABLE III. Scattering length.

	$P_{\rm III}$	Δm (MeV)	$\overline{a}$ (fm)	Δa (fm)	$\overline{r}$ (fm)	Δr (fm)
A	0.4	7.3	−17.9	0.79	2.42	−0.39
B	0.5	5.2	−17.9	0.52	2.46	−0.25
Expt. [3]			−18.1±0.5	1.5±0.5	2.80±0.12	0.10±0.12
B [18]		5.0	20.07	0.46		
CY [17]		6.0		2∼3.5		

cluded that the contribution of quarks to $\Delta A(\theta)$ is very small. But we will see that the contribution of quark spin-orbit interactions, Eq. (51), to $\Delta A(\theta)$ is large enough to reproduce the observed $\Delta A(\theta)$.

Using DWBA, we calculate the following matrix elements for $J=L\leqslant 3$:

$$\Delta T_{\rm CSB}=\langle {}^3L_J|V^{LS}_{\rm CSB}|{}^1L_J\rangle. \tag{55}$$

Then the total T matrix is given as follows:

$$T=\overline{T}_{\rm CS}+\Delta T_{\rm CSB}. \tag{56}$$

$\overline{T}_{\rm CS}$ is obtained by solving the RGM equation. T is regarded as a matrix based on the spin states and the analyzing power is given by

$$A_{\rm N}(\theta)=\frac{{\rm Tr}[T^\dagger\sigma_{\rm N}T]}{{\rm Tr}[T^\dagger T]}. \tag{57}$$

Then $\Delta A(\theta)$ is given in terms of $\overline{T}_{\rm CS}$ and $\Delta T_{\rm CSB}$,

$$\Delta A(\theta)=A_n(\theta)-A_p(\theta)=\frac{2\,{\rm Re}\,{\rm Tr}[\overline{T}^\dagger_{\rm CS}(\sigma_n-\sigma_p)\Delta T_{\rm CSB}]}{{\rm Tr}[\overline{T}^\dagger_{\rm CS}\overline{T}_{\rm CS}]}. \tag{58}$$

We show the explicit forms of the T matrix and of $\Delta A(\theta)$ in the Appendix.

IV. RESULTS

The parameters in our calculation are determined so as to reproduce the single nucleon property. In order to show explicitly how much the contribution of the III to the nucleon-Δ splitting is, we introduce a new parameter $P_{\rm III}$, which denotes the ratio of the contribution of III to the whole nucleon-Δ splitting. For example, when $P_{\rm III}=0.4$ the contribution of III to the Nucleon-Δ splitting is 40% of the whole one. $V_0^{(2)}$ is determined so as to reproduce the η and η' mass splitting. Our analysis shows that $P_{\rm III}\sim 0.4$–0.5 gives the right η-η' splitting. Here we try two values, $P_{\rm III}=0.4$ and 0.5. Using the nucleon mass formula Eq. (20), we obtain Δm for each $P_{\rm III}$. The results are given in Table I. The parameter b is the Gaussian size parameter for the internal wave function of the nucleon, which represents the nucleon size.

Another possible source of the N-Δ splitting is contribution of pion cloud around the baryon. For instance, the cloudy bag model predicts the N-Δ splitting of about 100 MeV [29]. This effect may reduce the roles of OGE and III, but it is not taken into account in this approach.

By increasing $P_{\rm III}$, we reduce α_s accordingly so that the N-Δ mass difference is fixed. For $P_{\rm III}=0.4$, α_s becomes 0.91, while $\alpha_s=1.52$ is necessary to reproduce the N-Δ mass difference only by OGE. In order to show the effect of the instanton induced interaction to ΔM, we show contribution of each term to ΔM in Table II, for various $P_{\rm III}$. The Kin, OGE, and EM represent the contributions of the kinetic energy, the one-gluon-exchange interaction, and the electromagnetic interaction to ΔM. It should be noted that when $P_{\rm III}=0$ we cannot reproduce the ΔM because OGE gives a large contribution, which goes to the opposite direction. This shows the essential role of the III, which reduces the OGE strength.

It is also found that the calculation of ‘‘strong hyperfine’’ for ‘‘p-n’’ in Table I of Ref. [28] is different from our calculation even if we use the same potential. This is because Isgur considers distortion of the quark wave function from the u-d quark mass difference. However, to be consistent the distortion of the wave function should not contribute to the energy in the first order of the perturbation theory.[1] The contribution of the ‘‘strong hyperfine’’ to ΔM should be $\frac{1}{3}\delta(\Delta m/\overline{m})$ instead of $\frac{5}{24}\delta(\Delta m/\overline{m})$ in Ref. [28], where δ is the nucleon-Δ mass splitting.

Next we calculate Δa using the parameters in Table I. The results are shown in Table III. $\overline{a}(\overline{r})$ and $\Delta a(\Delta r)$ are the average and the difference of the scattering lengths (effective ranges) of the p-p and n-n scatterings,

$$\overline{a}=\frac{a_{pp}+a_{nn}}{2}, \qquad \Delta a=a_{pp}-a_{nn}, \tag{59}$$

$$\overline{r}=\frac{r_{pp}+r_{nn}}{2}, \qquad \Delta r=r_{pp}-r_{nn}. \tag{60}$$

Our results, $\Delta a=0.79$ and 0.52 (fm) for $P_{\rm III}=0.4$ and 0.5, are somewhat smaller than the observed value ∼1.5 (fm). We, however, point out that Δa is sensitive to the parameters because it is given by a cancellation of positive and negative terms.

In Table IV, we show each contribution of CSB terms Eq. (48) to Δa. NMD, Kin, OGE, and EM are contributions of the first term of Eq. (48), the quark kinetic energy (including

[1]Chemtob and Yang also point out the mismatch with Isgur in their paper [17].

TABLE IV. The contributions to Δa of CSB terms (fm).

	NMD	Kin	OGE	EM
A	0.3	−2.6	2.9	0.2
B	0.3	−1.7	1.7	0.2

the $\Delta E_{\rm CSB}$ term), the one-gluon-exchange interaction, and the electromagnetic interaction, respectively.

We estimate Δa in our formulation for the parameters of Ref. [18] (*B*) and [17] (CY). The contributions of Kin and OGE should be given by

$$\Delta a_{\rm Kin} \propto \frac{\Delta m}{\bar{m}^2 b^2} \equiv \Delta b_{\rm Kin}, \tag{61}$$

$$\Delta a_{\rm OGE} \propto \frac{\alpha_s \Delta m}{\bar{m}^3 b^3} \equiv \Delta b_{\rm OGE}. \tag{62}$$

In Table V we show $\Delta b_{\rm Kin}$ and $\Delta b_{\rm OGE}$ for the parameters of *B* and CY. Using the values of Tables IV and V, we find

$$\Delta a_{\rm Kin} + \Delta a_{\rm OGE}|_B = -2.6 \times \frac{4.6}{8.0} + 2.9 \times \frac{5.2}{7.7} = -1.5 + 2.0 = 0.5, \tag{63}$$

$$\Delta a_{\rm Kin} + \Delta a_{\rm OGE}|_C = -2.6 \times \frac{5.6}{8.0} + 2.9 \times \frac{8.9}{7.7} = -1.8 + 3.4 = 1.6. \tag{64}$$

These estimates suggest that our results may become larger by changing the parameters. As $\Delta b_{\rm Kin}$ is larger than $\Delta b_{\rm OGE}$ in our parameter choice, the cancellation of $\Delta a_{\rm Kin}$ and $\Delta a_{\rm OGE}$ is stronger than the other cases. On the other hand, Δr is too large and has the wrong sign. More investigation should be done for Δr, which reflects not only the strength of the interaction but also its radial dependence.

Finally, we calculated $\Delta A(\theta)$ at two energy points, taking $P_{\rm III} = 0.4$. The results at $E_n = 183$ and 477 (MeV) are shown in Figs. 1 and 2. The results at $E_n = 183$ and 477 MeV are large enough to reproduce the data [1,2], which disagree with the conclusion of Bräuer *et al.* [19]. The difference mainly comes from two points. The first point is that they consider only the antisymmetric spin-orbit interaction of quarks (qALS), not the symmetric spin-orbit interaction of quarks (sSLS). The factor of qSLS is three times as large as that of qALS [see Eqs. (52) and (53)]. The remaining discrepancy might be attributed to their erroneous choice of the unit of γ_1 in Eq. (3.6) in their paper [19]. We convert their value of γ_1 in radian into that in degrees and obtain $\Delta A(\theta = 96°) = 5.4 \times 10^{-4}$, which is of the same order as our estimate. Our result at $E_n = 477$ (MeV) is too large. It is not surprising since we fit the phase shift of the *N*-*N* scattering up to $E_n = 400$ (MeV) and may not apply QCM at higher energy and we need higher partial waves.

TABLE V. $\Delta b_{\rm Kin}$ and $\Delta b_{\rm OGE}$.

	$\Delta b_{\rm Kin}$ (MeV)	$\Delta b_{\rm OGE}$ (Mev)
A	8.0	7.7
B	4.6	5.2
CY	5.6	8.9

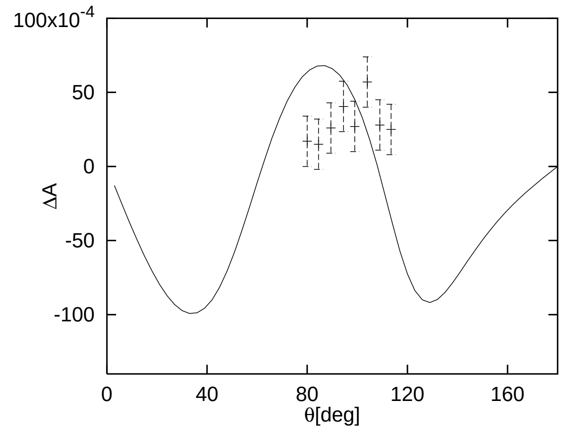

FIG. 1. $\Delta A(\theta)$ at $E_n = 183$ MeV.

Figures 3 and 4 show the contributions of $\langle ^1P_1|\hat{T}|^3P_1\rangle$, $\langle ^1{\rm D}_2|\hat{T}|^3{\rm D}_2\rangle$, and $\langle ^1{\rm F}_3|\hat{T}|^3{\rm F}_3\rangle$ to $\Delta A(\theta)$. It is found that the contribution of $\langle ^1P_1|\hat{T}|^3P_1\rangle$ is dominant in the observed θ region. But the other mixings of partial wave become important for the other θ region.

We also investigate each contribution of the one-gluon-exchange interaction and the electromagnetic interaction (Figs. 5 and 6). It is found that the contribution of OGE depends on the incident energy much strongly than that of the electromagnetic interaction does. This is because the dominant contribution of the EM interaction is the direct interaction, while OGE interaction contributes as the exchange interaction. Therefore their energy dependences are different from each other, which may be studied by future experiment at various energy points.

V. CONCLUSION

We have calculated the difference of the masses of the neutron and the proton, ΔM, the difference of the scattering

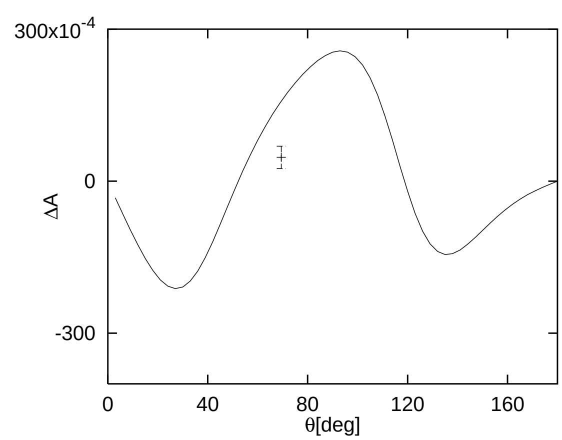

FIG. 2. $\Delta A(\theta)$ at $E_n = 477$ MeV.

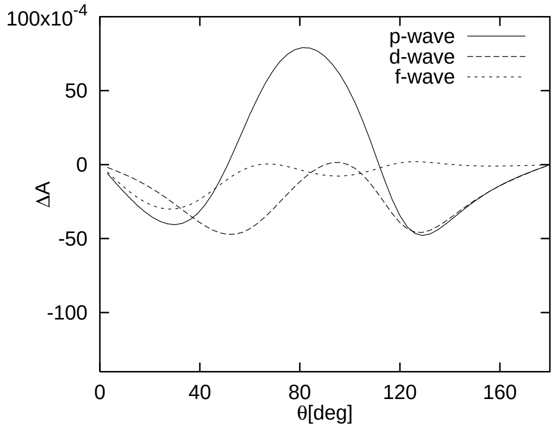

FIG. 3. The contribution of each partial wave mixing at $E_n = 183$ MeV.

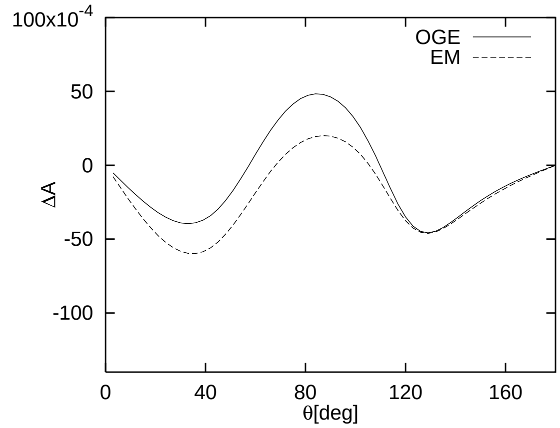

FIG. 5. The contribution of OGE and EM at $E_n = 183$ MeV.

lengths of the p-p and n-n scatterings, Δa, and the difference of the analyzing power of the proton and the neutron in the n-p scattering, $\Delta A(\theta)$, using the quark cluster model. In the calculation of ΔM, we treated the kinetic energy in the semirelativistic way and introduced the III. We have found that the contribution of the OGE interaction is suppressed by the introduction of the III and have determined the up-down quark mass difference $\Delta m = 7.3$ and 5.2 (MeV) for $P_{\rm III} = 0.4$ and 0.5.

We have calculated Δa for the CSB parameters fixed by ΔM. Our results are $\Delta a = 0.8$ and 0.5 (fm) for $P_{\rm III} = 0.4$ and 0.5, which are smaller than the observed value. It is found that the contribution of the u-d mass difference to Δa is comparable with that from EM interaction because the contributions of OGE and the quark kinetic energy cancel out each other. It is pointed out that Δa is sensitive to the choice of the quark model parameters because of this cancellation.

The P-wave CSB observable $\Delta A(\theta)$ is calculated for $P_{\rm III} = 0.4$. It is found that CSB of the short-range part in nuclear force is large enough to explain $\Delta A(\theta)$. This result is different from the conclusion of Bräuer *et al.* [19]. We have found that this discrepancy is attributed to the introduction of the quark symmetric spin-orbit interaction and the erroneous choice of the γ_1 in their paper. We also have investigated the importance of individual mixing matrix elements, $\langle {}^1P_1|\hat{T}|{}^3P_1\rangle$, $\langle {}^1\mathrm{D}_2|\hat{T}|{}^3\mathrm{D}_2\rangle$, and $\langle {}^1\mathrm{F}_3|\hat{T}|{}^3\mathrm{F}_3\rangle$ and also the relative importance of the OGE and EM interactions. It is found that the contributions of $\langle {}^1P_1|\hat{T}|{}^3P_1\rangle$ and OGE are dominant in the observed θ region. Future experiments for other angles as well as different energies may give us further information on the mixings of other partial waves and properties of the spin-orbit parts of the OGE and EM interactions. In fact, we have observed that at $E_n = 477$ (MeV) the contributions of the higher partial waves become more important than at $E_n = 183$ (MeV). The present quark model description is found to account for the short-range part of CSB. We would like to stress that the CSB for the single nucleon as well as the central and spin-orbit parts of the nuclear force are consistently described. There is a possible remaining short-range contribution introduced by Goldman *et al.* (GMS) in Ref. [30], which comes from interference between the QCD and QED effects. GMS pointed out that such an interference is necessary to explain the mass difference of the neutral and charged pions. Its effect on the NN scattering was studied by Kao and Yang [31]. Because this effect has much ambiguity, we have not included it in the present study in order to see how the current data can be explained without such complex effects.

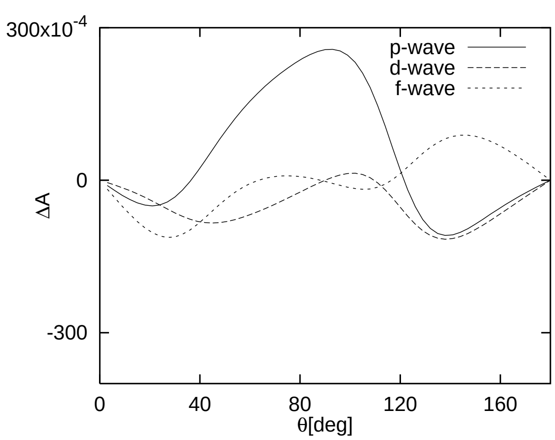

FIG. 4. The contribution of each partial wave mixing at $E_n = 477$ MeV.

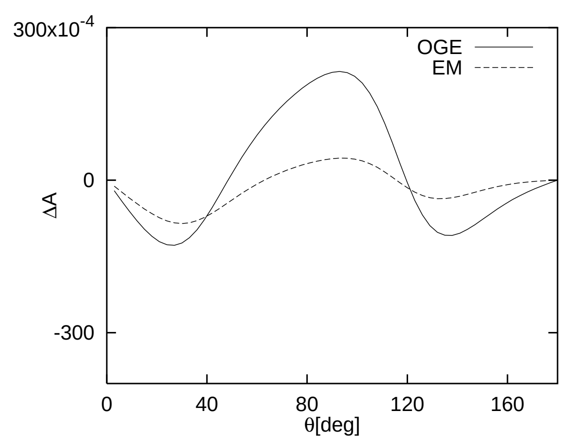

FIG. 6. The contribution of OGE and EM at $E_n = 477$ MeV.

Effects of longer range CSB may require further analysis. Approaches based on the chiral effective theory were performed in Refs. [32]. Although the applicability of the chiral perturbation theory at high-energy NN scattering phenomena is not established, its extension to the spin-orbit interaction might be interesting to pursue, which is a subject for future works.

APPENDIX: THE DECOMPOSITION OF THE *T* MATRICES

The representations of the T matrices in the basis of the nucleon spins are shown explicitly in the Appendix. First we expand the wave function of the two nucleons as

$$|\vec{p},s_z^a,s_z^b\rangle=\sqrt{4\pi}\sum_{L,S,J}\sum_{L_z+S_z=J_z}^{s_z^a+s_z^b=S_z}\langle L,L_z,S,S_z|J,J_z\rangle|^{2S+1}L_J\rangle Y_{L,L_z}(\hat{p})|s_z^a,s_z^b\rangle. \tag{A1}$$

Using the wave function Eq. (A1), we calculate the T matrix. For example, the T matrix of the ${}^3P_0\to{}^3P_0$ scattering is given by

$$\mathcal{T}_{^3\mathrm{P}_0\to{}^3\mathrm{P}_0}=4\pi\sum_{m,s_z}\langle 1,m,1,s_z|0,0\rangle^*\langle 1,0,1,0|0,0\rangle Y_{1,m}(\hat{k})^*Y_{1,0}(\hat{p})$$

$$\langle{}^3\mathrm{P}_0|T|{}^3\mathrm{P}_0\rangle\langle s_z^c,s_z^d|s_z^a,s_z^b\rangle|_{s_z^a+s_z^b=0,s_z^c+s_z^d=s_z}$$

$$=\frac{1}{2}T_{^3\mathrm{P}_0}\begin{pmatrix}0 & -s\ e^{-i\phi} & -s\ e^{-i\phi} & \\ & c & c & \\ & c & c & \\ & s\ e^{i\phi} & s\ e^{i\phi} & 0\end{pmatrix}, \tag{A2}$$

where $\hat{p}$ is the unit vector along the initial momentum $\vec{p}$ and we take it along the z axis. We show the T matrix of each partial wave in terms of $s\equiv\sin\theta, c\equiv\cos\theta$, and ϕ, where (θ,ϕ) is the scattering angle in the center-of-mass system:

$$\mathcal{T}_{^1\mathrm{S}_0\to{}^1\mathrm{S}_0}=\frac{1}{2}T_{^1\mathrm{S}_0}\begin{pmatrix}0 & & & \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0\end{pmatrix}, \tag{A3}$$

$$\mathcal{T}_{^3\mathrm{S}_1\to{}^3\mathrm{S}_1}=\frac{1}{2}T_{^3\mathrm{S}_1}\begin{pmatrix}2 & & & \\ & 1 & 1 & \\ & 1 & 1 & \\ & & & 2\end{pmatrix}, \tag{A4}$$

$$\mathcal{T}_{^1\mathrm{P}_1\to{}^1\mathrm{P}_1}=\frac{3}{2}T_{^1\mathrm{P}_1}c\begin{pmatrix}0 & & & \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0\end{pmatrix}, \tag{A5}$$

$$\mathcal{T}_{^3\mathrm{P}_0\to{}^3\mathrm{P}_0}=\frac{1}{2}T_{^3\mathrm{P}_0}\begin{pmatrix}0 & -s\ e^{-i\phi} & -s\ e^{-i\phi} & \\ & c & c & \\ & c & c & \\ & s\ e^{i\phi} & s\ e^{i\phi} & 0\end{pmatrix}, \tag{A6}$$

$$\mathcal{T}_{^3\mathrm{P}_1\to{}^3\mathrm{P}_1}=\frac{3}{4}T_{^3\mathrm{P}_1}\begin{pmatrix} 2c & 0 & 0 & 0 \\ s\ e^{i\phi} & 0 & 0 & -se^{-i\phi} \\ s\ e^{i\phi} & 0 & 0 & -se^{-i\phi} \\ 0 & 0 & 0 & 2c \end{pmatrix}, \tag{A7}$$

$$\mathcal{T}_{^3\mathrm{P}_2\to{}^3\mathrm{P}_2}=\frac{1}{4}T_{^3\mathrm{P}_2}\begin{pmatrix} 6c & 2s\ e^{-i\phi} & 2s\ e^{-i\phi} & 0 \\ -3s\ e^{i\phi} & 4c & 4c & 3s\ e^{-i\phi} \\ -3s\ e^{i\phi} & 4c & 4c & 3s\ e^{-i\phi} \\ 0 & -2s\ e^{i\phi} & -2s\ e^{i\phi} & 6c \end{pmatrix}, \tag{A8}$$

$$\mathcal{T}_{^1\mathrm{D}_2\to{}^1\mathrm{D}_2}=\frac{5}{4}T_{^1\mathrm{D}_2}(3c^2-1)\begin{pmatrix} 0 & & & \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0 \end{pmatrix}, \tag{A9}$$

$$\mathcal{T}_{^3\mathrm{D}_1\to{}^3\mathrm{D}_1}=\frac{1}{4}T_{^3\mathrm{D}_1}\begin{pmatrix} (3c^2-1) & -6sce^{-i\phi} & -6sce^{-i\phi} & 3s^2\ e^{-i2\phi} \\ 3sce^{i\phi} & 2(3c^2-1) & 2(3c^2-1) & -3sce^{-i\phi} \\ 3sce^{i\phi} & 2(3c^2-1) & 2(3c^2-1) & -3sce^{-i\phi} \\ 3s^2\ e^{2i\phi} & 6sce^{i\phi} & 6sce^{i\phi} & (3c^2-1) \end{pmatrix}, \tag{A10}$$

$$\mathcal{T}_{^3\mathrm{D}_2\to{}^3\mathrm{D}_2}=\frac{5}{4}T_{^3\mathrm{D}_2}\begin{pmatrix} (3c^2-1) & & & -s^2\ e^{-i2\phi} \\ sce^{i\phi} & 0 & 0 & -sce^{-i\phi} \\ sce^{i\phi} & 0 & 0 & -sce^{-i\phi} \\ -s^2\ e^{2i\phi} & & & (3c^2-1) \end{pmatrix}, \tag{A11}$$

$$\mathcal{T}_{^3\mathrm{D}_3\to{}^3\mathrm{D}_3}=\frac{1}{4}T_{^3\mathrm{D}_3}\begin{pmatrix} 4(3c^2-1) & 6sce^{-i\phi} & 6sce^{-i\phi} & 2s^2\ e^{-2i\phi} \\ -8sce^{i\phi} & 3(3c^2-1) & 3(3c^2-1) & 8sce^{-i\phi} \\ -8sce^{i\phi} & 3(3c^2-1) & 3(3c^2-1) & 8sce^{-i\phi} \\ 2s^2\ e^{2i\phi} & -6sce^{i\phi} & -6sce^{i\phi} & 4(3c^2-1) \end{pmatrix}, \tag{A12}$$

$$\mathcal{T}_{^1\mathrm{F}_3\to{}^1\mathrm{F}_3}=\frac{7}{4}T_{^1\mathrm{F}_3}(5c^3-3c)\begin{pmatrix} 0 & & & \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0 \end{pmatrix}, \tag{A13}$$

$$\mathcal{T}_{^3\mathrm{F}_2\to{}^3\mathrm{F}_2}=\frac{1}{2}T_{^3\mathrm{F}_2}\begin{pmatrix} c(5c^2-3) & -\frac{3}{2}s(5c^2-1)e^{-i\phi} & -\frac{3}{2}s(5c^2-1)e^{-i\phi} & 5s^2c\ e^{-2i\phi} \\ s(5c^2-1)e^{i\phi} & \frac{3}{2}c(5c^2-3) & \frac{3}{2}c(5c^2-3) & -s(5c^2-1)e^{-i\phi} \\ s(5c^2-1)e^{i\phi} & \frac{3}{2}c(5c^2-3) & \frac{3}{2}c(5c^2-3) & -s(5c^2-1)e^{-i\phi} \\ 5s^2ce^{2i\phi} & \frac{3}{2}s(5c^2-1)e^{i\phi} & \frac{3}{2}s(5c^2-1)e^{i\phi} & c(5c^2-3) \end{pmatrix}, \tag{A14}$$

$$\mathcal{T}_{^3\mathrm{F}_3\to{}^3\mathrm{F}_3}=\frac{1}{2}T_{^3\mathrm{F}_3}\begin{pmatrix} \frac{7}{2}c(5c^2-3) & & & -\frac{35}{4}s^2ce^{-2i\phi} \\ \frac{7}{8}s(5c^2-1)e^{i\phi} & 0 & 0 & -\frac{7}{8}s(5c^2-1)e^{-i\phi} \\ \frac{7}{8}s(5c^2-1)e^{i\phi} & 0 & 0 & -\frac{7}{8}s(5c^2-1)e^{-i\phi} \\ -\frac{35}{4}s^2ce^{2i\phi} & & & \frac{7}{2}c(5c^2-3) \end{pmatrix}, \tag{A15}$$

$$\mathcal{T}_{^3\mathrm{S}_1\to{}^3\mathrm{D}_1}=\frac{\sqrt{2}}{4}T_{^3\mathrm{S}_1\to{}^3\mathrm{D}_1}\begin{pmatrix} 3c^2-1 & 3sce^{-i\phi} & 3sce^{-i\phi} & 3s^2\ \ e^{-i\phi} \\ 3sce^{i\phi} & -(3c^2-1) & -(3c^2-1) & -3sce^{i\phi} \\ 3sce^{i\phi} & -(3c^2-1) & -(3c^2-1) & -3sce^{i\phi} \\ 3s^2\ \ e^{2i\phi} & -3sce^{i\phi} & -3sce^{i\phi} & 3c^2-1 \end{pmatrix}, \tag{A16}$$

$$\mathcal{T}_{^3\mathrm{D}_1\to{}^3\mathrm{S}_1}=\frac{\sqrt{2}}{2}T_{^3\mathrm{D}_1\to{}^3\mathrm{S}_1}\begin{pmatrix} 1 & & & \\ & -1 & -1 & \\ & -1 & -1 & \\ & & & 1 \end{pmatrix}, \tag{A17}$$

$$\mathcal{T}_{^3\mathrm{P}_2\to{}^3\mathrm{F}_2}=\frac{\sqrt{6}}{4}T_{^3\mathrm{P}_2\to{}^3\mathrm{F}_2}\begin{pmatrix} c(5c^2-3) & s(5c^2-1)e^{-i\phi} & s(5c^2-1)e^{-i\phi} & 5s^2ce^{-2i\phi} \\ s(5c^2-1)e^{i\phi} & -c(5c^2-3) & -c(5c^2-3) & -s(5c^2-1)e^{-i\phi} \\ s(5c^2-1)e^{i\phi} & -c(5c^2-3) & -c(5c^2-3) & -s(5c^2-1)e^{-i\phi} \\ 5s^2ce^{2i\phi} & -s(5c^2-1)e^{i\phi} & -s(5c^2-1)e^{i\phi} & c(5c^2-3) \end{pmatrix}, \tag{A18}$$

$$\mathcal{T}_{^3\mathrm{F}_2\to{}^3\mathrm{P}_2}=\frac{\sqrt{6}}{4}T_{^3\mathrm{F}_2\to{}^3\mathrm{P}_2}\begin{pmatrix} 2c & -se^{-i\phi} & -se^{-i\phi} & 0 \\ -se^{i\phi} & -2c & -2c & se^{-i\phi} \\ -se^{i\phi} & -2d & -2c & se^{-i\phi} \\ 0 & se^{i\phi} & se^{i\phi} & 2c \end{pmatrix}, \tag{A19}$$

$$\mathcal{T}_{^1\mathrm{P}_1\to{}^3\mathrm{P}_1}=\frac{3\sqrt{6}}{4}T_{^1\mathrm{P}_1\to{}^3\mathrm{P}_1}\begin{pmatrix} 0 & -se^{-i\phi} & se^{-i\phi} & 0 \\ & 0 & 0 & \\ & 0 & 0 & \\ 0 & -se^{i\phi} & se^{i\phi} & 0 \end{pmatrix}, \tag{A20}$$

$$\mathcal{T}_{^3\mathrm{P}_1\to{}^1\mathrm{P}_1}=\frac{3\sqrt{6}}{4}T_{^3\mathrm{P}_1\to{}^1\mathrm{P}_1}\begin{pmatrix} 0 & & & 0 \\ se^{i\phi} & 0 & 0 & se^{-i\phi} \\ -se^{i\phi} & 0 & 0 & -se^{-i\phi} \\ 0 & & & 0 \end{pmatrix}, \tag{A21}$$

$$\mathcal{T}_{^1\mathrm{D}_2\to{}^3\mathrm{D}_2}=\frac{5\sqrt{6}}{4}T_{^1\mathrm{D}_2\to{}^3\mathrm{D}_2}\begin{pmatrix} 0 & -sce^{-i\phi} & sce^{-i\phi} & 0 \\ & 0 & 0 & \\ & 0 & 0 & \\ 0 & -sce^{i\phi} & sce^{i\phi} & 0 \end{pmatrix}, \tag{A22}$$

$$\mathcal{T}_{^3\mathrm{D}_2\to{}^1\mathrm{D}_2}=\frac{5\sqrt{6}}{4}T_{^3\mathrm{D}_2\to{}^1\mathrm{D}_2}\begin{pmatrix} 0 & & & 0 \\ sce^{i\phi} & 0 & 0 & sce^{-i\phi} \\ -sce^{i\phi} & 0 & 0 & -sce^{-i\phi} \\ 0 & & & 0 \end{pmatrix}, \tag{A23}$$

$$\mathcal{T}_{^1\mathrm{F}_3\to{}^3\mathrm{F}_3}=\frac{7\sqrt{3}}{8}T_{^1\mathrm{F}_3\to{}^3\mathrm{F}_3}\begin{pmatrix} 0 & -s*(5c^2-1)e^{-i\phi} & s*(5c^2-1)e^{-i\phi} & 0 \\ & 0 & 0 & \\ & 0 & 0 & \\ 0 & -s*(5c^2-1)e^{i\phi} & s*(5c^2-1)e^{i\phi} & 0 \end{pmatrix}, \tag{A24}$$

$$\mathcal{T}_{{}^3\mathrm{F}_3\to{}^1\mathrm{F}_3}=\frac{7\sqrt{3}}{8}T_{{}^3\mathrm{F}_3\to{}^1\mathrm{F}_3}\begin{pmatrix} 0 & & & 0 \\ s(5c^2-1)e^{i\phi} & 0 & 0 & s(5c^2-1)e^{-i\phi} \\ -s(5c^2-1)e^{i\phi} & 0 & 0 & -s(5c^2-1)e^{-i\phi} \\ 0 & & & 0 \end{pmatrix}. \tag{A25}$$

Substituting the above T matrices into the denominator and the numerator of Eq. (58), we obtain

$$\begin{aligned} Tr[\overline{T}^{\dagger}\overline{T}]=&\tfrac{1}{8}\,|-2T_{{}^1\mathrm{S}_0}+2T_{{}^3\mathrm{S}_1}-2\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1}+2c(-3T_{{}^1\mathrm{P}_1}+T_{{}^3\mathrm{P}_0}+2T_{{}^3\mathrm{P}_2}-\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})+(3c^2-1)\\ &\times(-5T_{{}^1\mathrm{D}_2}+2T_{{}^3\mathrm{D}_1}+3T_{{}^3\mathrm{D}_3}-\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})+c(5c^2-3)(-7T_{{}^1\mathrm{F}_3}+3T_{{}^3\mathrm{F}_2}-\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})|^2+\tfrac{1}{8}\,|2T_{{}^1\mathrm{S}_0}+2T_{{}^3\mathrm{S}_1}\\ &-2\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1}+2c(3T_{{}^1\mathrm{P}_1}+T_{{}^3\mathrm{P}_0}+2T_{{}^3\mathrm{P}_2}-\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})+(3c^2-1)(5T_{{}^1\mathrm{D}_2}+2T_{{}^3\mathrm{D}_1}+3T_{{}^3\mathrm{D}_3}-\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})\\ &+c(5c^2-3)(7T_{{}^1\mathrm{F}_3}+3T_{{}^3\mathrm{F}_2}-\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})|^2+\tfrac{1}{8}\,|4T_{{}^3\mathrm{S}_1}+2\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1}+2c(3T_{{}^3\mathrm{P}_1}+3T_{{}^3\mathrm{P}_2}+\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})\\ &+(3c^2-1)(T_{{}^3\mathrm{D}_1}+5T_{{}^3\mathrm{D}_2}+4T_{{}^3\mathrm{D}_3}+\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})+c(5c^2-3)(2T_{{}^3\mathrm{F}_2}+7T_{{}^3\mathrm{F}_3}+\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})|^2+\tfrac{1}{8}\,s^4|3T_{{}^3\mathrm{D}_1}\\ &-5T_{{}^3\mathrm{D}_2}+2T_{{}^3\mathrm{D}_3}+3\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1}+c(10T_{{}^3\mathrm{F}_2}-\tfrac{35}{2}\,T_{{}^3\mathrm{F}_3}+5\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})|^2+\tfrac{1}{4}\,s^2|2T_{{}^3\mathrm{P}_0}-2T_{{}^3\mathrm{P}_2}+\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2}+3c\\ &\times(2T_{{}^3\mathrm{D}_1}-2T_{{}^3\mathrm{D}_3}-\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})+(5c^2-1)(3T_{{}^3\mathrm{F}_2}-\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})|^2+\tfrac{1}{4}\,s^2|3T_{{}^3\mathrm{P}_1}-3T_{{}^3\mathrm{P}_2}-\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2}\\ &+c(3T_{{}^3\mathrm{D}_1}+5T_{{}^3\mathrm{D}_2}-8T_{{}^3\mathrm{D}_3}+3\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})+(5c^2-1)(2T_{{}^3\mathrm{F}_2}+\tfrac{7}{4}\,T_{{}^3\mathrm{F}_3}+\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})|^2, \end{aligned} \tag{A26}$$

$$\begin{aligned} Tr[\overline{T}^{\dagger}(\sigma_n-\sigma_p)\Delta T_{CSB}]=&-\frac{1}{4}i\left(3\sqrt{6}sT_{{}^3\mathrm{P}_1\to{}^1\mathrm{P}_1}+5\sqrt{6}scT_{{}^3\mathrm{D}_2\to{}^1\mathrm{D}_2}+\frac{7\sqrt{3}}{2}s(5c^2-1)T_{{}^3\mathrm{F}_3\to{}^1\mathrm{F}_3}\right)\{4T_{{}^1\mathrm{S}_0}+4T_{{}^3\mathrm{S}_1}+2\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1}\\ &+2c(3T_{{}^3\mathrm{P}_1}+3T_{{}^3\mathrm{P}_2}+\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})+(3c^2-1)(10T_{{}^1\mathrm{D}_2}+T_{{}^3\mathrm{D}_1}+5T_{{}^3\mathrm{D}_2}+4T_{{}^3\mathrm{D}_3}+3\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})\\ &+(5c^2-3)c(14T_{{}^1\mathrm{F}_3}+2T_{{}^3\mathrm{F}_2}+7T_{{}^3\mathrm{F}_3}+\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})+s^2(3T_{{}^3\mathrm{D}_1}-5T_{{}^3\mathrm{D}_2}+2T_{{}^3\mathrm{D}_3}+3\sqrt{2}T_{{}^3\mathrm{S}_1\to{}^3\mathrm{D}_1})\\ &+s^2c(10T_{{}^3\mathrm{F}_2}-\tfrac{35}{2}\,T_{{}^3\mathrm{F}_3}+5\sqrt{6}T_{{}^3\mathrm{P}_2\to{}^3\mathrm{F}_2})\}. \end{aligned} \tag{A27}$$

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