

Flavor SU(3) breaking effects in the chiral unitary model for meson-baryon scatterings

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We examine flavor SU(3) breaking effects on meson-baryon scattering amplitudes in the chiral unitary model. It turns out that the SU(3) breaking, which appears in the leading quark mass term in the chiral expansion, cannot explain the channel dependence of the subtraction parameters of the model, which are crucial for reproducing the observed scattering amplitudes and resonance properties.

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Properties of baryonic excited states are investigated with great interest both theoretically and experimentally. Recently, the chiral unitary model has been successfully applied to this problem, especially to the first excited states of negative parity ($J^P = 1/2^-$) such as $\Lambda(1405)$ and $N(1535)$ [1–7]. In this method, based on the leading order interactions of the chiral Lagrangian and the unitarization of the S matrix, the baryon resonances are dynamically generated as quasibound states of ground state mesons and baryons. It reveals the importance of chiral dynamics not only in the threshold but also in the resonance energy region.

In the chiral unitary model for the meson-baryon scattering, we consider the coupled channel scatterings of the octet mesons and baryons. Imposing the unitarity condition on the scattering amplitudes T_{ij} in the N/D method, we obtain the scattering equation in the matrix form, Refs. [3,8],

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}, \quad (1)$$

where V_{ij} denotes the elementary interaction derived from the chiral Lagrangian at tree level. This equation can be solved algebraically. The loop integral G_i is the fundamental building block in the chiral unitary model and are regularized by the dimensional regularization:

$$\begin{aligned} G_i(\sqrt{s}) &= i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_i}{(P-q)^2 - M_i^2 + i\epsilon} \frac{1}{q^2 - m_i^2 + i\epsilon} \\ &= \frac{2M_i}{(4\pi)^2} \left[a_i(\mu) + \ln \frac{M_i^2}{\mu^2} + \frac{m_i^2 - M_i^2 + s}{2s} \ln \frac{m_i^2}{M_i^2} \right. \\ &\quad \left. + \frac{\bar{q}_i}{\sqrt{s}} (\text{Ln}_{+-} + \text{Ln}_{++} - \text{Ln}_{-+} - \text{Ln}_{--}) \right], \quad (2) \end{aligned}$$

with $\text{Ln}_{\pm\pm} \equiv \ln[\pm s \pm (M_i^2 - m_i^2) + 2\sqrt{s}\bar{q}_i]$, the masses of baryon and meson M_i and m_i , the three-momentum of the meson \bar{q}_i , the total energy in the center of mass system \sqrt{s} , and the regularization scale μ . In the present calculation, we follow the method shown in Refs. [5,6,9] and calculate only

the s -wave amplitudes since the contributions from the p wave (and higher partial waves) are much less important in energies considered here [10].

In actual calculations, it is necessary to determine the renormalization constants (a_i 's) in Eq. (2) so as to reproduce experimental data. The constants a_i are equivalent to the subtraction constants in the dispersion theory formulation [8] and, in fact, are free parameters of the model. As a consequence, they have depended very strongly on scattering channels, as shown in Table I.

In this work, we investigate whether such channel dependence of the subtraction constants could be explained by the SU(3) breaking terms of the chiral perturbation theory. By doing this, we expect that the free parameters of a_i 's could be controlled with suitable physics grounds, namely, the SU(3) breaking terms, in order to extend this model to various channels with predictive power. Here, we keep using just one subtraction constant a commonly in all channels to regularize the loop function G_i .

The use of only one subtraction constant is justified in the SU(3) symmetric limit [9,11]. Under the flavor SU(3) symmetry, the scattering amplitude should be expressed as a diagonal matrix in the SU(3) basis (1,8, . . .), which is transformed from the particle basis ($\pi N, \eta N, \dots$) by a fixed unitary matrix given by the SU(3) Clebsch-Gordan coefficients. Each component of the amplitude $T(D)$ separately satisfies the scattering equation such as Eq. (2) in each irreducible representation D . Therefore, on one hand, the function G represented in a matrix form becomes a diagonal matrix in the SU(3) basis. On the other hand, since the G function is given as a loop integral, as shown in Eq. (2), it is

TABLE I. Channel dependent subtraction constants a_i obtained in Refs. [5,6] with $\mu = 630$ MeV.

$S = -1$	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$\eta\Lambda$	$\eta\Sigma$	$K\Xi$
a_i	-1.84	-2.00	-1.83	-2.25	-2.38	-2.67
$S = 0$	πN	ηN	$K\Lambda$	$K\Sigma$		
a_i	0.711	-1.09	0.311	-4.09		

also diagonal in the particle basis. Therefore, the subtraction constants a_i 's are components of a diagonal matrix both in the SU(3) and particle bases. Such a matrix for the subtraction constants should be proportional to unity. Hence, it is concluded that there is only one subtraction constant a in the SU(3) limit.

Now let us show the Lagrangian with the flavor SU(3) breaking terms, which we use in the present work. The SU(3) breaking appears as the quark mass terms in the chiral expansion

$$\begin{aligned} \mathcal{L}_{SB} = & -\frac{Z_0}{2}\text{Tr}(d_m\bar{B}\{\xi\mathbf{m}\xi + \xi^\dagger\mathbf{m}\xi^\dagger, B\} \\ & + f_m\bar{B}[\xi\mathbf{m}\xi + \xi^\dagger\mathbf{m}\xi^\dagger, B]) \\ & -\frac{Z_1}{2}\text{Tr}(\bar{B}B)\text{Tr}(\mathbf{m}U + U^\dagger\mathbf{m}), \end{aligned} \quad (3)$$

where $f_m + d_m = 1$. Here, we employ the standard notation [12]: $\xi(\Phi) = \exp(i\Phi/\sqrt{2}f)$ and $U(\Phi) = \xi^2$. The 3×3 matrices B and Φ represent the baryon and meson fields. At this stage, we introduce one meson decay constant f , which is taken as an average value $f = 1.15f_\pi$ with $f_\pi = 93$ MeV. The quark mass matrix is defined as $\mathbf{m} = \text{diag}(\hat{m}, \hat{m}, m_s)$ with isospin symmetry $m_u = m_d \equiv \hat{m}$. The parameters Z_0, Z_1 , and f_m/d_m can be determined by the baryon masses and the pion-nucleon sigma term and, therefore, we have no new free parameters. Here, we have $Z_0 = 0.528$, $Z_1 = 1.56$, and $f_m/d_m = -0.31$ with $m_s/\hat{m} = 26$, which are determined in the chiral perturbation theory for meson masses. The terms in Eq. (5) are $O(p^2)$, based on the Gell-Mann-Oakes-Renner relation [13], which implies $m_q \propto m_\pi^2$. There are other chirally symmetric terms $O(p^2)$. Here, we do not take into account these terms, since we concentrate on the effects of the flavor SU(3) breaking.

Let us show the numerical results of the $\bar{K}N$ -induced scatterings. We use a single subtraction constant a and compare the results with and without the SU(3) breaking terms. In each case, the subtraction constant is determined by fitting threshold branching ratios [14,15]

$$\gamma = \frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-)}{\Gamma(K^- p \rightarrow \pi^- \Sigma^+)} \sim 2.36 \pm 0.04, \quad (4)$$

$$R_c = \frac{\Gamma(K^- p \rightarrow \text{charged particles})}{\Gamma(K^- p \rightarrow \text{all})} \sim 0.664 \pm 0.011, \quad (5)$$

$$R_n = \frac{\Gamma(K^- p \rightarrow \pi^0 \Lambda)}{\Gamma(K^- p \rightarrow \text{neutral particles})} \sim 0.189 \pm 0.015. \quad (6)$$

Without the symmetry breaking terms, we find the optimal value $a = -1.96$ (A). Now including the symmetry breaking term, the optimal value takes $a = -1.59$ (B). The calculated threshold values are presented in Table II. From the table, we see that the agreement with data is improved by including

TABLE II. Threshold branching ratios calculated by using $a = -1.96$ without the SU(3) breaking interaction (A), $a = -1.59$ with the SU(3) breaking interaction (B), and $a = -1.68$ with the SU(3) breaking interaction and the physical f (C). Experimental values are taken from Refs. [14,15].

	γ	R_c	R_n
Experiment	2.36 ± 0.04	0.664 ± 0.011	0.189 ± 0.015
(A)	1.80	0.624	0.225
(B)	2.19	0.623	0.179
(C)	2.35	0.626	0.172

the symmetry breaking effect. Note that this improvement is achieved without new free parameters.

Using these optimal values, we calculate the cross sections of $K^- p \rightarrow$ (various channels) and plot them in Fig. 1. Results of (A) are shown by dotted lines and those of (B) by dash-dotted lines. For (A), the agreement with data is still good, which is the well known result of the chiral unitary model [1,3,8]. Originally, the subtraction constants in the $S = -1$ channel are not very much dependent on the channels

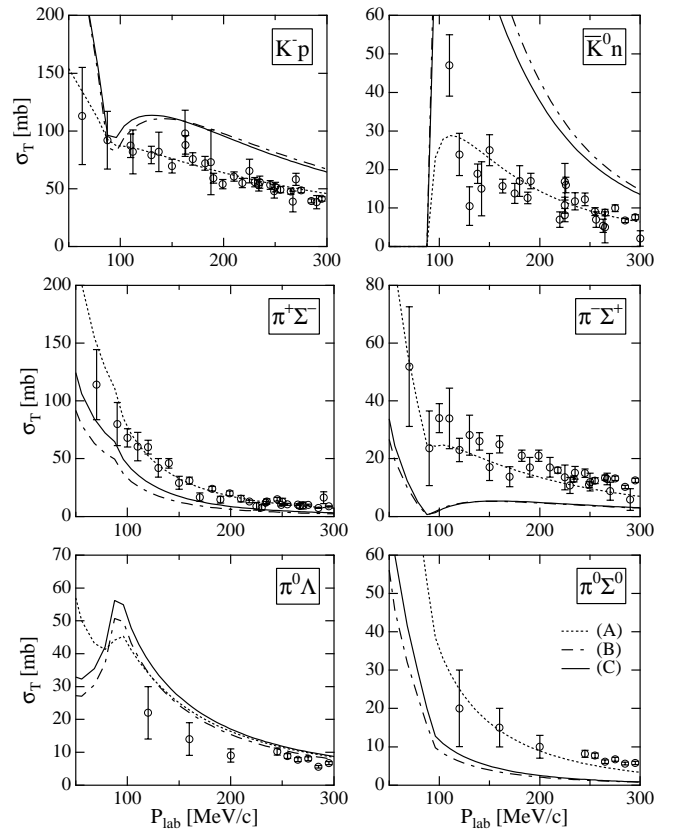


FIG. 1. Total cross sections of $K^- p$ scatterings ($S = -1$) as functions of P_{lab} , the three-momentum of initial K^- in the laboratory frame. Dotted lines show the results with $a = -1.96$ without SU(3) breaking (A), dash-dotted lines show the results including the SU(3) breaking with $a = -1.59$ (B), and solid lines show the results including the SU(3) breaking and the physical f with $a = -1.68$ (C). Open circles with error bars are experimental data taken from Refs. [16–24].

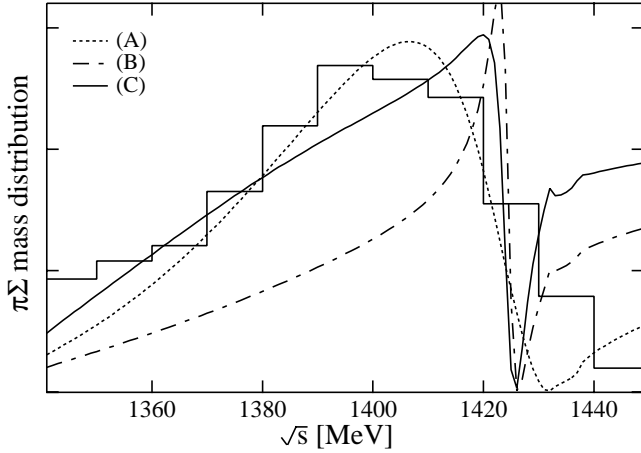


FIG. 2. Mass distributions of the $\pi\Sigma$ channel with $I=0$. Dotted line shows the result with $a=-1.96$ (A), dash-dotted line shows the result including the SU(3) breaking with $a=-1.59$ (B), and solid line shows the result including the SU(3) breaking and the physical f with $a=-1.68$ (C). Histogram are experimental data taken from Ref. [25].

and take values around $a_i \sim -2$ as shown in Table I. Now including the symmetry breaking terms (B), we find that agreement with data becomes worse (dash-dotted lines), contrary to our expectation, although the threshold branching ratios are better reproduced.

In Fig. 2, we show the $\pi\Sigma$ mass distribution in order to investigate the $\Lambda(1405)$ resonance. For (A) we obtained the dotted curve which agrees well with experimental data. If we include the symmetry breaking terms (B), once again the agreement becomes worse as shown by dash-dotted line. A sharp peak is pronounced around $\sqrt{s}=1420$ MeV, in obvious contradiction with the observed spectrum.

We also perform calculations with the inclusion of another source of the SU(3) flavor breaking, that is, the meson decay constants. We use the empirical values of the decay constants: $f_\pi=93\text{MeV}$, $f_K=1.22f_\pi$, and $f_\eta=1.3f_\pi$. The optimal value of the subtraction constant a in this case is $a=-1.68$ (C) to reproduce the threshold ratios. The results are shown in Figs. 1 and 2 with the solid lines. While the inclusion of the SU(3) breaking on the meson decay constants does not make a drastic improvement in the total cross sections of the K^-p scatterings as shown in Fig. 1, the shape of the peak in the $\pi\Sigma$ mass distribution becomes milder. However, the improvement is not enough to reproduce the experimental spectra.

We perform similar analyses for the πN scattering for the $S=0$ channel. At first, we use the common subtraction constant $a=-1.96$ obtained in the $S=-1$ channel without the symmetry breaking, since it reproduces the $\Lambda(1405)$ property well and we want to check the SU(3) flavor symmetry. Then, the attractive force between the mesons and baryons is so strong that an unexpected resonance has been generated at around $\sqrt{s}\approx 1250$ MeV. Therefore, we choose the values of a for $S=0$ by fitting the S_{11} scattering amplitudes of the πN channel up to the energy $\sqrt{s}\sim 1400$ MeV. We show the calculated scattering amplitudes of the S_{11} πN channel in Fig. 3

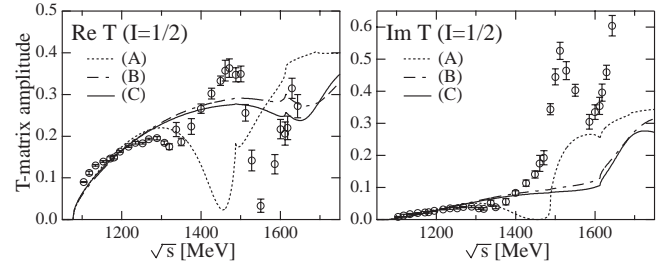


FIG. 3. Real and imaginary parts of the S_{11} T -matrix amplitudes of $\pi N \rightarrow \pi N$. Dotted lines show the results with $a=0.53$ (A), dash-dotted lines show the results including the SU(3) breaking interaction with $a=1.33$ (B), and solid lines show the results including the SU(3) breaking and the physical f with $a=2.24$ (C). Open circles with error bars are experimental data taken from Ref. [26].

for the following three cases: (A) $a=0.53$ without SU(3) breaking, (B) $a=1.33$ with SU(3) symmetry breaking, and (C) $a=2.24$ with physical meson decay constants. In all the cases, the scattering amplitudes and cross sections (we do not show the cross sections here) are not well reproduced. The result of (A) seems to have some structure around the $N(1535)$ energies, but it is far from the observed amplitude. A reasonable agreement with data is achieved only when channel dependent subtraction constants are introduced as shown in Table I [6].

In this work, motivated by the channel dependence of the parameters and symmetry consideration, we have tried to reproduce the observed cross sections and the resonance properties using a single subtraction constant. In the $S=-1$ channel, without the symmetry breaking terms, $a=-1.96$ is determined by the threshold branching ratios of the K^-p scatterings. With this parameter (A), the total cross sections of the K^-p scatterings are reproduced well as well as the mass distribution for $\Lambda(1405)$ (see Figs. 1 and 2). This value is close to $a\sim -2$ and corresponds to $\Lambda=630$ MeV in the three-momentum cutoff regularization [8]. The elementary interaction of the $\bar{K}N$ system is sufficiently attractive, and a resummation of the coupled channel interactions generates the $\Lambda(1405)$ resonance at the correct position, by imposing the unitarity condition with the natural value for the cutoff parameter. Hence, the wave function of $\Lambda(1405)$ is largely dominated by the $\bar{K}N$ component.

On the other hand, in the $S=0$ channel, if one uses the natural value for the subtraction constant as in the $S=-1$ channel, the attraction of the meson-baryon interaction becomes so strong that an unexpected resonance is generated at around $\sqrt{s}\approx 1250$ MeV. Therefore, a repulsive component is necessary to reproduce the observed πN scattering. However, with the fitted value $a\sim 0.5$, the $N(1535)$ resonance is not generated.

From the above observation, we see that the unitarized amplitudes are very sensitive to the attractive component of the interaction. Even including the SU(3) breaking terms, the interaction derived from the chiral Lagrangian alone does not describe all scattering amplitudes simultaneously. Both the fundamental interaction and the subtraction constants are important in order to reproduce proper results. For smaller a ,

the interaction becomes more attractive and for larger a , less attractive. For $S=0$, we need to choose $a\sim 0.5$ in order to suppress the attraction from the πN interaction in contrast to the natural value $a\sim -2$ in the $S=-1$ channel. Therefore, it is not possible to reproduce both the $\Lambda(1405)$ resonance properties and the low energy πN scattering with a common subtraction constant.

At this point, it is useful to discuss slightly in detail the structure of the $\Lambda(1405)$ resonance. Although the properties of $\Lambda(1405)$ have not been reproduced well with the SU(3) breaking terms as shown in the $\pi\Sigma$ mass distribution (Fig. 2), we have still found two poles for $\Lambda(1405)$ in the scattering amplitudes in the second Riemann sheet. The property of the two poles are investigated recently in detail and is related to the SU(3) structure of the meson and baryon states [8,11,27,28]. In the present study, we find $z_1(B)=1424+1.6i$ and $z_2(B)=1389+135i$ for the parameter (B). The pole z_1 , which is located very close to the real axis, is responsible for the sharp peak. When the SU(3) breaking of the meson decay constants is introduced (C), the poles are $z_1(C)=1424+2.6i$ and $z_2(C)=1363+87i$, where z_1 is still close to the real axis, while z_2 moves significantly.

The shape of the $\pi\Sigma$ mass distribution is strongly influenced by the location of the poles. In this case, the pole z_2 is sensitive to the pion decay constant. Since the resonance of $z_2(B)$ has a strong coupling to the $\pi\Sigma$ channel [11], the resonance properties are very much affected by the $\pi\Sigma$ interaction. In the chiral Lagrangian, the interaction is attractive as in the Weinberg-Tomozawa term, which contains a

coupling strength proportional to the inverse square of the pion decay constant. Therefore, by changing the decay constant from the SU(3) averaged value (107 MeV, case B) to the physical value (93 MeV, case C), the strength of the attractive $\pi\Sigma$ interaction is enhanced by $\sim 30\%$. This shifts the real part of z_2 to the lower side. At the same time this reduces the phase space and hence the imaginary part decreases.

To summarize, we have studied the flavor SU(3) symmetry breaking effect in the meson-baryon scatterings in the chiral unitary model. A reasonable method from the symmetry consideration by including the symmetry breaking mass terms, which appear in the next-to-leading order of the chiral expansion, makes theoretical predictions worse. So far, except for the use of channel dependent subtraction constants, we are not aware of what would resolve this problem. In the present framework, the role of the subtraction constants is very important.

A better understanding may be provided by introducing genuine resonance components. Very naively, such states could be quark originated as expected from the success of the quark model for baryon resonances. Full coupled channel studies of meson-baryon and quark degrees of freedom would be useful in order to resolve the problem discussed in the present study. Such an analysis will provide more microscopic understanding for the resonance structure.

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