PHYSICAL REVIEW C **68**, 015801 (2003)

Possibility of $\Lambda\Lambda$ pairing and its dependence on background density in a relativistic Hartree-Bogoliubov model

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(Received 8 August 2002; revised manuscript received 12 March 2003; published 22 July 2003)

We calculate a $\Lambda\Lambda$ pairing gap in binary mixed matter of nucleons and Λ hyperons within the relativistic Hartree-Bogoliubov model. Λ hyperons to be paired up are immersed in background nucleons in a normal state. The gap is calculated with a one-boson-exchange interaction obtained from a relativistic Lagrangian. It is found that at background density $\rho_N = 2.5\rho_0$ the $\Lambda\Lambda$ pairing gap is very small, and that a denser background makes it rapidly suppressed. This result suggests a mechanism, specific to mixed matter dealt with relativistic models, of its dependence on the nucleon density. An effect of weaker $\Lambda\Lambda$ attraction on the gap is also examined in connection with the revised information of the $\Lambda\Lambda$ interaction.

DOI: 10.1103/PhysRevC.68.015801 PACS number(s): 26.60.+c, 21.80.+a, 21.65.+f

I. INTRODUCTION

Pairing correlation in hadronic matter has been attracting attention due to a close relationship between properties of neutron stars and its interior superfluidity. Superfluidity inside neutron stars affects, for instance, heat capacity and neutrino emissivity. These quantities relate to the cooling processes of neutron stars.

In neutron stars, several types of baryon pairings appear. It is strongly believed that neutrons form the $^1S_0$ pairs in the inner crust region [1–3]. At the corresponding density $10^{-3}\rho_0 \lesssim \rho_B \lesssim 0.7\rho_0$, where ρ_0 is the saturation density of symmetric nuclear matter, the $^1S_0$ partial wave of the nucleon-nucleon (NN) interaction is attractive: In infinite matter an attraction, no matter how weak it is, brings about the BCS instability to the ground state. This type of pairing has been most extensively studied for decades using various models. Also important is the $^3P_2$ neutron pairing in the outer core region up to $\rho_B \sim 2\rho_0$. The $^3P_2$ partial wave of the NN interaction is attractive enough there for neutrons to be in a superfluid state [1,4]. On the other hand, the $^1S_0$ partial wave would become repulsive there so that the $^1S_0$ neutron pairs would disappear. Instead, the $^1S_0$ proton pairing is expected to be realized owing to its small fraction [1,3].

In the inner core region, baryon density becomes much larger ($\rho_B \gtrsim 2\rho_0$) and various hyperons may appear [5]. Some are expected to form pairs in the same way as the NN pairing owing to the attractive $^1S_0$ partial wave of the hyperon-hyperon (YY) interaction. Moreover, interspecies pairing such as Λ-neutron pairing may be realized at the total baryon densities higher than $\rho_B \gtrsim 4\rho_0$ where fractions of the two kinds of baryons are expected to be comparable. These kinds of pairings affect the properties of neutron stars through, say, suppression of the hyperon direct URCA processes. Whether Λ hyperons are in a superfluid state or not plays a decisive role for the microscopic understanding of neutron stars: Λ hyperons in a normal state would lead to too rapid cooling of the stars and force one to modify the cooling scenarios. Conversely, one can extract information on baryonic force and inner structure of neutron stars from these phenomena. Thus studying neutron stars is the driving force for the study on baryon superfluidity.

Unfortunately, the magnitude of the hyperon pairing gaps is still uncertain. More studies are needed exploiting available information from various sources such as the hypernuclear spectroscopy, direct observation of neutron stars, and so on.

Our aim of this study is twofold. One is to explore an effect of Dirac effective mass of Λ hyperons on the $\Lambda\Lambda$ pairing correlations in binary mixed matter composed of Λ hyperons and nucleons. In this respect, recognizing the significance of covariant representation led to the remarkable developments in nuclear/hadron physics in the past three decades. As is well known nowadays, cancellation between large Lorentz scalar and vector fields provides a proper saturation mechanism of nuclear matter. Typical examples are the phenomenological relativistic mean field (RMF) model and the microscopic Dirac-Brueckner-Hartree-Fock (DBHF) approach. Especially in the latter, self-consistent treatment of a nucleon spinor with a bare NN interaction brings the satura-

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tion points predicted by the nonrelativistic BHF approach towards the empirical one by a repulsive relativistic effect. This self-consistency is the key ingredient of the relativistic models. Then, we would like to ask a following question: What does the self-consistency bring to superfluidity in the composite hadronic matter? This is an important issue of the study of the neutron star matter with complex composition of baryons using relativistic models.

The other is to investigate an impact of the recent experimental finding on the $\Lambda\Lambda$ pairing. The KEK-PS experiment $E373$ (especially the NAGARA event [6]) refuted the "old" information on the $\Lambda\Lambda$ interaction, which has ruled hypernuclear systems for three decades. The event unambiguously determined the binding energy of the two Λ hyperons $B_{\Lambda\Lambda}$ in ${}^{6}_{\Lambda\Lambda}\mathrm{He}$. Most importantly, it suggests that the $\Lambda\Lambda$ interaction is weaker than it was thought before. If this is confirmed, the new information ought to have a significant impact on the microscopic understanding of the properties of neutron stars.

Unlike the NN pairing, there are only a few studies on the hyperon pairing. It was first studied in a nonrelativistic framework by Balberg and Barnea [7]. Then their results were applied to the study on cooling of neutron stars by Schaab, Balberg, and Schaffner-Bielich [8]. They obtained the $\Lambda\Lambda$ pairing gap in symmetric nuclear matter using an interaction based on the G matrix in symmetric nuclear matter and an approximation of nonrelativistic effective mass obtained from single-particle energies with first-order Hartree-Fock corrections, though their motivation was application to the physics of neutron stars. Their conclusion was that the maximal pairing gap became larger as the background density increased; at the same time, the effective mass of Λ hyperons became smaller. Since a smaller effective mass generally leads to a smaller pairing gap, this conclusion is against general expectations. Takatsuka and Tamagaki subsequently studied the problem using two types of bare $\Lambda\Lambda$ interactions and two types of hyperon core models [9]. Aiming at a better approximation of neutron star matter, they used the nonrelativistic effective mass which was obtained from the G matrix calculation for composite matter of neutrons and Λ hyperons, and was dependent on a total baryon density and a Λ fraction. Their gaps were somewhat smaller than Balberg and Barnea's due to a smaller effective mass and an appropriate choice of the interaction. They also showed that the result had a considerable dependence on the interactions and the hyperon core models owing to related uncertainties. An important thing common to these past studies is the use of the $\Lambda\Lambda$ interactions that are too attractive considering the consequence of the NAGARA event, which was unavailable at that time.

Therefore, we study the ${}^{1}S_{0}$ $\Lambda\Lambda$ pairing in binary mixed matter of nucleons and Λ hyperons using relativistic interactions, which reflects the new experimental information for the first time. The Λ hyperons are immersed in pure neutron matter or symmetric nuclear matter that is treated as a background. We use the relativistic Hartree-Bogoliubov (RHB) model in which density dependence of the interaction is automatically taken into account via the Lorentz structure. The density dependence that is an inherent mechanism in relativistic models may lead to a novel behavior of the pairing gap: Since pairs are formed in medium, medium effects on a particle-particle (p-p) channel interaction should be considered. In the RHB model, bare baryon masses are reduced by the scalar mean field. This decreased mass is the Dirac effective mass [10,5]. The mass decrease may change the pairing gap to some extent in comparison with that obtained with the bare masses. Although the two preceding studies also introduced the medium effects, each had a purely nonrelativistic origin. It has nothing to do with the Lorentz structure and the Dirac effective mass. We thus intend to compare with the results of the first study by Balberg and Barnea neglecting, for the time being, complexity of Λ-Σ^{0} mixing that probably occurs in asymmetric nuclear matter; this mixing will be discussed in Sec. III E. Besides, other constituents predicted to exist in neutron stars and equilibration, such as chemical equilibrium of neutron star matter, are ignored so that we narrow down arguments to the impact of the revision on the $\Lambda\Lambda$ pairing properties. Such a plain treatment should be taken as the very first step of our study on the hyperon pairing with the recently revised interactions in the neutron star matter.

This paper is organized as follows. In Sec. II, we illustrate the Lagrangian of the system and the gap equation for the ${}^{1}S_{0}$ $\Lambda\Lambda$ pairing. In Sec. III, we present results of the $\Lambda\Lambda$ pairing properties in the binary hadronic matter. Section IV contains a summary.

II. MODEL

A. Lagrangian

Our starting model Lagrangian of the system has the following expression:

$$\begin{aligned}\mathcal{L}=&\sum_{B=N,\Lambda}\bar{\psi}_B(i\gamma_\mu\partial^\mu-M_B)\psi_B\\&+\frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma)-\frac{1}{2}m_\sigma^2\sigma^2-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu\\&-\sum_{B=N,\,\Lambda}g_{\sigma B}\bar{\psi}_B\sigma\psi_B-\sum_{B=N,\Lambda}g_{\omega B}\bar{\psi}_B\gamma_\mu\omega^\mu\psi_B\\&+\frac{1}{2}(\partial_\mu\sigma^*)(\partial^\mu\sigma^*)-\frac{1}{2}m_{\sigma*}^2\sigma^{*2}-\frac{1}{4}S_{\mu\nu}S^{\mu\nu}+\frac{1}{2}m_\phi^2\phi_\mu\phi^\mu\\&-g_{\sigma*\Lambda}\bar{\psi}_\Lambda\sigma^*\psi_\Lambda-g_{\phi\Lambda}\bar{\psi}_\Lambda\gamma_\mu\phi^\mu\psi_\Lambda,\end{aligned}\tag{1}$$

where $\Omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$ and $S_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu$. The symbols M_N, M_Λ, m_σ, m_ω, $m_{\sigma*}$, and m_ϕ are the masses of nucleons, Λ hyperons, σ bosons, ω mesons, σ^* bosons, and ϕ mesons, respectively. Table I displays these masses, coupling constants, and their ratios used in this study. The model Lagrangian used in the study of the binding energy of double-Λ hypernuclei within the RMF model [11] was originally proposed by Schaffner *et al.* in the study of multiply strange hadronic systems including baryon species N, Λ, and Ξ [12,13]. The parameter set was determined by Marcos *et al.* [11] to reproduce the bulk properties of hadronic matter and finite nuclear systems including double-Λ hypernuclei

TABLE I. Parameter set HS-m2 [11]. We choose $M_N = 938.0$ MeV and $M_\Lambda = 1115.6$ MeV.

Mass (MeV)		(Coupling constant g)/(ratio α)	
m_σ	520.0	$g_{\sigma N}$	10.481
		$\alpha_\sigma = g_{\sigma\Lambda}/g_{\sigma N}$	0.623
m_ω	783.0	$g_{\omega N}$	13.814
		$\alpha_\omega = g_{\omega\Lambda}/g_{\omega N}$	2/3
$m_{\sigma*}$	975.0	$\alpha_{\sigma*} = g_{\sigma*\Lambda}/g_{\sigma N}$	Varied
m_ϕ	1020.0	$\alpha_\phi = g_{\phi\Lambda}/g_{\omega N}$	$-\sqrt{2}/3$

according to the old information. In Ref. [11], the original nucleon Lagrangian HS (an acronym of Horowitz-Serot) with "σ-ω" Lagrangian for the Λ sector, which is called "model 1," was supplemented with two additional boson fields. One is a scalar-isoscalar boson σ^* (975 MeV) and the other is a vector-isoscalar meson ϕ (1020 MeV). The Lagrangian that contains σ^* and ϕ is referred to as "model 2." Following Ref. [8], we call the model Lagrangian, Eq. (1), as "HS-m2" in short from now on. Details of model 1 and model 2 are described in Ref. [13]. These additional bosons were originally introduced to achieve strong attraction between Λ hyperons. Since it is, however, now probable that $\Lambda\Lambda$ interaction is weaker than what has been believed, we regard σ^* as the device for controlling the $\Lambda\Lambda$ attraction in this study. As a rough guide, we refer to Fig. 1 of Ref. [11] that shows the dependence of the bond energy,

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) - 2B_\Lambda({}^{A-1}_{\Lambda}Z), \tag{2}$$

on the coupling ratio $\alpha_{\sigma*} = g_{\sigma*\Lambda}/g_{\sigma N}$. On the contrary, the ratio $\alpha_\phi = g_{\phi\Lambda}/g_{\omega N}$ is fixed by the SU(6) relations.

B. Gap equation

Next, we explain the gap equation for the $\Lambda\Lambda$ pairing. The equations of motion are solved by the procedure illustrated in Ref. [14], except that pairing correlation is introduced by the Gor'kov factorization and hyperons other than Λ are absent in the present study. The Fock contribution is neglected and the so-called no-sea approximation is employed. This is the RHB model. As for the pairing gap, the gap equation,

$$\Delta(p) = -\frac{1}{8\pi^2}\int_0^\infty \frac{\Delta(k)}{\sqrt{(E_k^{(\Lambda)} - E_{k_{\mathrm{F}}}^{(\Lambda)})^2 + \Delta^2(k)}} \times \overline{v}(M_\Lambda^*; p,k) k^2 dk, \tag{3}$$

is solved numerically, where $E_k^{(\Lambda)}$ is the single-particle energy of Λ hyperons and $\overline{v}(M_\Lambda^*; p,k)$ is the p-p channel $\Lambda\Lambda$ interaction. Although the quality of bare $\Lambda\Lambda$ interactions is steadily getting higher, they still have room for improvement mainly due to sparsity of experimental data. Hence, also does predictability of the $\Lambda\Lambda$ pairing properties. Following the same prescription as in our previous studies of NN pairing [15,16], we therefore use the phenomenological interaction to study the possibility of the $\Lambda\Lambda$ pairing and its dependence on the density of background matter. We adopt to the p-p channel interaction the one-boson-exchange (OBE) interaction obtained from an RMF parameter set with the help of form factors. For convenience, we refer to this OBE interaction as "RMF interaction" in this study. The antisymmetrized matrix element of the RMF interaction V is defined by

$$\overline{v}(M_\Lambda^*; \mathbf{p},\mathbf{k}) = \langle \mathbf{p}s', \widetilde{\mathbf{p}s}' | V | \mathbf{k}s, \widetilde{\mathbf{k}s} \rangle - \langle \mathbf{p}s', \widetilde{\mathbf{p}s}' | V | \widetilde{\mathbf{k}s}, \mathbf{k}s \rangle, \tag{4}$$

where tildes denote time reversal. Since $\overline{v}$ depends on the Dirac effective mass of Λ hyperons in a baryon spinor, its dependence is explicitly indicated in Eqs. (3) and (4). Integration with respect to the angle between $\mathbf{p}$ and $\mathbf{k}$ has been performed in Eq. (3) to project out the S-wave component. Moreover, the form factors are included in $\overline{v}$ to regulate its high-momentum contributions. We use a Bonn-type form factor:

$$f(\mathbf{q}^2) = \frac{\Lambda_{\mathrm{c}}^2 - m_i^2}{\Lambda_{\mathrm{c}}^2 + \mathbf{q}^2}, \tag{5}$$

where $\mathbf{q}$ is the three-momentum transfer and m_i ($i = \sigma$, ω, σ^*, and ϕ) are the meson masses. The cutoff mass Λ_{c} is 7.26 fm^{-1} for all the mesons employed, whose value was determined in our study of the NN pairing for a form factor of a type different from Eq. (5) [16]; an effect of varying the cutoff mass will be examined later. We thereby aim at phenomenological construction of the effective $\Lambda\Lambda$ interaction usable in hadronic matter with a finite Λ fraction, such as the Gogny force in the nucleon sector. Note that the form factors are applied only to the p-p channel since we respect the fact that the Hartree part is unaffected by a monopole form factor [Eq. (9) shown later] for which the value of Λ_{c} was determined in our study of the NN pairing.

Combining the equations for the Dirac effective mass of nucleons,

$$M_N^* = M_N + g_{\sigma N}\langle\sigma\rangle, \tag{6}$$

that of Λ hyperons,

$$M_\Lambda^* = M_\Lambda + g_{\sigma\Lambda}\langle\sigma\rangle + g_{\sigma*\Lambda}\langle\sigma^*\rangle, \tag{7}$$

and Eq. (3), we obtain the coupled equations to be solved numerically.

Concerning the choice of the p-p channel interaction, it has been still open to argument whether the G matrix or a bare interaction is suitable for the gap equation. The p-p channel interaction in Ref. [7] is classified into the former and that in Ref. [9] into the latter. As described in Ref. [17], the gap equation itself handles short range correlations which the G matrix also does; this is the well-known double-counting of the correlations. Takatsuka and Tamagaki also argued that use of the G matrix itself or the interaction based on it in the gap equation cannot be justified [9]. Thus it is widely received to use a bare interaction in the gap equation with regard to NN pairing in infinite matter.

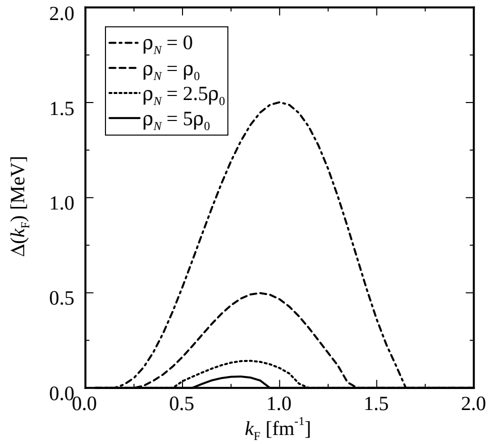

FIG. 1. $\Lambda\Lambda$ pairing gap at the Fermi surface of Λ hyperons, for pure neutron background densities $\rho_N = 0$, ρ_0, $2.5\rho_0$, and $5\rho_0$. The coupling ratio $\alpha_{\sigma*} = 0.5$ is used.

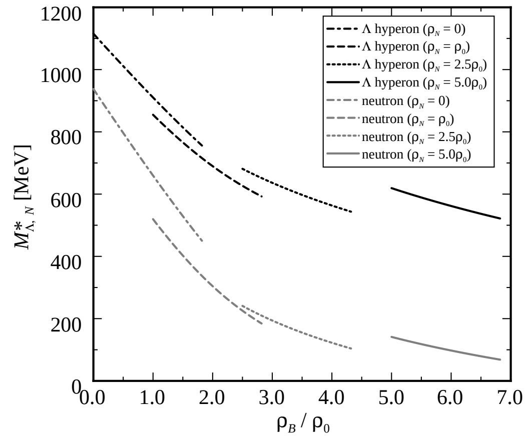

FIG. 2. Effective masses of Λ hyperons and neutrons for pure neutron background densities $\rho_N = 0$, ρ_0, $2.5\rho_0$, and $5\rho_0$. The coupling ratio $\alpha_{\sigma*} = 0.5$ is used.

Meanwhile, from a practical viewpoint such as application to finite nuclei, the Gogny force is often used as the *p-p* channel interaction as well as the particle-hole channel interaction. While it is regarded as a reasonable parametrization of the *G* matrix in the sense that it gives saturation properties of symmetric nuclear matter, it can reproduce the pairing gaps obtained from bare *NN* interactions. In fact, the Gogny force imitates bare interactions in the low-density limit [18,19]. With reference to practical usage, Matsuzaki and Tanigawa solved the gap equation of *NN* pairing using the RMF interactions [15,16]. In Ref. [16], phenomenological form factors were introduced to the *p-p* channel, with the Hartree part unchanged, so that the constructed interactions reproduced the results obtained from the *bare* Bonn potential. They successfully adjusted cutoff masses to reproduce both the pairing gaps and the coherence lengths at the same time. The cutoff masses independent of density were qualitatively similar to those of the Bonn potential. This shows the usefulness of the prescription. Nonetheless, we do not use the same procedure in the present study since we have no model of the *YY* interactions to follow, and our primary interest is behavior of the gap; we use the cutoff mass for the $\Lambda\Lambda$ interaction obtained from the study of the *NN* pairing [16] instead, which is qualitatively similar to those of the *YY* interactions.

Last but not least, we neglect effects beyond the mean field approximation, such as the dispersive effects [20,21] and the polarization effects [22,23], to concentrate on the effects that stem from the binary character of the matter. Both are, however, very important for the *NN* pairing since they turned out to reduce the pairing gap. Work in this direction is necessary in the future.

III. RESULTS AND DISCUSSIONS

A. Effect of Dirac effective mass decrease

Figure 1 shows the resulting $^1S_0$ $\Lambda\Lambda$ pairing gap at the Fermi surface in pure neutron matter of densities ρ_N at 0, ρ_0, $2.5\rho_0$, and $5\rho_0$, with $\alpha_{\sigma*} = 0.5$ chosen. This value of the coupling ratio can reproduce the bond energy, Eq. (2), of about 1 MeV in the RMF model, which is suggested by the NAGARA event. Contrary to the results obtained by Balberg and Barnea, the $\Lambda\Lambda$ pairing gap becomes suppressed as the neutron density increases. At $\rho_N = 2.5\rho_0$, where Λ hyperons already appear in some models of neutron stars [14], the maximal pairing gap is about 0.15 MeV. Since there are probably no Λ hyperons at $\rho_N = 0$ and ρ_0 in neutron star matter, the pairing gaps at these densities are quite hypothetical.

Figure 2 shows the density dependence of baryon effective masses, M_N^* and M_Λ^*, as functions of the total baryon density ρ_B. The background neutron densities are fixed here, so that variations in ρ_B correspond to those in the Fermi momentum of Λ hyperons. Since we ignore the chemical equilibrium here, the curves of the effective masses have discontinuous jumps; each piece corresponds to the fixed neutron densities, $\rho_N = 0$, ρ_0, $2.5\rho_0$, and $5.0\rho_0$. Consideration of the chemical equilibrium should connect them with each other. Nevertheless, we obtain the values qualitatively similar to the ones shown in, for example, Fig. 4 of Ref. [14]. It is therefore concluded that the in-medium property of the phenomenological $\Lambda\Lambda$ interaction used in this study is justifiable. The effective mass of neutrons decreases steeply as the total baryon density increases, while mildly does the effective mass of Λ hyperons due to the weaker coupling of Λ hyperons to the scalar bosons than the coupling of nucleons.

For the *p-p* channel, we use the RMF interaction as stated above. The interaction contains the Dirac effective mass of Λ hyperons, Eq. (7), through which the medium effects are introduced; the coupling of Λ hyperons to σ bosons, to which nucleons also couple, brings about the dependence on the background density. Figure 3 represents the $\Lambda\Lambda$ RMF interaction derived from the parameter set HS-m2. It is shown that increasing the background neutron density suppresses attractive contribution from low momenta. This is the main reason why the $\Lambda\Lambda$ pairing gap is smaller in denser background.

This new mechanism of the suppression is inherent in relativistic models that respect the Lorentz structure as

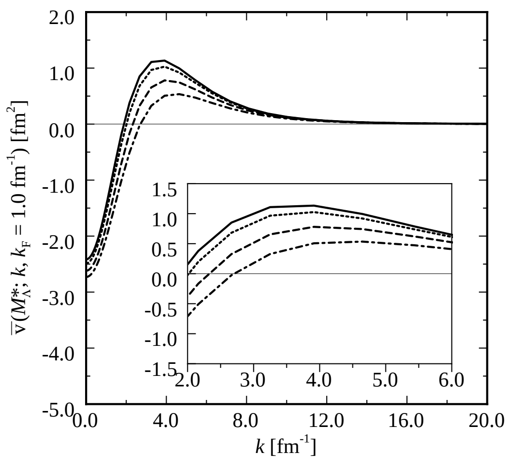

FIG. 3. ΛΛ RMF interaction $\overline{v}(M_\Lambda^*;k,k_\mathrm{F})$ at the Fermi momentum of Λ hyperons ($k_\mathrm{F}=1.0\ \mathrm{fm}^{-1}$), for pure neutron background densities $\rho_N=0$, ρ_0, $2.5\rho_0$, and $5\rho_0$, corresponding to $M_\Lambda^*=1068$, 813, 660, and 605 MeV, respectively. The coupling ratio $\alpha_{\sigma*}=0.5$ is used. The legend is the same as in Fig. 1. The inset shows a magnification of the region around the repulsive bumps.

shown in Eq. (7). It is shown that the decrease of the effective baryon mass plays an indispensable role when it is used self-consistently in the baryon spinor.

What is important is that relativistic models naturally lead to a density-dependent interaction through a self-consistent baryon spinor, where the bare mass in a free spinor is replaced with the Dirac effective mass. An apt example is the saturation of symmetric nuclear matter in the DBHF approach [24]. Requirement of the self-consistency for the nucleon spinor, that is, use of the Dirac effective mass in the nucleon spinor effectively gives repulsion to the binding energy of symmetric nuclear matter. Consequently, it pushes the saturation points predicted by nonrelativistic models toward the empirical one. It seems that our finding is similar to this repulsive effect. Furthermore, the mechanism is apparently not restricted to ΛΛ pairs. It is probable that other kinds of YY pairs have the same trend.

B. Effect of the NAGARA event

Next we explore the effect of the NAGARA event on the ΛΛ pairing. With relation to the revised information on the ΛΛ interaction, we vary the ratio $\alpha_{\sigma*}=g_{\sigma*\Lambda}/g_{\sigma N}$ between 0.4 and 0.6, referring to Fig. 1 of Ref. [11]: Thereby, we control the attractive component of the interaction. Figure 4 represents the maximal ΛΛ pairing gap at the Fermi surface of Λ hyperons as a function of the strength of ΛΛ attraction and the background density of pure neutron matter. From this figure as well as Fig. 1, one reads that the suppression of the gap occurs in denser background of neutrons. Moreover, it may even vanish in the end (though the result depends on the choice of RMF parameter sets and a cutoff mass as will be shown later). This varying $\alpha_{\sigma*}$ reveals likely closing of the gap at smaller $\alpha_{\sigma*}$ (i.e., weaker ΛΛ attraction) and its strong suppression in the denser neutron background. This result implies that the aforementioned mechanism acts in concert with the weakened attraction for closing the gap. Hence, the

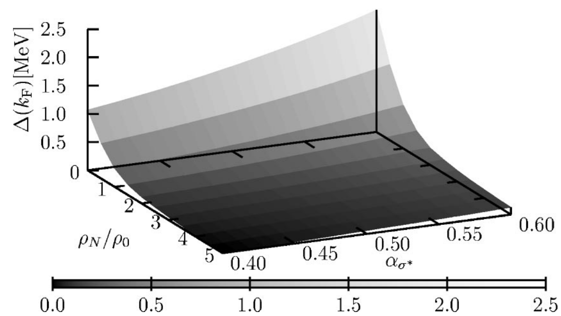

FIG. 4. Maximal ΛΛ pairing gap as a function of the strength of ΛΛ attraction and the background density in pure neutron matter.

ΛΛ pairing correlation in dense pure neutron matter becomes less likely than before. The result is almost the same with the background of symmetric nuclear matter.

In the light of the neutron star cooling, the absence of the ΛΛ pairing might call for the pairing of other hyperonic species and a modification of its scenarios. More realistic approximation of the internal composition of neutron stars needs a condition of chemical equilibrium, which plays a decisive role. Under the condition, other hyperons will emerge as the background density increases. Takatsuka *et al.* studied the $\Sigma^-\Sigma^-$ and $\Xi^-\Xi^-$ pairings and showed their possibility [25]. Nevertheless, the possibility of the ΛΛ pairing in *neutron star matter* stands unsettled in our model. As elucidated above, both the increase of the baryon density and the weakening of the ΛΛ attraction reduce the ΛΛ pairing gap in the binary matter. On the other hand, it is likely that complex composition of baryons in neutron star matter affects relevant scalar boson fields, namely, the Dirac effective masses of the baryons. In our present model of the binary matter, one cannot thereby estimate how the above two mechanisms toward closing the gap work in neutron star matter.

C. Comparison with nonrelativistic study

Now we make a comparison between relativistic and nonrelativistic predictions. For the comparison with the nonrelativistic results of Balberg and Barnea [7], we calculate the ΛΛ pairing gap in symmetric nuclear matter. Figure 5 represents our result. Slightly smaller gap than that obtained from the calculation of pure neutron matter (Fig. 1) reflects the smaller Dirac effective mass of Λ hyperons in symmetric nuclear matter than that in pure neutron matter.

We would like to note two remarkable differences between their result and ours. One difference is the dependence of the gap on the background density. Strikingly, ours is opposite to theirs (cf. Fig. 4 of Ref. [7]). This is brought about *directly* and *indirectly* by decrease of the Dirac effective mass. We intend by the word "directly" that we can grasp the decrease of the gap through an expression in the weak-coupling approximation,

$$\Delta(k_\mathrm{F})\propto\exp\left[-\frac{1}{N(k_\mathrm{F})|\overline{v}(k_\mathrm{F},k_\mathrm{F})|}\right], \tag{8}$$

where $N(k_\mathrm{F})=E_{k_\mathrm{F}}^{(\Lambda)}k_\mathrm{F}/2\pi^2\hbar^2$ is the density of states at the

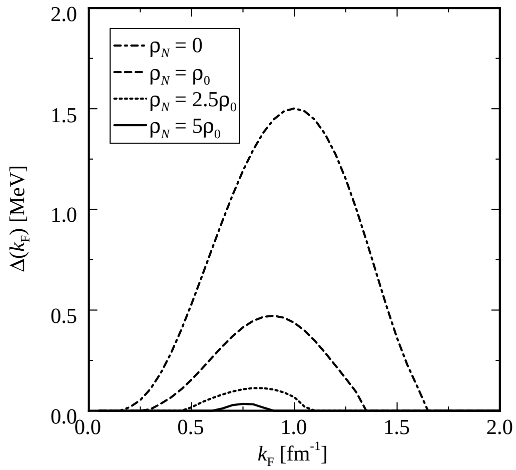

FIG. 5. $\Lambda\Lambda$ pairing gap at the Fermi surface of Λ hyperons, for nucleon background densities $\rho_N = 0$, ρ_0, $2.5\rho_0$, and $5\rho_0$. The coupling ratio $\alpha_{\sigma*} = 0.5$ is used.

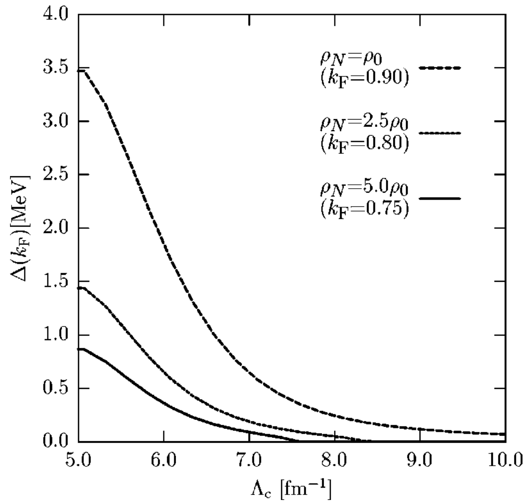

FIG. 6. Cutoff mass dependence of the $\Lambda\Lambda$ pairing gap at the Fermi surface in pure neutron matter. The coupling ratio $\alpha_{\sigma*} = 0.5$ is used.

Fermi surface. Equation (8) shows that the smaller the Dirac effective mass becomes, the smaller the density of states does, which makes the gap smaller. Note that we use the approximation for rough estimation here and the full integration of the gap equation (3) is done throughout the present study. Meanwhile, we intend by the word ''indirectly'' that the gap decreases as the density increases due to gradual weakening of the attraction in the *p-p* interaction, which is shown in Fig. 3. The other difference is the region of the Fermi momentum of Λ hyperons where the gaps are open. While the regions in their result are similar in all densities presented, Fig. 5 shows that the regions in our result narrow as the background density increases.

Finally, the result at $\rho_N = \rho_0$ may have relevance to the $\Lambda\Lambda$ pairing correlation around the center of hypernuclei [26]. We obtain the maximal gap $\Delta(k_F = 0.9\ \mathrm{fm}^{-1}) \simeq 0.5$ MeV.

Prior to the present study, Elgarøy *et al.* studied [27] relativistic effects on the neutron and proton pairing in neutron star matter, and made a comparison with a nonrelativistic result [28]. Their result shows a large effect of ''minimal relativity'' [29] on the $^3P_2$ neutron pairing while a small one on the $^1S_0$ proton pairing. They explained that using DBHF single-particle energies and factors of the minimal relativity are the causes of much smaller neutron pairing gap. As for our model, the factor corresponding to the minimal relativity is already included in the *p-p* interaction owing to the normalization of the Dirac spinor, $u^\dagger u = 1$.

D. Choice of form factor

Also noteworthy is a form factor: In this section, we investigate a dependence of the gap on the cutoff mass for each type of the form factor. We use the purely phenomenological form factor at each Λ hyperon-meson vertex to regulate the high-momentum components of the *p-p* interaction as in Ref. [16]. So far, we have chosen in this paper the Bonn-type form factor, Eq. (5), with the cutoff mass $\Lambda_c = 7.26\ \mathrm{fm}^{-1}$. In contrast to the *NN* pairing, there has yet been no proper guide to determine the cutoff mass in the form factors for the $\Lambda\Lambda$ pairing. Hence, we adopted the value from our previous study of the *NN* pairing [16].

However, the type of the form factor and the value of the cutoff mass significantly affect the magnitude of the pairing gap. We therefore calculate the dependence of the $\Lambda\Lambda$ pairing gap at the Fermi surface on the cutoff mass Λ_c in the form factors of the Bonn-type, Eq. (5). The cutoff mass is taken to be larger than 5 fm^{-1}, which roughly corresponds to mass of the heaviest meson employed (namely, ϕ); otherwise the interaction is unphysical. The result is shown in Fig. 6, in which the Fermi momenta of Λ hyperons are fixed to $k_F = 0.90$, 0.80, and 0.75 fm^{-1} for background density of pure neutron matter, $\rho_N = \rho_0$, $2.5\rho_0$, and $5.0\rho_0$, respectively. As expected, varying the cutoff mass changes the gaps steeply since it changes the balance of the attraction and the repulsion of the interaction. The peaks around $\Lambda_c \sim 5\ \mathrm{fm}^{-1}$ are due to consecutive suppression of the attraction (σ^* boson) and the repulsion (ϕ meson) by the form factor. Nonetheless, the importance of this result lies in the fact that the gaps become smaller in denser background for *any* cutoff mass. Thus the arbitrariness does not alter our conclusions.

On the other hand, a form factor of monopole type,

$$f(\mathbf{q}^2) = \frac{\Lambda_c^2}{\Lambda_c^2 + \mathbf{q}^2}, \tag{9}$$

with moderate cutoff masses does not give a finite pairing gap in our model with the HS-m2 set; using other RMF parameter sets may give finite gaps, and their gentle dependence on the cutoff mass is expected in the manner similar to the *NN* pairing [16].

We would like to stress that we do not intend to provide the optimal parameter sets for the description of $\Lambda\Lambda$ pairing for the time being; or rather, we intend to present its general trend of density dependence within the present model irrespective of a given set of parameters. Determining them precisely is inevitably deferred until the guide is available.

E. Λ-Σ mixing

Before concluding the discussions, we present relevant issues for further study. We have employed pure neutron matter and symmetric nuclear matter as background in this study. The physics of neutron stars requires isospin asymmetricity of the background matter, which should be considered in the next study. In connection with this, coherent and incoherent Λ-Σ couplings should be mentioned. Akaishi *et al.* argued that they are important to understand *s*-shell hypernuclei and, in particular, resolve the longstanding problem of overbinding in $^{5}_{\Lambda}$He [30]. Furthermore, the coherent Λ-Σ coupling predicts the coherent Λ-Σ^0 mixing in dense neutron-rich infinite matter [31]. As a consequence, the Λ-Σ^0 mixing shall come into play in asymmetric nuclear matter, which may change the critical density of hyperon emergence, and eventually scenarios of the evolution of neutron stars. It is therefore important to introduce it into the models of dense hadronic matter. Concerning relativistic models, the introduction into both infinite and finite systems has been performed using the quantum hadrodynamics along with the concept of effective field theory [32,33]; these works also show the importance of the mixing. On the other hand, the QCD sum rules predict that relatively weak mixing would be realized for Λ and Σ^0 of the positive energy state, while strong mixing would be realized for the negative energy state [34]. Hence, room for arguments over this issue still remains, and the effect of the mixing on the pairing is unknown so far. In all cases, we have ignored the mixing since it is beyond our scope of this study.

IV. SUMMARY

We have investigated the ΛΛ pairing in binary mixed matter of nucleons and Λ hyperons. Our theoretical framework is the RHB model combined with the RMF interaction both in the particle-hole and the particle-particle channels; we have used it to naturally incorporate the medium effect into the latter, as well as the former, via the Dirac effective mass of Λ hyperons in the Λ spinor. Two noteworthy conclusions are thereby drawn. First, we have found that the value of the ΛΛ pairing gap decreases as the background nucleon density increases. This result is opposite to that reported in Ref. [7]. It should be emphasized again that the origin of the medium effects is different from each other. Second, in concert with the effect of increasing the background density, the weaker the ΛΛ attraction becomes, the more the ΛΛ pairing gap gets suppressed. The present model shows the possibility that it may eventually disappear. Some arbitrariness of the form factors still remains since it has yet been virtually difficult to determine the cutoff mass precisely. The magnitudes of the ΛΛ pairing gap consequently remain uncertain because they have the strong dependence on the cutoff mass. We, however, have shown the essential result that the denser background reduces them gradually to be unchanged for different values of the cutoff mass. For transparency of the investigation, we have ignored the chemical equilibrium. This should be considered in the future. Unfortunately, our knowledge of the hyperon-hyperon interaction is somewhat limited at the moment. We notwithstanding expect that qualitative trends presented in this study will survive in more refined models, and also in pairing of another hyperonic species.

ACKNOWLEDGMENTS

One of us (T.T.) is grateful to the Japan Society for the Promotion of Science for research support and the members of the research group for hadron science at the Japan Atomic Energy Research Institute (JAERI) for fruitful discussions and their hospitality.

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