Triple-Pomeron matrix model for dispersive corrections to nucleon-nucleus total cross section

David R. Harrington*

Deparment of Physics and Astronomy, Rutgers University, 136 Frelinghuysen Road, Piscataway, New Jersey 08854-8019, USA

(Received 19 June 2002; revised manuscript received 28 March 2003; published 25 June 2003)

Dispersive corrections to the total cross section for high-energy scattering from a heavy nucleus are calculated using a matrix model, based on the triple-Pomeron behavior of diffractive scattering from a single nucleon, for the cross section operator connecting different states of the projectile nucleon. Energy-dependent effects due to the decrease in longitudinal momentum transfers and the opening of more channels with increasing energy are included. The three leading terms in an expansion in the number of inelastic transitions are evaluated and compared to exact results for the model in the uniform nuclear density approximation for the the scattering of nucleons from ^{40}Ca and ^{208}Pb for laboratory momenta ranging from 50 to 200 GeV/c.

DOI: 10.1103/PhysRevC.67.064904 PACS number(s): 13.85.−t, 24.10.Eq, 24.10.Ht, 25.40.−h

I. INTRODUCTION

One consequence of the composite nature of nucleons is a decrease in nucleon-nucleus total cross sections due to transitions between different internal states of the projectile nucleon [1,2]. This decrease can be calculated [3,4] using an operator to represent a generalized nucleon-nucleon total cross section, with the matrix elements representing the probability amplitudes for forward scattering transitions between different states of the nucleon.

The general problem of the propagation of high-energy composite particles through a nucleus has a long history [5–11]. If one has a simple model for the composite particle and the longitudinal momentum transfer due to the different masses of the different states of the composite system can be ignored the calculation of the dispersive corrections is equivalent to Glauber theory for composite-composite scattering and is relatively straightforward [11]. In general, however, simple models are not realistic and the mass differences cannot be ignored and the calculation becomes quite complicated. Because of the difficulty of this calculation, and uncertainties in the nature of the cross section operator, only the leading term in an expansion in the number of inelastic transitions has been evaluated [12]. (A closely related effect is involved in the analysis of color transparency in reactions on nuclei [13–15].) In this paper we test the accuracy of the leading order approximation, and the convergence of the expansion, using a simple finite matrix model for the cross section operator and taking the uniform nuclear density limit for which the exact result, including all orders in transitions, can also be calculated. This matrix model is consistent with the known triple-Pomeron behavior of high-energy, high-mass single-diffraction dissociation from a single nucleon, but cannot describe accurately the low-mass production which clearly contributes significantly to the dispersive corrections. Furthermore, the model has a high degree of arbitrariness. It is used only for the lack of a reliable microscopic model for the internal degrees of freedom of highly excited nucleons. It has some features in common with a ''simplified example'' used by Hove [9] for the limit of zero longitudinal momentum transfer. Using the uniform density model will also introduce errors since it cannot be accurate in the outer layers of the nucleus where most the dispersive correction originates. It is used partly because it simplifies the calculation of higher order terms in the expansion in the number of transitions but mainly because it allows the exact result to be calculated, allowing for a real test of the accuracy of the expansion.

Section II reviews the transition expansion for the dispersive corrections to the nucleon-nucleus total cross section, including the effects of the longitudinal momentum transfers due to the different masses of the different excited states of the nucleon. In Sec. III it is shown that the terms in this expansion simplify considerably in the uniform nuclear density limit, with each term in the expansion represented as a sum over products of transition amplitudes weighted by a function of the differences among cross sections and differences in the masses of the different nucleonic states. In the uniform density limit it is also possible to write the exact result, including all orders in the number of transitions, in terms of the exponential of a single position-independent operator, as shown in Sec. IV. The representation of the transition operator by a finite matrix, with a dimension which increases with energy, is discussed in Sec. V. This matrix is chosen to have a form consistent with the triple-Pomeron behavior of the nucleon-nucleon single-diffraction dissociation, but is otherwise highly arbitrary. Using this matrix the formulas are evaluated and the results are presented in Sec. VI. The results are summarized and discussed in Sec. VII.

II. EXPANSION IN INELASTIC TRANSITIONS

It has long been known that a version of the eikonal approximation holds for an infinite but restricted class of Feynman diagrams, which includes inelastic transitions among different states of the projectile [16]. The result is equivalent to the eikonal approximation in coupled-channel potential theory [17–19], and leads to an expression for projectile-nucleus total cross sections, which can be written as

$$\sigma(A)_{Total} = 2\,\mathrm{Re}\int d^2b \langle A|\langle 1|\hat{\Gamma}(\mathbf{b},\{\mathbf{r}_\alpha\})|1\rangle|A\rangle, \qquad (1)$$

*Email address: drharr@physics.rutgers.edu

where $|A\rangle$ is the ground state of the nucleus and $|1\rangle$ is the lowest-mass eigenstate of the projectile system. The profile function $\hat{\Gamma}$ is an operator in the internal space of the projectile,

$$\hat{\Gamma}(\mathbf{b},\{\mathbf{r}_\alpha\})=1-\mathcal{Z}\exp\left[-i\sum_\alpha \int dz\hat{u}_\alpha(\mathbf{r})\right], \tag{2}$$

where $\mathcal{Z}$ indicates a z-ordered product and

$$\hat{u}_\alpha(\mathbf{r})=(m/p_1)e^{-i\hat{p}z}\hat{v}(\mathbf{r}-\mathbf{r}_\alpha)e^{i\hat{p}z}, \tag{3}$$

with $\hat{v}(\mathbf{r}-\mathbf{r}_\alpha)$ the effective potential operator produced by a static target nucleon at $\mathbf{r}_\alpha$. Here $\hat{p}$ is the longitudinal momentum operator, diagonal in the mass eigenstates of the projectile, with

$$p_j\approx p_1-(m_j^2-m_1^2)/(2p_1), \tag{4}$$

where p_1 is the initial momentum of the projectile in the laboratory system where the nucleus is at rest and m_j is the mass of the jth excited state of the projectile.

Assuming the different $\hat{v}_\alpha$ do not overlap, ignoring nuclear correlations, and assuming a large nucleon number A, the expression for $\hat{\Gamma}$ simplifies to

$$\begin{aligned}\hat{\Gamma}(b)&=\langle A|\hat{\Gamma}(b);\{\mathbf{r}_\alpha\}|A\rangle\\ &\approx 1-\mathcal{Z}\exp\left[-(A/2)\int dz_1\rho(b,z_1)\hat{\sigma}(z_1)\right],\end{aligned} \tag{5}$$

where ρ is the nuclear density, normalized to 1, and

$$\hat{\sigma}(z_1)\equiv\exp(-i\hat{p}z_1)\hat{\sigma}\exp(i\hat{p}z_1), \tag{6}$$

with $\hat{\sigma}$ the cross section operator [4] for scattering of the projectile from a single target nucleon. Assuming that the corresponding elastic and diffractive scattering amplitudes are purely imaginary, the total projectile-nucleon cross section is

$$\sigma_T=\langle 1|\hat{\sigma}|1\rangle, \tag{7}$$

while the cross section for single-diffraction dissociation of the projectile interacting with a single nucleon, summed over all diffractively excited states of the projectile, at momentum transfer squared $t=0$, is

$$d\sigma_{diff}/dt=[\langle 1|\hat{\sigma}^2|1\rangle-\langle 1|\hat{\sigma}|1\rangle^2]/(16\pi). \tag{8}$$

If

$$\Gamma(b)\equiv\langle 1|\hat{\Gamma}(b)|1\rangle, \tag{9}$$

then

$$\sigma(A)_{Total}=2\mathrm{Re}\int d^2b\Gamma(b). \tag{10}$$

Since the dispersive corrections are small compared to the total cross section, it is useful to separate $\Gamma(b)$ into two parts:

$$\Gamma(b)=\Gamma_G(b)-\Gamma_D(b). \tag{11}$$

Defining the dimensionless absorption parameter

$$t(b)=(A/2)\sigma_T\int dz\rho(b,z), \tag{12}$$

the main Glauber contribution, which does not include dispersive corrections, is simply

$$\Gamma_G(b)=1-\exp[-t(b)], \tag{13}$$

while the diffractive correction is given by

$$\begin{aligned}\Gamma_D(b)&=\langle 1|\mathcal{Z}\exp\left[-(A/2)\int dz\rho(b,z)\hat{\sigma}(z)\right]|1\rangle\\ &\quad-\exp[-t(b)].\end{aligned} \tag{14}$$

Previous calculations taking into account longitudnal momentum transfers [12] of Γ_D have included only the leading second-order term in an expansion in the number of inelastic transition. While this is almost certainly accurate for light nuclei, it is not clear whether or not it is adequate for heavy nuclei. Below we develop expressions for the general terms in the expansion and evaluate them in a simple but possibly not completely unrealistic model.

We begin by separating the cross section operator $\hat{\sigma}$ into its diagonal and off-diagonal (transition) parts:

$$\hat{\sigma}=\hat{\sigma}_d+\hat{\sigma}_t, \tag{15}$$

where

$$\langle i|\hat{\sigma}_d|j\rangle=\delta_{ij}\langle j|\hat{\sigma}|j\rangle, \tag{16}$$

so that $\hat{\sigma}_t$ has only off-diagonal matrix elements. We now expand Γ_D in powers of the transition operator,

$$\Gamma_D(b)=\sum_{n=2}^{\infty}\Gamma_D^{(n)}(b). \tag{17}$$

Here the leading $n=2$ term contains contributions from processes in which the nucleon makes two transitions: one from the ground state to a higher-mass state, then another back to the ground state. The next $(n=3)$ term contains the contributions from processes with three transitions, with the two intermediate projectile states being neither the ground state nor equal to each other.

Using a derivation analogous to that for time-dependent perturbation theory [20], one can show that the $\Gamma_D^{(n)}(b)$ are given by the z-ordered integrals

$$\begin{aligned}\Gamma_D^{(n)}(b)&=(-A/2)^n e^{-t(b)}\int_{-\infty}^{\infty}dz_n\rho(b,z_n)\cdots\int_{-\infty}^{z_2}dz_1\rho(b,z_1)\\ &\quad\times\langle 1|\hat{\sigma}_{td}(z_n)\cdots\hat{\sigma}_{td}(z_2)\hat{\sigma}_{td}(z_1)|1\rangle,\end{aligned} \tag{18}$$

with

$$\hat{\sigma}_{td}(z)=\hat{U}(z)^{-1}\hat{\sigma}_t\hat{U}(z), \tag{19}$$

where

$$\hat{U}(z)=\exp\left[-(A/2)\int_{-\infty}^{z}dz_1\rho(b,z_1)\hat{\sigma}_d+i\hat{p}z\right] \tag{20}$$

is diagonal in mass eigenstates. These expressions can be simplified if we replace z by the dimensionless variable

$$u(b,z)\equiv(A/2)\sigma_T\int_{-\infty}^{z}dz_1\rho(b,z_1)/t(b), \tag{21}$$

so that $u(b,-\infty)=0$ and $u(b,\infty)=1$, if we also define the dimensionless cross section operators

$$\hat{x}\equiv\hat{\sigma}/\sigma_T, \tag{22}$$

$$\hat{x}_d\equiv\hat{\sigma}_d/\sigma_T, \tag{23}$$

$$\hat{x}_t\equiv\hat{\sigma}_t/\sigma_T, \tag{24}$$

and, for future use,

$$\tilde{x}\equiv\hat{x}-\hat{1}. \tag{25}$$

We can then write

$$\Gamma_D^{(n)}(b)=[-t(b)]^n\exp[-t(b)]\int_0^1du_n\cdots\int_0^{u_3}du_2\times\int_0^{u_2}du_1\langle 1|\hat{x}_{td}(u_n)\cdots\hat{x}_{td}(u_1)|1\rangle, \tag{26}$$

where

$$\hat{x}_{td}(u)=\hat{U}[z(b,u)]^{-1}\hat{x}_t\hat{U}[z(b,u)], \tag{27}$$

with $z(b,u)$ the inverse of $u(b,z)$ for fixed b.

Inserting complete sets of mass eigenstates between the $\hat{x}_{td}(u)$ operators in Eq. (20) gives

$$\Gamma_D^{(n)}(b)=\{[-t(b)]^n/n!\}\exp[-t(b)]\times\sum_{j_1,\ldots,j_{n-1}}\langle 1|\hat{x}_t|j_{n-1}\rangle\cdots\langle j_2|\hat{x}_t|j_1\rangle\times\langle j_1|\hat{x}_t|1\rangle f^{(n)}(b;j_1,j_2,\ldots,j_{n-1}), \tag{28}$$

where

$$\begin{aligned}f^{(n)}(b;j_1,j_2,\ldots,j_{n-1})&\equiv n!\int_0^1du_n\cdots\int_0^{u_3}du_2\\&\times\int_0^{u_2}du_1\exp[t(b)u_n(x_1-x_{j_{n-1}})\\&-iz_n(p_1-p_{j_{n-1}})]\cdots\exp[t(b)u_2(x_{j_2}-x_{j_1})\\&-iz_2(p_{j_2}-p_{j_1})]\exp[t(b)u_1(x_{j_1}-x_1)\\&-iz_1(p_{j_1}-p_1)],\end{aligned} \tag{29}$$

with $z_j=z(b,u_j)$ and $x_j=\langle j|\hat{x}|j\rangle$. Since $\hat{x}_t$ has only off-diagonal matrix elements, terms with equal successive $j_i's$ do not contribute to the sum in Eq. (22).

The functions $f^{(n)}$ defined above depend only on the differences between successive $x_j's$ and $p_j's$, and thus are unchanged if these variables are replaced by

$$\tilde{x}_j\equiv x_j-1 \tag{30}$$

and

$$\tilde{p}_j\equiv p_j-p_1. \tag{31}$$

Then $f^{(n)}$ can be written as

$$\begin{aligned}f^{(n)}(b;j_1,j_2,\ldots,j_{n-1})&=n!\int_0^1du_n\cdots\int_0^{u_3}du_2\int_0^{u_2}du_1\\&\times\exp[-t\tilde{x}_{j_{n-1}}(u_n-u_{n-1})\\&+i\tilde{p}_{j_{n-1}}(z_n-z_{n-1})]\cdots\exp[-t\tilde{x}_{j_1}(u_2-u_1)\\&+i\tilde{p}_{j_2}(z_2-z_1)],\end{aligned} \tag{32}$$

where, because of the u (and z) ordering, the differences between $u's$ and $z's$ in the parenthesis are never negative, and the real parts of $f^{(n)'}s$, needed to calculate the total cross sections, are always less than their limits as $\tilde{p}_j's$ approach zero. Each exponential in this expression acts as a propagator for the projectile from the location of one transition to that of the next, and includes absorptive and phase-changing parts depending upon the state of the projectile at this stage of its journey through the nucleus.

III. UNIFORM DENSITY LIMIT

The evaluation of the expressions for the $f^{(n)'}s$ is complicated by the fact that z and u are in general not simple functions of one another. The relation between them is determined by the shape of the nuclear density function ρ and depends upon the impact parameter. For heavy nuclei, ρ is well approximated by the simple Woods-Saxon form, but the relation between z and u is still not simple. For the heaviest

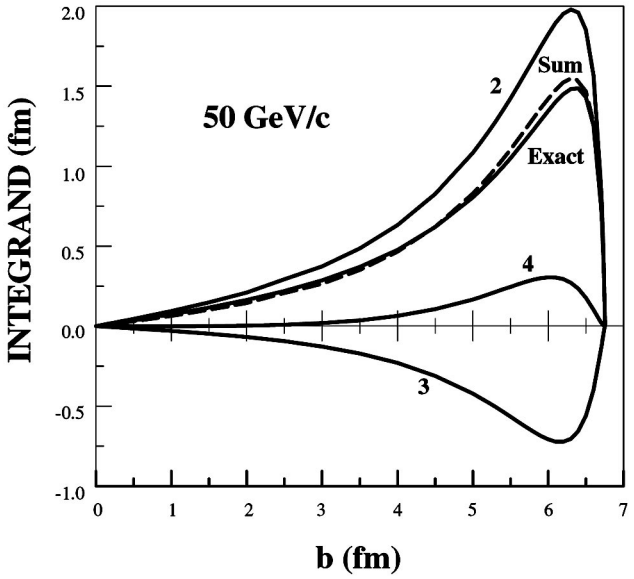

FIG. 1. Integrands for diffractive reductions in nucleon-^{208}Pb total cross sections as a function of impact parameter b at 50 GeV. The solid curves show the exact result along with the contributions from second through fourth order in the number of inelastic transitions. The dashed curve shows the sum of the second- through fourth-order terms.

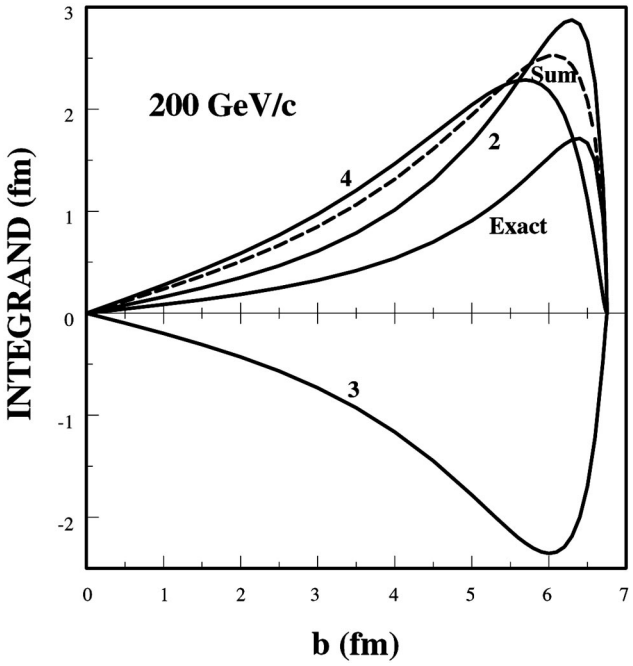

FIG. 2. Integrands for diffractive reductions in nucleon-^{208}Pb total cross sections as a function of impact parameter b at 200 GeV. The solid curves show the exact result along with the contributions from second through fourth order in the number of inelastic transitions. The dashed curve shows the sum of the second- through fourth-order terms.

nuclei the surface thickness is much less than the nuclear radius so that it may not introduce excessive errors to replace ρ by its uniform density limit, especially at small momentum transfers and, in particular, for evaluating the dispersive contribution to the total cross section. As can be seen in Figs. 1 and 2, however, even for the total cross section the dispersive corrections originate mainly in the outer layers where the uniform density differs most from the actual density, so some errors are certainly introduced by this simplification.

In the uniform density limit,

$$\rho(r)=\rho_0\Theta(R-r), \tag{33}$$

giving

$$t(b)=A\sigma_T\rho_0\sqrt{R^2-b^2}\Theta(R-b), \tag{34}$$

where

$$\rho_0=1/(4\pi R^3/3) \tag{35}$$

and $R\approx r_0A^{1/3}$, with $r_0\approx 1.14$ fm. [With this expression for $t(b)$, there is an analytic expression [6] for the main Glauber contribution to $\sigma(A)_{Total}$.] In this limit z and u are linearly related,

$$z=\sqrt{R^2-b^2}(2u-1), \tag{36}$$

for $0\leqslant u\leqslant 1$. The arguments of the exponentials in Eq. (26) then simplify considerably,

$$-\tilde{x}_{j_n}t(b)(u_{n+1}-u_n)+i\tilde{p}_{j_n}(z_{n+1}-z_n)$$
$$=-y_{j_n}\sqrt{R^2-b^2}(u_{n+1}-u_n), \tag{37}$$

where the complex number

$$y_j\equiv\tilde{x}_jA\sigma_T\rho_0-2i\tilde{p}_j \tag{38}$$

is independent of b and the u's. In the uniform density limit, then, the effects of longitudinal momentum transfers are taken into account simply by adding an imaginary part to each diagonal matrix element $\tilde{x}_j$, and modifying the calculations of the functions $f^{(n)}$ accordingly.

IV. EXACT RESULT

In the uniform density limit an exact expression for the dispersive correction to the total cross section can be found in terms of the exponential of a z-independent operator. The simplest derivation of this result starts with Eq. (5) and removes the z dependence of $\hat{\sigma}(z)$ by adding a term proportional to $\hat{p}$ to the operator in the exponent. For a given impact parameter, a single z-independent matrix is involved so that the z ordering in Eq. (2) can be ignored, giving

$$\Gamma_D(b)=\langle 1|\exp[-\hat{M}(b)]|1\rangle-\exp[-t(b)], \tag{39}$$

with the z-independent operator

$$\hat{M}(b)=[A\rho_0\hat{\sigma}-2i(\hat{p}-p_1)]\sqrt{R^2-b^2}. \tag{40}$$

This can also be written as

$$\Gamma_D(b)=\exp[-t(b)]\{\langle 1|\exp[-\tilde{M}(b)]|1\rangle-1\}, \tag{41}$$

where

$$\tilde{M}(b)=\hat{M}(b)-t(b)\hat{1}=[A\rho_0\sigma_T\tilde{x}-2i\tilde{p}]\sqrt{R^2-b^2}. \tag{42}$$

Either of the expressions (39) or (41) can be evaluated by expanding the exponential of the z-independent operator in a power series, with the second converging somewhat more rapidly. (Using the matrix model below, one has to include of the order of 50 terms in the expansion, and there is considerable cancellation, so the individual terms must be calculated to high accuracy.)

Another approach for evaluating $\Gamma_D(b)$ depends upon the fact that, since the nucleon always enters the reaction in its ground state, the full operator $\exp(-\tilde{M})$ is not needed. It is sufficient to work with the reaction-modified state,

$$|V\rangle\equiv\exp(-\tilde{M})|1\rangle, \tag{43}$$

with

$$\Gamma_D(b)=\exp[-t(b)][\langle 1|V\rangle-1]. \tag{44}$$

Expanding the exponential

$$|V\rangle=\sum_{n=0}^{\infty}|Vn\rangle, \tag{45}$$

where

$$|Vn\rangle\equiv(1/n!)(-\tilde{M})^n|1\rangle=-(1/n)\tilde{M}|V(n-1)\rangle \tag{46}$$

can be calculated recursively starting with $|V0\rangle=|1\rangle$.

V. MATRIX MODEL

The expressions for $\Gamma_D^{(n)}$ above involve the matrix elements of the dimensionless cross section operator $\hat{x}$ and the longitudinal momentum operator $\hat{p}$. In this section a model for these operators is developed, which, although highly arbitrary, is consistent with the experimental high-energy behavior of diffraction dissociation, which is in turn approximately consistent with that expected from the leading triple-Pomeron behavior. There is no reason why this model should be accurate for the important low-mass diffractively produced states: two-component duality [21] suggests that in the resonance region it could give a smooth background but not the contribution due to the resonances themselves. The model, however, should give some idea of the importance of high-mass states and the convergence of the expansion in the number of inelastic transitions. Ignoring contributions from secondary Regge poles, and taking the Pomeron intercept $\alpha(0)=1$, this leads to the simple behavior at momentum transfer squared $t=0$ [22,23]

$$d\sigma/(dtdM^2)=\sigma_T^{3/2}g_{PPP}/(16\pi M^2), \tag{47}$$

where σ_T is the nucleon-nucleon total cross section and $g_{PPP}\approx 0.364$ mb$^{1/2}$ [24] is the triple-Pomeron vertex, while the mass square of the diffractively excited nucleon runs from $M_{min}^2\approx 1.5$ (GeV/c^2)^2 to $M_{max}^2\approx m_1^2+2p_1m_\pi$, the latter condition following from the requirement that the longitudinal momentum transfer should be less than m_π, the inverse of the range of the strong force. (At very high energies the effective Pomeron intercept is greater than 1, and the above must be modified. Details can be found in Refs. [21,25,26].)

The continuous range of m^2 between M_{min}^2 and M_{max}^2 can be approximately replaced by a finite number of ''bins'' of width Δm_j^2 centered at m_j^2. The operator $\hat{x}$ is then represented by a finite matrix with elements $\langle j|\hat{x}|i\rangle$ constrained by

$$d\sigma_j/dt\approx\sigma_T^2\langle j|\hat{x}|1\rangle^2/(16\pi)\approx[d\sigma/(dtdM^2)]_{m_j^2}\Delta m_j^2, \tag{48}$$

or

$$\langle j|\hat{x}|1\rangle^2\approx g_{PPP}\Delta m_j^2/(\sigma_T^{1/2}m_j^2). \tag{49}$$

To complete the model, one must also have a prescription for the sizes of the mass bins. For simplicity, equal spacing in m^2 is used below:

$$m_j^2=m_1^2+m_0^2(j-1), \tag{50}$$

with m_0 a parameter determined by the spacing of the low-energy diffractively produced resonances. Equation (35) then takes the simple form

$$\langle j|\hat{x}|1\rangle^2\approx(g_{PPP}/\sigma_T^{1/2})/[(m_1/m_0)^2+j-1]. \tag{51}$$

With the expression above for M_{max}^2, the dimension N of the matrix is given by

$$N\approx 2p_1m_\pi/m_0^2, \tag{52}$$

which increases linearly with p_1, the momentum of the incident proton in the rest frame of the target nucleus. This expression for N is clearly only a rough estimate, but fortunately excited states with j near N do not contribute much to the dispersive correction compared to lower states. Changing N slightly does not affect the results below appreciably. Unfortunately, diffraction dissociation constrains only one row (and column, from the assumed symmetry) of the matrix $\langle j|\hat{x}|i\rangle$. For small m_i^2 and large m_j^2, one can argue that the triple-Pomeron behavior should still be valid and the $\Delta m_j^2/m_j^2$ dependence should still hold. For simplicity, here it is assumed that *every* off-diagonal element of the matrix is given by

$$\langle j|\hat{x}|i\rangle=\sqrt{g_{PPP}/[\sigma_T^{1/2}(a^2+|j-i|)]}, \tag{53}$$

where $a=m_1/m_0$. This expression is consistent with both experiment and the triple-Pomeron behavior for $i=1$ and large j, but is only a guess elsewhere, especially when i and j are comparable in size. Furthermore, although the matrix is in general complex, it will below be assumed real. This is done mainly because the phases of the matrix elements other than x_{11} are unknown, and is consistent with the fact that

TABLE I. Dispersive reductions in the total cross sections for scattering of high-energy nucleons from ^{40}Ca. The figures in parentheses are obtained if the longitudinal momentum transfers are set to zero.

Lab momentum (GeV)	Dispersive reductions in $\sigma_{Total}(A=40)$(mb) $(\Delta p_L=0)$				
	Order in inelastic transitions				
	2	3	4	Sum	Exact
50	20.8 (24.2)	−7.4(−11.8)	2.9 (6.3)	16.4 (18.7)	15.7 (16.7)
100	26.4 (28.5)	−15.2(−19.7)	9.4 (14.9)	20.6 (23.7)	17.4 (17.7)
150	29.0 (30.5)	−20.5(−24.2)	15.7 (21.4)	24.2 (27.6)	17.8 (18.0)
200	30.6 (31.7)	−24.4(−27.4)	21.2 (26.6)	27.5 (30.8)	18.0 (18.1)

the real part of the forward proton-proton scattering amplitude at high energies is known to be small.

Finally, we need an expression for the diagonal elements $x_j=\langle j|\hat{x}|j\rangle$,

$$x_j=1+d(j-1), \tag{54}$$

which allows the cross section for nucleons to scatter from excited nucleons to increase with the degree of excitation. This means that highly excited states will be absorbed more strongly than lower states while propagating between transitions. (If $d=0$, all diagonal elements of x are unity, and the expressions above simplify considerably.) It would probably be more reasonable for x_j to approach a limiting value as j increases, but this would introduce still more parameters and assumptions into the model.

With these assumptions both x_j and m_j^2 are linear in j, and so are the complex numbers y_j defined in Eq. (34),

$$y_j=(j-1)y_2, \tag{55}$$

where

$$y_2=A\sigma_T\rho_0 d+im_0^2/p_1. \tag{56}$$

For small j's the influence of the longitudinal momentum transfer, given by the imaginary part of y_2, decreases as $1/p_1$, but the dimension of the matrix increases as p_1, so that for the heaviest excited nucleon included we have

$$y_N=2A\sigma_T\rho_0 d m_\pi p_1/m_0^2+i2m_\pi, \tag{57}$$

with an imaginary part which is independent of p_1 and a real part which increases linearly with momentum.

VI. RESULTS

The formulas above have been evaluated for scattering of nucleons from ^{40}Ca and ^{208}Pb for incident laboratory momenta from 50 to 200 GeV/c, and for n, the number of inelastic diffractive transitions, ranging from 2 to 4. The corresponding reductions in the total cross sections can be written as

$$\sigma_D^{(n)}=\int db 4\pi b\ \mathrm{Re}\Gamma_D^{(n)}(b). \tag{58}$$

Using $g_{PPP}=0.363$ mb$^{1/2}$ and $\sigma_T=38.5$ mb gives the coefficient in Eq. (39) $(g_{PPP}/\sigma_T^{1/2})^{1/2}=0.24$. We also take $m_0^2=1.5$ GeV^2, $a^2=0.5$, and $d=0.1$, although any values of the same order of magnitude would be just as reasonable.

The shapes of the integrands for ^{208}Pb for 50 and 200 GeV are shown in Figs. 1 and 2, respectively, while the values of the diffractive reductions in the total cross sections are given in Tables I and II. The most surprising feature of these results is that, although the contributions are all small compared to the uncorrected Glauber cross section of about 2582 mb for ^{208}Pb and 737 mb for ^{40}Ca, at high energies the expansion in the number of inelastic transitions does not converge well at all. As noted above, as the energy increases more and heavier excited nucleons are included and the correction at each order increases. Because of the poor conver-

TABLE II. Dispersive reductions in the total cross sections for scattering of high-energy nucleons from ^{208}Pb. The figures in parentheses are obtained if the longitudinal momentum transfers are set to zero.

Lab Momentum (GeV)	Dispersive reductions in $\sigma_{Total}(A=208)$(mb) $(\Delta p_L=0)$				
	Order in inelastic transitions				
	2	3	4	Sum	Exact
50	45.4 (57.9)	−16.4(−39.2)	5.6 (31.4)	34.6 (50.1)	34.2 (37.1)
100	59.6 (66.9)	−40.6(−61.4)	29.6 (65.7)	48.6 (71.2)	37.6 (38.5)
150	66.0 (70.6)	−56.9(−73.0)	54.4 (88.2)	63.5 (85.8)	38.5 (38.9)
200	69.7 (72.9)	−68.2(−80.6)	75.3 (104.8)	76.8 (97.1)	38.9 (39.2)

gence, however, the expansion in the number of inelastic transitions is not very useful, especially at higher energies. It gives only an order of magnitude for the exact result, with the individual terms oscillating in sign. The leading second-order term in particular is always of the right order of magnitude but larger than the exact result, with the error increasing from 33% at 50 GeV/c to nearly 80% at 200 GeV/c.

These results can be compared with the results of high-energy neutron-nucleus total cross section measurements [2]. The experimental values of the total cross sections are somewhat larger than our calculations, presumably reflecting the errors introduced by the uniform density approximation. The experimental results are definitely lower than the results of careful Glauber theory calculation, and the differences are comparable to the leading order dispersive reductions calculated from experimental diffractive dissociation cross sections. These reductions are about twice as large as those calculated here, presumably because the model here does not include properly the contributions from the important low-mass resonant states.

The influence of the longitudinal momentum transfer was studied by comparing the result calculated from the formulas above with those with the longitudinal momentum transfers dropped (so that y_2 becomes a real number). The results in this limit are given as the numbers in parentheses in Tables I and II. Although the longitudinal momentum transfers reduce significantly the magnitudes of the individual terms in the expansion, they have a relatively small effect, decreasing with increasing energy, on the exact results.

VII. CONCLUSION

Previous calculations of the dispersive corrections considered here have considered only a small number of channels, ignored the longitudinal momentum transfer, or included only the lowest order term in the the transition expansion. In the context of our model it has been shown that all of these can lead to large errors. It should be remembered, however, that the matrix model used above has many arbitrary features, even though it is roughly consistent with what is known about high-mass single-diffraction dissociation from a single nucleon. In particular, the amplitudes for transitions from one highly excited nucleon state to another are essentially unknown, and the expressions used in the model are simply guesses based on the known behavior of the amplitudes for excitation from the nucleon itself. It would be useful to repeat the calculations with other assumptions for these amplitudes in order to get some idea of the dependence of the results on the assumptions: a better treatment of the contributions from low-mass resonances should be included if possible.

In the model used here, the diffractive corrections to the total cross sections are all small compared to the total cross section itself, but their expansion in the number of inelastic transitions does not converge well at higher energies. In particular, the leading second-order correction, which has been used to estimate the size of the diffractive correction, is too large by nearly 80% at a laboratory momentum of 200 Gev/c.

It would be interesting to extend these calculations to single-diffraction dissociation from nuclei, since for these processes there is no zeroth-order term, corresponding to the large Glauber contribution to the total cross section, so that the corrections due to higher order terms might be relatively quite large. A preliminary investigation suggests that it should also be possible to do an exact calculation in this case in the uniform density limit. (A calculation of single-diffraction dissociation from the deuteron would also be very interesting, and might put additional constraints on the assumptions that go into the matrix model.) ''Coherent'' diffraction dissociation, where the nucleus remains in its ground state, would be particularly simple to calculate, but probably experimentally challenging. One could also calculate a ''nuclear inclusive'' cross section in which all nuclear excited states are summed over.

ACKNOWLEDGMENT

The author would like to thank Professor Dieter Drechsel and his colleagues in the Theory Group at the Institut für Kernphysik at the Univerität Mainz, where this work was completed, for their hospitality.

[1] V.N. Gribov, Zh. Eksp. Teor. Fiz. **56**, 892 (1969) [Sov. Phys. JETP **29**, 483 (1969)].
[2] P.V.R. Murthy, C.A. Ayre, H.R. Gustafson, L.W. Jones, and M.J. Longo, Nucl. Phys. **B92**, 269 (1975).
[3] B. Blaettel, G. Baym, L.L. Frankfurt, H. Heiselberg, and M. Strikman, Phys. Rev. D **47**, 2761 (1993).
[4] D.R. Harrington, Phys. Rev. C **52**, 926 (1995).
[5] M.L. Good and W.D. Walker, Phys. Rev. **120**, 1857 (1960).
[6] S.D. Drell and J.S. Trefil, Phys. Rev. Lett. **16**, 552 (1966).
[7] M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966).
[8] K.S. Koelbig and B. Margolis, Nucl. Phys. **B6**, 85 (1968).
[9] L.V. Hove, Nucl. Phys. **B46**, 75 (1972).
[10] J. Huefner and B. Kopeliovich, Phys. Rev. Lett. **76**, 192 (1996).
[11] D.R. Harrington, Phys. Rev. D **1**, 260 (1970).
[12] B.K. Jennings and G.A. Miller, Phys. Rev. C **49**, 2637 (1994).
[13] B. Kopeliovich and J. Nemchik, Phys. Lett. B **368**, 187 (1996).
[14] L. Frankfurt, G.A. Miller, and M. Strikman, Phys. Rev. Lett. **71**, 2859 (1993).
[15] L. Frankfurt, V. Guzey, and M. Strikman, J. Phys. G **27**, R23 (2001).
[16] D.R. Harrington, Phys. Rev. D **4**, 840 (1971).
[17] D.R. Harrington, Phys. Rev. **170**, 1460 (1968).
[18] G.V. Bachmann and B. Margolis, Nucl. Phys. **B14**, 609 (1969).
[19] H. Feshbach and J. Huefner, Ann. Phys. (N.Y.) **56**, 268 (1970).
[20] R. Shankar, *Principles of Quantum Mechanics* (Plenum, New York, 1994).

[21] S. Donnachie, G. Dosch, P. Landshoff, and O. Nachtmann, *Pomeron Physics and QCD* (Cambridge University Press, Cambridge, 2002).
[22] K. Goulianos, Phys. Rep. **101**, 169 (1983).
[23] V. Barone and E. Predazzi, *High-Energy Particle Diffraction* (Springer, New York, 2002).
[24] R.L. Cool, Phys. Rev. Lett. **47**, 701 (1981).
[25] C.-I. Tan, Phys. Rep. **315**, 175 (1999).
[26] K. Goulianos and J. Montanha, Phys. Rev. D **59**, 114017 (1999).