

Low-energy direct capture in the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions

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The cross sections of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ capture reactions have been analyzed using the direct capture model. At low energies, which are the astrophysically relevant regions, the capture process is dominated by $E1$ transitions from incoming s waves to bound p states. The cross sections of both mirror reactions can be described simultaneously with consistent potential parameters, whereas previous calculations have overestimated the capture cross sections significantly. However, the parameters of the potential have to be chosen very carefully because the calculated cross section of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction depends sensitively on the potential strength.

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I. INTRODUCTION

Nucleon capture in the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ mirror reactions has attracted much attention in the recent years. The low-energy behavior of both reactions is of astrophysical relevance. Nucleosynthesis of light nuclei is hindered by the gaps at $A=5$ and $A=8$, where no stable nuclei exist. However, these gaps may be bridged by reactions involving the unstable $A=8$ nuclei ${}^8\text{Li}$ ($T_{1/2}=840$ ms) and ${}^8\text{B}$ ($T_{1/2}=770$ ms).

The ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction is important in inhomogeneous big bang models. Here the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction [1] is in competition with the ${}^8\text{Li}(\alpha, n){}^{11}\text{B}$ reaction where much effort has been spent recently [2–5]. Typical temperatures are around $T_9 \approx 1$ [5–7], with T_9 being the temperature around 1×10^9 K. The role of light neutron-rich nuclei in the r process, e.g., in type-II supernovae, was analyzed in Ref. [8]; here a temperature range of $0.5 \leq T_9 \leq 4$ is relevant. Because of the missing Coulomb barrier for neutron-induced reactions, astrophysically relevant energies for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction are around $E \approx kT$. Hence, for both scenarios the cross section has to be determined for energies below $E \leq 500$ keV. In this paper all energies are given in the center-of-mass system.

The ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction leads to a hot part of the pp chain as soon as the proton capture of ${}^8\text{B}$ is faster than the competing β^+ decay [9]. Then a breakout to the hot carbon-nitrogen-oxygen cycle and to the rp process is possible with the ${}^9\text{C}(\alpha, p){}^{12}\text{N}$ reaction [9]. The ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction is especially relevant in low-metallicity stars with high masses where such a proton-rich reaction chain can be faster than the triple- α process [9,10], and furthermore, the reaction may become important under nova conditions [11]. The typical temperature range in both astrophysical scenarios is around several times 10^8 K, which corresponds to energies of the Gamow window around $50 \text{ keV} \leq E \leq 300 \text{ keV}$.

There are many common properties of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ mirror reactions in both experimental and theoretical points of view. Because of the unstable ${}^8\text{Li}$ and

${}^8\text{B}$ nuclei, direct experiments are extremely difficult at astrophysically relevant energies below 500 keV. However, indirect experiments have been performed successfully. A stringent limit for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ capture cross section has been derived from the Coulomb breakup reaction ${}^{208}\text{Pb}({}^9\text{Li}, {}^8\text{Li} + n){}^{208}\text{Pb}$ at MSU [12,1]. The astrophysical S factor at zero energy for the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction (usually referred to as S_{18}) has been derived from the asymptotic normalization coefficient method using transfer reactions. The ${}^2\text{H}({}^8\text{B}, {}^9\text{C})n$ reaction was measured at RIKEN [13], and one-proton removal reactions on carbon, aluminum, tin, and lead targets were used at Texas A&M University [14].

From the theoretical point of view, the astrophysical reaction rate of both reactions is dominated by direct (nonresonant) $E1$ transitions from incoming s waves to bound p waves. However, because of the larger Q value of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction ($Q=4064$ keV) and because of the missing Coulomb repulsion, the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction is not purely peripheral as expected for the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction with its small Q value ($Q=1296$ keV).

This paper is restricted to an analysis of the s -wave capture to the ground states of ${}^9\text{Li}$ and ${}^9\text{C}$. The total reaction rate for both reactions is slightly enhanced by resonant contributions, by p -wave and d -wave capture, and by the transition to the first excited state in ${}^9\text{Li}$ in the case of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction. The level scheme of the mirror nuclei ${}^9\text{Li}$ and ${}^9\text{C}$ (combined from Refs. [15–17]) is shown in Fig. 1.

Various models have been used to predict the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction cross sections. However, practically all predictions overestimated the experimentally determined values for both reactions [6,19–22]. Especially for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction, the predictions vary between a factor of 3 and up to a factor of 50 higher than the present upper limit [6,19–22] (see also Table I in Ref. [1]). It is the aim of the present work to analyze the peculiarities of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions at low energies.

II. DIRECT CAPTURE MODEL AND RESULTS

The cross section for direct capture (DC) σ^{DC} , is proportional to the spectroscopic factor C^2S and to the square of the overlap integral of the scattering wave function χ_{scatt} , the

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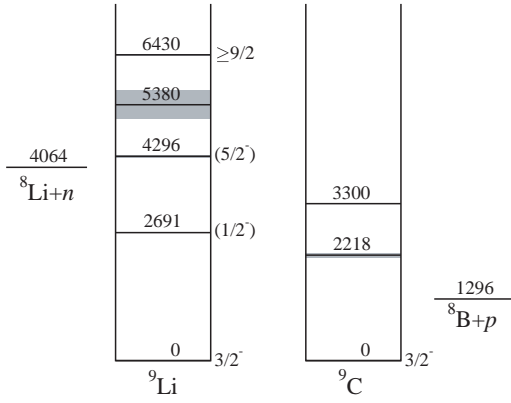


FIG. 1. Level scheme of the mirror nuclei ${}^9\text{Li}$ and ${}^9\text{C}$ [15–17]. Spin and parity of the $(5/2^-)$ state at $E=4296$ keV in ${}^9\text{Li}$ are taken from theory [18,6]. The widths of broad levels are indicated by gray shadings.

electric dipole operator O^{E1} , and the bound state wave function u_{bound} :

$$\sigma^{\text{DC}} \sim C^2S \left| \int \chi_{\text{scatt}}(r) O^{E1} u_{\text{bound}}(r) dr \right|^2. \quad (1)$$

The full formalism can be found, e.g., in Ref. [23]. The relation between this simple two-body model and microscopic models has been recently studied in Refs. [24,25].

The essential ingredients are the potentials which are needed to calculate the wave functions χ_{scatt} and u_{bound} . In the following a real folding potential $V_F(r)$ is used, which is calculated from an approximate density for the $A=8$ nuclei (taken as the weighted average of the measured charge density distributions of ${}^7\text{Li}$ and ${}^9\text{Be}$ [26]) and an effective nucleon-nucleon interaction of $M3Y$ type [27]. The imaginary part of the potential can be neglected at low energies. The resulting potential is adjusted by a strength parameter λ which has been found close to unity in many cases:

$$V(r) = \lambda V_F(r). \quad (2)$$

For mirror reactions, it is usually accepted that the potentials $V(r)$ and the spectroscopic factors C^2S are very similar. The folding potential (with $\lambda=1$) is shown in Fig. 2. The volume integral per interacting nucleon pair is $J = -616.57$ MeV fm^3 , and the root-mean-square radius is $r_{\text{rms}} = 3.0114$ fm.

As usual, the parameter λ for the bound state wave function is adjusted to the binding energies of a $1p_{3/2}$ neutron (proton) in the ${}^9\text{Li}$ (${}^9\text{C}$) residual nuclei. The resulting values are $\lambda_{\text{bound}} = 1.065$ (1.045) for ${}^9\text{Li}$ (${}^9\text{C}$). The deviation between both values for λ_{bound} is very small; thus the above assumption of similar parameters for mirror nuclei is confirmed.

For scattering waves, the potential strength parameter λ can be adjusted to reproduce experimental phase shifts. For s waves, which are relevant for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions, an adjustment to thermal neutron scattering lengths is also possible (and should be preferred because of the dominance of s waves at thermal energies). Suc-

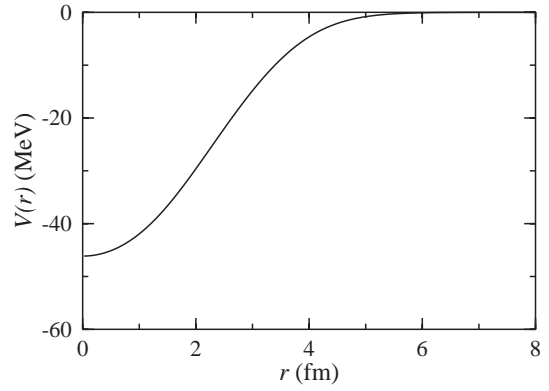


FIG. 2. Folding potential for the interactions ${}^8\text{Li}-n$ and ${}^8\text{B}-p$ (with $\lambda=1$).

cessful examples of this procedure can be found in Refs. [28,29]. Unfortunately, for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions phase shifts or neutron scattering lengths are not available from experiment, and therefore no experimental restriction from experimental scattering data exists for λ_{scatt} .

As soon as the potential parameters λ_{bound} and λ_{scatt} are fixed, the capture cross sections can be calculated from Eq. (1); the model contains no further adjustable parameters. A Woods-Saxon potential can also be used, and similar results will be obtained. But because of the larger number of adjustable parameters, the conclusions cannot be drawn as clearly as in the case of the folding potential with the only adjustable parameter λ .

For the spectroscopic factors of the ground states of ${}^9\text{Li} = {}^8\text{Li} \otimes n$ and ${}^9\text{C} = {}^8\text{B} \otimes p$, we use $C^2S = 1.0$ close to the calculated values of $C^2S = 0.81-0.97$ [13,19]. But also larger values up to $C^2S = 2.5$ have been obtained [9]; this large value was not used in the present work. It has been shown further [13] that the dominating contribution to C^2S comes from the nucleon transfer to the $1p_{3/2}$ orbit whereas the contribution of the $1p_{1/2}$ orbit remains below 5%.

To fix the potential strength parameter λ_{scatt} , and to see whether it is possible to reproduce the experimental values of both reactions simultaneously within this simple model, the theoretical capture cross sections of both reactions were calculated in the energy range $E \leq 1$ MeV. A spectroscopic factor $C^2S = 1.0$ was used in all the calculations. In Fig. 3, the capture cross section $\sigma(E)$ for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction is shown as a function of energy with $\lambda_{\text{scatt}} = 0.55$ and 1.20 as parameters. In the case of the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction, the energy dependence of the astrophysical S factor $S_{18}(E)$ is shown in Fig. 4 with $\lambda_{\text{scatt}} = 0.55$, 1.50, and 1.55 as parameters. As can be seen, for both reactions the results depend sensitively on the choice of the potential strength parameter λ_{scatt} . Therefore for both reactions the cross section dependence on the potential strength parameter λ_{scatt} has been calculated at a fixed energy $E = 25$ keV. The interesting result of the cross section dependence on the potential strength parameter λ_{scatt} is shown in Fig. 5. Note that the range of the values for λ_{scatt} has to be restricted to realistic values with the Pauli-forbidden $1s$ state below the respective threshold ($\lambda_{\text{scatt}} \geq 0.5$) and the Pauli-allowed $2s$ state far above the

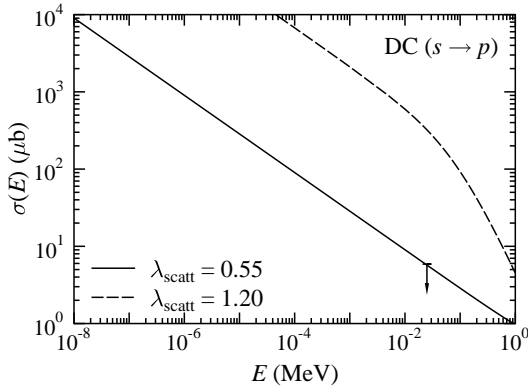


FIG. 3. Direct capture cross section $\sigma^{\text{DC}}(E)$ for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction with $C^2S=1.0$. The full line is obtained using $\lambda_{\text{scatt}}=0.55$; it shows the usual $1/v$ behavior. Significant differences from the $1/v$ behavior are found for $\lambda_{\text{scatt}}=1.20$ (dashed line). The arrow shows the experimental upper limit taken from Ref. [1]. For further discussion, see Sec. III.

threshold ($\lambda_{\text{scatt}} \leq 1.2$). Additionally, the ratio $r = \sigma(25 \text{ keV})/S_{18}(25 \text{ keV})$ is plotted in Fig. 5. If one makes the usual assumption that the spectroscopic factors are equal in mirror reactions, this ratio does not depend on the chosen spectroscopic factor.

The results shown in Figs. 3–5 have quite surprising features. It is difficult to fit the experimental results $\sigma(25 \text{ keV}) \leq 5.88 \mu\text{b}$ for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction (derived from [1] using a standard $1/v$ energy dependence) and $S_{18}(25 \text{ keV}) \approx S_{18}(0) = 45.5 \pm 5.5 \text{ eV b}$ for the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction (weighted average from Refs. [13,14]). Only in a narrow range of $\lambda_{\text{scatt}} \approx 0.55$, both experimental results are reproduced simultaneously. Higher values of λ (closer to the expected $\lambda \approx 1$) lead to a significant overestimation of both cross sections. From the calculated ratio r in Fig. 5, again the allowed range of λ_{scatt} is very narrow and around λ_{scatt}

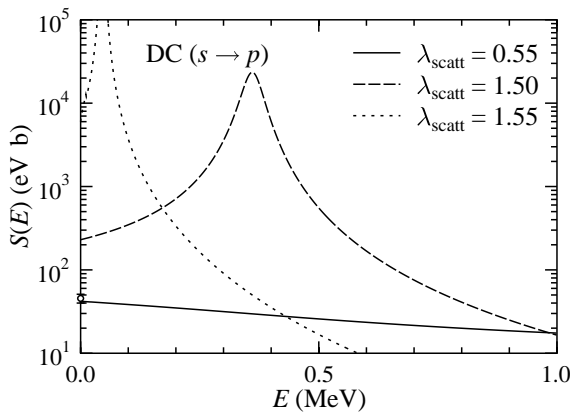


FIG. 4. Astrophysical S factor $S_{18}(E)$ for the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction, derived from the direct capture cross section with $C^2S=1.0$. The experimental point at $E=0$ is taken from Refs. [13,14]. The full line is obtained using $\lambda_{\text{scatt}}=0.55$; as expected, the S factor is almost constant. However, resonances are obtained for $\lambda_{\text{scatt}}=1.50$ (dashed line) and $\lambda_{\text{scatt}}=1.55$ (dotted line). For further discussion, see Sec. III.

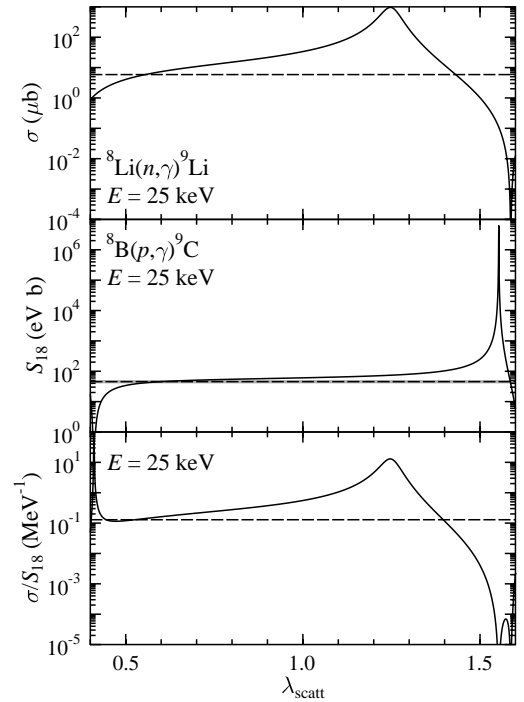


FIG. 5. Direct capture cross section σ^{DC} for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction (upper diagram), S_{18} for the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction (middle), and the ratio $r = \sigma/S_{18}$ (lower), depending on the potential strength parameter λ_{scatt} . All values have been calculated at $E=25 \text{ keV}$ using $C^2S=1.0$. The horizontal lines show the experimental results: $\sigma \leq 5.88 \mu\text{b}$ for ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ [1], $S_{18} = 45.5 \pm 5.5 \text{ eV b}$ for ${}^8\text{B}(p, \gamma){}^9\text{C}$ [13,14], and the ratio $r \leq 0.129 \text{ MeV}^{-1}$. For further discussion, see the text.

≈ 0.55 . Larger values of λ_{scatt} do not reproduce the ratio r because of the different sensitivities of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction cross sections on the potential strength. Hence it is possible to determine λ_{scatt} from experimental capture data for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions.

A more detailed view on both ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions follows. In Figs. 3 and 4, the full line represents $\lambda_{\text{scatt}}=0.55$ which was derived from the ratio of both reactions. Additionally, the dashed lines show calculations with λ_{scatt} values, which lead to extreme cross sections (see Fig. 5).

In the case of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction, the cross section is very sensitive to the chosen value of λ . Increasing λ_{scatt} from 0.55 to 0.75 leads to an increase in the cross section by more than a factor of 2, and decreasing λ_{scatt} from 0.55 to 0.50 reduces the cross section by about 30%. With $\lambda_{\text{scatt}}=0.55$ an energy dependence of the cross section proportional to $1/v$ is observed, whereas with $\lambda_{\text{scatt}}=1.20$ a clear deviation from the usual $1/v$ behavior can be seen in Fig. 3.

The ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction is mainly peripheral and does not depend sensitively on the chosen potential strength. Changing λ_{scatt} from 0.55 to 0.75 (0.50) increases (decreases) the S factor by 28% (17%). The relatively weak dependence on the potential strength parameter λ_{scatt} can also be seen from Fig. 5, where S_{18} changes only by roughly a factor of 2 from $\lambda_{\text{scatt}}=0.60$ to $\lambda_{\text{scatt}}=1.4$. Furthermore, the S

factor of this reaction depends only weakly on the chosen energy; for $\lambda_{\text{scatt}}=0.55$, one finds that $S_{18}(E=0)$ is roughly 2% larger than the quoted $S_{18}(25 \text{ keV})$.

When λ_{scatt} is increased, resonances are observed in both the reactions (see Fig. 5). In the case of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction, the parameter $\lambda_{\text{scatt}}=1.25$ leads to a resonance at $E=25 \text{ keV}$. In the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction, a resonance appears at $E=360 \text{ keV}$ with a width of $\Gamma=50 \text{ keV}$ using $\lambda_{\text{scatt}}=1.50$; for $\lambda_{\text{scatt}}=1.55$, this resonance is shifted to lower energies, and the width is much smaller (dotted line in Fig. 4). These resonances will be interpreted in the following section.

III. DISCUSSION

The results of the calculations now can be summarized.

(i) There is a possibility to fit the experimental data of both ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions within this simple model simultaneously when a potential strength parameter $\lambda_{\text{scatt}} \approx 0.55$ and a spectroscopic factor $C^2S \approx 1$ are used. Any other combinations lead to discrepancies with the experimental results. λ_{scatt} is determined from the ratio r without ambiguity (independent of the spectroscopic factors). And for much smaller or much larger values of C^2S , a simultaneous description of both reactions is not possible.

(ii) A surprisingly large difference for the potential parameters $\lambda_{\text{bound}} \approx 1$ and $\lambda_{\text{scatt}} \approx 0.55$ is found. This might be an indication that the simple $M3Y$ interaction fails to describe systems with extreme neutron-to-proton ratio N/Z ; here $N/Z=2.0$ for ${}^9\text{Li}$ and $N/Z=0.5$ for ${}^9\text{C}$. It is interesting to note that a calculation using a variant of the $M3Y$ interaction including a spin-orbit and a tensor part is able to predict $S_{18}(0)=53 \text{ eV b}$ for the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction [30] which is close to the experimental results [13,14]. Unfortunately, there is no prediction for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction in Ref. [30].

(iii) Using $\lambda_{\text{scatt}} \geq 1$ leads to a so-called ‘‘potential resonance.’’ This phenomenon has been discussed in detail in Ref. [29], and resonances have been described successfully within a potential model, e.g., in Refs. [23,31]. For ${}^9\text{Li}$ and ${}^9\text{C}$, the $2s$ orbit is shifted to lower energies by an increased λ_{scatt} , and the low-energy tail of this resonance influences the cross sections at 25 keV significantly. For $\lambda_{\text{scatt}} \approx 1.25$ ($\lambda_{\text{scatt}} \approx 1.55$), the $2s$ resonance appears at energies around 25 keV in the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ [${}^8\text{B}(p, \gamma){}^9\text{C}$] reaction (see Figs. 3–5). Probably this resonant behavior is the reason why most of the previous calculations using standard potentials failed to predict the experimental data for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions correctly.

Many nuclei in the $1p$ shell with $N \approx Z$ show such low-lying s -wave resonances with a large reduced width corresponding to the $2s$ level. One example is the $1/2^+$ state at $E_x=2365 \text{ keV}$ in ${}^{13}\text{N}$, which appears as a resonance in the ${}^{12}\text{C}(p, \gamma){}^{13}\text{N}$ reaction at $E=421 \text{ keV}$ [32]. This resonance (and its mirror state in ${}^{13}\text{C}$ which is located below the ${}^{12}\text{C}-n$ threshold; this state plays an important role in the ${}^{12}\text{C}(n, \gamma){}^{13}\text{C}$ reaction [33,34]) can be described within the present model using $\lambda \approx 1$ [31]. Another example is the 1^- resonance at $E_x=5173 \text{ keV}$ in ${}^{14}\text{O}$, which appears as a resonance in the ${}^{13}\text{N}(p, \gamma){}^{14}\text{O}$ reaction at $E=545 \text{ keV}$ [35].

The experimental data [1,13,14] indicate for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions that there are no low-lying s -wave resonances close above the threshold. What is the difference between the nonresonant ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions and the resonant ${}^{12}\text{C}(p, \gamma){}^{13}\text{N}$ and ${}^{13}\text{N}(p, \gamma){}^{14}\text{O}$ reactions? As discussed in the following, the nonresonant behavior of ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ can be understood from basic shell model considerations. For simplicity, the following paragraph discusses the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction and properties of ${}^9\text{C}$. Similar conclusions are reached for ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^9\text{Li}$ by exchanging protons and neutrons.

For light nuclei with extreme neutron-to-proton ratio N/Z , one has to distinguish between neutron and proton orbits. The $J^\pi=3/2^-$ ground state of ${}^9\text{C}$ corresponds to a neutron $1p_{3/2}$ orbit. Two neutrons and two protons in the $1s_{1/2}$ orbits couple to an inert α core with $J^\pi=0^+$, and four protons fill the $1p_{3/2}$ subshell and couple to $J^\pi=0^+$. Excited states with $J^\pi=1/2^-$ ($1/2^+$) correspond to neutron $1p_{1/2}$ ($2s_{1/2}$) orbits. The $1/2^-$ state is probably the first excited state (see Fig. 1). The $1/2^+$ state (not known experimentally) is expected at low energies (as usual for $1p$ shell nuclei) with a large reduced neutron width. But the proton $2s_{1/2}$ orbit in ${}^9\text{C}$ must be located at much higher energies because a proton pair must be broken for a ${}^9\text{C}={}^8\text{B}_{\text{g.s.}} \otimes p$ configuration (g.s. represents ground state). Spin and parity of such a proton $2s_{1/2}$ orbit in ${}^9\text{C}$ are $J^\pi=3/2^+$ and $5/2^+$ because $J^\pi({}^8\text{B}_{\text{g.s.}})=2^+$.

Only shallow potentials with $\lambda_{\text{scatt}} \approx 0.55$ can describe such a high-lying proton $2s_{1/2}$ orbit in a correct way. A standard potential (with $\lambda_{\text{scatt}} \approx 1$) would shift this proton $2s_{1/2}$ orbit to lower energies leading to an s -wave resonance relatively close above the threshold. Because of the low-energy tail of this resonance, the cross sections are strongly overestimated in this case.

The low-lying $1/2^+$ state from the neutron $2s_{1/2}$ orbit cannot contribute to the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction as s -wave resonance because the excitation of a $J^\pi=1/2^+$ resonance in the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction requires a d wave. Additionally, only a small reduced proton width is expected for such a neutron single-particle state.

The derivation of the equation for the overlap integral (1) makes use of Siegert’s theorem [36]. Therefore, Eq. (1) is exactly valid only if the same Hamiltonians, i.e., the same potentials, are used for the calculation of the bound state and scattering wave functions. Otherwise, the nonorthogonality of the bound state and scattering wave functions may lead to considerable theoretical uncertainties [37].

At first view, the huge discrepancy between the bound state potential ($\lambda_{\text{bound}} \approx 1.05$) and the scattering potential ($\lambda_{\text{scatt}} \approx 0.55$) seems to indicate a strong violation of the Siegert’s theorem. However, this is not the case for the considered $E1$ transitions from incoming s waves in the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions. Practically identical wave functions for the $p_{3/2}$ bound state are derived from (i) a central potential with the strength parameter λ_{bound} , and (ii) a combination of central and spin-orbit potential with strength parameters $\lambda_{\text{bound}}^{\text{central}}$ and $\lambda_{\text{bound}}^{\text{LS}}$. The shape of the spin-

orbit potential $V_{\text{LS}}(r) = \lambda_{\text{bound}}^{\text{LS}} (1/r)(dV_F/dr)\vec{L} \cdot \vec{S}$ is practically identical to the central folding potential $V_F(r)$ because $V_F(r)$ has an almost Gaussian shape. Hence, an increased $\lambda_{\text{bound}}^{\text{central}}$ can be compensated by a decreased $\lambda_{\text{bound}}^{\text{LS}}$ and vice versa, leading to the same total potential and bound state wave function. A proper combination of $\lambda_{\text{bound}}^{\text{central}}$ and $\lambda_{\text{bound}}^{\text{LS}}$ has to be chosen to reproduce the binding energies. The quoted $\lambda_{\text{bound}} \approx 1.05$ were obtained without spin-orbit potential ($\lambda_{\text{bound}}^{\text{LS}} = 0$). Alternatively, using $\lambda_{\text{bound}}^{\text{central}} = \lambda_{\text{scatt}} = 0.55$ leads to $\lambda_{\text{bound}}^{\text{LS}} = -3.89 \text{ fm}^2$ for the ${}^9\text{Li}$ ground state ($\lambda_{\text{bound}}^{\text{LS}} = -3.71 \text{ fm}^2$ for ${}^9\text{C}$). The potential for the incoming s wave is not affected by the additional spin-orbit potential. The bound state and scattering wave functions are now orthogonal because they are calculated from the same potential. Therefore, the overlap integral, Eq. (1), is exact for the $E1$ transitions from incoming s waves to bound p waves if one uses the above combination of central and spin-orbit potentials in the entrance and exit channels. Equation (1) remains a good approximation if one uses only the central potential with the different λ_{scatt} and λ_{bound} because there are only minor deviations of the order of 10% between the different bound state wave functions; hence, the orthogonality of the wave functions remains approximately fulfilled. For simplicity, the calculations in this work were performed without spin-orbit potentials.

A possible core polarization (sometimes also called semidirect capture) may lead to asymmetries in the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ mirror reactions. The core polarization can be taken into account by modifications of the $E1$ operator in the nuclear interior. Following the formalism of Ref. [38] one finds that the modification of the $E1$ operator leads to a negligible change of the cross section in the case of the ${}^8\text{B}(p, \gamma){}^9\text{C}$ reaction because of its extremely peripheral character; but also in the case of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ reaction the modified $E1$ operator changes the cross section by significantly less than 10%. Therefore, the modification of the

$E1$ operator by semidirect capture was neglected in this work.

IV. CONCLUSIONS

A consistent description of the capture cross sections of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions has been obtained using the direct capture model. However, a surprisingly strong difference between the potential parameters λ_{scatt} for the scattering wave function and λ_{bound} for the bound state wave function was found. The small value of $\lambda_{\text{scatt}} \approx 0.55$ is well defined from the ratio of the cross sections of both reactions and leads to a shallow potential. The value of $\lambda_{\text{bound}} \approx 1.05$ is derived from the binding energies of the ${}^9\text{Li}$ and ${}^9\text{C}$ ground states, and is close to the usual values $\lambda \approx 1$. The strong difference between λ_{scatt} and λ_{bound} indicates limitations of the simple $M3Y$ interaction for nuclei with extreme neutron-to-proton ratios N/Z . On the other hand, the parameters λ_{scatt} and λ_{bound} are practically equal for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions, as is expected for mirror reactions. The sensitivity of the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ cross section and, to a lesser extent, the ${}^8\text{B}(p, \gamma){}^9\text{C}$ S factor to the potential strength parameter λ_{scatt} is strong, which can be explained by resonances in the potential model. Consequently, the choice of potential parameters for direct capture calculations has to be done very carefully. This problem is not particular for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions, but has to be taken into account in any direct capture calculation. The big discrepancies between previous predictions and recently obtained experimental results for the ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ and ${}^8\text{B}(p, \gamma){}^9\text{C}$ reactions are probably a consequence of the neglect of these potential resonances.

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