Screened α decay in dense astrophysical plasmas and superstrong magnetic fields

Theodore E. Liolios*
Hellenic Naval Academy of Hydra, School of Deck Officers, Department of Science, Hydra Island 18040, Greece
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This paper shows that ultrastrong magnetic fields (such as those of magnetars) and dense astrophysical plasmas can reduce the half-life of α-decaying nuclei by many orders of magnitude. In such environments, the conventional Geiger-Nuttall law is modified so that all relevant half-lives are shifted to dramatically lower values.

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I. INTRODUCTION

α radioactivity has been known for a long time in heavy nuclei. This process, which is described in various textbooks of physics (see, for example, Ref. [1]), can be described by the nuclear reaction

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + {}^{4}_{2}\mathrm{He}_2 + Q, \tag{1}$$

where the usual textbook notation for the parent, the daughter, and the α nuclei have been used. The value of the Q energy, which is released during the decay, can be derived by the application of the conservation-of-energy principle which demands that

$$m_X c^2 = m_{X'} c^2 + m_\alpha c^2 + Q_n, \tag{2}$$

where m_i stands for the mass of nuclei (i) and c is the speed of light. This approach, adopted in most textbooks, yields the energy released due to the rearrangement of nucleons which takes place during the decay. Actually, the subscript n has been used in order to indicate that this energy is purely nuclear. However, if one wants to be precise in his application of the conservation-of-energy principle, then the atomic nature of the reactants should also be taken into account. Thus, the actual Q energy released in the emission process is given by

$$Q = Q_n + B_e(Z) - B_e(Z-2) - B_e(2), \tag{3}$$

where $B_e(Z), B_e(Z-2), B_e(2)$ are the total electron binding energies of the parent, the daughter, and the α atoms, respectively. Although in typical terrestrial conditions such atomic corrections are of little importance, they still have to be taken into account if an accurate experimental value of the half-life $T_{1/2}$ of the decay is to be obtained. On the other hand, in certain astrophysical environments, screened α decay can present spectacular properties which have never been investigated before. One of the main features of this paper is to investigate those properties and their possible implications on some long standing theories. The layout of the paper is as follows.

In Sec. II, we study the effects of the electron cloud when α decay occurs in a usual terrestrial environment. We derive new formulas which, unlike others, can also take into account the degree of ionization of the atomic cloud. The new formulas agree perfectly well with other less sophisticated ones. In Sec. III, the parent nucleus is considered to be under the influence of a superstrong magnetic field such as the one encountered in magnetars. In Sec. IV, we study the α decay of nuclei in a dense astrophysical plasma where the slow (s) and rapid (r) processes take place. In Sec. V, the usual Geiger-Nuttall law is modified appropriately for magnetars and dense astrophysical plasmas. Finally, Sec. VI presents briefly the conclusions of the present paper.

II. SCREENED α DECAY IN A TERRESTRIAL ENVIRONMENT

Let us assume that the parent nucleus is fully ionized (unscreened). During alpha decay, outside the range of the nuclear forces, the α particle (${}^{4}_{2}\mathrm{He}_2$) experiences only the repulsive Coulomb potential of the daughter nucleus (${}^{A-4}_{Z-2}X'_{N-2}$) so that the interaction energy will be

$$V_c(r) = \frac{2(Z-2)e^2}{r}. \tag{4}$$

The maximum height of the barrier will of course be

$$V_0 = \frac{2(Z-2)e^2}{R}, \tag{5}$$

where R is the minimum distance between the daughter nucleus and the α particle roughly given by

$$R = 1.3[(A-4)^{1/3} + 4^{1/3}] \text{ fm}. \tag{6}$$

The α-decay half-life $T_{1/2}^{NSC}$ of an uscreened heavy nucleus is inversely proportional to the penetration factor $P(E_\alpha)$ given by the WKB method:

$$P(E_\alpha) = \exp\left[-\frac{2\sqrt{2\mu}}{\hbar} \int_R^{r_c(E_\alpha)} \sqrt{V_c(r) - E_\alpha}\, dr \right], \tag{7}$$

where the kinetic energy of the α particle is

$$E_\alpha = \frac{A-4}{A} Q_n \tag{8}$$

and the classical turning point is given by

*URL: www.liolios.info

$$V(r_c)=E_\alpha. \tag{9}$$

We will now define the two major limits of our study whose validity and plausibility have been firmly established [2] in the study of multielectron screening effects in astrophysical reactions. Namely, if the nucleus belongs to a neutral atom, we should distinguish two modes of decay.

The adiabatic limit (AL). This limit assumes that the atomic clouds around the daughter and the α nuclei rapidly adjust themselves during the decay so that all the participants of the decay (parent, daughter, α) are always in a neutral atomic form.

The sudden limit (SL). Here we assume that throughout the decaying process, the atomic cloud of the parent nucleus remains undisturbed so that the daughter nucleus is screened by the same cloud as the parent one and the α particle is emitted fully ionized. Note that in that limit, the neutral daughter atom will be assumed to have $Z-2$ electrons so that the Thomas-Fermi (TF) theory can be used.

In the adiabatic limit, the kinetic energy E_α will be

$$E_\alpha^{AL}=E_\alpha+U_e, \tag{10}$$

where the energy shift will be given by

$$U_e=\left(\frac{A-4}{A}\right)[B_e(Z)-B_e(Z-2)-B_e(2)]. \tag{11}$$

The energy shift is usually much smaller than the kinetic energy E_α imparted on the α particle due to the rearrangement of nucleons and can be calculated in the framework of the Thomas-Fermi theory [2]. According to previous studies [3], we can always define a screening enhancement factor (SEF) so that

$$f_\alpha(Z,A,Q_n)=\frac{P^{SC}(E_\alpha+U_e)}{P^{NSC}(E_\alpha)}\geqslant 1, \tag{12}$$

where $P^{SC}(E_\alpha+U_e)$ is the screened penetration factor and $P^{NSC}(E_\alpha)$ is the unscreened one. Note that the kinetic energy E_α in Eq. (12) refers to the unscreened nucleus.

Since $T_{1/2}\sim P^{-1}(E_\alpha)$, we can write the following for the screened $T_{1/2}^{SC}(Z,A,Q_n)$ and the unscreened $T_{1/2}^{NSC}(Z,A,Q_n)$ half-lives:

$$T_{1/2}^{SC}(Z,A,Q_n)=\frac{T_{1/2}^{NSC}(Z,A,Q_n)}{f_\alpha(Z,A,Q_n)}. \tag{13}$$

On the other hand, if the screening energy shift U_e is much smaller than the kinetic energy E_α of the α particle, then

$$f_\alpha(Z,A,Q_n)=\exp\left(\pi n\frac{U_e}{E_\alpha}\right). \tag{14}$$

The screened half-life will, therefore, be given by

$$T_{1/2}^{SC}(Z,A,Q_n)=\exp\left(-\pi n\frac{U_e}{E_\alpha}\right)T_{1/2}^{NSC}(Z,A,Q_n), \tag{15}$$

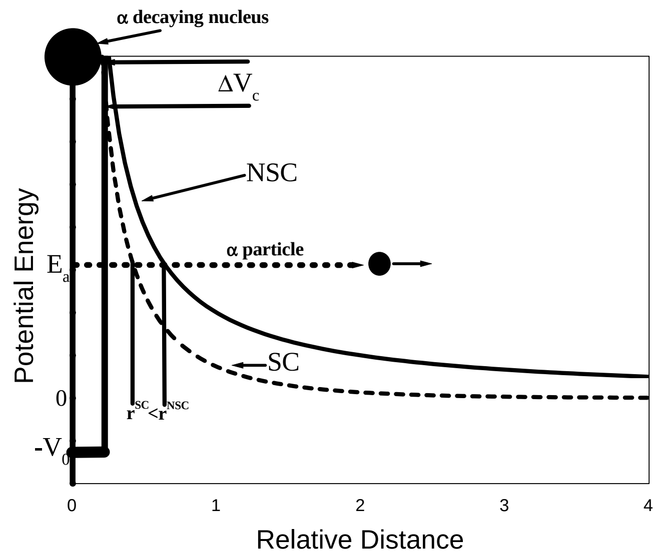

FIG. 1. A simplified picture of screened α decay. The alpha particle is emitted with a (relative) kinetic energy E_α, while the screened (r^{SC}) and unscreened (r^{NSC}) classical turning points are also shown. Note that the maximum height of the Coulomb barrier in the screened case will be shifted downwards by ΔV_c while the nuclear state of the parent nucleus is described by a potential well of depth $-V_0$. The relative distance is measured in screening radii while the mantissa has been modified (exaggerated) in certain points to help visualization of the effect.

where n is the Sommerfeld parameter for the interaction between the daughter and the helium nuclei.

Obviously, the screening effect reduces the half-life of the decaying nucleus. This is of course as expected, since the screening cloud reduces the Coulomb barrier thus easing the way of the α particle out of the parent nucleus. In Fig. 1, we have drawn a simplified picture of the screened α decay. According to that figure, the Coulomb potential practically vanished at distances further than three screening radii (see following sections and Ref. [2]).

The SEFs for heavy nuclei have been calculated in a paper [2] dealing with astrophysical nuclear reaction experiments. Actually, the derived formulas are particularly relevant here, since the atoms involved in alpha decay are always multielectronic. We can easily adjust those formulas appropriately in order to describe the relevant screening effect in α decay:

Sudden limit.

$$f_\alpha^{SL}(E)\simeq\exp\left\{-\frac{3.856(Z-2)^{7/3}}{Q_n^{3/2}}\left(\frac{A}{A-4}\right)\left[S(q)+\frac{q}{x_0(q)}\right]\right\}, \tag{16}$$

where the degree of ionization is defined by

$$q=1-\frac{\text{electrons}}{\text{protons}} \tag{17}$$

and the quantities $S(q),x_0(q)$ are defined in Ref. [2]. For neutral atoms, $q=0,x_0(0)<\infty$, and $S(0)=-1.588$ so that

$$f_{TF}^{SL}(E) \simeq \exp\left[\frac{6.1233(Z-2)^{7/3}}{Q_n^{3/2}}\left(\frac{A}{A-4}\right)\right]. \tag{18}$$

Note that the quantity Q_n is measured in keV throughout this paper.

Adiabatic limit.

$$f_{TF}^{AL}(E) \simeq \exp\left[-\frac{62(Z-2)[F(q_{12}F)Z^{7/3}-F_1(q_1)(Z-2)^{7/3}-F_2(q_2)2^{7/3}]}{Q_n^{3/2}}\frac{A}{A-4}\right], \tag{19}$$

where $F(q)$ is defined in Ref. [5]:

$$F(q)=\frac{12}{7}\left(\frac{2}{9\pi^2}\right)^{1/3}\frac{e^2}{a_H}\left[S(q)+\frac{q^2}{x_0(q)}\right], \tag{20}$$

and a_H is the Bohr radius.

If we assume, according to the AL, that the parent, the daughter, and the α nuclei are all in a neutral atomic state, then $F(q_{12})=F(q_1)=F(q_2)=-20.98$ eV and the relevant SEF is written

$$f_{\alpha}^{AL}(E) \simeq \exp\left[1.297(Z-2)\frac{[Z^{7/3}-(Z-2)^{7/3}-2^{7/3}]}{Q_n^{3/2}}\frac{A}{A-4}\right]. \tag{21}$$

We have compared Eq. (18) to Eq. (21) and have found that their results practically coincide for all α-decaying nuclei. This remarkable coincidence proves the validity of the present method and allows us to use the simple SL formula for the description of the screening effect in terrestrial α decay.

Usually in α-decay studies, experimentalists use a semiempirical formula for screening energy,

$$U_e=65.3(Z-2)^{7/5}-80(Z-2)^{2/5}\ \text{eV}, \tag{22}$$

which when inserted in Eq. (15) gives roughly the same results as Eq. (18).

Thus we have derived alternative formulas for the accurate description of the screening effect in α decay. Those formulas, which are based on the solid mathematical framework of the TF theory, are the only ones available that can take into account the degree of ionization of the participant nuclei.

III. MAGNETICALLY CATALYZED α DECAY IN MAGNETARS

Nowadays, there is a growing body of evidence (see Ref. [4] for a review) for a population of neutron stars with magnetic fields of the order of 10^{15} G, which is much larger than the typical magnetic field of a neutron star (i.e., 10^{12} G). These "magnetars" are distinguished from radio pulsars and accreeting binary neutron stars not only by the strength of their field but also by the fact that their decaying magnetic field is their primary energy source. Moreover, recent observations [4] provide strong evidence for the validity of the old hypothesis that two separate classes of astronomical x-ray sources—the soft gamma repeaters and the anomalous x-ray pulsars—are actually different manifestations of this peculiar type of star. The giant magnetic field of magnetars has a significant and observable effect on quantum electrodynamic processes operating near the star. It can also support strong and persistent electrical currents, which alter the spin down of the star and contribute to the continuous glow of x rays and optical light observed in between outbursts. In this section, we will investigate its effects on the abundances of α-decaying heavy elements which may find themselves in the neighborhood of a magnetar.

In large magnetic fields, such as those existing in the atmospheres of neutron stars, atomic clouds are compressed both perpendicular and parallel to the magnetic field direction [5]. The effects of giant magnetic fields ($B \geqslant 10^{12}$ G) on hydrogen and helium atoms have been extensively studied by many authors. Various studies have appeared focusing on such topics as the formation of molecules and chains [6] (and references therein) and nuclear fusion [7]. However, no author has ever considered the effects of such a magnetic field on α-decay processes.

Let us consider the heavy neutral atom of an α-decaying element, which is under the influence of such an ultrastrong magnetic field. We will disregard all exchange, thermal, and relativistic effects as a first approximation and adopt the usual supermagnetic field notation [6] $B_{12}=(B/10^{12})$ G, $B_0 = 2.351\times 10^9$ G, $b=B/B_0$. Moreover, the parent and the daughter nuclei are considered spinless (e.g., U-238,Th-234), just like the α particle, so that we can disregard any coupling with the external magnetic field. Note that the effect of a superstrong magnetic field on nuclear properties has also been disregarded in the study of magnetically catalyzed fusion reactions [6,7]. However, in such cases where the fusing nuclei are not always spinless, coupling effects may play a non-negligible role.

In any case, the present study will exclusively focus on the perturbation of half-lives due to atomic (tunneling) effects allowing for an extra perturbation term due to purely nuclear effects. This assumption is based on the Born-Oppenheimer (BO) approximation according to which there is a complete decoupling between electronic and nuclear degrees of freedom. (The BO approximation has been used frequently in screening studies [8–10].)

Sudden limit. The magnetic TF screened Coulomb potential will be given by

$$\Phi_{sc}(r)=\frac{Ze}{r}\phi\left(\frac{r}{R_B}\right), \tag{23}$$

where the scaling parameter is $R_B=55\,133Z^{1/5}b^{-2/5}$ fm and the universal function $\phi(x)$ is given by Kadomtsev's [11] differential equation with the initial conditions of Ref. [12],

$$\frac{d^2\phi(x)}{dx^2}=(x\phi)^{1/2},\quad \phi(0)=1,\phi'(0)=-0.938\,965, \tag{24}$$

where we have set $x=r/R_B$.

The above model is valid for neutral atoms when the condition $Z^{4/3}\ll b\ll 2Z^3$ (or according to another study [13], $Z^{4/3}\ll b\ll 4.25Z^3$) is satisfied.

In the sudden limit approximation, the α particle, on its way out of the parent nucleus, will have to penetrate the screened Coulomb potential given by Eq. (23) so that the tunneling will involve an interaction potential energy given by

$$V_{sc}(r,B)=\frac{2(Z-2)e^2}{r}\phi\left(\frac{r}{R_B}\right), \tag{25}$$

where $R_B=55{,}133(Z-2)^{1/5}b^{-2/5}$ fm and the respective SEF will of course be given by the screened versus the unscreened penetration factor:

$$f_\alpha^{SL}(Z,A,B)=\exp\left[-\frac{2\sqrt{2\mu}}{\hbar}\left(\int_R^{r_c(E_\alpha,B)}\sqrt{V_{sc}(r,B)-E_\alpha}dr-\int_R^{r_c(E_\alpha)}\sqrt{V_c(r)-E_\alpha}dr\right)\right], \tag{26}$$

where the classical turning point in the magnetized α-decay is given as usual by

$$V_{sc}(r_c,B)=E_\alpha. \tag{27}$$

We might follow the treatment of Sec. II where we derived approximate analytic SL SEFs for conventional α decay, assuming that there exists a constant screening energy shift (much smaller than E_α). This method, which actually replaces Eq. (26) with Eq. (14), would indeed yield very elegant analytic SEFs but we cannot afford to make any approximations yet. This is due to the fact that we are studying a completely novel effect and thus we must be certain about the accuracy of our results. Thus we will numerically evaluate the SL SEFs given by Eq. (26).

Moreover, in some cases, relativistic corrections to the TF atom may become important. In order to investigate relativistic effects, we will employ the equation derived by Hill, Grout, and March [14] and Shivamoggi and Mulser [15],

$$\frac{d^2\phi(x)}{dx^2}=(x\phi)^{1/2}\left(1+\Lambda\frac{\phi}{x}\right)^{1/2},\quad \phi(0)=1,$$

$$\phi'(0)=-0.938\,965, \tag{28}$$

where the relativistic parameter Λ stands for

$$\Lambda=\frac{Ze^2}{2m_ec^2R_B}\ll 1 \tag{29}$$

or else

$$Z^{4/5}b^{2/5}\ll 38\,286. \tag{30}$$

We have run extensive numerical integrations of Eq. (26) applying the above relativistic model to various magnetic fields and heavy nuclei. Provided that the conditions $\Lambda\ll 1$ and $Z^{4/3}\ll b\ll 2Z^3$ are valid, we have concluded that relativistic corrections to $f_\alpha^{SL}(Z,A,B)$ are negligible.

Adiabatic limit. In an ultrastrong magnetic field, due to the multielectron nature of an α-decaying atom, the sudden limit is expected to yield practically the same results as the adiabatic limit. This has been shown in the preceding section for conventional screened α decay and common sense demands that this is the case when supermagnetized atoms are considered. It is obvious that subtracting two electrons from the large number of them which orbit the parent nucleus will induce a very small perturbation to the charge distribution around it. This of course means that the sudden limit is expected to be very accurate just as was shown in the preceding section. We can use the total binding energy of a supermagnetized heavy atom $E\simeq -13.6Z^{9/5}b^{2/5}$ eV, in order to obtain the screening shift yielded by the adiabatic limit which, according to Eq. (11), reads

$$U_{TF}^{AL}=0.0136\,b^{2/5}[Z^{9/5}-(Z-2)^{9/5}-2^{9/5}]\ \text{keV} \tag{31}$$

and after some algebra the relevant AL SEF is found to be given by the formula

$$f_\alpha^{AL}(Z,A,Q,B)\simeq\exp\left[\frac{0.85}{Q_n^{3/2}}(Z-2)\left(\frac{A}{A-4}\right)b^{2/5}\times[Z^{9/5}-(Z-2)^{9/5}-2^{9/5}]\right]. \tag{32}$$

In Fig. 2 ,we have numerically integrated Eq. (26) in order to plot the magnetic SL SEF for the α decay of ^{238}U ($T_{1/2}=4.46\times 10^9$ yr), and ^{235}U($T_{1/2}=0.7\times 10^9$ yr) with respect to the magnetic field strength (measured in units of 2.351×10^9 G). We have also included the AL SEFs given by Eq. (32). The solid vertical bar signifies the upper limit of our model for the nuclei in question, while the lower limit is actually that field for which the SEF becomes roughly unity. The results of both limits are very close to each other just as predicted.

We have particularly chosen these two uranium isotopes as they are thoroughly used as cosmochronological tools [16,17]. By observing the reduction in the half-lives of those α-decaying isotopes in Fig. 1, we can argue that ultrastrong magnetic fields act as giant transformers of ^{238}U,^{235}U into their respective daughters ^{234}Th and ^{231}Th.

According to Fig. 2, magnetars can reduce the half-life of uranium by four orders of magnitude. The effect is of a simi-

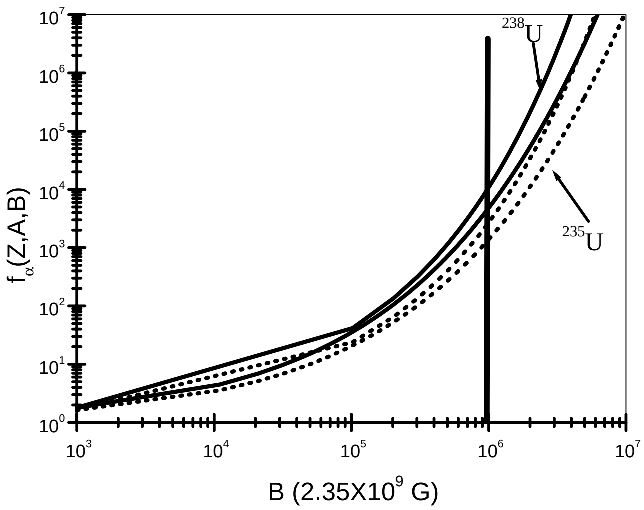

FIG. 2. The ratio of the unscreened half-life $T_{1/2}^{NSC}$ to the screened one $T_{1/2}^{SC}$ (i.e., the SEF) for two important α-decaying isotopes with respect to the magnetic field strength (measured in units of 2.351×10^9 G): ^{238}U (upper/lower solid curves), ^{235}U (upper/lower dotted curves). The upper (lower) curves stand for the AL (SL) SEFs for each isotope.

lar order of magnitude for other heavy α-decaying nuclei as well. Although the mathematics of our model forbids its use at fields larger than 10^{15} G, it is more than obvious that half-lives will be further reduced at ever stronger fields where our model is invalid.

Another interesting fact about ultramagnetized α decay is that the compression of the electron cloud is particularly large in the direction perpendicular to the magnetic field, while it is very small in the parallel direction. Thus, the emission of α particles will not be isotropic as is usually the case in terrestrial process but it will occur in such a way that it peaks in the perpendicular direction. The phenomenon of anisotropically enhanced α decay has never been investigated before. In Sec. IV, we will prove that it also appears in the *s* and *r* processes in stellar plasmas, although in such sites the screened half-lives can be up to nine orders of magnitude smaller than the unscreened ones.

Finally, we note that the screened half-life (the SEF) is an increasing (decreasing) function of the decay energy Q_n. This is due to the fact that the classical turning point is a decreasing function of the energy Q_n so that the smaller the Q_n the thicker the barrier that the α particle will have to cross and thus the stronger the screening effect. Our tests have shown that the TF screened Coulomb potential exhibits a marked deviation from the unscreened one mainly at large distances from the nucleus. Thus, large turning points allow the screening effect to play a more important role in the tunneling process.

IV. SCREENED α DECAY IN DENSE ASTROPHYSICAL PLASMAS

Although various authors have studied the effects of a very dense astrophysical plasma on fusion reaction rates, no author has ever studied such effects on the α-decay process. Actually, heavy nuclei which decay by α particle emission exist only in the form of seeds in ordinary massive stars where the zero metallicity scenario is usually valid for most stellar evolution calculations. For example, in population I stars the uranium abundance is roughly [18] 11 (6) orders of magnitude smaller than that of hydrogen (silicon). Nevertheless, there are stellar processes such as the *s* and *r* ones which generate a significant number of heavy nuclei which are then ejected into space via a supernova explosion. Admittedly, the production of such nuclei does not play any significant role in stellar evolution which is governed by light element production-destruction processes. However, the abundances of heavy elements give important information about the formation of the universe and therefore all factors which influence them deserve special attention. In this section, we will prove that α decay in dense stellar plasmas can play a much more important role in the destruction of heavy elements than initially thought [20].

Let us consider a heavy α-decaying nucleus ${}^A_Z M_N$ in a fully ionized multicomponent plasma which is at thermodynamic equilibrium. We will modify Mitler's model [21] for screened thermonuclear reactions in order to derive screening corrections in our α-decay study. Actually, this modification is perfectly legitimate, since all plasma screening models are concerned with the perturbation of the penetration factor $P(E)$ which is the same for both fusion and decay.

Sudden limit. In that limit, we assume that the plasma which screens the nucleus ${}^A_Z M_N$ remains undisturbed by the emission of the α particle. According to Secs. II and III, we model this process by considering the interaction between the daughter nucleus and the α particle inside the plasma. If we modify Mitler's model, the screened Coulomb potential is given by

$$V_{sc}^M(r)=\frac{2(Z-2)}{r}-C_o+C_1r^2, \quad r<r_0, \tag{33}$$

$$V_{sc}^M(r)=\frac{2(Z-2)C}{r}, \quad r>r_0, \tag{34}$$

where

$$x=\frac{r_0}{R_D}=\left(\frac{3(Z-2)}{4\pi n_e R_D^3}+1\right)^{1/3}-1. \tag{35}$$

R_D is the usual Debye-Huckel radius (corrected of course for electron degeneracy), n_e is the average electron number density in the plasma, and the constants C_0, C_1 are given by

$$C_1=\frac{2}{3}\pi e n_e, \quad C_0=2\pi e n_e R_D^2 x(x+2). \tag{36}$$

In order to derive a simple analytic formula for the SL SEF, let us first assume that the screening energy due to the stellar plasma is much smaller than the decay energy Q_n, which is usually a few MeV. To the extent that this assumption is wrong, our calculation would yield a conservative estimate of the associated SEF (i.e., the SEF will certainly be larger).

However, as we will prove, this assumption is perfectly legitimate in most stellar plasmas away from solidification. In such a case, the screening energy will be the properly modified Mitler's shift:

$$U_e^M = \frac{2(Z-2)e^2}{R_D} g(x), \tag{37}$$

where

$$g(x) = \left(\frac{1+x_1/2}{1+x_1+x_1^2/2} \right), \tag{38}$$

then using Eq. (14) we obtain

$$f_\alpha^M = \exp\left(\pi n \frac{U_e^M}{Q_n} \right)$$

or else

$$f_\alpha^M(Z,A,\rho,T) = \exp\left(\frac{2(Z-2)e^2 \pi n g(x)}{Q_n R_D} \right). \tag{39}$$

Adiabatic limit. In order to be more precise, we have to take into account the screening effects induced by the α particle as well as that the assumption of a very small screening energy is not necessarily true for all cases. Both these factors are taken into account by the adiabatic limit. If we further assume that the stellar plasma where the α decay takes place has not reached the solidification point, which is the case in *s* and *r* process environments, then the screening enhancement factor will be the respective Mitler's [21] SEF modified appropriately for an α-decay process:

$$f_M = (f_S)^{g(\zeta_1,\zeta_2)}, \tag{40}$$

where f_S is the usual Salpeter's [22] SEF and the parameter g is [23]

$$g(\zeta_1,\zeta_2) = \frac{9}{10}\left(\frac{1}{\zeta_1 \zeta_2} \right)[(\zeta_1+\zeta_2+1)^{5/3} - (\zeta_1+1)^{5/3} - (\zeta_2+1)^{5/3} + 1], \tag{41}$$

where ζ_1, ζ_2 are dimensionless parameters which for the α-decay process are given by

$$\zeta_1 = \frac{3(Z-2)}{4\pi N_e R_D^3}, \quad \zeta_2 = \frac{3\times 2}{4\pi N_e R_D^3}. \tag{42}$$

Thus, the screened half-life of a particular α-decaying heavy nucleus will now be a function of plasma composition, density, and temperature:

$$T_{1/2}^{SC}(\rho,T) = (f_\alpha^M)^{-1} T_{1/2}^{NSC}. \tag{43}$$

We have compared the results given by Eqs. (40) and (39) and have found that they are practically the same for all relevant stellar environments.. Therefore the simple formula given by Eq. (39) describes accurately the reduction of the half-life of the screened α-decaying nuclei. In any case, the SEF is well constrained by Eqs. (39) and (40).

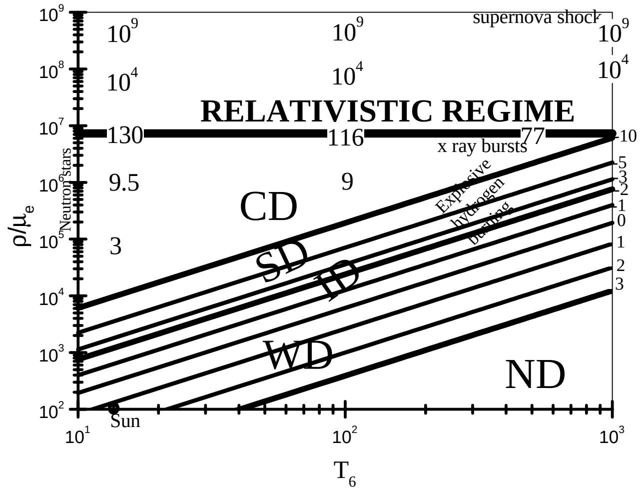

FIG. 3. The ratio of the unscreened half-life $T_{1/2}^{NSC}$ to the screened one $T_{1/2}^{SC}$ (i.e., the SEF) for the isotope ^{238}U in various stellar plasmas. The vertical column of values $(-10,-5$, etc.) on the right-hand-side mantissa stands for the well-known degeneracy parameter a which is related to the electron chemical potential μ_e via the formula $a=-\mu_e/kT$. In the plot, five electron degeneracy regimes are shown: ND, nondegenerate; WD, weakly degenerate; ID, intermediate degeneracy; SD, strong degeneracy; CD, complete degeneracy (defined in Ref. [2]). The numbers in the whited-out areas of the plot correspond to the SEFs for ^{238}U calculated according to the theory of Sec. IV. Various stellar sites are shown while the thick horizontal line at $\rho = 7.3\times 10^6$ g/cm^3 defines the relativistic domain of the the equation of state.

The whited-out figures of Fig. 3 represent the ratio of the unscreened half-life $T_{1/2}^{NSC}$ versus the screened one $T_{1/2}^{SC}$ (i.e., the SEF) for the isotope ^{238}U in various stellar environments. The screening reduction of half-lives is not very sensitive to temperature for completely degenerate environments. This is due to the fact that, as can be shown after some algebra, in such ultradense environments the screening energy is approximately given for both limits (see Fig. 2) by the simple formula

$$U_e^{AL} = 0.0176\left(\frac{\rho}{\mu_e} \right)^{1/3} [Z^{5/3} - (Z-2)^{5/3} - 2^{5/3}] \ \text{keV}, \tag{44}$$

which is independent of temperature. Equation (44) is actually Salpeter's [22] formula for completely degenerate electron gases modified appropriately for our study. The relevant SEF is of course still given by Eq. (14). Note that, according to Fig. 3, in supernova shocks, where the *r* process takes place, the screening effect is particularly accentuated.

Most heavy nuclei, which undergo α decay, are produced [24] during the *s* and *r* processes of stellar evolution either during a long epoch of thousands of years or during short pulses and shocks of milliseconds. Let us assume that such a nucleus of abundance $N(t)$ is produced in a dense stellar plasma. We know that this abundance will actually follow the usual law of exponential decay that is

$$N(t)=N(0)\exp\left(-\frac{\ln 2}{T_{1/2}}t\right). \tag{45}$$

According to the evolutionary stage of the star, there are various mechanisms which generate or destroy the heavy nucleus in question with the paramount ones being neutron capture (i.e., *s* and *r* processes), β decay, and photodisintegration. It is a very important finding of the present paper that α-decay half-lives in dense astrophysical plasmas can become so small that α decay can play an equally significant role in the evolution of heavy element abundances. Instead of presenting a detailed analysis of this effect, we can give a fair approximation to the actual extent of the new effect by comparing the screened half-lives to the time scale of the destruction/production mechanisms: First we note that if we disregard all other factors, then α decay alone can reduce the stellar abundance of a nucleus by three orders of magnitude within ten half-lives. Since the half-life itself in a screened environment can be many orders of magnitude smaller than the unscreened one, it is obvious that a new important mechanism of destruction has been discovered which so far has been considered negligible for a lot of heavy elements. In fact, if τ is the time scale for a certain process which produces or destroys a heavy nucleus, then the abundances of all nuclei whose unscreened half-life is of the order of

$$T_{1/2}^{NSC}\sim f_{\alpha}^{M}(Z,A,\rho,T)\tau \tag{46}$$

will be considerably affected by the α-decay process. Considering that the time scales of the *s* and *r* processes vary [19] from seconds to several hundred thousands of years, the importance of the present findings is now obvious.

V. THE GEIGER-NUTTALL LAW FOR MAGNETARS AND DENSE THERMONUCLEAR PLASMAS

The success of the quantum mechanical description of α decay has been established by the Geiger-Nuttall (GN) law [25], which is described in most textbooks dealing with α-decay theory. According to that law, in an unscreened environment, a good fit to the half life data $T_{1/2}$ of a large number of α emitters is obtained with the formula

$$\log_{10}T_{1/2}(Z,A,Q_n)=C_1(Z)Q_n^{-1/2}+C_2, \tag{47}$$

where $C_2$is a constant and $C_1(Z)$ is a slowly varying parameter of the atomic number *Z*.

These relationships have been proved more effective than most microscopically based calculations in the prediction of α-decay half-lives. Their application to the decays of all isotopic sequences of the heaviest elements with neutron number $N>126$ has long been known [26] to yield spectacularly straight line plots. The validity of this linear correlation has been established [27,28] for lighter nuclei, as well.

According to the new findings of the present paper, the GN law in magnetar atmospheres and dense thermonuclear plasmas should be modified. Therefore, if the GN law in an unscreened environment is given as a plot of the half-life with respect to the atomic number and the decay energy, then in the previously studied screened environments all such plots should be modified so that the readings on the mantissa should be shifted by $\log_{10}f_{\alpha}^{-1}$. Thus, in our study of α decay in magnetars and dense plasmas, we can use all conventional GN plots and data currently available provided we apply the following rules:

$$\log_{10}T_{1/2}^{SC}(Z,A,B)=\log_{10}T_{1/2}^{NSC}(Z,A)-\log_{10}f_{\alpha}^{TF}(Z,A,B) \tag{48}$$

for magnetars and

$$\log_{10}T_{1/2}^{SC}(Z,A,\rho,T)=\log_{10}T_{1/2}^{NSC}(Z,A)-\log_{10}f_{\alpha}^{M}(Z,A,\rho,T) \tag{49}$$

for dense stellar plasmas.

A final argument concerning heavy element production/destruction should be expressed: α decay is a nuclear process which bears a lot of physical similarities to fission. Since fission is also important (e.g., the californium hypothesis [24]) in the evolution of heavy element abundances, we argue that similar strong screening effects are bound to appear when fissionable nuclei exist in the astrophysical environments discussed in the present paper.

VI. CONCLUSIONS

We have studied electron screening effects in α-decay processes applying a formalism which so far has been exclusively used in the study of astrophysical fusion reactions. We have derived alternative analytic SEF formulas for terrestrial α-decay processes which can also take into account the degree of ionization of the decaying atom.

More importantly, this paper also studies the effects of superstrong magnetic fields (such as those of magnetars) on α decay proving that the relevant half-life can be reduced by several orders of magnitude. The whole effect, which is expressed in the form of a very handy formula, namely Eq. (32), may possibly have notable implications on heavy element abundances and the cosmochronological models that rely on them.

Finally, it has been shown, for the first time, that α decay half-lives in dense astrophysical plasmas can be reduced by many orders of magnitude due to plasma screening. Those results may have significant implications on the evolution of heavy element abundances during the *s* and *r* processes. A very simple analytical formula has been produced [i.e., Eq. (39)], which can take into account all those novel effects.

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