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Anomalous Coulomb matrix elements in the $f_{7/2}$ shell

S. J. Williams,[1,*] M. A. Bentley,[1] D. D. Warner,[2] A. M. Bruce,[3] J. A. Cameron,[4] M. P. Carpenter,[5] P. Fallon,[6] L. Frankland,[3] W. Gelletly,[7] R. V. F. Janssens,[5] D. T. Joss,[2] C. D. O'Leary,[8] C. J. Lister,[5] A. Poves,[9] P. H. Regan,[7] P. Reiter,[10] B. Rubio,[11] D. Seweryniak,[5] C. E. Svensson,[12] and S. M. Vincent[7]

[1]*School of Chemistry and Physics, Keele University, Staffordshire ST5 5BG, United Kingdom*
[2]*CLRC Daresbury Laboratory, Daresbury, Warrington WA4 4AD, United Kingdom*
[3]*School of Engineering, University of Brighton, Brighton BN2 4GJ, United Kingdom*
[4]*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4K1*
[5]*Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439, USA*
[6]*Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[7]*School of Physics and Chemistry, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom*
[8]*Department of Physics, University of York, Heslington, York YO10 5DD, United Kingdom*
[9]*Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, E-28049, Spain*
[10]*Ludwig-Maximillians-Universitat, D-85748 Garching, Germany*
[11]*CSIC, Universitat de València, E-46071 València, Spain*
[12]*Department of Physics, University of Guelph, Guelph, Ontario, Canada N1G 2W1*

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γ decays from high-spin states in the $N=Z-1$ nucleus ${}^{53}_{27}Co_{26}$ have been identified for the first time. Level energies and Coulomb energy differences between these states and their analogs in its mirror nucleus ^{53}Fe have been compared with large-scale pf shell-model calculations, which offer excellent agreement. New information has been obtained on two-proton Coulomb matrix elements needed in the interpretation. These have been extracted from the data via a number of methods and are shown to exhibit an anomalous behavior for the $J=2$ coupling.

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One of the fundamental tenets of nuclear structure is the charge independence of the nuclear force. This gives rise to the concept of the neutron-proton exchange symmetry, which mandates that the energies of excited states in two nuclei described by the exchange of neutron and proton numbers (mirror nuclei) differ only because of the effect of the Coulomb force. Until the last decade, studies of these Coulomb energy differences (CED) focused almost exclusively on the ground states of nuclei. However, the study of high-spin states in mirror nuclei has become one of significant interest in recent years due to the massive increases in sensitivity afforded by large arrays of γ-ray detectors. This has enabled the study of nuclei with $N<Z$ (which are invariably the least accessible) to excitation energies over 10 MeV. As the Coulomb force is well understood, the CED—which arise from the spatial rearrangement of the protons—provide a particularly delicate probe that offers considerable insight into the changing structure of the quantum states.

The study of high-spin states in mirror nuclei has been primarily centered around nuclei where the last nucleons are filling the $1f_{7/2}$ orbit of the nuclear shell model (e.g., Refs. [1–6]). This orbit holds the unique position of being relatively well separated in energy from other orbits, whilst having sufficient degeneracy to allow the development of a significant degree of collectivity. It is thus an ideal arena in which to study the interplay of single-particle and collective effects as the lowest states of each spin are built upon relatively pure $f_{7/2}$ configurations. However, the appearance of collectivity signals the need for additional degrees of freedom and recent advances in the shell model have enabled calculations incorporating the full set of pf orbits to be made in this region (e.g., Refs. [7,8]).

Considerable success has now been achieved (e.g., Refs. [4,5]) in using the predictions of the shell model to help interpret the CED in odd-A mirror pairs in terms of the structural changes that accompany increasing angular momentum. This requires the definition of a set of two-proton Coulomb matrix elements (CME) to be used in conjunction with the shell-model wave functions to determine the Coulomb energy contribution for each level. The choice of CME to be used in the model calculations is crucial, and a number of different methods of obtaining these quantities can be proposed. Here we discuss these methods and present new results on CME extracted from experimental data which reveal a consistent anomaly in their behavior as a function of spin. Specifically, the $J=2$ value is *higher* than the $J=0$ value, rather than the steady reduction in the CME anticipated on intuitive physical grounds as a pair of $f_{7/2}$ protons recouples from $J=0$ to $J=6$. Nevertheless, calculations of the CED incorporating these anomalous CME have been shown to reproduce well the general trends of experimentally determined CED in the $f_{7/2}$ shell (e.g., Refs. [4,5]). A number of possible origins of the anomaly will be considered in more detail later, including that proposed in a recent detailed shell-model study by Zuker *et al.* [9], in which an additional isospin-nonconserving part of the nuclear interaction was introduced to account for the observed CED.

The new results stem from an experimental study that has determined, for the first time, a detailed high-spin level

*Electronic address: s.j.williams@phys.keele.ac.uk

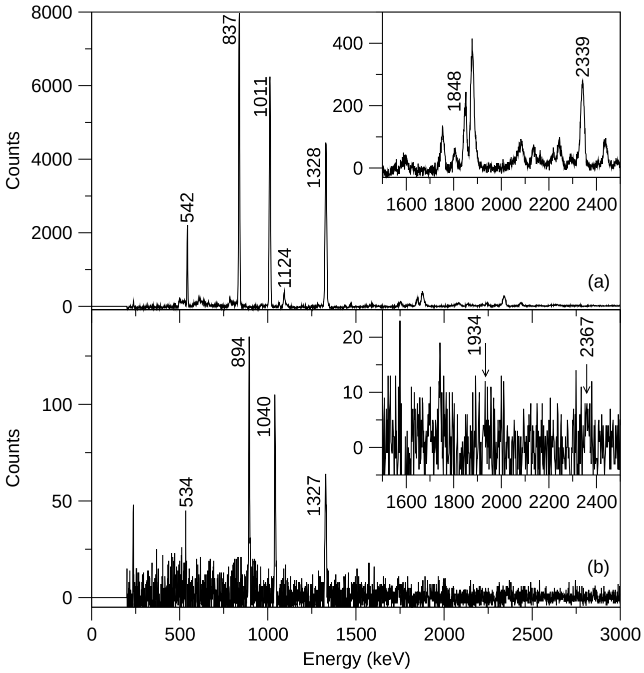

FIG. 1. (a) A spectrum of ^{53}Fe created by requiring double coincidences between pairs of yrast transitions (see text). Strong unmarked transitions in the inset correspond to higher-lying transitions in ^{53}Fe. (b) A spectrum of ^{53}Co created in an identical manner to (a), by gating on the equivalent analog transitions.

scheme of ^{53}Co, one member of the $A=53$, $N=Z\pm1$ mirror pair, and a comparison of the data with the results of shell-model calculations within pure-$f_{7/2}$ and large-scale pf model spaces. Of the $A=53$ mirror pair, $^{53}_{27}$Co$_{26}$ and $^{53}_{26}$Fe$_{27}$, only the latter has been studied through γ-ray spectroscopy (e.g., Ref. [10]). Prior to the current work, two proton-decaying excited states have been identified in ^{53}Co [11]. One of these, which is important for the work presented here, is an isomeric state at (3179 ± 30) keV ($T_{1/2}=260\pm20$ ms), which is presumed to be the analog of the isomeric $J^{\pi}=\frac{19}{2}^{-}$ band terminating state in ^{53}Fe. The large error on the energy comes from the uncertainties in the ground-state masses and measured proton energies.

An experiment was performed at the ATLAS facility at Argonne National Laboratory. The mirror nuclei were populated using the ^{32}S(^{24}Mg,$2pn$)^{53}Fe and ^{32}S(^{24}Mg,$p2n$)^{53}Co reactions with a ^{32}S beam energy of 95 MeV and a 500 μg cm^{-2} ^{24}Mg target. γ rays were detected using the GAMMASPHERE array of 101 Compton-suppressed HpGe detectors, and the resulting γ-ray coincidences were sorted into $\gamma\gamma$ matrices and $\gamma\gamma\gamma$ cubes. A spectrum created by requiring double coincidences between a γ ray at 287 keV and any one of the 837, 1011, and 1328 keV known γ decays in ^{53}Fe [10] is shown in Fig. 1(a), with the level scheme shown in Fig. 2(b). The resulting spectrum of ^{53}Fe consists of *four* strong coincident γ rays, with only a weak contribution from other ^{53}Fe transitions [see inset of Fig. 1(a)]. This characteristic spectrum provides an ideal fingerprint to aid in the search for γ decays between states in ^{53}Co. This search was performed in a two- and three-dimensional coincidence analysis, placing wide coincidence gates at a range of energies close to those expected for the mirror transitions and searching for transitions of the expected intensity distributions. By this method weak γ rays of energies 1327, 1040, 894, 534, and 320 keV were found to be in coincidence and the resulting spectrum, generated in an identical manner as for ^{53}Fe, is shown in Fig. 1(b).

A level scheme up to $J^{\pi}=\frac{17}{2}^{-}$ was obtained from the coincidence analysis and is shown in Fig. 2(a). This is assigned to ^{53}Co through mirror symmetry arguments and the similarity of the spectra with respect to the fingerprint discussed. Figure 2(b) shows the level scheme of ^{53}Fe, and is in agreement with Ref. [10]. Due to the low statistics, spins and parities for ^{53}Co were not measured and are assigned on the basis of mirror symmetry arguments alone. In Fig. 2(c) we present the predicted ^{53}Fe scheme from a large-scale pf shell-model calculation allowing up to five excitations into non-$f_{7/2}$ pf levels. Calculations performed on this basis have been reported in Refs. [5,8] and have been shown [8] to give results virtually indistinguishable from the full-pf calculations.

The experimental CED, calculated as $E_x(^{53}\mathrm{Co};J)-E_x(^{53}\mathrm{Fe};J)$ are shown in Fig. 3(a), where the most striking aspect is the smooth evolution of the curve throughout the

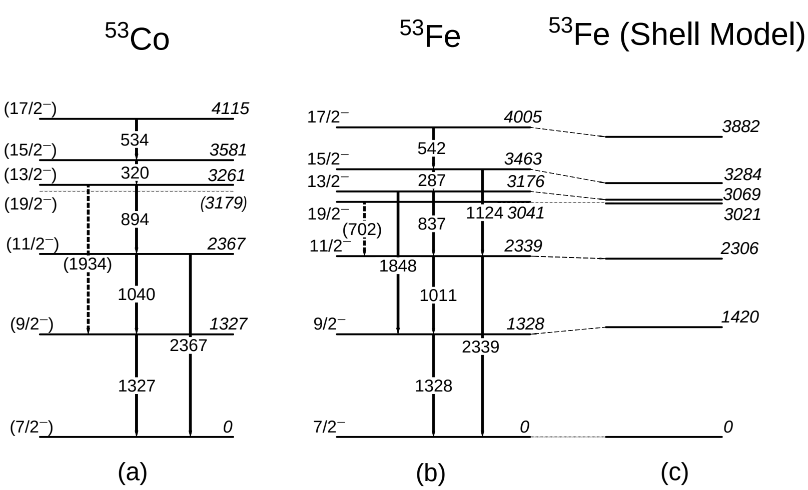

FIG. 2. (a) The level scheme of ^{53}Co from this work. The isomeric $J^{\pi}=\frac{19}{2}^{-}$ level was not observed, and is taken from Ref. [11]. (b) The known level scheme of ^{53}Fe [10]. (c) Prediction of the large-scale pf shell model for ^{53}Fe.

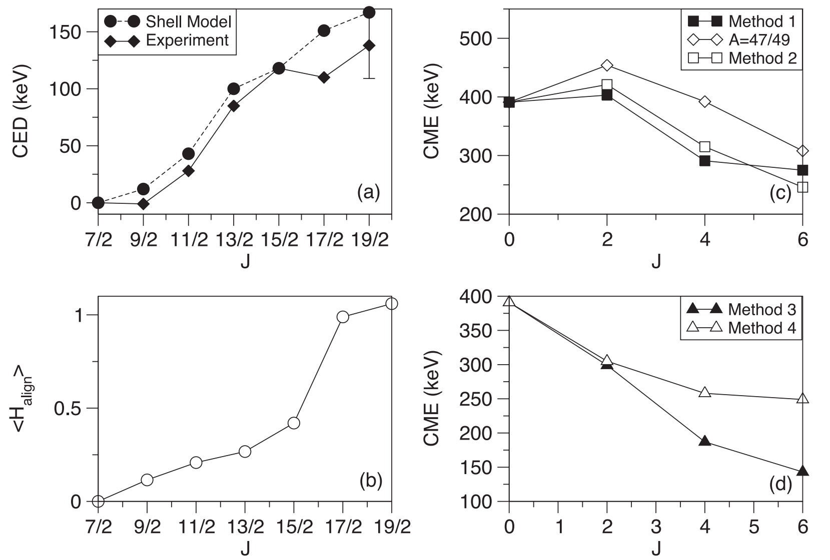

FIG. 3. (a) The $A=53$ experimental CED compared with the CED determined from the large-scale pf shell-model calculations. The error bar on the final data point arises from the uncertainty (see text) on the energy of the $J^\pi=\frac{19}{2}^-$ state in ^{53}Co. (b) Difference in the expectation value of H_{align}, plotted as ^{53}Fe-^{53}Co (see text). (c) The Coulomb matrix elements (CME) determined from this work using methods 1 and 2 and from a fit to the A=47 and 49 mirror pairs, see text. (d) CME calculated from methods 3 and 4 [4], see text. For ease of comparison, all the CME shown here are normalized at $J=0$ to the harmonic oscillator value of 391 keV [4].

spin range. The Coulomb effect of rotational alignments of pairs of particles has been used previously (e.g., Refs. [1–5,12]) to explain similar trends in the CED of other mirror pairs nearer to midshell, but here the small valence number precludes collective arguments. However, considering a pure $f_{7/2}$ space, the effect becomes clear. In this basis the yrast sequences up to $J^\pi=\frac{19}{2}^-$ have the configurations $\nu(f_{7/2})^{-2}\pi(f_{7/2})^{-1}$ for ^{53}Co and $\nu(f_{7/2})^{-1}\pi(f_{7/2})^{-2}$ for ^{53}Fe—i.e., three "holes" in the closed-shell nucleus ^{56}Ni. The dominant contribution to the ground-state wave function in ^{53}Fe is therefore expected to be $\nu(f_{7/2})^{-1}_{j_\nu=\frac{7}{2}}\otimes\pi(f_{7/2})^{-2}_{j_\pi=0}$, while the band-terminating state at $J^\pi=\frac{19}{2}^-$ has a pure $\nu(f_{7/2})^{-1}_{j_\nu=\frac{7}{2}}\otimes\pi(f_{7/2})^{-2}_{j_\pi=6}$ structure. The intermediate states result from a gradual recoupling of the proton-hole pair from $J=0$ to the maximum angular momentum of $J=6$. This recoupling involves a gradual reduction in the spatial overlap between the pair of protons, yielding a reduction of the Coulomb repulsion between them. The overall effect is a compression of the excited states in ^{53}Fe with respect to ^{53}Co and as a result the CED should show a smooth increase up to the band-terminating state.

To confirm this interpretation of the behavior of the proton pairs, we have used the large-scale pf shell wave functions to determine the expectation value of an operator (called H_{align}) which, in effect, "counts" the number of fully aligned $J=6$, $T=1$ pp pairs contributing to each state (see Ref. [5] for details). The result is plotted as ^{53}Fe-^{53}Co in Fig. 3(b), and confirms that the $J=6$ contribution gradually increases through the spin range. A notable feature of this plot is the large increase from $J=\frac{15}{2}$ to $J=\frac{17}{2}$, which indicates that the gradual re-coupling of the proton-hole pair in ^{53}Fe has occurred fully by $J=\frac{17}{2}$. While in the pure $f_{7/2}$ space, both the $J^\pi=\frac{17}{2}^-$ and $J^\pi=\frac{19}{2}^-$ states can be built only from the configuration $\nu(f_{7/2})^{-1}_{j_\nu=\frac{7}{2}}\otimes\pi(f_{7/2})^{-2}_{j_\pi=6}$, Figs. 3(a) and 3(b) show that both the calculated CED and $\langle H_{\mathrm{align}}\rangle$ increase slightly from $J=\frac{17}{2}$ to $J=\frac{19}{2}$, indicating contributions from outside the $f_{7/2}$ space.

As mentioned initially, an essential ingredient in reproducing the empirical CED with the shell-model calculations is the set of CME for the allowed couplings of the valence protons. The pf-basis calculations for this $A=53$ mirror pair are presented here in Fig. 3(a). Here, as in previous work [4,5], the CME have been determined empirically from the level energies of the $A=42$, $T=1$ mirror pair for the $f_{7/2}$ protons and calculated from harmonic oscillator wave functions otherwise. Figure 3(a) shows that the resultant shell-model CED, using these matrix elements, provides excellent agreement with the data on a state-by-state basis.

Given their importance in interpreting the structural changes implied by the measured CED, it is valuable to compare alternative methods to extract the CME.

Method 1. We obtain the CME in this work by exploiting the simplicity of the $A=53$ system and through fitting the experimental CED. The pure $f_{7/2}$ wave functions of Ref. [13] have been used which employ the jj-coupling scheme in the proton-neutron representation. (Calculations with this basis reproduce the energies of the states quite reasonably [13]). Within this formalism the 1 neutron, 2 proton wave function of ^{53}Fe is written as

$$|J^\alpha\rangle=\sum_{J_p} a^{J,\alpha}_{J_p}|j_n=7/2,j_p^2=J_p;J^\alpha\rangle,$$

where $J_p=0,2,4,6$ and α labels different states with equal J.

We assume, in this simple scenario, that the CED is entirely attributable to pp effects in ^{53}Fe. The CED for each state can then be determined from

$$\mathrm{CED}(J)=\sum_{J_p} a_{J_p}^2 V_C^{J_p},$$

where $V_C^{J_p}$ is the Coulomb matrix element for two protons coupled to J_p and the labels J,α on the amplitudes have been dropped. The CME were allowed to vary independently and the best fit to the experimental $A=53$ CED was obtained. Only states up to $J^\pi=\frac{17}{2}^-$ were included in the fit, due to the large error on the energy of the $J^\pi=\frac{19}{2}^-$ state. The results are shown in Fig. 3(c). It was found that the fit was largely insensitive to the magnitude of the $J_p=0$ element, due to the fact that the CED is determined from excitation energies. It is only the *relative* values of the CME that contribute to the CED curve and hence the $J_p=0$ point has been fixed at the harmonic oscillator value of 391 keV (taken from Ref. [4]) for *all* sets of CME shown in Figs. 3(c) and 3(d).

Method 2. The CME can be obtained from the $A=42$ nuclei assuming *charge symmetry*, i.e., from the energies of the 2^+,4^+, and 6^+ states in the mirror nuclei ^{42}Ti and ^{42}Ca, which are, respectively, two protons and two neutrons added to the ^{40}Ca core. The difference in Coulomb energy can be represented as

$$E_C(J)=\mathrm{BE}_J(^{42}\mathrm{Ti})-\mathrm{BE}_J(^{42}\mathrm{Ca})+V_C^{\mathrm{g.s.}},$$

where BE is binding energy (negative) and $V_C^{\mathrm{g.s.}}$ accounts for the ground state energy difference from the neutron-proton mass difference and the Coulomb interaction between the valence protons and the core. Using the excitation energies from these nuclei effectively eliminates $V_C^{\mathrm{g.s.}}$ as long as the core interaction remains constant as a function of J. The CME obtained in this manner are plotted in Fig. 3(c).

Method 3. The CME can be obtained from the $A=42$ isobaric triplet assuming *charge independence* [14]. In this case, the difference in Coulomb energies is

$$E_C(J)=\mathrm{BE}_J(^{42}\mathrm{Ti})+\mathrm{BE}_J(^{42}\mathrm{Ca})-2\mathrm{BE}_J(^{42}\mathrm{Sc}).$$

The difference in nucleon mass and the interaction of the $f_{7/2}$ protons with the core are eliminated for each state by including the odd-odd ^{42}Sc system. Only the Coulomb interaction between the last two valence protons should remain, and these CME are shown in Fig. 3(d).

Method 4. The CME can be calculated using harmonic oscillator wave functions. These calculations [14] are shown in Fig. 3(d).

Also shown in Fig. 3(c) are the CME derived from a global fit of the CED from the $A=47$ and 49 mirror pairs using the full-pf shell model [4] (open diamonds)—a similar approach to method 1. Examination of the three sets of CME in Fig. 3(c) (derived from CED in mirror nuclei) immediately reveals an anomaly at $J=2$. The values consistently increase in going from the $J=0$ to the $J=2$ coupling in each case. This is, of course, counterintuitive, since breaking a proton pair must decrease the Coulomb energy between them, and hence indicates that these CME must include other hidden effects. Thus, the CME derived from the new $A=53$ data, taken together with those obtained from the fit to the $A=49$ and 47 systems and the empirical values from the $A=42$ mirror pair, reveal a consistent picture emerging across the *entire* shell. That is, an anomalously high $J=2$ matrix element is always present when the CME are derived from experimental data on mirror pairs. In contrast, Fig. 3(c) shows that matrix elements extracted from the $A=42$ isobaric triplet (method 3) behave as expected, as do the pure harmonic oscillator values (method 4).

In order to understand this anomaly, we turn first to the CME obtained directly from the $A=42$ mirror nuclei using method 2 [see Fig. 3(c)]. These values (charge symmetric) will contain a contribution from the interaction of the valence protons with the core which, if such an interaction changes with J, will affect the CME derived from the relative energies of the excited states. If this was the source of the anomaly, it would be absent in the $A=42$ values in Fig. 3(d) from method 3 (charge independent) as this core interaction is eliminated on *a state-by-state basis*. This J-dependent interaction could be attributed either to a changing deformation or to changing admixtures of configurations other than pure $(f_{7/2})^2$ (e.g., four-particle–two-hole) in the $A=42$ states. However, whilst these effects could contribute to the anomaly in the $A=42$ values, it is difficult to imagine how they can contribute consistently across the *entire* shell, especially when deformation is known to change drastically with the number of $f_{7/2}$ valence particles. Nevertheless, the data presented here suggest the existence of an effect occurring at low spin across the shell which results in an anomalous $J=2$ CME value.

Another phenomenon that can contribute to small variations in the observed CED at low spins is associated with changing deformation among the yrast states with increasing spin. This was first treated in a geometrical framework in Ref. [4] and has been investigated in the shell-model approach for the $A=50$ mirror pair ^{50}Fe/^{50}Cr [15] and more recently for a range of $f_{7/2}$ isobaric multiplets [9]. It was shown in these latter studies that the deformation effect associated with partial occupation of the $p_{3/2}$ orbital can produce significant contributions to the CED at low spins, where the anomaly is observed. However, these contributions are of the wrong sign to account for the $J=2$ anomaly. The recent shell-model study of Zuker *et al.* [9] has investigated the various monopole (i.e., radial/deformation effects) and multipole contributions to the CED as a function of spin in isobaric analog nuclei in the middle of the $f_{7/2}$ shell. This study concludes that, when CME derived from the harmonic oscillator are used in the calculations, good agreement with the CED data on mirror nuclei can only be obtained when an additional multipole term for the $f_{7/2}$ $J=2$ coupling is included. This observation is, of course, consistent with the conclusions presented here regarding the $J=2$ anomaly. Reference [9] derives the magnitude of the additional $J=2$ term from a comparison of the harmonic oscillator CME with those derived from the $A=42$ mirror pair, and attributes its origin to an isospin-nonconserving contribution to the nuclear interaction, i.e., a charge-symmetry breaking term in the case of mirror nuclei.

In summary, therefore, it is now becoming established that a *quantitative* study of CED allows a much more reliable insight into nuclear structure effects than might previously have been assumed. Indeed, the surprise is that an analysis of Coulomb energies at the level of tens of keV is, in fact, possible. Moreover, it has now been shown that one can fit the measured CED to the predictions of the shell model and obtain results with an astonishing consistency. This process has, however, revealed a striking $J=2$ anomaly in the low-spin CED of mirror nuclei which, although its origin remains unclear, has now been shown to occur consistently throughout the $f_{7/2}$ shell.

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