## Generalization of the $N_p N_n$ scheme to nonyrast levels of even-even nuclei

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In this Brief Report we present the systematics of excitation energies for even-even nuclei in two regions: the  $50 < Z \le 66$ ,  $82 < N \le 104$  region, and the 66 < Z < 82,  $82 < N \le 104$  region. Using the  $N_p N_n$  scheme, we obtain compact trajectories for the ground band as well as quasi- $\beta$  and quasi- $\gamma$  bands. This suggests that the  $N_p N_n$  scheme is useful even if one extends it to nonyrast levels, and thus can serve as a general tool to disclose new types of structural evolution for higher excitations, besides the yrast states which have been investigated extensively. It is highlighted that deformations in nonyrast quasibands of nuclei with  $Z \sim 80$  and  $N \sim 104$  are often very different from those in the ground bands.

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The importance of the residual valence p-n interaction in the development of collectivity, phase/shape transitions and deformation has been stressed and discussed by many authors, such as deShalit and Goldhaber [1], Talmi [2], and Federman and Pittel [3]. This idea was further simplified by Casten [4]. Suppose that a simple product of valence proton number and valence neutron number,  $N_pN_n$ , is a reasonable estimate to gauge this interaction. Then there must be some correlations between the collective observables (such as excited energies, deformations, etc.) and  $N_pN_n$ .

The  $N_pN_n$  plots helped a lot in the past decades for the classification and a better understanding of the increasingly rich data [5]. However, almost all the plots based on the  $N_pN_n$  scheme were applied to the ground or yrast bands. One curious question is whether these simple and naive plots are helpful in any way for the nonyrast levels. In this paper we shall find that the  $N_pN_n$  plots are actually applicable to these higher excited states.

The clue lies in a concept-"quasiband," which was proposed by Sakai [6] in 1967-for nearly spherical and/or transitional nuclei. It might be inadequate to use the terms such as  $\beta$  or  $\gamma$  band(s) for levels of nuclei near closed shell nuclei, because the physical content of bands in transitional and nearly spherical nuclei is different from those of deformed nuclei. Nevertheless, for the sake of convenience, one may introduce terms such as quasiground, quasi- $\beta$ , and quasi- $\gamma$ bands in spherical and transitional nuclei, which are regarded as the counterparts of collective bands in deformed regions. The quasiground bands and quasi- $\beta$  bands thus have spin sequences  $0^+$ ,  $2^+$ ,  $4^+$ ,  $6^+$ , etc., and the quasi- $\gamma$  bands have spin sequences  $2^+$ ,  $3^+$ ,  $4^+$ ,  $5^+$ , etc. The data for even-even nuclei using "quasibands" have been compiled and revised many times [7], and have been used extensively by many authors.

The question relevant to this paper is when we go from spherical regions with vibrational motion to the transition regions with complicated modes, and finally to deformed regions with ground rotational and  $\beta$  and  $\gamma$  vibrational bands, do the excited energies of nonyrast levels evolve smoothly with their  $N_p N_n$  values? Even if the evolution is smooth, those data versus  $N_p N_n$  might be very scattered and not so useful, because the behaviors of these nonyrast levels are *not* necessarily the same as those of the yrast band.

It is therefore interesting to investigate whether the excitations based on these "bandheads" evolve similarly as the yrast band, and whether the  $N_pN_n$  scheme is relevant to classify the levels for nonyrast quasibands, in particular, whether the  $N_pN_n$  scheme is able to highlight some anomalies which reflect some interesting mechanism in the relevant levels.

We first look at the situation for the yrast levels. We focus on two regions which were studied extensively:  $50 \le Z \le 68$ ,  $84 \le N \le 104$ , and  $68 < Z \le 80$ ,  $84 \le N \le 104$ . For the former region, we use the effective numbers of valence protons for nuclei which are affected by the Z = 64 subshell. The effective numbers were tabulated in a recent paper by a fit of ground-state deformations using the  $N_pN_n$  scheme [8]. For the latter region, no effective numbers are used but the nuclei with  $Z \sim 78$  were found to exhibit anomalies of ground-state deformations which are maximized around N = 104 [9], and their  $E_{2_1}^{+}$ 's are also abnormal in the  $N_pN_n$  plots. We thus discriminate the Z > 76 and  $Z \le 76$  cases in the  $68 < Z \le 80$ ,  $84 \le N \le 104$  region. We use solid squares for  $Z \le 76$ and open squares for Z > 76.

All the data used in this Brief Report are taken from Ref. [7]. Figure 1 shows the systematics of  $E_{2_1^+}$  versus  $N_p N_n$ . We see that there exists a very strong correlation between the  $E_{2_1^+}$ 's and the  $N_n N_p$  values, if we reasonably offset the subshell effect and exclude the anomalies of  $Z \sim 78$  and  $N \sim 104$ .

For nonyrast states, we define  $E'_{3^+}(\gamma_1) = E_{3^+}(\gamma_1)$  $-E_{2^+}(\gamma_1)$ , and  $E'_{2^+}(\beta_1) = E_{2^+}(\beta_1) - E_{0^+}(\beta_1)$ . It is noted that some authors would not use the above appellation of " $\beta_1$  band" (which may imply a very specific structure that may not be true in *any* given nucleus), but would use the term of the "lowest excited K=0 band." However, we keep to Sakai's notation for short in this paper. The behavior of these E' values is then assumed to reflect the structural evolution of the nonyrast states.



FIG. 1. The  $E_{2_1^+}$  versus  $N_pN_n$  for nuclei with (a)  $50 < Z \le 66$  and  $82 < N \le 104$ ; (b) 66 < Z < 82 and  $82 < N \le 104$ . In (a), effective numbers of valence protons are taken from Ref. [8]. In (b), the nuclei with Z = 78 and 80 are labeled with open squares. See the text for details.

Figure 2 plots  $E'_{3^+}(\gamma_1)$  and  $E'_{2^+}(\beta_1)$  versus their  $N_pN_n$  values. Here the same  $N_nN_p$  numbers as in the Fig. 1 are used. Despite of the fact that the behaviors of excited states in the ground band may be different from those in nonyrast bands (say, the  $\beta_1$  or  $\gamma_1$  band here), very interestingly, the  $N_nN_p$  scheme continues to work very well. A strong correlation between the  $E'_{3^+}(\gamma_1)$  or  $E'_{2^+}(\beta_1)$  values and their corresponding  $N_nN_p$  is easily seen.

The above  $N_pN_n$  plots not only provide a naive correlation between the excited energies of nonyrast levels and their  $N_pN_n$  values, but also provide more insights into their structural evolution. For the ground band, nuclei with Z=78 and 80 are much more deformed than they "should" be, with  $N \sim 104$  nuclei deviating farthest from the correlation, which was shown in Ref. [9]. Figure 1(b) highlights the same anomalies in their  $2_1^+$  energy levels. Therefore, one would easily have the intuition that the nonyrast levels of these nuclei deviate from the correlation in a similar manner. However, Fig. 2(d) shows that the  $E'_{3+}(\gamma_1)$  values of Z=78 and N=102, 104 are quite "normal." If it is assumed that a large deformation is associated with smaller excitation energies within a band, as is well known in the ground band, the present observation indicates a very drastic change in defor-



FIG. 2. The excited energy values of quasi- $\beta$  and quasi- $\gamma$  bands versus  $N_p N_n$ . In (a) and (b)  $50 < Z \le 68$ ,  $84 \le N \le 104$ . In (c) and (d)  $68 < Z \le 80$ ,  $84 \le N \le 104$ . The  $E'_{2_1}(\beta_1)$  are given in (a) and (c), and  $E'_{3_1}(\gamma_1)$  are given in (b) and (d), respectively.

mation when we go from the ground band to the  $\gamma_1$  band for the nuclei  ${}^{180}_{78}\text{Pt}_{102}$  and  ${}^{182}_{78}\text{Pt}_{104}$ . It would be very interesting to check the data for  $E'_{3^+}(\gamma_1)$  for the Hg isotopes with  $N \leq 104$  when they will be available in the future. The deformation of  ${}^{184}_{80}\text{Hg}_{104}$  in the  $2^+(\beta_1)$  state in the  $\beta_1$  band, as discussed above, is suggested to be quite close to that of the ground state, while the deformation of  ${}^{182}_{78}\text{Pt}_{104}$  in the  $2^+(\beta_1)$ state is substantially smaller than in the ground state. Therefore, based on the  $N_pN_n$  plots of this paper and assuming a correlation between the values of deformation and excited energies in the quasibands discussed in Refs. [6,7], we suggest that the deformations of nonyrast levels of nuclei with  $N \sim 104$  and  $Z \sim 78$  are complicated, and very state dependent or band dependent even in the low-lying and low-spin states.

To summarize, compact trajectories of excitation energies of both the ground band and the quasi- $\beta$  and quasi- $\gamma$  bands for even-even nuclei have been obtained in the  $N_pN_n$  plots, showing that the  $N_pN_n$  scheme is also useful for nonyrast states, despite of the fact that the behavior of the quasi- $\beta$  and quasi- $\gamma$  bands might be different from that of the yrast one. These plots highlight the anomalies of both the ground band and higher excited bands. We suggest, using this simple scheme, that the deformations of nonyrast (but very low) levels of nuclei with  $N \sim 104$  and  $Z \sim 78$  are very state or banddependent. The  $N_pN_n$  scheme is thus very useful to disclose anomalies of structural evolution for nonyrast states without rich data.

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