Charge-symmetry breaking and the two-pion-exchange two-nucleon interaction

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Charge-symmetry breaking in the nucleon-nucleon force is investigated within an effective field theory, using a classification of isospin-violating interactions based on power-counting arguments. The relevant charge-symmetry-breaking interactions corresponding to the first two orders in the power counting are discussed, including their effects on the ^{3}He-^{3}H binding-energy difference. The static charge-symmetry-breaking potential linear in the nucleon-mass difference is constructed using chiral perturbation theory. Explicit formulas in momentum and configuration spaces are presented. The present work completes previously obtained results.

I. INTRODUCTION

Significant advances in understanding isospin violation in the nuclear force have been made in the past decade. Experimental progress (which is reviewed and summarized in Refs. [1–3]) has been supplemented recently by the advent of chiral perturbation theory (ChPT) [4–8]. This powerful technique casts the symmetries of QCD into effective interactions of the traditional, low-energy degrees of freedom of nuclear physics (viz., nucleons and pions). These building blocks (in the Lagrangian) can then be combined in a systematic way to produce nuclear forces that violate isospin in exactly the same way as in QCD.

An important feature of effective field theories is power counting, which is the technique used to organize calculations [4–6,8–10]. A well-defined ordering of terms in the Lagrangian according to scales intrinsic to QCD and nuclei is used to generate all terms of a particular size. Terms in the Lagrangian are labeled (in the conventional way, $\mathcal{L}^{(n)}$) by the number of implicit powers (n) of the inverse of the large-mass scale of QCD, $\Lambda \sim 1$ GeV. When considering processes where the typical momentum is of the order of the pion mass, $Q \sim m_\pi$, amplitudes are expanded in powers of Q/Λ. The limitation of this approach is that it is appropriate only for low energies. The expansion breaks down as the energy or momentum becomes a significant fraction of Λ.

Much work has been done in the past few years regarding the derivation of the isospin-symmetric part of the nuclear potential [8]. The components of short ($\sim \Lambda^{-1}$) range in the potential come from (renormalized) contact interactions whose strengths are not determined by symmetry. The components of longer range arise from pion exchanges, and are

determined in terms of pion couplings to the nucleon, which are fewer and in many cases determined from processes involving a single nucleon. Perhaps the most significant result of this method is the first derivation of a two-pion-range potential consistent with the approximate chiral symmetry of QCD [5–7]. This two-pion-exchange potential (TPEP) has been incorporated in the Nijmegen phase-shift analysis [11,12]. Pion-nucleon couplings determined from two-nucleon data were found to be in good agreement with other determinations based on pion-nucleon scattering. We note that there is a significant body of older work that properly emphasized the chiral-symmetry aspects of this problem, but that older work did not systematically accommodate broken chiral symmetry and effective multipion vertices.

In effective theories, isospin-breaking interactions can be classified [5] according to whether their origin is the mass difference between u and d quarks or hard electromagnetic (EM) interactions at the quark level. The soft EM interactions (such as the Coulomb force) can be constructed in the usual way [13,14].

The power counting for isospin-violating interactions was developed by one of us [5], and it explains the sizes of the various isospin structures present in the nuclear force. A convenient and universal [1] classification for nuclear isospin is as follows: class (I)—isospin conserving; class (II)—charge-independence breaking (CIB) of isospin, but charge symmetric; class (III)—charge-symmetry breaking (CSB) of isospin; class (IV)—isospin mixing in the np system between $T=0$ and $T=1$. Power counting can be used to demonstrate [5] that a class (N) force appears only at order $n=N-1$ or higher. We, therefore, deduce on the basis of QCD a result that was noticed before on an empirical basis: class (I) forces

are stronger than class (II), which are stronger than class (III), which are stronger than class (IV).

Using this formalism, several of the various isospin components of the nuclear potential have been calculated [5,15,14,16–18]. In particular, we have computed the long-range components of the class (I) [4–7] and class (II) [5,15,14,16] two-nucleon potentials, up to order $n = 3$. The class (III) isospin violation is the purview of this work.

Charge symmetry [19] can be loosely defined as invariance under turning neutrons into protons and protons into neutrons. It has long been considered a particularly interesting aspect of isospin violation because the difference in quark masses is a source of CSB. On the other hand, a significant interaction that violates this symmetry is the electromagnetic interaction, which is large between two protons and very small between two neutrons. Moreover, the Coulomb interaction between protons is long ranged, while it has a nuclear range for two neutrons and is, therefore, indistinguishable from nuclear interactions. There are corrections of the order of $1/M^2$ to the static Coulomb interaction (where M is the nucleon mass), which are part of the Breit interaction. Although these familiar electromagnetic interactions dominate the CSB in nuclei, we will largely ignore them and concentrate on the nuclear mechanisms [15]. In the nuclear force, the comparison that interests us is the difference in forces between two protons and two neutrons, which is restricted to the $T = 1$ channel for two nucleons.

The long standing interest in CSB [19] has been highlighted recently by two exciting new experiments. One experiment, carried out at TRIUMF and currently undergoing final stages of analysis, measures the front-back asymmetry of the pion produced in the reaction $np \rightarrow d\pi^0$ close to threshold [20]. Another experiment has just been completed at IUCF, and measures the near-threshold cross section for the reaction $dd \rightarrow \alpha\pi^0$ [21]. It has been argued in Ref. [22] that the chiral properties of two different contributions to the nucleon-mass difference can give a relatively large effect in $np \rightarrow d\pi^0$, of opposite sign to more well-known mechanisms. A similar phenomenon might exist in $dd \rightarrow \alpha\pi^0$ [22]. There is reason to hope that these data will allow a model-independent (albeit crude) determination of both quark-mass and electromagnetic components of the nucleon-mass difference.

An issue that arises naturally is the role of the nucleon-mass difference in the two-pion-exchange nucleon-nucleon potential. Reference [17] calculated the crossed-box contribution, and it contains a comprehensive discussion of the older literature [23,24], where aspects of chiral symmetry are not emphasized. More recently, Ref. [18] calculated a CSB seagull contribution using the formalism of Ref. [5].

In this paper, we discuss the relative sizes of CSB interactions in effective field theories, and calculate the two-nucleon potential of two-pion range that arises from the nucleon-mass difference. Pieces of this potential have appeared before [17,18]. Here we complete the calculation of the long-range component of the class (III) potential up to (and including) order $n = 3$.

II. SIZES

In the chiral Lagrangian, the largest isospin-violating terms are those that determine the mass differences of nucleons and pions [5]. In order to compare these and various other contributions, it is useful to estimate relative sizes of EM and quark-mass effects. The pion-mass splitting provides a good starting point.

The pion-mass splitting (squared), $\delta m_\pi^2 = (m_\pi^\pm)^2 - (m_\pi^0)^2$, receives contributions from both the difference between the masses of the u and d quarks and EM interactions. Because of the chiral transformation properties of the pion, the contribution from the quark-mass difference is proportional to $(m_\pi^2/\Lambda)^2$. The proportionality constant (ϵ^2) is of the order of the square of the difference of quark masses divided by their sum ($\epsilon \sim 0.3$). The pion-mass difference is, therefore, mostly due to EM interactions. It is nominally of the order of $n = -2$, but the large strength implied by this counting is compensated by the small fine-structure constant ($\alpha \sim 1/137$), which reduces the strength by slightly more than two orders of magnitude. The size of the pion-mass splitting is then rather accurately described by $\delta m_\pi^2 \simeq \alpha\Lambda^2/\pi$, with $\Lambda \sim m_\rho$, the mass of the ρ meson [5]. This suggests that the relevant dimensionless parameter for the EM-induced isospin violation is $\delta m_\pi^2/\Lambda^2 = \alpha/\pi \cong 2 \times 10^{-3}$, which numerically is close to $\epsilon m_\pi^3/\Lambda^3 \sim 2 \times 10^{-3}$. Since the latter would ordinarily correspond in power counting to $n = 3$, we deduce a convenient mnemonic of adding 3 to the order of the EM-induced isospin-violating Lagrangian when comparing sizes with quark-mass-induced mechanisms. Henceforth, our power counting for EM-induced interactions will contain this additional factor of 3 (e.g., $n = -2$ as counted above will be counted as $n = 1$ below).

The power counting has been used in Ref. [5] to classify various contributions to the nuclear potential. The leading isospin-breaking interaction appears at $n = 1$, when the pion-mass difference is inserted in the one-pion-exchange potential (OPEP). It gives a class (II) force, which is of relative strength $\delta m_\pi^2/m_\pi^2$ (times OPEP) [5]. Many other class (II) forces appear up to order $n = 3$. There are three EM mechanisms of roughly the same relative strength, α/π (times the usual OPEP) produced by $n = 3$ terms in the Lagrangian or from loops. They are the two-pion-exchange potential with different charged and neutral pion masses [16], CIB in the pion-nucleon coupling constant [15] and consequently in OPEP, and the π-γ-exchange potential [14]. Moreover, there are OPEP corrections that arise from higher-order isospin-conserving pion-nucleon interactions (such as recoil in the usual pion-nucleon coupling), and from second-order effects due to the pion- and nucleon-mass differences [5].

Here we concentrate on the class (III) mechanisms. It is worthwhile to discuss briefly the various mechanisms that contribute to CSB in the NN system and in the trinucleon system (³He-³H), their relative sizes, and their relationship in the context of power counting.

The bulk of the CSB is due to the soft EM interactions. The 764 keV binding-energy difference of ³He and ³H is largely accounted for by the 648 keV Coulomb energy difference [25] plus ≈ 29 keV from the small Breit corrections

plus vacuum polarization [26,13,27–29]. Calculations of these contributions appear to be rather robust, and we henceforth ignore them. The remaining mechanisms that generate ≈ 87 keV are contained in ChPT, and we discuss them below.

The most obvious signature of CSB due to nuclear forces is the difference in the scattering lengths of two neutrons and two protons, once all EM mechanisms are accounted for and the effect of the different nucleon masses on the kinetic energy [see Eq. (1c) below] is treated. The resulting "experimental" $a_{nn} - a_{pp}$ scattering-length difference [2,3,15] of $-1.5(5)$ fm is then attributed to CSB in the nuclear force. Typically, one treats only the short-range (as opposed to pion-range) CSB force, by adjusting the force to produce the desired difference in scattering lengths. This CSB modification of the force then produces a contribution to the ^{3}He-^{3}H binding-energy difference $\approx 65(22)$ keV, a number that also appears robust [28–30] and can accommodate the 87 keV binding-energy difference missing after soft EM processes are taken into account. Note that this scattering-length difference is generated by the total CSB nuclear force, and does not differentiate between components of different ranges.

The mass of a single nucleon can be expressed in terms of isospin operators (the Pauli isospin operator $\boldsymbol{\tau}$ satisfies $\boldsymbol{\tau} = 2\,\mathbf{t}$, where t_z gives $+1/2$ for a proton and $-1/2$ for a neutron):

$$M_N = \tfrac{1}{2}(M_p + M_n) + \tfrac{1}{2}(M_p - M_n)\tau_z \equiv M + \tfrac{1}{2}\delta M_N \tau_z, \tag{1a}$$

where $M = \tfrac{1}{2}(M_p + M_n)$ is the average nucleon mass, $\delta M_N = (M_p - M_n)$ is the nucleon-mass difference and $\tfrac{1}{2}\delta M_N \tau_z$ is a CSB nucleon-mass interaction. The nucleon-mass difference δM_N receives contributions from both the quark-mass difference δM_N^{qm} and EM interactions at the quark level, δM_N^{EM},

$$\delta M_N = \delta M_N^{\mathrm{qm}} + \delta M_N^{\mathrm{EM}}. \tag{1b}$$

(We neglect here mixed contributions of higher order.) The piece due to the quark-mass difference is the sole CSB contribution of the order of $n = 1$, implying that this quantity is proportional to m_π^2 / Λ. The parameter $\epsilon m_\pi^2 / \Lambda^2 \sim 10^{-2}$ plays an important role in the power counting for isospin violation, and would lead to a nucleon-mass difference of the order of 8 MeV (a factor of 3 higher than most model estimates, and therefore reasonable). The corresponding nucleon-mass term from EM interactions at the quark level is of order $n = 2$, but is numerically comparable to the quark-mass term (somewhat smaller and of opposite sign) because it implicitly contains a power of the fine-structure constant.

The largest CSB interactions in the Lagrangian thus come from the nucleon-mass difference of the orders of $n = 1$ and 2 for the two different mechanisms. The effects of the nucleon-mass difference on nuclear amplitudes are somewhat suppressed, however. Summing Eq. (1a) over all nucleons produces an overall constant (determined by the average nucleon mass), which can be removed by a shift in the zero of energy, and a term proportional to the z component of the

total nuclear isospin, T_z. The latter term will contribute to a CSB shift in the nuclear kinetic energy [for simplicity, we restrict ourselves to only two nucleons in their center-of-mass (c.m.) frame],

$$\delta T = -T \frac{\delta M_N}{2M} T_z, \tag{1c}$$

where T is the usual c.m. kinetic energy for two nucleons with equal masses, and $T_z = t_z(1) + t_z(2)$ for two nucleons. The CSB nucleon-mass difference will also lead to a modification of the nuclear potential energy. The CSB part of Eq. (1a) ($\delta M_N T_z$) will commute with an isospin-conserving Hamiltonian and, therefore, will play no role to order δM_N, except inside certain loops (viz., crossed-box and triangle diagrams) that involve exchanging two charged pions. In the latter case for a process initiated by two identical nucleons of one type, the intermediate state would contain two nucleons of the other type. Exchanging neutral mesons clearly does not invoke this mechanism, nor does ladder approximation (sequential exchanges of neutral mesons) for two protons or two neutrons. Because this effect requires a loop, it is of the order of $n = 3$. Thus although nominally of the order of $n = 1$ (by itself), the nucleon-mass difference actually first contributes in the much higher order $n = 3$, which is a reduction in size typical of loop contributions. In addition to these loop insertions, there are associated triangle-graph interactions from seagulls (discussed below).

Other contributions in the Lagrangian begin at order $n = 2$ for the quark-mass terms (i.e., they are the largest) and arise from two types of CSB nuclear forces: short-range forces (such as those arising from ρ-ω [31] and a_1-f_1 [32] mixing), and from isospin violation in the pion-nucleon coupling constant (a model for the latter is provided by π-η mixing [24]). We denote the former by $V_{\mathrm{SR}}^{\mathrm{CSB}}$ and the latter by V_π^{CSB}. To leading order [5] for two nucleons, one has

$$V_{\mathrm{SR}}^{\mathrm{CSB}} = (\gamma_s + \bar{\gamma}_s) V_0(r) T_z + (\gamma_\sigma + \bar{\gamma}_\sigma) V_1(r) \vec{\sigma}(1) \cdot \vec{\sigma}(2) T_z, \tag{2}$$

where $\vec{r}$ is the separation of nucleons 1 and 2, and V_0 and V_1 have unit volume integral. Constants γ_s and γ_σ stem from the quark-mass difference and are of the order of $\epsilon m_\pi^2 / f_\pi^2 \Lambda^2$, while $\bar{\gamma}_s$ and $\bar{\gamma}_\sigma$ are EM corrections of the order of $\alpha / \pi f_\pi^2$. This implies a potential strength $V_{\mathrm{SR}}^{\mathrm{CSB}} \sim (\epsilon m_\pi^2 / \Lambda^2) V_{\mathrm{SR}}^{NN}$, where V_{SR}^{NN} is the expectation value of the short-range interaction between either the pp pair in ^{3}He or the nn pair in ^{3}H. A 34-channel calculation using the AV18 potential [13] gives -7.6 MeV for the latter, or an estimate of about 45 keV for the contribution of the CSB short-range interaction to the ^{3}He-^{3}H binding-energy difference. This interaction has traditionally always been a part of CSB studies.

The OPEP contains a CSB (coupling constant) modification [5,15]

$$V_\pi = v_\pi \left(\mathbf{t}(1) \cdot \mathbf{t}(2) - \frac{(\beta_1 + \bar{\beta}_3)}{2 g_A} T_z \right) \equiv V_\pi^0 + V_\pi^{\mathrm{CSB}}, \tag{3}$$

where $V_\pi^0 = v_\pi \mathbf{t}(1) \cdot \mathbf{t}(2)$ is the usual isospin-conserving OPEP, and we do not write (higher-order) recoil corrections explicitly. Here, $\beta_1 \sim \epsilon m_\pi^2/\Lambda^2$ is the (quark-mass induced) CSB pion-nucleon coupling constant and $\bar\beta_3 \sim \alpha/\pi$ is the (EM-induced) CSB pion-nucleon coupling constant. The size of V_π^{CSB} is constrained only by an upper limit on $\beta_1 + \bar\beta_3$, arising from the Nijmegen phase-shift analysis of NN data [15]. A numerical estimate of V_π^{CSB} in the trinucleon system can be obtained from Eq. (3), which shows that the contribution to the trinucleon binding-energy difference is approximately $\langle V_\pi^{CSB} \rangle \cong -4(\beta_1 + \bar\beta_3) V_\pi^{NN}/g_A$, where V_π^{NN} is the expectation value of the potential energy due to OPEP between either the pp pair in ^{3}He or the nn pair in ^{3}H. A 34-channel Faddeev calculation using the AV18 potential [13] produces -1.67 MeV for the latter. Using the experimental value of $(\beta_1 + \bar\beta_3) = 0(9) \times 10^{-3}$ from Ref. [15] produces $\langle V_\pi^{CSB} \rangle \sim 0(50)$ keV of uncertain sign. The dimensional estimate [5,15] using $\beta_1 \sim \epsilon m_\pi^2/\Lambda^2 \sim 10^{-2}$ produces $\langle V_\pi^{CSB} \rangle \sim 50$ keV, and also of uncertain sign. Thus a substantial part of the "short-range" contribution to CSB [65(22) keV] in the trinucleons could come from CSB in OPEP, rather than from a shorter-range interaction. In any event, the two mechanisms are predicted by power counting to be the dominant hadronic contributions and to be roughly comparable in size.

The next order in ChPT for CSB is $n = 3$. The modification of the nuclear kinetic energy induced by the different nucleon masses [Eq. (1c)] is of this order, and is always taken into account, both in the NN interaction and in the trinucleon CSB, where it contributes a robust 14 keV [13,26–29]. A power-counting estimate of its size can be made by using m_π to estimate the average value of the nucleon momentum in the trinucleons, which produces a value $\sim (m_\pi^2/M^2)\delta M_N \sim 25$ keV (and within a factor of 2 of the actual value). The size of this contribution illustrates the general caveat about power counting: even though this kinetic-energy modification is an order smaller in the Lagrangian than the potential couplings ($n = 3$ vs $n = 2$), they can make actual contributions in a nucleus that are not very different in size. This contribution added to the soft-EM mechanisms discussed earlier leaves about 73 keV to be explained by the various CSB nuclear potentials.

The other mechanism of the order of $n = 3$ is the two-pion-exchange CSB arising from the nucleon-mass difference in certain loops, which we discussed above and will treat below. It depends not simply on the full proton-neutron mass difference, but on δM_N^{qm} and δM_N^{EM} separately. Since these two components of the nucleon-mass difference are presently unknown, we can only estimate the impact of this potential on observable quantities. Power counting suggests that this contribution is one order smaller than the (three) previously discussed mechanisms. We can make a numerical estimate of its contribution to the ^{3}He-^{3}H binding-energy difference using naive dimensional analysis. Again assuming that the typical momentum in the nucleus is $Q \sim m_\pi$ and that the loop integral gives the usual factor of $(4\pi)^{-2}$, we expect in coordinate space $V_{2\pi}^{CSB} \sim (m_\pi^4/64\pi f_\pi^2\Lambda^2)\delta M_N$, which is more than an order of magnitude smaller than the estimate
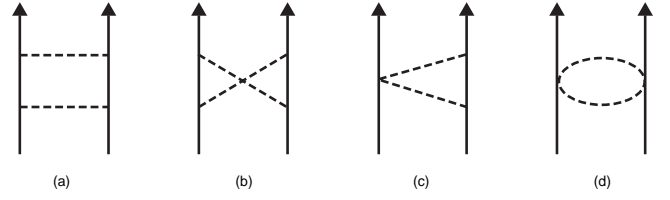

FIG. 1. Two-pion-exchange graphs that contribute to isospin-conserving nucleon-nucleon scattering.

for δT of 25 keV. As we will see below, however, the loop integrals in this case only give one power of $(4\pi)^{-1}$. The CSB TPEP then likely supplies a larger contribution than expected on the basis of power counting.

Note that the CSB TPEP contribution is also a part of the traditional short-range nuclear CSB mechanism, and its effect is, therefore, included in the 65(22)-keV short-range part of the trinucleon binding-energy difference. One can, of course, accommodate all of these short-range mechanisms using models [31,30]. Alternatively, one can hope that a sophisticated partial-wave analysis of nucleon-nucleon scattering, such as that carried out by the Nijmegen group [12,11], may be able to disentangle the CSB interactions of one-pion range, two-pion range, and short range, each of which contains an unknown parameter that must be fitted to the data. Incorporating these forces into their procedure should be straightforward, although there may not be enough sensitivity in the data to distinguish between the three mechanisms. A full analysis of few-nucleon systems within effective field theory [6,33,34] will eventually include all these CSB mechanisms.

In summary, there are two mechanisms for CSB of the order of $n = 2$ at the Lagrangian level [CSB short-range forces in Eq. (2), and CSB OPEP in Eq. (3)] and one of the order of $n = 3$ [nuclear CSB kinetic energy in Eq. (1c)] that should make roughly comparable contributions in a nucleus. The TPEP modified by δM_N^{qm} and δM_N^{EM}, together with the associated CSB seagull interactions [all given in Eqs. (9) below], should be somewhat smaller.

III. CSB TPEP

Isospin-conserving TPEPs are an old problem with a new twist. In static order (containing only terms that remain when the nucleon mass M or the large-mass scale of QCD, Λ, becomes very large), the diagrams of Fig. 1 (properly symmetrized) contribute to the TPEP. The vertices and propagators follow from the leading-order Lagrangian for pions and nucleons,

$$\mathcal{L}^{(0)} = \frac{1}{2}[\dot{\boldsymbol{\pi}}^2 - (\vec\nabla\boldsymbol{\pi})^2 - m_\pi^2\boldsymbol{\pi}^2] + N^\dagger\left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})\right]N$$

$$+ \frac{g_A}{2f_\pi}N^\dagger\vec\sigma\cdot\vec\nabla(\boldsymbol{\tau}\cdot\boldsymbol{\pi})N, \tag{4}$$

where the $\pi\pi N$ term is the Weinberg-Tomozawa (WT) interaction [35] and the πN term is the usual interaction that depends on the axial-vector coupling constant g_A and the
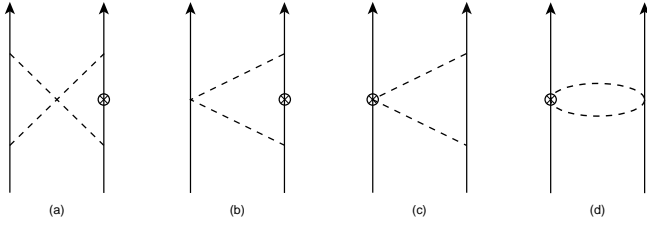
FIG. 2. Two-pion-exchange graphs that contribute to charge-symmetry breaking in nucleon-nucleon scattering. Graph (d) vanishes because of the symmetry of isospin operators. The $\otimes$ symbol indicates a CSB vertex, either a seagull in (c) and (d), or a mass insertion in (a) and (b).

pion-decay constant f_π. Terms with additional pions or nucleons are neglected here, as they only contribute to the nuclear force at higher orders. The WT term has a specific normalization $(-1/4f_\pi^2)$ required by the underlying chiral symmetry.

The nucleon-mass difference corresponds to Lagrangian terms that arise from both quark-mass differences $\delta M_N^{\rm qm}$ and from EM interactions at the quark level, $\delta M_N^{\rm EM}$. The largest term of the first type is [5]

$$\mathcal{L}_{\rm qm}^{(1)} = -\delta M_N^{\rm qm} N^\dagger \left[\tfrac{1}{2}\tau_3 - \frac{1}{4f_\pi^2}\,\pi_3 \boldsymbol{\tau}\cdot\boldsymbol{\pi} \right] N, \qquad (5)$$

and the leading-order term of the EM type is [5]

$$\mathcal{L}_{\rm EM}^{(-1)} = -\delta M_N^{\rm EM} N^\dagger \left[\tfrac{1}{2}\tau_3 - \frac{1}{4f_\pi^2}(\boldsymbol{\pi}^2\tau_3 - \pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}) \right] N. \quad (6)$$

As with the WT term, the $\pi\pi N$ interactions in Eqs. (5) and (6) are *required* by chiral symmetry and have a fixed strength $(-1/4f_\pi^2)$ relative to the mass terms. We again drop terms that involve more pion fields and do not contribute to the nuclear potential in low orders.

The CSB TPEP can be computed in a straightforward way. We simply consider all insertions of the mass and interaction terms above into the diagrams of Fig. 1, including external lines. Alternatively, we implement the p-n mass-difference mechanism by including the simple p-n mass difference [i.e, the sum of the first terms in each Lagrangian in Eqs. (5) and (6)] in the initial and final nuclear states, and then compensate for this addition by a subtraction. That is, we write for a single nucleon

$$\delta M_{\rm CSB} = \tfrac{1}{2}\delta M_N(\tau_z - \tau_z^0), \qquad (7)$$

where τ_z^0 is the reference value of τ_z for that nucleon. The expectation value of this contribution for all the nucleons will vanish, as does the contribution from uncrossed-box diagrams. The crossed-box diagram shown in Fig. 2(a) and the triangle diagram in Fig 2(b) can also be easily computed. In addition, there are seagull terms involving two pions [from the second parts of the two Lagrangians in Eqs. (5) and (6)], which will generate triangle diagrams [Fig. 2(c)], and in principle, "football" diagrams [Fig. 2(d)]. The latter vanish

for this problem because of the isospin symmetries of the WT (antisymmetric) and CSB seagull (symmetric) vertices.

We find a result that is a pure class (III) potential. Our results in momentum space are given below, with both pion momenta $\vec{q}_1 = \vec{k} + \tfrac{1}{2}\vec{q}$ and $\vec{q}_2 = -\vec{k} + \tfrac{1}{2}\vec{q}$ pointing into the seagull vertex. The loop momentum is $\vec{k}$ and $\vec{q}$ is the momentum transfer. Writing

$$V_{\rm CSB}^{a,b,c}(\vec{q}) = \frac{1}{4\pi}\left(\frac{g_A}{(2f_\pi)^2}\right)^2 [\tau_z(1) + \tau_z(2)]v^{a,b,c}(\vec{q}) \quad (8)$$

and

$$v^{a,b,c}(\vec{q}) = 8\pi \int \frac{d^3k}{(2\pi)^3}u^{a,b,c}(\vec{q}_1,\vec{q}_2), \qquad (9)$$

graph 2(a) gives

$$u^a(\vec{q}_1,\vec{q}_2) = -g_A^2\delta M_N \left(\frac{1}{q_1^2 + m_\pi^2} + \frac{1}{q_2^2 + m_\pi^2}\right)$$

$$\times \frac{(\vec{q}_1\cdot\vec{q}_2)^2 + \vec{\sigma}(1)\cdot\vec{q}_1\times\vec{q}_2\,\vec{\sigma}(2)\cdot\vec{q}_1\times\vec{q}_2}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)},$$

$$(9a)$$

while graph 2(b) generates

$$u^b(\vec{q}_1,\vec{q}_2) = -\delta M_N \frac{\vec{q}_1\cdot\vec{q}_2}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)}, \qquad (9b)$$

and finally graph 2(c) produces

$$u^c(\vec{q}_1,\vec{q}_2) = (\delta M_N - \tfrac{1}{2}\delta M_N^{\rm qm})\frac{\vec{q}_1\cdot\vec{q}_2}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)}. \qquad (9c)$$

The integrals in these expressions are divergent, requiring regularization and renormalization by the spin-independent nucleon-nucleon contact terms given in Eq. (2). The loop integration over $\vec{k}$ gives for the nonanalytic terms

$$v^a(\vec{q}) = g_A^2\delta M_N\left[2\frac{q^2 + 2m_\pi^2}{q}\arctan\left(\frac{q}{2m_\pi}\right) + 6m_\pi\right.$$

$$\left. - \frac{2m_\pi^3}{q^2 + 4m_\pi^2} - \frac{\vec{\sigma}(1)\times\vec{q}\cdot\vec{\sigma}(2)\times\vec{q}}{q}\arctan\left(\frac{q}{2m_\pi}\right)\right],$$

$$(8a)$$

and

$$v^{b+c}(q) = -\delta M_N^{\rm qm}\left[\frac{q^2 + 2m_\pi^2}{2q}\arctan\left(\frac{q}{2m_\pi}\right) + m_\pi\right]. \qquad (8b)$$

The long-range part of the potential in configuration space is independent of the regularization procedure. It is simplest

to derive it by introducing a cutoff ("form") factor $F(q^2)$ for each pion line carrying a momentum $\vec{q}$ in or out of a vertex. The three nucleon-nucleon potentials depicted in Figs. 2(a–c) are

$$V_{\mathrm{CSB}}(\vec{r}) = \frac{1}{(4\pi)^2}\left(\frac{g_A m_\pi^2}{(2f_\pi)^2}\right)^2 [\tau_z(1) + \tau_z(2)]v^{a,b,c}(m_\pi\vec{r}) \tag{10}$$

with

$$\begin{aligned}
v^a(\vec{x}) = {} & 2g_A^2\delta M_N(\{h(x)\nabla^2 h(x) + x\,h'(x)\nabla^2 h(x) \\
& - 2[h'(x)]^2\} + \vec{\sigma}(1)\cdot\vec{\sigma}(2)\{h(x)\nabla^2 h(x) \\
& - h(x)h'(x)/x + [h'(x)]^2\} \\
& - \vec{\sigma}(1)\cdot\hat{r}\vec{\sigma}(2)\cdot\hat{r}\ \{h(x)\nabla^2 h(x) - 3h(x)h'(x)/x \\
& + [h'(x)]^2\}),
\end{aligned} \tag{10a}$$

and

$$v^{b+c}(\vec{x}) = \delta M_N^{\mathrm{qm}}[h'(x)]^2. \tag{10b}$$

In these potentials we have defined $\vec{x} = m_\pi\vec{r}$, all derivatives are with respect to x, and $h(x)$ is the regulated Yukawa function,

$$h(x) \equiv 4\pi\int \frac{d^3 l}{(2\pi)^3}\frac{F^2(l^2 m_\pi^2)}{(l^2+1)}e^{i\vec{l}\cdot\vec{x}}\to\frac{e^{-x}}{x}, \tag{11}$$

where the last form holds only for $F\equiv 1$. Note that $\nabla^2 h(x)$ generates a purely short-range contribution (i.e., a δ function if $F\equiv 1$) that is indistinguishable from other short-range contributions. It is, therefore, permissible to ignore such terms and replace $\nabla^2 h(x)$ by $h(x)$.

Equations (10), (10a), and (10b) form our main result: the model-independent, long-range part of the CSB TPEP. This result can now be used, for example, as added input to the Nijmegen phase-shift analysis [12].

The contribution from the crossed-box diagram, Eq. (9a), is proportional to the total p-n mass difference δM_N. Our results agree with the functional form of Ref. [17] in momentum space. A potential in configuration space was not presented there. The contribution from the CSB seagull, Eq. (9c), was first calculated in Ref. [18]. The triangle with mass insertion, Eq. (9b), has not been previously calculated. There are cancellations between the latter two terms, which make them proportional to the quark-mass component δM_N^{qm} of the nucleon-mass difference. While the quantity δM_N is a pre-

cisely known observable, the quantity δM_N^{qm} is not (yet). Note that our expressions (8a) and (8b) contain only one power of $(4\pi)^{-1}$, while usually a loop contributes two powers. Ignoring δM_N^{qm} and treating the dimensionless $h(x)$ and any of its derivatives as order 1, we get $V_{2\pi}^{\mathrm{CSB}} \sim (m_\pi^4/8f_\pi^2\Lambda^2)\,\delta M_N$, which is about half the estimate for δT of 25 keV. Numerical estimates of the effect of terms (9a) and (9c) have already been published [17,18]. At present, only the effect of the sum of the one-pion, (the just-calculated) two-pion, and short-range CSB potentials is constrained by experiment, while none of the individual terms are uniquely specified.

In summary, we have calculated the additional two-pion-exchange CSB nucleon-nucleon force that is the same order as (and complements) the two-pion-exchange two-nucleon CSB force previously calculated in Refs. [17,18]. This force has been developed using ChPT, and should be somewhat smaller than the other CSB interactions that we discussed. We have also discussed the interplay between the CSB OPEP and short-range CSB force, both of which are larger than the CSB TPEP. The observed CSB in the ^{3}He-^{3}H system is consistent with modern calculations. No model-independent resolution of the CSB nuclear potentials into components of different range has yet been made, because each force of one-pion, two-pion, or short range contains an (as yet) undetermined constant. It is hoped that by completing the one-pion-range and two-pion-range parts of these potentials in ChPT, a phase-shift analysis of nucleon-nucleon scattering data that incorporates this information, may be able to differentiate the components of various ranges. In addition to the two-pion-exchange nucleon-nucleon force derived here and in Refs. [17,18], there will be two-pion-exchange three-nucleon CSB forces of the order of $n=3$, which we have not treated herein.

[1] E. M. Henley, in *Isospin in Nuclear Physics*, edited by D.H. Wilkinson (North-Holland, Amsterdam, 1969), p. 15; E. M. Henley and G. A. Miller, in *Mesons and Nuclei*, edited by M. Rho and G. E. Brown (North-Holland, Amsterdam, 1979), Vol. I, p. 405.

[2] G.A. Miller, B.M.K. Nefkens, and I. Šlaus, Phys. Rep. **194**, 1 (1990); G. A. Miller and W. T. H. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, edited by W. Haxton and E. M. Henley (World Scientific, Singapore, 1995).

[3] S.A. Coon, nucl-th/9903033, invited talk given at XIIIth Inter-

national Seminar on High Energy Physics Problems (ISHEPP 13), "Relativistic Nuclear Physics and Quantum Chromodynamics," Joint Institute for Nuclear Research, Dubna, Russia, September, 1996.

[4] S. Weinberg, Physica A **96**, 327 (1979); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. B **251**, 288 (1990); **295**, 114 (1992).

[5] U. van Kolck, Ph.D. Thesis, University of Texas, 1993; U. van Kolck, Few-Body Syst., Suppl. **9**, 444 (1995).

[6] C. Ordóñez and U. van Kolck, Phys. Lett. B **291**, 459 (1992); C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994); U. van Kolck, Phys. Rev. C **49**, 2932 (1994); C. Ordóñez, L. Ray, and U. van Kolck, *ibid.* **53**, 2086 (1996).

[7] J.L. Friar and S.A. Coon, Phys. Rev. C **49**, 1272 (1994); N. Kaiser, R. Brockmann, and W. Weise, Nucl. Phys. **A625**, 758 (1997); J.-L. Ballot, M.R. Robilotta, and C.A. da Rocha, Phys. Rev. C **57**, 1574 (1998); E. Epelbaoum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. **A637**, 107 (1998); J.L. Friar, Phys. Rev. C **60**, 034002 (1999).

[8] P.F. Bedaque and U. van Kolck, Annu. Rev. Nucl. Part. Sci. **52**, 339 (2002). This is a comprehensive current review of the application of ChPT to nuclear physics.

[9] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); H. Georgi, Phys. Lett. B **298**, 187 (1993).

[10] J.L. Friar, Few-Body Syst. **22**, 161 (1997).

[11] M.C.M. Rentmeester, R.G.E. Timmermans, J.L. Friar, and J.J. de Swart, Phys. Rev. Lett. **82**, 4992 (1999).

[12] M.C.M. Rentmeester, R.G.E. Timmermans, and J.J. de Swart, Phys. Rev. C **67**, 044001 (2003).

[13] R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).

[14] U. van Kolck, M.C.M. Rentmeester, J.L. Friar, T. Goldman, and J.J. de Swart, Phys. Rev. Lett. **80**, 4386 (1998).

[15] U. van Kolck, J.L. Friar, and T. Goldman, Phys. Lett. B **371**, 169 (1996).

[16] J.L. Friar and U. van Kolck, Phys. Rev. C **60**, 034006 (1999).

[17] S.A. Coon and J.A. Niskanen, Phys. Rev. C **53**, 1154 (1996).

[18] J.A. Niskanen, Phys. Rev. C **65**, 037001 (2002).

[19] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1990); G.A. Miller, Nucl. Phys. **A578**, 345 (1990); Chin. J. Phys. (Taipei) **32**, 1075 (1994). These are fairly recent reviews of charge symmetry. See also Refs. [1–3].

[20] A.K. Opper *et al.*, nucl-ex/0306027.

[21] E.J. Stephenson *et al.*, nucl-ex/0305032.

[22] U. van Kolck, J.A. Niskanen, and G.A. Miller, Phys. Lett. B **493**, 65 (2000).

[23] D.O. Riska and Y.H. Chu, Nucl. Phys. **A235**, 499 (1974); J.V. Noble, in *The Interaction Between Medium Energy Nucleons in Nuclei—1982*, edited by H.-O. Meyer, AIP Conf. Proc. No. **97** (AIP, New York, 1983), p. 83; P.G. Blunden and M.J. Iqbal, Phys. Lett. B **385**, 25 (1996).

[24] S.A. Coon and M.D. Scadron, Phys. Rev. C **26**, 562 (1982).

[25] J.L. Friar, B.F. Gibson, and G.L. Payne, Phys. Rev. C **35**, 1502 (1987).

[26] R.A. Brandenburg, S.A. Coon, and P.U. Sauer, Nucl. Phys. **A294**, 305 (1978).

[27] Y. Wu, S. Ishikawa, and T. Sasakawa, Phys. Rev. Lett. **64**, 1875 (1990); **66**, 242(E) (1991).

[28] S.C. Pieper, V.R. Pandharipande, R.B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001) (see Table XI); S.C. Pieper and R.B. Wiringa, Annu. Rev. Nucl. Part. Sci. **51**, 53 (2001).

[29] A. Nogga, A. Kievsky, H. Kamada, W. Glöckle, L.E. Marcucci, S. Rosati, and M. Viviani, Phys. Rev. C **67**, 034004 (2003).

[30] R. Machleidt and H. Müther, Phys. Rev. C **63**, 034005 (2001); G.Q. Li and R. Machleidt, *ibid.* **58**, 1393 (1998).

[31] S.A. Coon and R.C. Barrett, Phys. Rev. C **36**, 2189 (1987).

[32] S.A. Coon, B.H.J. McKellar, and V.G.J. Stoks, Phys. Lett. B **385**, 25 (1996).

[33] E. Epelbaum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. **A671**, 295 (2000); D.R. Entem and R. Machleidt, Phys. Lett. B **524**, 93 (2002).

[34] E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, U.-G. Meißner, and H. Witała, Phys. Rev. C **66**, 064001 (2002).

[35] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966); Y. Tomozawa, Nuovo Cimento A **46**, 707 (1966).