Microscopic calculation of the electric decay properties of low-energy vibrational states in even $^{110-120}$Cd isotopes

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(Received 12 March 2003; published 25 July 2003)

Reduced electric quadrupole decay strengths, $B(E2)$, of even $^{110-120}$Cd isotopes have been calculated using a recently developed theoretical framework suited for dynamical microscopic description of two-phonon-like states and their energy splitting due to interaction with low-lying one-phonon states. This model starts from a realistic single-particle valence space and a microscopic many-body Hamiltonian, which is used to generate the one-phonon states by the use of the Quasiparticle Random-Phase Approximation. The anharmonic vibrational effects, contained in the calculated energies and $B(E2)$ values, have been studied along the $^{110-120}$Cd chain of isotopes. Comparison of the calculated energies and $B(E2)$ values with data points to mixing between anharmonic vibrations and deformed intruder degrees of freedom.

DOI: 10.1103/PhysRevC.68.014307 PACS number(s): 21.60.Ev, 21.60.Jz, 23.20.Lv, 27.60.+j

I. INTRODUCTION

Spherical and nearly spherical superfluid nuclei have usually low-lying spectra with a vibrationaltype behavior. These vibrational states typically include one-, two- or even three-phonon excitations. A systematic phenomenological analysis of data has confirmed the existence of many-phonon states in even-even nuclei [1]. Microscopically the vibrational states are described as coherent collective combinations of proton and/or neutron one-particle–one-hole (1p-1h), two-particle–two-hole (2p-2h), or three-particle–three-hole (3p-3h) excitations in closed-shell nuclei. In the case of open-shell nuclei, the one-, two- and three-phonon states are described by linear combinations of two-, four- or six-quasiparticle states, respectively. The ground state and pairing correlations, taken into account by the use of the Quasiparticle Random-Phase Approximation (QRPA), are the most important ingredients necessary to build microscopically the multiphonon vibrational states [2,3] of open-shell nuclei. In the case of low-energy excitations the phonons are the lowest (collective) solutions of the QRPA equations of motion. The multiphonon states can be constructed from the QRPA phonons, using several methods of various degrees of complexity. One interesting approach to this direction is the multistep shell model (MSM), which was used to study the structure of the multiquasiparticle states for the Sn [4] and Pb isotopes [5]. Another interesting formalism is the Quasiparticle-Phonon Model (QPM) used by Soloviev and collaborators [6] to study a large variety of spectra for vibrational as well as for deformed nuclei. It is also possible to extend the particle-hole basis by a direct inclusion of the 2p-2h terms within the extended QRPA [7], but in most applications it is preferred, for the sake of simplicity, to adopt the multistep technique in which building blocks are the QRPA phonons.

The even cadmium isotopes $^{106-116}$Cd are known to contain clear indications of vibrational excitations in their low-energy spectra. A systematic experimental survey of the two-phonon-like and three-phonon-like states in these nuclei was performed in Refs. [8–13]. Further studies for the heavier cadmiums, $^{116-122}$Cd, were performed, e.g., in Ref. [14–17]. In these studies the cadmium isotopes were seen as anharmonic vibrators where the anharmonicities are in a position to push the three-phonon 0^+ state below the two-phonon 0^+ state for the heavier $^{116-120}$Cd. A contradicting interpretation, considering ^{118}Cd to be a nearly perfect harmonic vibrator up to three phonons, was given in Ref. [18]. For these excitations the major contribution was assumed to come from the open neutron shell, but also the deformed states, coming from the proton 2p-2h excitations across the $Z=50$ shell gap were looked for. Surprisingly enough, the two-phonon-type of 0^+ state turned out to start also a rotational band typical of a deformed nucleus. This observation, in turn, contradicted the interpretation of [19]. Thus, according to [8,9,11], the first excited 0^+ in the $^{106-114}$Cd nuclei and the second excited 0^+ state in $^{116-120}$Cd nuclei played a double role: on one hand it behaved like a member of an anharmonic spherical two-phonon triplet, on the other hand it seemed to be a bandhead of a deformed intruding rotational band.

Theoretical description of the low-energy excited states and their electromagnetic decays in the cadmium nuclei has been a major challenge already for a long time. Large anharmonic contributions in the vibrator picture have been introduced [20–22] in order to explain the lowering of the energies of the states belonging to the alleged three-phonon quintuplet. The vibrational features of the open-shell neutron excitations and the deformed intruder features of the combined proton 2p-2h and neutron degrees of freedom have envoked the configuration-mixing approach [23] to explain the rather contradictory experimental data in a natural way. Since then the study of the vibrational vs intruder aspects of the cadmium nuclei has been vigorous [10,19,24–31]. In the above-mentioned works the IBA-1 model [32] has been used

as a basic framework describing the anharmonic vibrational features near the U(5) dynamical symmetry limit. The low energy of the 0^+ member of the possible three-phonon quintuplet has been associated with the γ-unstable features of the cadmium isotopes. This is why the authors above have considered the mixing of the U(5) vibrational limit with the O(6) γ-unstable limit of the IBA-1 by using a schematic mixing Hamiltonian simulating 2p-2h excitations from the proton core (thus introducing two extra IBA-1 bosons for the intruder O(6) configurations). In Refs. [30,31] the $^{110-114}$Cd nuclei were treated by diagonalizing the mixing Hamiltonian numerically, and fairly good results were obtained both for the energies and $B(E2)$ values in these nuclei. The emerging conclusion was that the $^{110-114}$Cd nuclei resemble quadrupole vibrators in their $E2$ decay properties (global weak mixing), whereas the energy spectrum has strong contributions of the γ-unstable character (local strong mixing). Later, the three-phonon states were studied in the more advanced IBA-2 framework for ^{110,112}Cd nuclei in Refs. [12,13].

The above IBA-1 studies are based on phenomenological Hamiltonians whose forms were guided by physical intuition based on experimental observations in the cadmium chain of isotopes. On the other hand, it would be rewarding to be able to access the two-phonon-like states in these nuclei by using a completely microscopic approach based on two-nucleon interactions of a realistic many-body Hamiltonian. As a step to this direction, in Ref. [33] we proposed a microscopic method, similar to the MSM, to describe multiphonon states starting from a realistic mean-field basis and a realistic microscopic many-body Hamiltonian. It was first time introduced at the Tamm-Dancoff approximation level in Ref. [34]. In this method we use directly the QRPA equations in deriving the equation of motion for one- and two-phonon states. In this way the Hamiltonian matrix elements, connecting two-phonon components, become proportional to the metric matrix. In Sec. II we give theoretical details necessary to understand the basic ingredients of our model. In Sec. III we define our nuclear Hamiltonian to be used for the discussed cadmium isotopes, and in Sec. IV we analyze the systematics of the low-lying spectra and $B(E2)$ values of $^{110-120}$Cd against the available data. In the last section we draw conclusions.

II. SHORT REVIEW OF THE FORMALISM

In the present formalism the two-phonon states are built in terms of the QRPA degrees of freedom, using a spherical single-particle mean field as a starting point. The most simple way to define a low-energy two-phonon state in the vibrational picture would be to write a degenerate triplet of states based on the two lowest collective quadrupole phonons of the QRPA, i.e., $\Gamma^\dagger_J = 1/\sqrt{2}(Q^\dagger_{2_1^+} Q^\dagger_{2_1^+})_J$. However, these wave functions are too simple to explain the observed splitting of the two-phonon triplet and the decay rates of its states to the one-phonon 2_1^+ state and the ground state. In fact, in this extremely simplified picture the transition from the two-phonon 2^+ state to the ground state is completely forbidden. Thus it is necessary to consider a more general superposition of one- and two-phonon components as starting point. Therefore, the resulting excitation operator must be defined as

$$\Gamma^\dagger_{a_4\alpha_4\mu_4} = \sum_{a_2} Z_1(a_2;a_4\alpha_4) Q^\dagger_{a_2\alpha_4\mu_4} + \sum_{a_2\alpha_2 \leqslant b_2\beta_2} Z_2(a_2\alpha_2 b_2\beta_2;a_4\alpha_4) \times (Q^\dagger_{a_2\alpha_2} Q^\dagger_{b_2\beta_2})_{\alpha_4\mu_4}, \quad (1)$$

where $a_4\alpha_4$ denotes the eigenvalue index and total spin-parity of the state and we denote by $a_2\alpha_2$ the two-particle quantum numbers, namely, the energy eigenvalue and angular momentum (and parity). The eigenstates can be found by using the equation-of-motion technique, i.e.,

$$[\hat{H}, \Gamma^\dagger_{a_4\alpha_4\mu_4}] = \mathcal{E}_{a_4\alpha_4} \Gamma^\dagger_{a_4\alpha_4\mu_4}, \quad (2)$$

leading to the following system of equations:

$$\begin{pmatrix} E_{a_2\alpha_2}\delta_{a_2a_2'} & \mathcal{H}_{12}(a_2;a_2'\alpha_2'b_2'\beta_2') \\ \mathcal{H}_{21}(a_2\alpha_2 b_2\beta_2;a_2') & \mathcal{H}_{22}(a_2\alpha_2 b_2\beta_2;a_2'\alpha_2'b_2'\beta_2') \end{pmatrix} \times \begin{pmatrix} Z_1(a_2';a_4\alpha_4) \\ Z_2(a_2'\alpha_2'b_2'\beta_2';a_4\alpha_4) \end{pmatrix} = \mathcal{E}_{a_4\alpha_4} \begin{pmatrix} \delta_{a_2a_2'} & 0 \\ 0 & \mathcal{I}_{\alpha_4}(a_2\alpha_2 b_2\beta_2;a_2'\alpha_2'b_2'\beta_2') \end{pmatrix} \times \begin{pmatrix} Z_1(a_2';a_4\alpha_4) \\ Z_2(a_2'\alpha_2'b_2'\beta_2';a_4\alpha_4) \end{pmatrix}. \quad (3)$$

Here the metric matrix is defined as

$$\mathcal{I}_{\alpha_4}(a_2\alpha_2 b_2\beta_2;a_2'\alpha_2'b_2'\beta_2') = \langle 0|[(Q_{b_2\beta_2}Q_{a_2\alpha_2})_{\alpha_4}, (Q^\dagger_{a_2'\alpha_2'}Q^\dagger_{b_2'\beta_2'})_{\alpha_4}]|0\rangle, \quad (4)$$

and its explicit form, together with the expressions for the submatrices $\mathcal{H}_{22}$, $\mathcal{H}_{12}$, and $\mathcal{H}_{21}$, are given in Ref. [33]. Here the metric matrix takes care of the Pauli principle when forming the two-phonon states (for a more extensive discussion about these aspects see Refs. [33,35]).

Considering the electromagnetic decay rates of the low-lying one- and two-phonon-like states the corresponding reduced matrix elements, connecting two eigenstates of form (1), or an eigenstate (1) with the ground state, are given explicitly in Ref. [33]. For the electromagnetic transitions, as also for the energies, the metric matrix, taking care of the Pauli principle, is the most important ingredient in our approach.

TABLE I. Basic data for the discussed cadmium nuclei. The pairing gaps (Δ) and the corresponding pairing strengths ($g_{\rm pair}$) for protons and neutrons are given in columns 2–5. In columns 6 and 7 the particle-hole parameters of the QRPA are given for the 2^+ and 4^+ multipoles. In column 8 the experimental $B(E2;2_1^+\rightarrow 0_1^+)$ are given in Weisskopf units (W.u.), and finally, in column 9 we list the values of the corresponding polarization parameter for the effective proton and neutron charges.

A	Δ_p	Δ_n	$g_{\rm pair}^{(p)}$	$g_{\rm pair}^{(n)}$	$g_{\rm ph}(2^+)$	$g_{\rm ph}(4^+)$	$B(E2)_{\rm exp}$(W.u.)	χ
110	1.5058	1.3825	1.1163	0.9136	0.687	0.500	27.4(3)[a]	0.47
112	1.5190	1.3200	1.1205	0.8814	0.710	0.500	30.2(3)[b]	0.54
114	1.4413	1.3511	1.0699	0.9098	0.744	0.700	31.0(19)[c]	0.54
116	1.4373	1.3710	1.1300	0.9950	0.796	0.850	33.56(213)[d]	0.44
118	1.4095	1.4157	1.1170	1.0060	0.889	0.750	33(3)[e]	0.35
120	1.4925	1.4900	1.1430	1.0900	0.836	0.700	27[f]	0.28

[a]Data from Ref. [41].
[b]Data from Ref. [42].
[c]Data from Ref. [43].
[d]Data from Ref. [45].
[e]Data from Ref. [44].
[f]Calculated using $B(E2)=\{0.05659/[t_{1/2}E_\gamma^5(1+\alpha_{\rm tot})]\}e^2b^2$.

III. NUMERICAL APPLICATION TO CADMIUM ISOTOPES

We start our microscopic approach from a single-particle basis of a suitable size. In the present work we use eigenvalues of the spherical Woods-Saxon nuclear mean field with the Coulomb terms included using the parametrization of Ref. [36]. The single-particle wave functions are taken, however, to be eigenstates of a spherical harmonic-oscillator with a suitable oscillator constant, which is a good approximation for bound states in nuclei. We have chosen for the studied $^{110-120}$Cd isotopes a rather large basis of ten proton and 15 neutron single-particle levels around the proton and neutron Fermi surfaces, spanning the following valence space: pf-sdg-$h_{11/2}$ shells for the protons and pf-sdg-pfh shells for the neutrons. The BCS occupation amplitudes and the QRPA eigenstates were calculated using as a residual two-body interaction the G-matrix elements of the Bonn one-boson-exchange interaction [37]. Different channels of this interaction are scaled by constants as described in Refs. [38,39].

The pairing strength for protons and neutrons was adjusted by requiring the calculated pairing gaps to reproduce the empirical ones obtained from the proton and neutron separation energies [40]. The resulting pairing parameters, along with the used pairing gaps, have been listed in Table I. In this case the values $g_{\rm pair}^{(\rm p)}=1.0$ and $g_{\rm pair}^{(\rm n)}=1.0$ correspond to pairing matrix elements coming from the bare monopole part of the G matrix.

The G-matrix elements for the $J^\pi=2^+,4^+$ multipoles in the QRPA calculations have been parametrized by two parameters [39], namely, the particle-hole parameter, $g_{\rm ph}$, and the particle-particle parameter, $g_{\rm pp}$. Here the particle-particle part has practically no effect on the physical observables so that its value has been set to $g_{\rm pp}=1.0$, corresponding to the bare G matrix. The value of the particle-hole parameter controls the energy of the lowest 2^+ and 4^+ states in the QRPA calculation. Thus, this parameter can be used to control the position of the first 2^+ and second 4^+ states in the final theoretical spectrum, except in the case of ^{120}Cd where the collectivity of the QRPA 2^+ phonon made it impossible to adjust the 2_1^+ energy and, at the same time, to obtain a reasonable excitation energy for the 0^+ two-phonon state. In this case a compromise was made to produce reasonable 2_1^+ and $0_{2\text{-ph}}^+$ energies.

The adopted values of the $g_{\rm ph}$ parameters are listed in Table I. It has to be mentioned that the results for the five lowest calculated states (2_1^+, $0_{2\text{-ph}}^+$, $2_{2\text{-ph}}^+$, $4_{2\text{-ph}}^+$, and 4_2^+) depend on the number of the QRPA 2^+ and 4^+ phonons included into the diagonalization of the eigenvalue problem (3). According to our calculations it is enough to take five lowest QRPA phonons of both of these angular momenta to achieve stable energies and wave functions for the above-mentioned states. All the results presented in the following section have been calculated using this number of the QRPA phonons.

The experimental $B(E2;2_1^+\rightarrow 0_1^+)$ value (see Table I) was used to fix the proton and neutron effective charges, $e_{\rm p}$ and $e_{\rm n}$, by using the relations $e_{\rm p}=(1+\chi)e$, $e_{\rm n}=\chi e$ (see, e.g., Ref. [36]). The polarization parameter χ is listed in the last column of Table I.

IV. RESULTS AND DISCUSSION

We present our results for the energies and the $B(E2)$ values in Tables II–IV. Table II shows the theoretical and experimental energies of the two-phonon-type of levels in $^{110-120}$Cd. The two-phonon-type of states are of great interest and the key in the many discussions about the vibrational/intruder features of the low-energy spectra of the $^{106-120}$Cd isotopes along the years [8–19,23–31]. In our calculations for the cadmium chain the two-phonon-type of 0^+ state is always below the other two two-phonon-like states, coincid-

TABLE II. Experimental and theoretical energies of two-phonon states in the $^{110-120}$Cd isotopes. The IBA-1 results are taken from Ref. [10]. For the energy of the $0^+_{2\text{-ph}}$ state in $^{116-120}$Cd isotopes two possible experimental values are given (for more discussion see the text).

	$0^+_{2\text{-ph}}$			$2^+_{2\text{-ph}}$			$4^+_{2\text{-ph}}$		
A	exp	th	IBA-1	exp	th	IBA-1	exp	th	IBA-1
110	1.4731	1.250	1.641	1.4758	1.479	1.473	1.5425	1.425	1.582
112	1.2245	1.218	1.401	1.31241	1.505	1.321	1.4156	1.369	1.406
114	1.1345	1.034	1.243	1.2097	1.361	1.248	1.2837	1.236	1.292
116	1.2826	1.111		1.2130	1.204		1.2194	1.145	
	1.3803								
118	1.2856	0.887	1.214	1.2696	1.286	1.248	1.1649	1.282	1.292
	1.6151								
120	1.3889	0.995		1.3231	1.373		1.2037	1.347	
	1.7449								

ing the interpretation of Refs. [8,9,11] for the $^{110-114}$Cd nuclei. However, for the $^{116-120}$Cd nuclei the theoretical result contradicts the view of Refs. [8,9,11] where for the heavier cadmiums the deformed intruder state becomes the lowest excited 0^+ state and the vibrational state becomes the second excited 0^+ state, already at rather high an energy for the ^{118,120}Cd nuclei. For comparison, in Table II we also quote the IBA-1 results of Kern *et al.* [10]. These results are roughly of the same quality as our results concerning comparison with the data. In Ref. [10] it is also implied that the two-phonon 0^+ state could be the first excited 0^+ state. To allow for the two possible interpretations of the two-phonon 0^+ state, we have indicated in Table II the two possible values for the $0^+_{2\text{-ph}}$ energy starting from ^{116}Cd on. For $^{110-116}$Cd our theoretical spectrum corresponds rather nicely to the experimental one, but for ^{118,120}Cd the calculated 0^+ state tends to drop in energy due to the increasing collectivity of its basic building block, the lowest 2^+ phonon of the QRPA.

Our model does not produce the correct ordering of all the measured two-phonon members in the discussed cadmium nuclei. For the heavier cadmiums, ^{118,120}Cd, the reason being the increase in collectivity of the first 2^+ phonon of the QRPA, leading to a strong drop in the $0^+_{2\text{-ph}}$ energy. For the lighter $^{110-116}$Cd the reason could lie in the omission of the three-phonon and/or the intruder degrees of freedom, in our formalism. These surely affect the relative energies of states in the two-phonon triplet for these nuclei. Thus, the simple energetics of our model calculation can be interpreted as missing three-phonon and/or intruder degrees of freedom which could perturb our calculated wave functions. However, one has to bear in mind that although our calculation does not contain these degrees of freedom, it contains anharmonicities in any wanted amount, so that we are in a position to analyze the effects of such anharmonicities in a consistent way for the energies and the $B(E2)$ values.

The anharmonicities can be clearly seen in Figs. 1 and 2 where we have decomposed the five lowest theoretical levels

TABLE III. Experimental and theoretical ratios $R_{if}(E2)$ of Eq. (5) for the $^{110-114}$Cd isotopes.

	^{110}Cd [a]		^{112}Cd [b]		^{114}Cd	
Transition	th	exp	th	exp	th	exp
$2^+_1 \to 0^+_1$	1	1	1	1	1	1
$0^+_{2\text{-ph}} \to 2^+_1$	1.463		1.349	1.69(49)	1.333	0.88(12) [c]
$2^+_{2\text{-ph}} \to 0^+_{2-\text{ph}}$	0.007		0.021		0.025	0.058(21) [c]
$2^+_{2\text{-ph}} \to 2^+_1$	1.535	1.09(20)	1.551	0.50(11)	1.306	0.92(12) [c]
$4^+_{2\text{-ph}} \to 2^+_{2-\text{ph}}$	0.049		0.094		0.174	0.13(2) [d]
$4^+_{2\text{-ph}} \to 2^+_1$	1.546	1.69(25)	1.506	2.02(23)	1.590	2.16(3) [e]
$2^+_{2\text{-ph}} \to 0^+_1$	0.025	0.049(8)	0.059	0.020(4)	0.108	0.016(3) [c]
$4^+_2 \to 4^+_{2\text{-ph}}$	0.000		0.000		0.010	0.41(14) [c]
$4^+_2 \to 2^+_{2\text{-ph}}$	0.006		0.003		0.017	1.03(49) [c]
$4^+_2 \to 2^+_1$	0.169		0.218		0.161	0.013(4) [c]

[a]Data from Ref. [41].
[b]Data from Ref. [42].
[c]Data from Ref. [43].
[d]Data from Ref. [30].
[e]Data from Ref. [9].

TABLE IV. The same as Table III for the $^{116-120}$Cd isotopes.

	^{116}Cd		^{118}Cd [a]		^{120}Cd	
Transition	th	exp	th	exp	th	exp
$2_1^+ \rightarrow 0_1^+$	1	1	1	1	1	1
$0_{2\text{-ph}}^+ \rightarrow 2_1^+$	1.429	0.92(36) [b]	1.402		1.402	
$2_{2\text{-ph}}^+ \rightarrow 0_{2-\text{ph}}^+$	0.040		0.023		0.011	
$2_{2\text{-ph}}^+ \rightarrow 2_1^+$	1.097	1.00(7) [c]	1.484	1.00(10)	1.549	
$4_{2\text{-ph}}^+ \rightarrow 2_{2-\text{ph}}^+$	0.180		0.126		0.054	
$4_{2\text{-ph}}^+ \rightarrow 2_1^+$	1.590	1.70(56) [b]	1.651		1.583	
$2_{2\text{-ph}}^+ \rightarrow 0_1^+$	0.138	0.051(4) [c]	0.083	0.058(6)	0.036	
$4_2^+ \rightarrow 4_{2-\text{ph}}^+$	0.001	0.0565(4) [c]	0.000	0.09(9)	0.000	
$4_2^+ \rightarrow 2_{2-\text{ph}}^+$	0.008	1.00(7) [c]	0.003	1.00(10)	0.000	
$4_2^+ \rightarrow 2_1^+$	0.035	0.005(1) [c]	0.019	0.01(1)	0.043	

[a]Data from Ref. [18].
[b]Data from Ref. [45].
[c]Data from Ref. [16].

into their one- and two-phonon components. From these figures one can see that the members of the two-phonon triplet $0_2^+, 2_2^+, 4_1^+$ have a large $(2_1^+ 2_1^+)_J$ component, thus supporting the picture of a two-phonon vibrational state with anharmonicities showing up as other nonzero components of one-phonon and two-phonon type. As can also be seen, the two one-phonon-like states (2_1^+ and 4_2^+) have as the major component the corresponding QRPA phonon. Anharmonicities show up as appreciable mixing of two-QRPA-phonon components into these states. For the 4_1^+ state also the 4_2^+ phonon plays an important role.

In Tables III and IV we summarize the experimental information about the values of the ratio

$$R_{if}(E2) = \frac{B(E2; J_i^+ \rightarrow J_f^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \tag{5}$$

for the cadmium isotopes under discussion. The relevant theoretical results concerning the five lowest theoretical states have also been given. Comparing our calculated $B(E2)$ values with the experimental ones for the $^{110-114}$Cd nuclei, the by far best agreement between theory and data is achieved when the experimental 0_2^+, 2_2^+, and 4_1^+ states are taken to correspond to our calculated triplet of anharmonic two-phonon-like states, in agreement with Refs. [8,9,11,15]. This choice of correspondence leads to the numbers of Table III. For ^{116}Cd a similar comparison of theory and data suggests

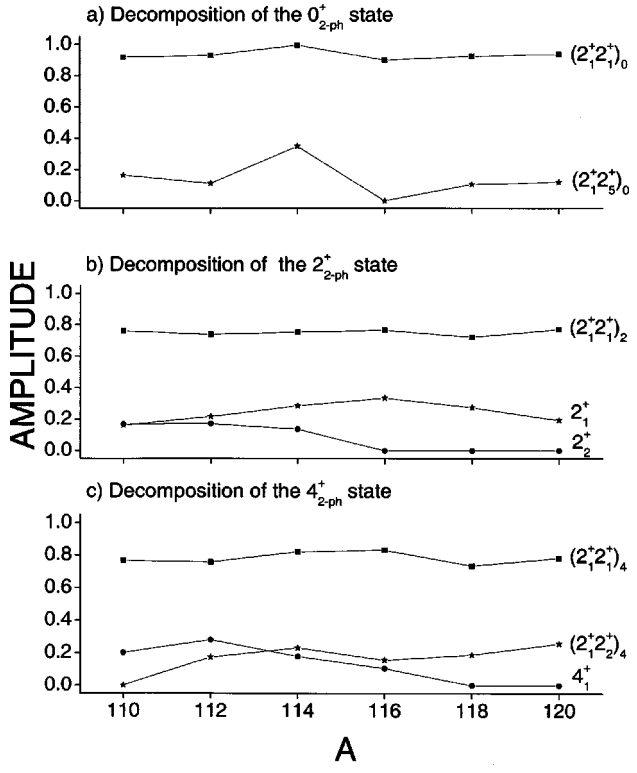

FIG. 1. Decomposition of the calculated wave function into one- and two-QRPA-phonon amplitudes for the $0^+_{2\text{-ph}}$ state (panel a), $2^+_{2\text{-ph}}$ state (panel b), and $4^+_{2\text{-ph}}$ state (panel c) for the discussed $^{110-120}$Cd nuclei.

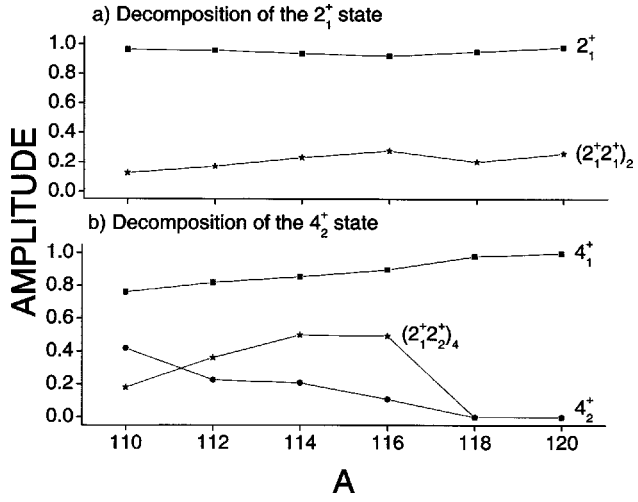

FIG. 2. Decomposition of the calculated wave function into one- and two-QRPA-phonon amplitudes for the 2_1^+ state (panel a) and 4_2^+ state (panel b) for the discussed $^{110-120}$Cd nuclei.

that the choice 0_3^+, 2_2^+, and 4_1^+ would best correspond to the calculated two-phonon triplet, again in agreement with Refs. [8,9,11,15]. The experimental numbers given in Table IV correspond to this interpretation. For the two heaviest cadmiums of Table IV the experimental data is scarce and no definite conclusions about the correspondence can be drawn.

Let us look closer at the correspondence of the calculated and experimental $B(E2)$ values of Tables III and IV. From Table III one can see that judging by the transitions of the two-phonon-type states to the 2_1^+ state one can say that ^{112}Cd seems to be more pure vibrator than the theory predicts, whereas for ^{114}Cd the reverse happens. Notable exceptions are the $2_{2\text{-ph}}^+ \rightarrow 2_1^+$ transition in ^{112}Cd (some earlier data compilations give a value 1.86 for this transition) and the $4_{2\text{-ph}}^+ \rightarrow 2_1^+$ transition in ^{114}Cd. For ^{110}Cd and ^{116}Cd the available data correspond rather well to the calculated $B(E2)$ values. For the other cadmiums there are not enough data to make definite conclusions. What one can say is that for the transition $4_{2\text{-ph}}^+ \rightarrow 2_1^+$ the experimental value is always larger than the theoretical one in the case of $^{110-116}$Cd nuclei. For the very weak $2_{2\text{-ph}}^+ \rightarrow 0_1^+$ transition, the theoretical $B(E2)$ value increases when going from $A=110$ to $A=116$, but then decreases towards $A=120$, whereas the opposite happens for the experimental values. From the theoretical point of view this transition can proceed mainly through the mixing of the lowest 2^+ phonon of the QRPA into the two-QRPA-phonon state. Square of the mixing amplitude determines the theoretical $B(E2)$ value for this transition.

The availability of experimental data allows us to make some observations about the decay of the theoretical 4_2^+ state to the $4_{2\text{-ph}}^+$, $2_{2\text{-ph}}^+$, and 2_1^+ states for the $^{114-118}$Cd nuclei. The theoretical 4_2^+ state is a typical two-quasiparticle state described by a noncollective QRPA phonon. As can be seen from Tables III and IV, the agreement between the data and the calculations is bad for the decay of this state, much worse than for the transitions involving the other theoretical low-energy states. An explanation for this discrepancy could be that the experimental 4_2^+ state is not of two-quasiparticle character, but rather of three-phonon (as suggested, e.g., in Ref. [10]) or intruder (as interpreted, e.g., in Ref. [29]) character. In any case, it seems that the 4_2^+ state and maybe the 4_3^+ state also [10,29] are not appropriate to be associated with our theoretical 4_2^+ state. Our calculated 4_2^+ state should then rather be associated with some higher-lying experimental 4^+ state. This conclusion we have taken into account in our calculations by adjusting the $g_{\text{ph}}(4^+)$ parameter of Table I such as to yield to the energy of the theoretical 4_2^+ state a value somewhat above the experimental 4_2^+ energy, if possible. It has to be stressed here that the above conclusion is only indirect since we have not calculated the intruder or three-phonon contributions to the $B(E2)$ branchings.

According to our calculated $B(E2)$ values the two-phonon triplet of states seems to follow the systematics postulated in Refs. [8,9,11,15], whereas according to the calculated anharmonicities in the energies of these states the interpretation of Refs. [8,9,11,15] differs from our results for the excited 0^+ states in $^{116-120}$Cd. It thus seems that the deformed intruder states have a small effect on the $B(E2)$ values but a considerable effect on the energies, especially for the heavier $^{116-120}$Cd. The picture of strong-mixing (for the 0_2^+ and 0_3^+ states) vs weak-mixing (for the 2^+ and 4^+ states) advocated in Refs. [30,31] seems to support this conclusion.

V. CONCLUSIONS

In this paper we have used a microscopic formalism to study the low-energy excited states of $^{110-120}$Cd, in particular, the feasibility of certain states to be interpreted as two-phonon states of vibrational origin. The adopted theory is based on a large single-particle valence space and a realistic nuclear Hamiltonian, using phenomenologically renormalized two-body interaction based on the Bonn one-boson-exchange G matrix. The used theoretical formalism naturally embraces vibrational degrees of freedom starting from the QRPA collective phonons. The above-mentioned nuclear Hamiltonian is used to introduce unharmonicities into the description of the low-lying excited states leading to dynamical splitting of the energies of the two-phonon vibrational states. At the same time the Hamiltonian is also allowed to mix the one-phonon and two-phonon collective degrees of freedom.

We have applied our model to describe low-lying excited states in the $^{110-120}$Cd isotopes. When comparing our results with the data, one has to be careful since experimentally the cadmium isotopes have low-lying three-phonon-type of states and deformed intruder degrees of freedom arising from the proton 2p-2h excitations across the $Z=50$ closed major shell. We have found that our calculated $B(E2)$ values for the two-phonon-like triplet of states well reproduce the measured $B(E2)$ values of the lowest excited 0^+ and 4^+ states, the second lowest 2^+ state for the $^{110-114}$Cd isotopes and the lowest excited 4^+ state, and the second lowest excited 0^+ and 2^+ states for the ^{116}Cd isotope. This is in agreement with some earlier empirical studies of the systematics of the $^{106-120}$Cd isotopes. However, our result contradicts this systematics when the excitation energies of the 0^+ states are considered. This would indicate that the intruder degrees of freedom only weakly affect the electric decay properties of these states but cause strong perturbation in the 0^+ energies for the heavier $^{116-120}$Cd isotopes. This scheme seems to support the strong-mixing vs weak-mixing picture propagated in Refs. [30,31]. Additionally, based on the calculated $B(E2)$ values and excitation energies, one may indirectly deduce that the experimental 4_2^+ state is not likely to have a noncollective two-quasiparticle character, but rather a three-phonon or deformed intruder origin.

ACKNOWLEDGMENTS

This work was supported by the Academy of Finland under the Finnish Center of Excellence Programme 2000-2005 (Project No. 44875, Nuclear and Condensed Matter Programme at JYFL).

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