Two-proton overlap functions in the Jastrow correlation method and cross section of the $^{16}O(e,e'pp)^{14}C$ reaction

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The two-proton overlap functions for the $^{16}O(e,e'pp)^{14}C$ reaction are calculated on the basis of a two-body density matrix obtained within the Jastrow correlation method which incorporates short-range correlations. The resulting overlap functions are applied to calculate cross sections of the $^{16}O(e,e'pp)^{14}C$ reactions leading to the 0^+ ground and the 1^+ (11.3-MeV) excited states of the residual nucleus. The results are compared with the cross sections calculated with different theoretical treatments of the two-nucleon overlap functions.

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I. INTRODUCTION

Since a long time, electromagnetically induced two-nucleon knockout has been devised as the most direct tool to study the properties of nucleon pairs within nuclei at short distance and thus the dynamical short-range correlations (SRC) in a nucleus [1,2]. Correlations in the nuclear wave function beyond the mean-field approximation are very important to describe the properties of nuclear structure [3,4].

Two nucleons can be naturally ejected by two-body currents, which effectively take into account the influence of subnuclear degrees of freedom such as mesons and isobars. Direct insight into SRC can be obtained from the process where the real or virtual photon hits, through a one-body current, either nucleon of a correlated pair and both nucleons are then ejected from the nucleus. The role and relevance of these two competing processes can be different in different reactions and kinematics. It is thus possible to envisage situations where either process is dominant and various specific effects can be disentangled and separately investigated.

Various theoretical models for cross-section calculations have been developed in recent years in order to explore the effects of ground-state NN correlations on $(e,e'NN)$ [5–10] and (γ,NN) [11–17] knockout reactions. It appears from these studies that the most promising tool for investigating SRC in nuclei is represented by the $(e,e'pp)$ reaction, where the effect of the two-body currents is less dominant as compared to the $(e,e'pn)$ and (γ,NN) processes. Measurements of the exclusive $^{16}O(e,e'pp)^{14}C$ reaction performed at NIKHEF in Amsterdam [18–20] and MAMI in Mainz [21,22] have confirmed, in comparison with the theoretical results, the validity of the direct knockout mechanism for transitions to low-lying states of the residual nucleus and have given clear evidence of SRC for the transition to the ground state of ^{14}C. This result opens up good perspectives that further theoretical and experimental efforts on the two-nucleon knockout will be able to determine SRC.

One of the main ingredients in the transition matrix elements of exclusive two-nucleon knockout reactions is the two-nucleon overlap function (TOF). The TOF contains information on nuclear structure and correlations and allows one to write the cross section in terms of the two-hole spectral function [2]. The TOF's and their properties are widely reviewed, e.g., in Ref. [23]. In an inclusive reaction, integrating the spectral function over the whole energy spectrum produces the two-body density matrix (TDM).

In Ref. [8], the TOF's for the $^{16}O(e,e'pp)^{14}C$ reaction are given by the product of a coupled and fully antisymmetrized pair function of the shell model and a Jastrow-type correlation function which incorporates SRC. Only the central term of the correlation function is retained in the calculation.

A more sophisticated treatment is used in Ref. [9], where the TOF's are obtained from an explicit calculation of the two-proton spectral function of ^{16}O [24], which includes, with some approximations but consistently, both SRC and long-range correlations (LRC). The numerical predictions of this model are in reasonable and in some cases in good agreement with data [19–22].

Although satisfactory, these first comparisons with data have also raised problems that require further theoretical investigation and a more refined treatment of nuclear structure aspects in the calculation of the spectral function. The study of the two-hole spectral functions including different types of correlations, however, requires substantial efforts in computational many-body physics and represents a very difficult task.

A different method to calculate the TOF's has been suggested in Ref. [25] using the established general relationships connecting the TOF's with the ground-state TDM. The procedure is based on the asymptotic properties of the TOF's in coordinate space, when the distance between two of the particles and the center of mass of the remaining core becomes very large. This procedure can be considered as an extension of the method suggested in Ref. [26], where the relationship between the one-body density matrix and the one-nucleon overlap function is established. The latter has been applied [27–35] to calculate the one-nucleon overlap functions, spectroscopic factors and to make consistent the calculations of the cross sections of different one-nucleon removal reactions, such as (p,d), $(e,e'p)$, and (γ,p) [27,30–35] on ^{16}O [31,32,34] and ^{40}Ca [33,34], (p,d) on ^{24}Mg, ^{28}Si, and ^{32}S

[35], as well as $(e,e'p)$ on ^{32}S [35]. Various correlation methods, such as the Jastrow method, the Green function method, the correlated basis function method, and the generator coordinate method, have been used to obtain the one-nucleon overlap functions which are necessary for cross section calculations.

The first aim of the present paper is to apply the procedure suggested in Ref. [25] to calculate TOF's for ^{16}O using the TDM calculated in Ref. [36] with the Jastrow correlation method (JCM), which incorporates the nucleon-nucleon SRC. As a second aim, the resulting two-proton overlap functions are used to calculate the cross section of the $^{16}\mathrm{O}(e,e'pp)$ reaction for the transition to the 0^+ ground and the 1^+ excited at 11.3-MeV states of ^{14}C. The cross sections are calculated on the basis of the theoretical approach developed in Refs. [5,8,9].

The reliability of the TOF's obtained in our method depends strongly on the availability of realistic TDM's. We use the Jastrow TDM, though incorporating only SRC (and using harmonic-oscillator single-particle wave functions in the Slater determinant). Our choice is determined by the convenience of its analytical form obtained in Ref. [36], which makes practically possible the calculations of the TOF's. There exist more sophisticated methods aiming at calculating realistic TDM including not only SRC [4,37–39]. So far, however, it is a difficult task to obtain explicit forms of these TDM's and, consequently, to use them for calculations of the TOF's. So, the usage in our work of the Jastrow TDM must be considered as a first attempt to use an approach which fulfills the general necessity of the TOF's to be extracted from the TDM and to apply them to cross-section calculations of the two-nucleon knockout reactions.

The method to calculate the TOF's on the basis of the TDM is briefly outlined in Sec. II. The results of the calculations of the TOF's and the cross sections of the $^{16}\mathrm{O}(e,e'pp)^{14}$C reaction are presented and discussed in Sec. III. Some conclusions are drawn in Sec. IV.

II. TWO-BODY DENSITY MATRIX AND OVERLAP FUNCTIONS

In this section, we present shortly the definitions and some properties of the TDM and related quantities in both natural orbital (geminal) and overlap function representations. The method to extract the TOF's from the TDM [25] used in this work is also given.

The TDM is defined in coordinate space as

$$\rho^{(2)}(x_1,x_2;x_1',x_2')=\langle\Psi^{(A)}|a^\dagger(x_1)a^\dagger(x_2)a(x_2')a(x_1')|\Psi^{(A)}\rangle, \tag{1}$$

where $|\Psi^{(A)}\rangle$ is the antisymmetric A-fermion ground-state wave function normalized to unity and $a^\dagger(x)$, $a(x)$ are creation and annihilation operators at position x. The coordinate x includes the spatial coordinate $\mathbf{r}$ and spin and isospin variables. The TDM $\rho^{(2)}$ is trace normalized to the number of pairs of particles:

$$\mathrm{Tr}\,\rho^{(2)}=\frac{1}{2}\int\rho^{(2)}(x_1,x_2)dx_1dx_2=\frac{A(A-1)}{2}. \tag{2}$$

Since $\rho^{(2)}$ is a Hermitian matrix, its eigenstates $\psi_\alpha^{(2)}$ form a complete orthonormal set in terms of which $\rho^{(2)}$ can be decomposed as

$$\rho^{(2)}(x_1,x_2;x_1',x_2')=\sum_\alpha\lambda_\alpha^{(2)}\psi_\alpha^{(2)*}(x_1,x_2)\psi_\alpha^{(2)}(x_1',x_2'). \tag{3}$$

The eigenfunctions $\psi_\alpha^{(2)}(x_1,x_2)$ are called natural geminals and the associated real eigenvalues $\lambda_\alpha^{(2)}$ are called natural geminal occupation numbers [40]. As a consequence of the antisymmetry of the nuclear ground state, the eigenvalues $\lambda_\alpha^{(2)}$ obey the inequalities

$$0\leqslant\lambda_\alpha^{(2)}\leqslant(A-1)/2 \quad \text{for} \quad A \quad \text{odd},$$
$$0\leqslant\lambda_\alpha^{(2)}\leqslant A/2 \quad \text{for} \quad A \quad \text{even}. \tag{4}$$

The upper bound in Eq. (4) is actually reached only for systems which are maximally correlated, as, e.g., the occupation number of zero-coupled pairs in the seniority formalism in the limit of large shell degeneracy.

Of direct physical interest is the decomposition of the TDM in terms of the overlap functions between the A-particle ground state and the eigenstates of the $(A-2)$-particle systems, since the TOF's can be probed in exclusive knockout reactions.

The TOF's are defined as the overlap between the ground state of the target nucleus $\Psi^{(A)}$ and a specific state $\Psi_\alpha^{(C)}$ of the residual nucleus $(\mathrm{C}=A-2)$ [23]:

$$\Phi_\alpha(x_1,x_2)=\langle\Psi_\alpha^{(C)}|a(x_1)a(x_2)|\Psi^{(A)}\rangle. \tag{5}$$

Inserting a complete set of $(A-2)$ eigenstates $|\alpha(A-2)\rangle$ into Eq. (1), one gets

$$\rho^{(2)}(x_1,x_2;x_1',x_2')=\sum_\alpha\Phi_\alpha^*(x_1,x_2)\Phi_\alpha(x_1',x_2'). \tag{6}$$

The norm of the two-body overlap functions defines the spectroscopic factors

$$S_\alpha^{(2)}=\langle\Phi_\alpha|\Phi_\alpha\rangle. \tag{7}$$

As in the case of the single-particle spectroscopic factors, where the latter cannot exceed the maximal natural occupation number [26], one can find that $S_\alpha^{(2)}\leqslant\lambda_{\max}^{(2)}$.

A procedure for obtaining the TOF's on the basis of the TDM has been suggested in Ref. [25]. It is due to the particular asymptotic properties of the TOF's and is similar to the one suggested in Ref. [26] for deriving the one-body overlap functions from the one-body density matrix.

In the case when two like nucleons (neutrons or protons) unbound to the rest of the system are simultaneously transferred, the following hyperspherical type of asymptotics is valid for the two-body overlap functions [23,41,42]:

$$\Phi(r,R)\rightarrow N\exp\left\{-\sqrt{\frac{4m|E|}{\hbar^2}\left(R^2+\frac{1}{4}r^2\right)}\right\}\times\left(R^2+\frac{1}{4}r^2\right)^{-5/2}, \quad (8)$$

where r and R are the magnitudes of the relative and center-of-mass (c.m.) coordinates, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$, respectively, m is the nucleon mass, and $E=E^{(A)}-E^{(C)}$ is the two-nucleon separation energy.

For a target nucleus with $J^{\pi}_{tar}=0^+$, the TOF in Eq. (5) can be written in the form

$$\Phi_{\nu JM}(x_1,x_2)=\sum_{LS}\{\Phi_{\nu JLS}(\mathbf{r}_1,\mathbf{r}_2)\otimes\chi_S(\sigma_1,\sigma_2)\}_{JM}, \quad (9)$$

where ν is the number of the state of the residual nucleus with a given total momentum J,

$$\chi_{SM_S}(\sigma_1,\sigma_2)=\{\chi_{1/2}(\sigma_1)\otimes\chi_{1/2}(\sigma_2)\}_{SM_S}=\sum_{m_{s_1}m_{s_2}}\left(\frac{1}{2}m_{s_1}\frac{1}{2}m_{s_2}|SM_S\right)\times\chi_{1/2m_{s_1}}(\sigma_1)\chi_{1/2m_{s_2}}(\sigma_2), \quad (10)$$

and $\Phi_{\nu JLSM_L}(\mathbf{r}_1,\mathbf{r}_2)$ is the spatially dependent part of the overlap function. Performing a decomposition into angular momenta $\mathbf{l}=\mathbf{l}_r$ and $\mathbf{L}_R$ ($\mathbf{L}=\mathbf{l}+\mathbf{L}_R$) corresponding to the relative and c.m. coordinates, one obtains

$$\Phi_{\nu JSLM_L}(\mathbf{r},\mathbf{R})=\sum_{lL_R}\Phi_{\nu JSLlL_R}(r,R)\{Y_{L_R}(\hat{R})\otimes Y_l(\hat{r})\}_{LM_L}. \quad (11)$$

Then the TDM can be written as

$$\rho^{(2)}(x_1,x_2;x_1',x_2')=\sum_{JM}\sum_{LSL'S'}\sum_{lL_Rl'L_R'}\rho^{(2)}_{JSLlL_RS'L'l'L_R'}(r,R;r',R')\times A^{JM*}_{SLlL_R}(\sigma_1,\sigma_2;\hat{r},\hat{R})A^{JM}_{S'L'l'L_R'}(\sigma_1',\sigma_2';\hat{r}',\hat{R}'), \quad (12)$$

where the radial part of the density matrix is

$$\rho^{(2)}_{JSLlL_RS'L'l'L_R'}(r,R;r',R')=\sum_{\nu}\Phi^*_{\nu JSLlL_R}(r,R)\times\Phi_{\nu JS'L'l'L_R'}(r',R') \quad (13)$$

and the spin-angular function is

$$A^{JM}_{SLlL_R}(\sigma_1,\sigma_2;\hat{r},\hat{R})=\{\{Y_{L_R}(\hat{R})\otimes Y_l(\hat{r})\}_{LM_L}\otimes\chi_{SM_S}(\sigma_1,\sigma_2)\}_{JM}. \quad (14)$$

We will consider the diagonal part of the radial TDM in Eq. (13):

$$\rho^{(2)}_{JSLlL_R}(r,R;r',R')=\sum_{\nu}\Phi^*_{\nu JSLlL_R}(r,R)\Phi_{\nu JSLlL_R}(r',R'). \quad (15)$$

For large $r'=a$ and $R'=b$ a single term with ν_0, corresponding to the smallest two-nucleon separation energy, will dominate the sum on the right-hand side of Eq. (15). Then, according to Eq. (8), the radial part of the TOF $\Phi_{\nu_0 JSLlL_R}(r,R)$ can be expressed in terms of the TDM as

$$\Phi_{\nu_0 JSLlL_R}(r,R)=\frac{\rho^{(2)}_{JSLlL_R}(r,R;a,b)}{\Phi_{\nu_0 JSLlL_R}(a,b)}=\frac{\rho^{(2)}_{JSLlL_R}(r,R;a,b)}{N\exp\{-k\sqrt{(b^2+\frac{1}{4}a^2)}\}(b^2+\frac{1}{4}a^2)^{-5/2}}, \quad (16)$$

where $k=(4m|E|/\hbar^2)^{1/2}$ is constrained by the experimental values of the two-nucleon separation energy E.

The relationship obtained in Eq. (16) makes it possible to extract TOF's with quantum numbers $JSLlL_R$ from a given TDM. The coefficient N and the constant k can be determined from the asymptotics of $\rho^{(2)}_{JSLlL_R}(r,R;r,R)$.

III. RESULTS

A. The two-proton overlap functions

The procedure described in Sec. II has been applied to calculate the two-proton overlap functions in the ^{16}O nucleus for the transition to the 0^+ ground and the 1^+ state of ^{14}C, at 11.3 MeV excitation energy. The TDM obtained in Ref. [36] in the framework of the low-order approximation (LOA) of the Jastrow correlation method has been used. The latter incorporates the nucleon-nucleon SRC in terms of the wave-function ansatz [43]:

$$\Psi^{(A)}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=(C_A)^{-1/2}\prod_{1\leqslant i<j\leqslant A}f(|\mathbf{r}_i-\mathbf{r}_j|)\times\Phi^A_{SD}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A), \quad (17)$$

where C_A is a normalization constant and Φ^A_{SD} is a single Slater determinant wave function built from harmonic-oscillator (HO) single-particle wave functions which depend on the oscillator parameter α_{osc}, having the same value for both protons and neutrons. Only central correlations are included in the correlation factor $f(r)$, which is state independent and has a simple Gaussian form

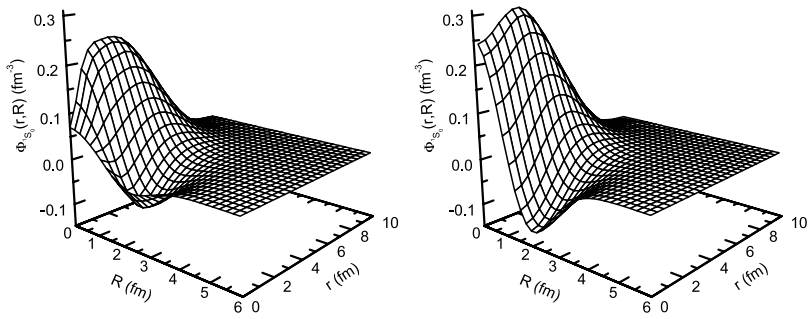

FIG. 1. The $^1S_0$ two-proton overlap functions for the nucleus ^{16}O leading to the 0^+ ground state of ^{14}C extracted from the JCM (left) and uncorrelated (right) two-body density matrices.

$$f(r)=1-c\exp(-\beta^2 r^2), \tag{18}$$

where the correlation parameter β determines the healing distance and the parameter c accounts for the strength of the SRC. The LOA keeps all terms up to the second order in $h=f-1$ and up to the first order in $g=f^2-1$ in such a way that the normalization of the density matrices is ensured order by order [44].

The values of the parameters α_{osc} and β have been obtained [45] phenomenologically by fitting the experimental elastic form factor data for ^{4}He, ^{16}O, and ^{40}Ca nuclei. The value of the parameter c has been determined [36] under the additional condition of the relative pair density distribution

$$\rho^{(2)}(\mathbf{r})=\int \rho^{(2)}(\mathbf{r},\mathbf{R};\mathbf{r},\mathbf{R})d\mathbf{R} \tag{19}$$

to reproduce at $r=0$ the associated value obtained within the variational Monte Carlo approach [37]. Thus, in the present calculations the following values of the parameters are used for ^{16}O: $\alpha_{osc}=0.61$ fm^{-1}, $\beta=1.30$ fm^{-1}, $c=0.77$.

In order to obtain the radial part of the TDM, $\rho^{(2)}(r,R;r',R')$ of Eq. (13), we use the analytical expression for the TDM obtained in Ref. [36] substituting the coordinates of the two particles $\mathbf{r}_1$ and $\mathbf{r}_2$ by the c.m. $\mathbf{R}$ and relative $\mathbf{r}$ coordinates. Then, the radial part of Eq. (15) can be obtained by multiplying the TDM by $A^{JM}_{S''L''l''L''_R}(\sigma_1,\sigma_2;\hat{r},\hat{R})A^{JM*}_{S'''L'''l'''L'''_R}(\sigma_1',\sigma_2';\hat{r}',\hat{R}')$ and then integrating over the angles and summing over the spin variables.

In order to obtain the values of the parameters k and N in Eq. (16) simultaneously, we look for such a radial contribution $\rho^{(2)}_{\nu_0 JSLlL_R}(r,R;r',R')=\Phi^*_{\nu_0 JSLlL_R}(r,R)\times\Phi_{\nu_0 JSLlL_R}(r',R')$ whose diagonal part $\rho^{(2)}_{\nu_0 JSLlL_R}(r,R)$ minimizes the trace

$$\mathrm{Tr}\{[\rho^{(2)}_{JSLlL_R}(r,R)-\rho^{(2)}_{\nu_0 JSLlL_R}(r,R)]^2\}=\min. \tag{20}$$

The correct determination of these parameters requires a proper definition of the asymptotic region where the trace in Eq. (20) has to be minimized. If we denote the point in which $\rho^{(2)}_{JSLlL_R}(r,R)$ has a maximum with $(r_{\max},R_{\max})$, the starting point of the asymptotic region (r_0,R_0) is obtained looking for a point r_0, at $R=R_{\max}$, for which $\rho^{(2)}_{JSLlL_R}(r_0,R_{\max})\leqslant 10\%$ of $\rho^{(2)}_{JSLlL_R}(r_{\max},R_{\max})$. When r_0 has been determined, we look for a point R_0, at $r=r_0$, for which $\rho^{(2)}_{JSLlL_R}(r_0,R_0)\leqslant 10\%$ of $\rho^{(2)}_{JSLlL_R}(r_0,R_{\max})$. The length of the asymptotic region over r and R is determined by the requirement to obtain the separation energy which is maximally close to the experimental one. The asymptotic point (a,b) is chosen to be that one which gives the minimal least-squared deviation expressed by Eq. (20).

When all the parameters are determined, Eq. (16) can be used to calculate the radial part $\Phi_{\nu_0 JSLlL_R}(r,R)$. Then, including also the spin-angular part in Eqs. (11) and (9) we obtain the TOF's.

For a given set of quantum numbers $JSLlL_R$, the TOF is calculated by minimizing the trace of the corresponding part of the TDM. Thus, for a particular final state J^π of the residual nucleus, different TOF's can independently be calculated using this procedure for each set of quantum numbers, and each one of them is fully responsible for the two-proton knockout process and the transition to the state J^π.

The TOF's obtained in the JCM for the $^1S_0$ and $^3P_1$ states are presented in Figs. 1 and 2, respectively. They are compared with the uncorrelated TOF's obtained by applying the same procedure to the uncorrelated TDM, i.e., with $c=0$ in the correlation factor of Eq. (18). The notation for the partial waves in our case is $^{2S+1}l_L$. It differs from the generally accepted one $^{2S+1}l_j$ because we have a different coupling scheme of spin and angular momenta.

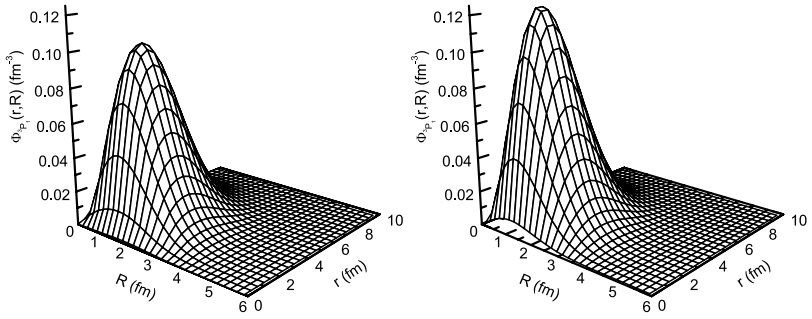

FIG. 2. The $^3P_1$ two-proton overlap functions for the nucleus ^{16}O leading to the 0^+ ground state of ^{14}C extracted from the JCM (left) and uncorrelated (right) two-body density matrices.

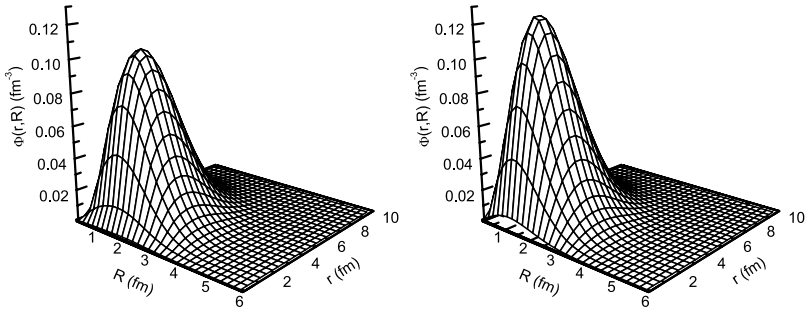

FIG. 3. The two-proton overlap functions for the nucleus ^{16}O leading to the 1^+ excited state of ^{14}C extracted from the JCM (left) and uncorrelated (right) TDM's.

It can be seen from the figures that SRC affect both size and shape of the TOF's. Their role, however, is different in the two states and, as it was already found in previous and different calculations (see, e.g., Refs. [9,24]), is much more important when the two protons are in the $^1S_0$ than in the $^3P_1$ state.

The spectroscopic factors corresponding to the $^1S_0$ and $^3P_1$ overlap functions are 0.958 and 0.957, respectively. Also the D wave can contribute for the transition to the 0^+ ground state of ^{14}C, but the corresponding TOF is very small and is not considered in the present study.

As a next step, we derive the total TOF $\Phi_{\nu JM}(x,X)$ in terms of a sum over all possible partial components, i.e.,

$$\Phi_{\nu JM}(x,X)=\sum_{LSlL_R}\Phi_{\nu JSLlL_R}(r,R)A^{JM}_{SLlL_R}(\sigma_1,\sigma_2;\hat{r},\hat{R}). \tag{21}$$

We integrate the squared modulus of the total TOF in Eq. (21) over the angles and sum over the spin variables. The result can be written in the form (for the smallest value of $\nu=\nu_0$)

$$\overline{|\Phi_{JM}(x,X)|^2}\equiv|\tilde{\Phi}_{JM}(r,R)|^2=\sum_{LSlL_R}\rho^{(2)}_{JLSlL_R}(r,R), \tag{22}$$

where the bar denotes the integration over the angles and summation over the spin variables, and $\tilde{\Phi}_{JM}(r,R)$ is the radial part of the total TOF obtained after the integration and summation. Using the asymptotics of $\tilde{\Phi}_{JM}(r,R)$ at $r\to a$, $R\to b$ one can write

$$\tilde{\Phi}_{JM}(r,R)=\frac{\sum_{LSlL_R}\rho^{(2)}_{JSLlL_R}(r,R;a,b)}{N\exp\{-k\sqrt{(b^2+\frac{1}{4}a^2)}\}(b^2+\frac{1}{4}a^2)^{-5/2}}. \tag{23}$$

The parameters N, k, a, b in Eq. (23) can be redetermined from the asymptotics of $\Sigma_{LSlL_R}\rho^{(2)}_{JLSlL_R}(r,R;r,R)$ using the procedure already explained in the first part of this section. Then, each partial radial component $\Phi_{JSLlL_R}(r,R)$ in Eq. (21) can be separately calculated from Eq. (16) using for each one of them the same coefficients N, k, a, b which correspond to the asymptotics of the total TOF. The asymptotic point (a,b) determines the individual contribution of each partial overlap function to the total TOF. This prescription allows us to combine, with some approximations, the different radial components in Eq. (21).

The results for the $^1S_0$ and $^3P_1$ partial components have a similar behavior as in Figs. 1 and 2, the main difference is that they are somewhat reduced in magnitude. The reduction is determined by the contribution of each component to the total TOF. The spectroscopic factor corresponding to the total TOF is equal to unity in the uncorrelated case and 0.965 in the Jastrow case.

The Jastrow TDM (including only SRC) is not ''rich'' enough to be able to explain realistically transitions to all the excited states of ^{14}C. Therefore, only the transition to the 1^+ state is considered in the present paper as an example of the applicability of the method.

In the case of the transition to the 1^+ excited state of ^{14}C at 11.3 MeV, pp pairs in the $^3P_{0,1,2}$ and $^1D_1$ states can contribute to the process. The $^3P_0$, $^3P_2$, and $^1D_1$ TOF's, however, are very small and their contributions to the cross sections were found to be negligible, so that the total TOF practically coincides with the $^3P_1$ TOF.

In our method, the radial TOF generally depends on J and M. In the 1^+ case, however, the results with different M practically coincide: for $M=\pm1$ they are exactly the same and very close results are obtained for $M=0$. The Jastrow and the uncorrelated TOF's, averaged over M, are presented in Fig. 3. They are similar to those displayed in Fig. 2 for the $^3P_1$ TOF for the transition to the 0^+ ground state of ^{14}C. The shapes of the correlated and uncorrelated TOF's are similar and the magnitude of the result with correlations at the maximum is lower than that in the uncorrelated case. The value of the spectroscopic factor is 0.967 in the Jastrow case and unity in the uncorrelated one.

B. The ^{16}O$(e,e'pp)^{14}$C reaction

The TOF's obtained from the TDM within the Jastrow correlation method have been used to calculate the cross section of the ^{16}O$(e,e'pp)^{14}$C knockout reaction.

The coincidence cross section for the reaction induced by an electron, with momentum $\mathbf{p}_0$ and energy E_0 with $E_0=|\mathbf{p}_0|=p_0$, where two nucleons, with momenta $\mathbf{p}_1'$ and $\mathbf{p}_2'$ and energies E_1' and E_2', are ejected from a nucleus is given, in the one-photon exchange approximation and after integrating over E_2', by [2,5]

$$\frac{d^8\sigma}{dE_0'\mathrm{d}\Omega dE_1'd\Omega_1'd\Omega_2'}=K\Omega_{\mathrm{f}}\,f_{\mathrm{rec}}|j_\mu J^\mu|^2. \tag{24}$$

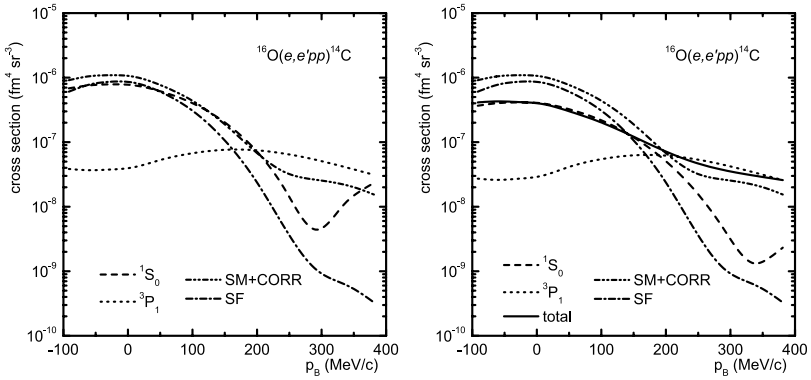

FIG. 4. The differential cross section of the $^{16}\mathrm{O}(e,e'pp)$ reaction as a function of the recoil momentum p_B for the transition to the 0^+ ground state of ^{14}C in the superparallel kinematics with $E_0=855$ MeV, $\omega=215$ MeV, and $q=316$ MeV/c. Positive (negative) values of p_B refer to situations where $\mathbf{p}_\mathrm{B}$ is parallel (antiparallel) to $\mathbf{q}$. The curves are obtained with different treatments of the TOF: $^1S_0$ (dashed line) and $^3P_1$ (dotted line) as ''independent'' TOF's in the JCM (i.e., those drawn in the left panel of Figs. 1 and 2, respectively) in the left panel and as partial components (see the explanations given in Sec. III A) in the right panel, the total TOF (solid line), the TOF from the spectral function (SF) [9,24] (dot-dashed line), the product of a pair function of the shell model and the correlation function of Eq. (18) (SM+CORR) (dot-dot-dashed line).

In Eq. (24), E_0' is the energy of the scattered electron with momentum $\mathbf{p}_0'$, $K=e^4p_0'^2/4\pi^2Q^4$ where $Q^2=\mathbf{q}^2-\omega^2$, with $\omega=E_0-E_0'$ and $\mathbf{q}=\mathbf{p}_0-\mathbf{p}_0'$, is the four-momentum transfer. The quantity $\Omega_\mathrm{f}=p_1'E_1'p_2'E_2'$ is the phase-space factor and integration over E_2' produces the recoil factor

$$f_\mathrm{rec}^{-1}=1-\frac{E_2'\,\mathbf{p}_2'\cdot\mathbf{p}_\mathrm{B}}{E_\mathrm{B}\,|\mathbf{p}_2'|^2}, \tag{25}$$

where E_B and $\mathbf{p}_\mathrm{B}$ are the energy and momentum of the residual nucleus. The cross section is given by the square of the scalar product of the relativistic electron current j^μ and of the nuclear current J^μ, which is given by the Fourier transform of the transition matrix elements of the charge-current density operator between initial and final nuclear states,

$$J^\mu(\mathbf{q})=\int\langle\Psi_\mathrm{f}|\hat{J}^\mu(\mathbf{r})|\Psi_\mathrm{i}\rangle\exp(i\mathbf{q}\cdot\mathbf{r})d\mathbf{r}. \tag{26}$$

If the residual nucleus is left in a discrete eigenstate of its Hamiltonian, i.e., for an exclusive process, and under the assumption of a direct knockout mechanism, Eq. (26) can be written as [5,8]

$$J^\mu(\mathbf{q})=\int\Phi_\mathrm{f}^*(x_1,x_2)J^\mu(\mathbf{r},x_1,x_2)\Phi_\mathrm{i}(x_1,x_2)\times\exp(i\mathbf{q}\cdot\mathbf{r})d\mathbf{r}dx_1dx_2. \tag{27}$$

The TOF's extracted from the TDM have been used for the calculation of the radial part of the two-nucleon overlap function Φ_i. The other theoretical ingredients of the integral in Eq. (27) have been calculated within the theoretical framework of Refs. [5,8,9]. Therefore, the nuclear current operator J^μ is the sum of a one-body and a two-body part. The one-body part contains a Coulomb, a convective, and a spin term. For the pp knockout, the two-body current contains only the contributions of non charge-exchange processes with intermediate Δ-isobar configurations [9,16]. In the final-state wave function Φ_f, the mutual interaction between the two outgoing nucleons is neglected and the scattering state is given by the product of two uncoupled single-particle distorted wave functions, eigenfunctions of a complex phenomenological optical potential which contains a central, a Coulomb, and a spin-orbit term [46].

The differential cross sections calculated for the transition to the 0^+ ground state of ^{14}C are shown in Figs. 4 and 5 for two kinematical settings considered in the experiments performed at NIKHEF [18–20] and MAMI [21,22]. In Fig. 4,

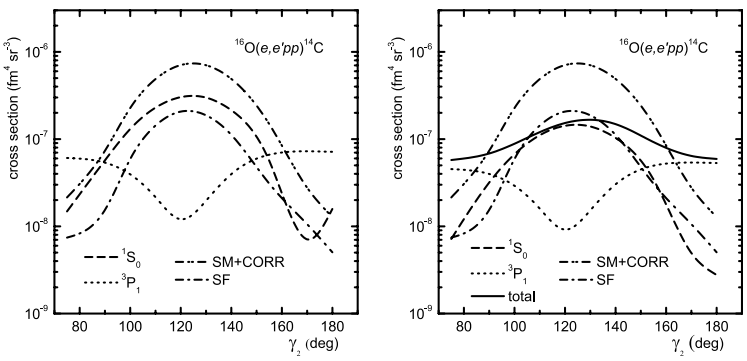

FIG. 5. The differential cross section of the $^{16}\mathrm{O}(e,e'pp)$ reaction as a function of the angle γ_2 for the transition to the 0^+ ground state of ^{14}C in a kinematics with $E_0=584$ MeV, $\omega=212$ MeV, $q=300$ MeV/c, $T_1'=137$ MeV, and $\gamma_1=-30°$, on the opposite side of the outgoing electron with respect to the momentum transfer. Line convention as in Fig. 4.

the cross section is calculated in the superparallel kinematics of the MAMI experiment, where the two nucleons are ejected parallel and antiparallel to the momentum transfer and, for a fixed value of ω and q, it is possible to explore, for different values of the kinetic energies of the outgoing nucleons, all the possible values of p_B. In the calculations, the incident electron energy is fixed at $E_0=855$ MeV, $\omega=215$ MeV, and $q=316$ MeV/c. In Fig. 5, a specific kinematical setting included in the experiments carried out at NIKHEF is considered, with $E_0=584$ MeV, $\omega=212$ MeV, and $q=300$ MeV/c. The kinetic energy of the first outgoing proton T_1' is 137 MeV and the angle γ_1, between the first outgoing proton and **q**, is 30° on the opposite side of the outgoing electron with respect to the momentum transfer. Changing the angle γ_2, between the second outgoing proton and **q**, on the other side, different values of the recoil momentum p_B are explored in the range between -250 and 300 MeV/c, including the zero values at $\gamma_2 \simeq 120°$.

The cross sections calculated with the $^1S_0$ and $^3P_1$ TOF's as independent and fully responsible for the knockout process presented above in Figs. 1 and 2 are displayed in the left panels of Figs. 4 and 5. In the right panels, the cross sections obtained with the total TOF, Eq. (23), from the Jastrow TDM are plotted and compared with the contributions given by the $^1S_0$ and $^3P_1$ partial components, i.e., calculated using Eq. (16).

These results are compared in the figures with the cross sections already shown in Ref. [9], where the TOF is taken from a calculation of the two-proton spectral function (SF) [24], where a two-step procedure has been adopted to include both SRC and LRC. LRC are calculated in a shell-model space large enough to incorporate the corresponding collective features which influence the pair removal amplitude. The single-particle propagators used for this dressed random phase approximation (RPA) description of the two-particle propagator also include the effect of both LRC and SRC. In the second step, that part of the pair removal amplitudes, which describes the relative motion of the pair, is supplemented by defect functions obtained from the same G matrix which is also used as the effective interaction in the RPA calculation. Different defect functions are produced by different realistic NN potentials. The results shown in Figs. 4 and 5 are obtained with the Bonn-A potential. The explicit expression of the TOF's is given in a form of the same kind as in Eq. (21), in terms of a combination of the c.m. and relative wave functions. The $^1S_0$ and $^3P_1$ relative waves give the main contribution for the transition to the 0^+ ground state, while only a negligible contribution is given by the D wave. The results of this model are able to give a proper description of available data [19–22].

In the figures are also shown for a comparison the results obtained with a simpler approach, where the two-nucleon wave function is given by the product of the pair function of the shell model and of a Jastrow-type central and state-independent correlation function. In this approach (SM +CORR), the ground state of ^{14}C is described as a pure $(1p_{1/2})^{-2}$ hole in ^{16}O. In order to allow a more direct and clear comparison with the TOF's from the Jastrow TDM, HO single-particle wave functions and the same correlation function as in Eq. (18), with the same parameters as in the calculation of the TDM, have been adopted.

The shape of the calculated cross sections is determined by the value of the c.m. orbital angular momentum L_R, that is, $L_R=0$ for $^1S_0$, which gives the major contribution at lower values, and $L_R=1$ for $^3P_1$, which dominates at higher values of the recoil momentum.

SRC are quite strong and even dominant for the $^1S_0$ state and much weaker for the $^3P_1$ state. The role of the isobar current is strongly reduced for the $^1S_0$ pp knockout, since there the magnetic dipole $NN \leftrightarrow N\Delta$ transition is suppressed [17,47]. As a consequence, in the figures, the $^1S_0$ results are dominated by the one-body current and thus by SRC, while the Δ current gives the main contribution to the $^3P_1$ pp knockout.

One of the main results of the previous theoretical investigations, which has been clearly confirmed in comparison with data, is the dominance of the $^1S_0$ pp knockout in the $^{16}\mathrm{O}(e,e'pp)^{14}\mathrm{C}_{\mathrm{g.s.}}$ reaction. It can be seen from Fig. 4 that the cross section calculated with the Jastrow TOF for the $^1S_0$ state is close to the SF and also to the SM+CORR results at low values of p_B, up to $\sim$150–200 MeV/c, that is, just in the region where the $^1S_0$ contribution is dominant. For $p_B \geqslant 200$ MeV/c, the $^3P_1$ knockout becomes dominant with all the different treatments of the TOF. The results with the $^3P_1$ TOF from the Jastrow TDM are however, much larger than the SF result and also larger than the SM+CORR cross section. It can be noted that even the $^1S_0$ curve in Fig. 4 is, at large values of the momentum, higher that the SF result. This is an indication that SRC in the JCM produce a stronger enhancement of the high-momentum components.

The behavior of the pure $^1S_0$ result in the left panel of Fig. 5 is somewhat similar to that of the SF and SM+CORR cross sections, which appear driven by the $^1S_0$ contribution. There are anyhow significant differences in the shape and large differences in the size of the various results.

The cross sections calculated with the total TOF, obtained from the combination of the $^1S_0$ and $^3P_1$ partial components, are shown in the right panels of Figs. 4 and 5. In both kinematical settings, the $^1S_0$ component dominates at low values of p_B, while the $^3P_1$ component produces a strong enhancement at high momenta. The contributions of the partial components are reduced with respect to the results in the left panels, where each one of them is fully responsible for the knockout process. Thus, the cross sections calculated with the total TOF from the Jastrow TDM are somewhat reduced at low recoil momenta. The contribution of the $^3P_1$ component to the total TOF is much more relevant than with the other theoretical treatments and the enhancement at high momenta turns out to be much larger. Thus, the shape of the cross sections with the total TOF from the JCM is flatter than that with the SF and SM+CORR results.

In the right panels of Figs. 4 and 5, the cross sections with the Jastrow TOF are lower at low recoil momenta and much larger at high momenta than the SF calculations. This result is due to the larger contribution of the $^3P_1$ component in the Jastrow TOF. The SM+CORR cross sections are higher than the other results at low recoil momenta. This is an indication of a stronger contribution of SRC in this calculation. This

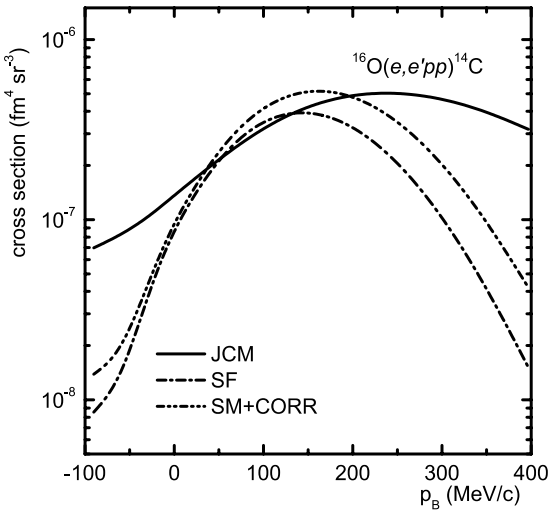

FIG. 6. The differential cross section of the $^{16}O(e,e'pp)$ reaction as a function of the recoil momentum p_B for the transition to the 1^+ excited state of ^{14}C in the same kinematics as in Fig. 4. The results are obtained with the Jastrow TOF (solid line), the TOF from the spectral function (SF) (dot-dashed line) and the product of a pair function of the shell model and the correlation function of Eq. (18) (SM+CORR) (dot-dot-dashed line).

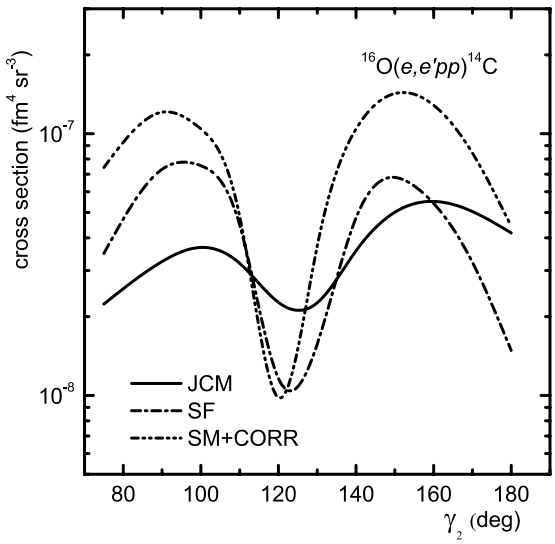

FIG. 7. The differential cross section of the $^{16}O(e,e'pp)$ reaction as a function of the angle γ_2 for the transition to the 1^+ excited state of ^{14}C in the same kinematics as in Fig. 5. Line convention as in Fig. 6.

contribution, however, depends on the particular expression adopted for the correlation functions, which in the calculations of Figs. 4 and 5 is exactly the same as in the calculation of the TDM. At high momenta, the SM+CORR cross sections remain always higher than the SF results, but generally lower than the results given by the TOF from the JCM.

Although obtained from a calculation of the TDM within the JCM where only SRC are included, the TOF's used in our calculations are able to reproduce the main qualitative features which were found in previous theoretical investigations. This means that the procedure suggested in Ref. [25] to calculate the TOF's from the TDM can be applied and exploited in the study of the two-nucleon knockout reactions.

The large differences found in Figs. 4 and 5 indicate that the calculated cross sections are very sensitive to the different approaches used and to the theoretical treatment of nuclear structure and correlations in the TOF. It would be interesting to apply the procedure used in this work for the calculation of the TOF's to more refined treatments of the TDM [4,37–39].

The differential cross sections calculated for the transition to the 1^+ state of ^{14}C, at 11.3 MeV excitation energy, are shown in Figs. 6 and 7 in the same kinematical settings already considered for the 0^+ state in Figs. 4 and 5. In both kinematics, large differences in the size and also in the shape of the calculated cross sections are given by the different treatments of the TOF. With respect to the other results, the Jastrow TOF produces in the superparallel kinematics a strong enhancement at high momenta, which makes the shape of the cross sections larger and flatter than that with the SF and SM+CORR TOF's. A similar effect is found also in the kinematics of Fig. 7. In this case, however, larger differences are produced by the three different TOF's in the maximum region, where the cross section calculated with the Jastrow TOF is much lower than the other results. This result is presumably due to the simple two-body density, used as a first step to test the validity of the method.

IV. CONCLUSIONS

The results of the present work can be summarized as follows.

(i) The two-nucleon overlap functions (and their norms, the spectroscopic factors) corresponding to the knockout of two protons from the ground state of ^{16}O and the transition to the ground and 1^+ (11.3-MeV) excited states of ^{14}C are calculated using the recently established relationship [25] between the TOF's and the TDM. In the calculations, the TDM obtained within the JCM [36] is used. Though only SRC are accounted for in the Jastrow TDM, the results can be considered as a first attempt to use an approach which fulfills the general necessity of the TOF's to be extracted from theoretically calculated TDM's corresponding to realistic wave functions of the nuclear states. Of course, the quality of the results will depend heavily on the availability of a realistic TDM incorporating all necessary types of NN correlations.

(ii) The contributions of the two-proton overlap functions leading to the ground state of ^{14}C corresponding to the removal of $^1S_0$ and $^3P_1$ pp pairs from ^{16}O are calculated in two manners: (1) when each one is fully responsible for the knockout process, and (2) when they are partial components of the total TOF. The $^1S_0$ and $^3P_1$ results obtained in the two manners are similar, the main difference being that the partial components in case (2) are reduced in magnitude. The comparison between the results for the TOF's in the correlated (Jastrow) case and in the uncorrelated case shows that SRC affect both size and shape of the $^1S_0$ and $^3P_1$ overlap functions. The effects of SRC, however, are much stronger when the two protons are in an $^1S_0$ state. Only one partial component $^3P_1$ gives the main contribution to the transition to the

1^+ excited state of ^{14}C. The TOF's derived for this state are similar to the $^3P_1$ TOF's calculated for the transition to the 0^+ ground state.

(iii) The TOF's extracted from the Jastrow TDM are included in the theoretical approach of Refs. [5,8,9] to calculate the cross section of the ^{16}O$(e,e'pp)^{14}$C knockout reaction. Numerical results, in different kinematics, are presented and compared with the cross sections calculated, within the same theoretical model for the reaction mechanism, with different treatments of the TOF, in particular with the more refined approach of Refs. [9,24], where the TOF's are obtained from a calculation of the two-proton spectral function of ^{16}O where both SRC and LRC are included. The calculated cross sections are very sensitive to the theoretical treatment and different results are produced by the different TOF's. The cross sections calculated in the present work, where the TOF's are extracted from the Jastrow TDM, confirm the dominant contribution of the $^1S_0$ pp knockout at low values of recoil momentum, up to $\simeq$150–200 MeV/c. The $^3P_1$ contribution is mainly responsible for the high-momentum part of the cross section at $p_B \geqslant 200$ MeV/c.

(iv) Our method is applied in the present work only to the 0^+ ground and the 1^+ (11.3-MeV) excited states of ^{14}C. The main aim was to check the practical application of all steps of the method to a given state of the residual nucleus. Therefore, the results obtained for the ^{16}O$(e,e'pp)^{14}$C reaction, which are able to reproduce the main qualitative features of the cross sections calculated with different treatments of the TOF's, can serve as an indication of the reliability of the method, which can be applied to a wider range of situations and, as an alternative to an explicit calculation of the two-hole spectral function, to more refined approaches of the TDM [4,37–39].

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[1] K. Gottfried, Nucl. Phys. **5**, 557 (1958); Ann. Phys. (N.Y.) **21**, 29 (1963).
[2] S. Boffi, C. Giusti, F. D. Pacati, and M. Radici, *Electromagnetic Response of Atomic Nuclei*, Oxford Studies in Nuclear Physics Vol. 20 (Clarendon, Oxford, 1996).
[3] A. N. Antonov, P. E. Hodgson, and I. Zh. Petkov, *Nucleon Correlations in Nuclei* (Springer-Verlag, Berlin, 1993).
[4] H. Müther and A. Polls, Prog. Part. Nucl. Phys. **45**, 243 (2000).
[5] C. Giusti and F.D. Pacati, Nucl. Phys. **A535**, 573 (1991).
[6] J. Ryckebusch, M. Vanderhaeghen, K. Heyde, and M. Waroquier, Phys. Lett. B **350**, 1 (1995).
[7] J. Ryckebusch, V. Van der Sluys, K. Heyde, H. Holvoet, W. Van Nespen, M. Waroquier, and M. Vanderhaeghen, Nucl. Phys. **A624**, 581 (1997).
[8] C. Giusti and F.D. Pacati, Nucl. Phys. **A615**, 373 (1997).
[9] C. Giusti, F.D. Pacati, K. Allaart, W.J.W. Geurts, W.H. Dickhoff, and H. Müther, Phys. Rev. C **57**, 1691 (1998).
[10] C. Giusti, H. Müther, F.D. Pacati, and M. Stauf, Phys. Rev. C **60**, 054608 (1999).
[11] C. Giusti, M. Radici, and F.D. Pacati, Nucl. Phys. **A546**, 607 (1992).
[12] S. Boffi, C. Giusti, M. Radici, and F.D. Pacati, Nucl. Phys. **A564**, 473 (1993).
[13] J. Ryckebusch, M. Vanderhaeghen, L. Machenil, and M. Waroquier, Nucl. Phys. **A568**, 828 (1994).
[14] J. Ryckebusch, L. Machenil, M. Vanderhaeghen, V. Van der Sluys, and M. Waroquier, Phys. Rev. C **49**, 2704 (1994).
[15] J. Ryckebusch, Phys. Lett. B **383**, 1 (1996).
[16] P. Wilhelm, H. Arenhövel, C. Giusti, and F.D. Pacati, Z. Phys. A **359**, 467 (1997).
[17] C. Giusti and F.D. Pacati, Nucl. Phys. **A641**, 297 (1998).
[18] C.J. Onderwater *et al.*, Phys. Rev. Lett. **78**, 4893 (1997).
[19] C.J. Onderwater *et al.*, Phys. Rev. Lett. **81**, 2213 (1998).
[20] R. Starink *et al.*, Phys. Lett. B **474**, 33 (2000).
[21] G. Rosner, in *Proceedings of the Tenth Mini-Conference on Studies of Few-Body Systems with High Duty-Factor Electron Beams*, edited by H. P. Blok, E. Jans, and G. van der Steenhoven (NIKHEF, Amsterdam, 1999), p. 95.
[22] G. Rosner, Prog. Part. Nucl. Phys. **44**, 99 (2000).
[23] J.M. Bang, F.A. Gareev, W.T. Pinkston, and J.S. Vaagen, Phys. Rep. **125**, 253 (1985).
[24] W.J.W. Geurts, K. Allaart, W.H. Dickhoff, and H. Müther, Phys. Rev. C **54**, 1144 (1996).
[25] A.N. Antonov, S.S. Dimitrova, M.V. Stoitsov, D. Van Neck, and P. Jeleva, Phys. Rev. C **59**, 722 (1999).
[26] D. Van Neck, M. Waroquier, and K. Heyde, Phys. Lett. B **314**, 255 (1993).
[27] M.V. Stoitsov, A.N. Antonov, and S.S. Dimitrova, Phys. Rev. C **53**, 1254 (1996).
[28] D. Van Neck, A.E.L. Dieperink, and M. Waroquier, Phys. Rev. C **53**, 2231 (1996); Z. Phys. A **355**, 107 (1996).
[29] D. Van Neck, L. Van Daele, Y. Dewulf, and M. Waroquier, Phys. Rev. C **56**, 1398 (1997).
[30] S.S. Dimitrova, M.K. Gaidarov, A.N. Antonov, M.V. Stoitsov, P.E. Hodgson, V.K. Lukyanov, E.V. Zemlyanaya, and G.Z. Krumova, J. Phys. G **23**, 1685 (1997).
[31] M.K. Gaidarov, K.A. Pavlova, S.S. Dimitrova, M.V. Stoitsov, A.N. Antonov, D. Van Neck, and H. Muether, Phys. Rev. C **60**, 024312 (1999).
[32] M.K. Gaidarov, K.A. Pavlova, A.N. Antonov, M.V. Stoitsov, S.S. Dimitrova, M.V. Ivanov, and C. Giusti, Phys. Rev. C **61**, 014306 (2000).
[33] M.V. Ivanov, M.K. Gaidarov, A.N. Antonov, and C. Giusti, Phys. Rev. C **64**, 014605 (2001).
[34] M.V. Ivanov, M.K. Gaidarov, A.N. Antonov, and C. Giusti, Nucl. Phys. **A699**, 336 (2002).
[35] M.K. Gaidarov, K.A. Pavlova, A.N. Antonov, C. Giusti, S.E. Massen, Ch.C. Moustakidis, and K. Spasova, Phys. Rev. C **66**, 064308 (2002).

[36] S.S. Dimitrova, D.N. Kadrev, A.N. Antonov, and M.V. Stoitsov, Eur. Phys. J. A **7**, 335 (2000).
[37] S.C. Pieper, R.B. Wiringa, and V.R. Pandharipande, Phys. Rev. C **46**, 1741 (1992).
[38] J.H. Heisenberg and B. Mihaila, Phys. Rev. C **59**, 1440 (1999).
[39] A. Fabrocini, F. Arias de Saavedra, and G. Co', Phys. Rev. C **61**, 044302 (2000).
[40] F.D. Peat, in *Physical Chemistry Vol. XI A* (Academic, San Francisco, 1975), pp. 429–486.
[41] S.P. Merkuriev, Sov. J. Nucl. Phys. **19**, 447 (1974).
[42] J. Bang, Phys. Scr. **22**, 324 (1980).
[43] R. Jastrow, Phys. Rev. **98**, 1479 (1955).
[44] M. Gaudin, J. Gillespie, and G. Ripka, Nucl. Phys. **A176**, 237 (1971); M. Dal Rí, S. Stringari, and O. Bohigas, *ibid.* **A376**, 81 (1982).
[45] M.V. Stoitsov, A.N. Antonov, and S.S. Dimitrova, Phys. Rev. C **47**, R455 (1993); **48**, 74 (1993); Z. Phys. A **345**, 359 (1993).
[46] A. Nadasen *et al.*, Phys. Rev. C **23**, 1023 (1981).
[47] P. Wilhelm, J.A. Niskanen, and H. Arenhövel, Nucl. Phys. **A597**, 613 (1996).