Measurement of the $\pi^- p \to 3\pi^0 n$ total cross section from threshold to 0.75 GeV/c

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We report a new measurement of the $\pi^- p \to 3\pi^0 n$ total cross section from threshold to $p_\pi = 0.75$ GeV/c. The cross section near the $N(1535)\frac{1}{2}^-$ resonance is only a few μb after subtracting the large $\eta \to 3\pi^0$ background associated with $\pi^- p \to \eta n$. A simple analysis of our data results in the estimated branching fraction $\mathcal{B}[S_{11} \to \pi N(1440)\frac{1}{2}^+] = (8 \pm 2)\%$. This is the first such estimate obtained with a three-pion production reaction.

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In this paper, we present a new measurement of the $\pi^- p \to 3\pi^0 n$ cross section from threshold to $p_{\pi^-} = 750$ MeV/c. If this reaction is s-channel dominated, it is expected to proceed via sequential N^* decays:

$$\pi^- p \to N^* \to \pi^0 N^0(1440)\frac{1}{2}^+ \to \pi^0 \pi^0 \Delta^0 \to 3\pi^0 n$$

and

$$\pi^- p \to N^* \to \pi^0 N^0(1440)\frac{1}{2}^+ \to \pi^0 f_0 n \to 3\pi^0 n,$$

where f_0 denotes the strong s-wave isoscalar $\pi\pi$ interaction. The first process is expected to dominate because the branching fraction for $N(1440)\frac{1}{2}^+ \to \pi\Delta$ is about double that for $N(1440)\frac{1}{2}^+ \to f_0 n$ [1]. The dominant resonances in our energy range are $N(1520)\frac{3}{2}^-$ and $N(1535)\frac{1}{2}^-$. For the first resonance, the decay $N^* \to \pi N(1440)\frac{1}{2}^+$ involves a d-wave transition and should therefore be suppressed. Thus, the favored first stage should be $\pi^- p \to N(1535)\frac{1}{2}^- \to \pi^0 N^0(1440)\frac{1}{2}^+$, which involves an s-wave transition. Within a simple model consistent with this picture of the reaction, we have extracted the branching fraction $\mathcal{B}[S_{11} \to \pi N(1440)\frac{1}{2}^+]$, where S_{11} denotes the $N(1535)\frac{1}{2}^-$ resonance.

Two factors determine the complexity of the measurement.

(i) The detection of six photons in the final state requires a nearly 4π sr solid angle photon spectrometer with good energy and angular resolution.

(ii) The threshold for $\pi^- p \to 3\pi^0 n$ is at $p_\pi = 463$ MeV/c. The $\pi^- p \to \eta n$ channel opens up at $p_\pi = 685$ MeV/c, introducing a large background due to the $\eta \to 3\pi^0$ decays. The total cross section for η production grows very rapidly, reaching about 2.5 mb at $p_{\pi^-} = 750$ MeV/c [2]. This value is about 100 times larger than the expected $3\pi^0$ production. Good resolution of the $3\pi^0$ invariant mass is necessary to reject this background.

The Crystal Ball multiphoton spectrometer readily fulfills these needs.

The Crystal Ball (CB) is constructed of 672 optically isolated NaI(Tl) crystals that cover 93% of 4π sr. Electromag-

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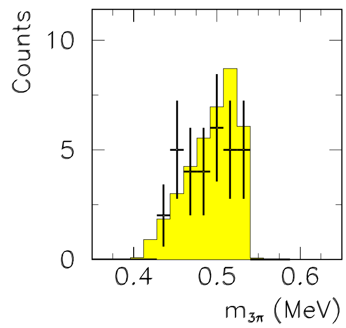

FIG. 1. Invariant mass distribution of the selected $3\pi^0$ events at $p_{\pi^-}=665$ MeV/c. Crosses show the data. The shaded histogram indicates the Monte Carlo results normalized to the number of experimental events.

netic showers in the spectrometer are measured with an energy resolution $\sigma_E/E\sim2.0\%/[E\ (\mathrm{GeV})]^{0.36}$; the angular resolution for photon showers at energies 0.05–0.5 GeV is $\sigma_\theta=2°-3°$ for the polar angle and $\sigma_\phi=2°/\sin\theta$ for the azimuthal angle. The experiment was performed in the C6 beam line of the Brookhaven National Laboratory AGS. The centroid of the beam momentum distribution is known to better than 1%. Further details on the experiment and analysis can be found in Ref. [3].

For the Monte Carlo simulation of our experiment, we implemented the GEANT 3.21 package available from CERN [4]. The $3\pi^0n$ acceptance was calculated with four-body phase space. We also simulated the sequential process $\pi^-p\to\pi^0N^0(1440)$ followed by $N^0(1440)\to\pi^0\Delta^0(1232)$, and then $\Delta^0(1232)\to\pi^0n$. The result of the simulation for the acceptance is consistent with that of the four-body phase-space distribution. For the interaction of neutrons with NaI we used the FLUKA and GHEISHA packages [4]. The combination FLUKA/GHEISHA gives good agreement between the simulated and measured efficiency of the CB for neutrons with kinetic energies above 20 MeV [6].

The event selection procedure is described in Ref. [3]. There are 15 possible combinations of two-cluster pairs to form $3\pi^0$ event candidates from six clusters. We tested all 15 combinations. An event candidate was used when at least one combination satisfied the $3\pi^0$ hypothesis at the 80% confidence level; i.e., with probability $>20\%$. The $\eta\to3\pi^0$ background was suppressed by rejecting events that satisfy the $\eta\to3\pi^0$ hypothesis at the 99% confidence level. The resulting acceptance varies from about 17% for beam momenta below η threshold to about 9% for momenta above η threshold.

The invariant mass of $3\pi^0$ events selected for $p_{\pi^-}=665$ MeV/c is shown in Fig. 1 and compared to the Monte Carlo results. The width of the beam momentum distribution (σ) is about 1% (see first column in Table I). A small fraction of photons produce more than one cluster in the detector. Because of this effect the data above 653 MeV/c are contaminated with 5–10% $2\pi^0$ background, which was subtracted for the cross section calculations. The $2\pi^0$ background vanishes for momenta of 653 MeV/c and below because of the strong decline in the $2\pi^0$ production cross section [7]. Above 685 MeV/c, the analysis becomes more complex due to the $\eta\to3\pi^0$ contribution. The left panel of Fig. 2 shows the $3\pi^0$ invariant mass for $p_{\pi^-}=747$ MeV/c before subtracting the $\eta\to3\pi^0$ background. The experimental results are compared to the $\eta\to3\pi^0$ Monte Carlo events that satisfied all the above selection criteria for $3\pi^0$ events from resonance production. The normalization for the $\eta\to3\pi^0$ Monte Carlo distribution is calculated with the total number of η's produced and the Crystal Ball acceptance for the $\eta\to3\pi^0$ background events. The total number of η's is determined from the number of $\eta\to2\gamma$ events detected in our experiment. In our calculations, we used the η branching ratios listed in Ref. [5]. Above η production threshold, the number of $\eta\to3\pi^0$ events produced in the experiment is $40-60$ times higher than the number of $3\pi^0$ events not from

TABLE I. $\pi^-p\to3\pi^0n$ total cross section. Only statistical uncertainties are listed. At 471 MeV/c, the upper limit for $3\pi^0$ production is listed at the 90% confidence level. Column 1 lists the average beam momentum, column 2 lists the number of raw $3\pi^0n$ events detected, and column 3 lists the calculated number of $\eta\to3\pi^0$ background events. The number of good events was calculated as the difference between the number of $3\pi^0n$ events and the $\eta\to3\pi^0$ background. A (5–10)% contribution from $2\pi^0$ is not included in this table.

p_{π^-} (MeV/c)	$3\pi^0n$ detected (data)	$\eta\to3\pi^0$ background (MC)	$\sigma_{total}(3\pi^0)$ (μb)
471 ± 5	0	0 ± 0	<0.25
547 ± 5	1	0 ± 0	0.11 ± 0.11
653 ± 6	36	0 ± 0	2.5 ± 0.5
665 ± 7	31	0 ± 0	3.1 ± 0.9
677 ± 7	29	3 ± 0	5.8 ± 1.8
689 ± 7	52	25 ± 1	10.3 ± 3.3
704 ± 7	226	164 ± 4	6.5 ± 2.4
718 ± 7	175	101 ± 2	19.8 ± 4.7
726 ± 8	229	129 ± 3	22.7 ± 4.4
747 ± 7	197	122 ± 3	14.2 ± 5.2

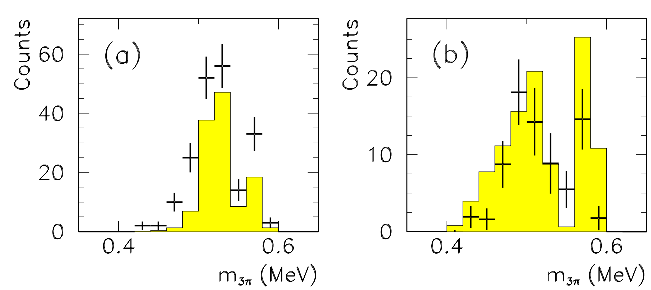

FIG. 2. (a) $3\pi^0$ invariant mass distribution at $p_{\pi^-}=747$ MeV/c before subtracting the $\eta\to3\pi^0$ background. The experimental points shown by the crosses are compared to the normalized $\eta\to3\pi^0$ Monte Carlo results shown by the shaded histogram. See the text for a discussion on the $\eta\to3\pi^0$ background. (b) $3\pi^0$ invariant mass after the $\eta\to3\pi^0$ background subtraction. Crosses show the data. The $\pi^-p\to3\pi^0n$ simulation is indicated by the shaded histogram. The Monte Carlo results were normalized to the number of experimental events. The dip at $m_{\pi\pi}=547$ MeV is due to the removal of the $\eta\to3\pi^0$ events.

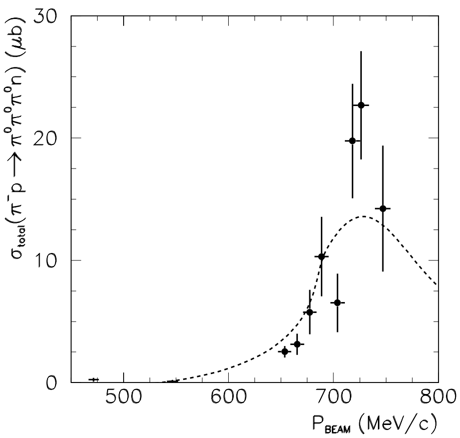

FIG. 3. The incident beam momentum dependence of the $\pi^- p \to 3\pi^0 n$ total cross section is shown by solid circles. The dashed line shows the S_{11} reaction cross section normalized to our data below η production threshold.

η decays. While most of this large $\eta \to 3\pi^0$ background is eliminated by the confidence-level cut described above, about 60% of events in the remaining $3\pi^0$ sample (before background subtraction) still originate from η decays. The right panel of Fig. 2 shows the $3\pi^0$ invariant mass after the $\eta \to 3\pi^0$ background subtraction compared to the Monte Carlo results. The experimental distribution agrees with the result of the simulation within the statistical accuracy.

The resulting $\pi^- p \to 3\pi^0 n$ total cross section was normalized to the number of incident pions. The beam normalization was verified by comparing our $\pi^- p \to \pi^0 n$ differential cross sections to the preliminary results of a special calibration run. We estimate the accuracy of our normalization to be $\pm 9\%$.

The beam-momentum dependence of the $\pi^- p \to 3\pi^0 n$ total cross section is shown in Fig. 3 (see also Table I). Only statistical uncertainties are shown. Systematic uncertainties below η threshold are dominated by the uncertainty in the beam normalization. Above 685 MeV/c, an additional uncertainty arises from the $\eta \to 3\pi^0$ background subtraction. The total systematic uncertainty above the η threshold is 40%. No events were detected at $p_{\pi^-} = 471$ MeV/c, where we quote the upper limit for $3\pi^0$ production at the 90% confidence level.

The S_{11} reaction cross section [8], normalized to our data below η production threshold, is shown in Fig. 3 for comparison. The cross section reaches its maximum of 23 ± 4 μb at 726 MeV/c. This value is 60−80 times smaller than results obtained in previous experiments [9–11]. We believe that much of this discrepancy may be due to the large $\eta \to 3\pi^0$ contribution that was not always recognized in the early measurements.

We can use a simple model and our measurements of $3\pi^0$ production to estimate the branching ratio $\mathcal{B}[S_{11} \to \pi N(1440)\frac{1}{2}^+]$. Consider the reasonable assumption that $3\pi^0$ production occurs predominantly via a sequential decay involving three intermediate baryon resonances,

$$\pi^- p \to N(1535)\frac{1}{2}^- \to \pi^0 N^0(1440)\frac{1}{2}^+ \to \pi^0\left[\pi^0 \Delta^0(1232)\frac{3}{2}^+\right] \to 3\pi^0 n.$$

The cross section for this process can be written as

$$\sigma(\pi^- p \to \pi^0\pi^0\Delta^0 \to 3\pi^0 n) = 4\pi\lambda\!\!\!^-{}^2\left(J+\frac{1}{2}\right)|T[\pi^- p \to \pi^0 N^0(1440)]|^2 \mathcal{B}[N^0(1440) \to \pi^0\Delta^0]\mathcal{B}(\Delta^0 \to \pi^0 n),$$

where $J = 1/2$ is the spin of the intermediate $N(1535)\frac{1}{2}^-$ resonance and $\lambda\!\!\!^- = \hbar/k$ with k the center of mass momentum for the reaction. At the energy of the $N(1535)\frac{1}{2}^-$ resonance, $\lambda\!\!\!^-{}^2 = 1.78$ mb and the square of the T-matrix amplitude can be written as

$$|T[\pi^- p \to \pi^0 N^0(1440)]|^2 = \frac{2}{9}\mathcal{B}(S_{11} \to \pi N)\mathcal{B}[S_{11} \to \pi N(1440)],$$

where 2/9 is the square of an isospin Clebsch-Gordon coefficient (CGC). The first branching fraction is known: $\mathcal{B}(S_{11} \to \pi N) = 0.51 \pm 0.05$ [1]. Other branching fractions needed for the calculation are also known [1]: $\mathcal{B}[N^0(1440) \to \pi^0\Delta^0] = \frac{1}{3}\mathcal{B}[N(1440) \to \pi\Delta] = \frac{1}{3}(0.22 \pm 0.03)$ and $\mathcal{B}(\Delta^0 \to \pi^0 n) = \frac{2}{3}\mathcal{B}(\Delta \to \pi N) = 0.67$. The numerical factors preceding the charge-independent branching fractions are squares of isospin CGCs.

We may obtain our first estimate for the branching fraction $\mathcal{B}[S_{11} \to \pi N(1440)]$ by ignoring other (smaller) contributions to $\sigma(\pi^- p \to 3\pi^0 n)$. In order to reduce the statistical uncertainty of our data, we calculated an average cross section for our four higher-momenta points. If we set $\sigma(\pi^- p \to 3\pi^0 n) = \sigma(\pi^- p \to \pi^0\pi^0\Delta^0 \to 3\pi^0 n)$ and then use our average cross section and uncertainty, $\sigma(\pi^- p \to 3\pi^0 n) = 16 \pm 2$ μb to solve for the branching fraction, we obtain $\mathcal{B}[S_{11} \to \pi N(1440)] = 0.13 \pm 0.03$. This value overestimates the branching fraction because we have, so far, ignored the $N^0(1440) \to f_0 n$ decay mode.

Now let us consider the contribution from the sequential decay process,

$$\pi^- p \to N(1535)\frac{1}{2}^- \to \pi^0 N^0(1440)\frac{1}{2}^+ \to \pi^0[f_0 n] \to 3\pi^0 n,$$

where again f_0 is the strong S-wave isoscalar $\pi\pi$ interaction. The cross section for this process can be written as

$$\sigma(\pi^- p \to \pi^0 f_0 n \to 3\pi^0 n) = 4\pi\lambda\!\!\!^-{}^2\left(J+\frac{1}{2}\right)|T[\pi^- p \to \pi^0 N^0(1440)]|^2 \mathcal{B}[N^0(1440) \to f_0 n]\mathcal{B}(f_0 \to \pi^0\pi^0).$$

The two new branching fractions needed are $\mathcal{B}[N^0(1440) \to f_0 n] = \mathcal{B}[N(1440) \to f_0 N] = 0.09 \pm 0.02$ [1] and $\mathcal{B}(f_0 \to \pi^0\pi^0) = \frac{1}{3}\mathcal{B}(f_0 \to \pi\pi) = 0.33$. Now we set $\sigma(\pi^- p \to 3\pi^0 n) = \sigma(\pi^- p \to \pi^0\pi^0\Delta^0 \to 3\pi^0 n) + \sigma(\pi^- p \to \pi^0 f_0 n \to 3\pi^0 n)$, where interference effects between the two decay modes of the $N(1440)$ have been ignored. Our final estimated branching fraction is $\mathcal{B}[S_{11} \to \pi N(1440)] = 0.08 \pm 0.02$. This result does not include any contribution from the model dependence of our calculation or from the systematical uncertainty of our cross-section measurements. The value we obtain agrees with the limit <0.07 quoted in Ref. [5]. Our result is the first estimate of the $S_{11} \to \pi N(1440)$ branching fraction obtained with a three-pion production reaction. Older determinations are based on isobar-model analyses of $\pi N \to \pi\pi N$ and are of very low precision [1,12].

We summarize our results as follows. The new Crystal Ball measurements for the total cross section of $\pi^- p \to 3\pi^0 n$ above η threshold (excluding $\eta \to 3\pi^0$ events) yield a few μb, a value $60-80$ times smaller than the results of the previous measurements. A simple model was used to extract the $S_{11} \to \pi N(1440)$ branching fraction. Our model assumes isospin invariance and that the peak cross section for $\pi^- p \to 3\pi^0 n$ is saturated by s-wave production of the $N(1535)\frac{1}{2}^-$ resonance; i.e., we ignore contributions from nonresonant background and other resonances. We further assumed that the reaction proceeds through $N(1535)\frac{1}{2}^- \to \pi N(1440)\frac{1}{2}^+$, where the different decay modes of the $N(1440)$ are incoherent with each other. Finally, we assumed that the peak cross section can be approximated as the average of our four highest momentum points. Within this model, the measured total cross section is consistent with the branching fraction $\mathcal{B}[S_{11} \to \pi N(1440)\frac{1}{2}^+] = (8 \pm 2)\%$. The quoted uncertainty does not include the systematical uncertainty of our cross-section measurement.

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