Wobbling motion: A γ-rigid or γ-soft mode?

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For even-even nuclei, it is shown that the predicted $B(E2)$ values from the odd spin states of the quasi-γ band in a γ-soft nucleus to the yrast band are quite similar to those predicted for the one-phonon wobbling mode of a rigidly triaxial nucleus. This suggests that the observation of wobbling points to axial asymmetry, but not necessarily to rigid triaxiality. However, another observable that does distinguish γ-soft from γ-rigid structure is identified.

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I. INTRODUCTION

One of the interesting recent developments in nuclear structure has been the discovery of wobbling motion, in the form of bands characterized by one and two wobbling phonons. The wobbling phonon was first predicted decades ago [1] (in the context of even-even nuclei) but has only recently been found experimentally at medium spins in the level schemes of odd-A nuclei, first in ^{163}Lu [2,3] and subsequently in ^{165}Lu [4] and ^{167}Lu [5]. The theoretical predictions describing this mode have been worked out in detail [1,6–8] with specific predictions of energies, electromagnetic selection rules, and $B(E2)$ and $B(M1)$ values as a function of spin.

The wobbling concept has been discussed as a collective mode in deformed nuclei that develop stable triaxial shape, with increasing angular momentum, by which is meant that there is a relatively sharp minimum in the energy surface for a large finite value of γ in some spin range.

Though many predictions of γ-soft [9,10] and γ-rigid [11] models in even-even nuclei are known to be very similar [12,13], there is one well-known, easy-to-measure characteristic that distinguishes γ softness and rigidity at low spin [14], specifically, the energy staggering of the odd and even spin members of the γ band. This observable indicates that axial asymmetry at low spin arises from γ softness [14], and not from rigid triaxiality. Therefore, the wobbling phenomenon raises two important questions.

The first is whether a model based on a γ-soft shape would produce similar predictions in the wobbling spin regime (roughly $I\sim 8$–20) as a rigidly triaxial one. If so, then the issue arises of searching for definitive signatures of γ softness or rigidity at medium to high spins. If not, and if, therefore, the new data specifically point to rigid triaxiality at medium-to-high spin, this implies a shape transition as a function of angular momentum. The second question then is how and where (in what spin range) the shape (phase?) transition from γ soft to γ rigid would occur in deformed nuclei. Understanding the answer to the first question, which we address here, is a prerequisite for attempting to deal with the second.

It is the purpose of this paper to present calculations using a γ-soft model for comparison with those using the rigid triaxial model. The aim is both to test if the characteristic signatures of wobbling definitely point to γ rigidity, and also to identify simple observables that can be used to determine the γ dependence of the potential.

Although the experimental evidence for wobbling motion has focused so far on odd-A nuclei, the mode should exist in even-even nuclei as well, where, indeed, it was first proposed [1]. Here, for simplicity and convenience, we will focus on the latter since the calculations for γ-soft even-even nuclei can be done analytically, the results are more transparent, and there is no ambiguity due to coupling of a particle to a rotor core. Also, in these model calculations we assume no band crossing or backbending for both the γ-rigid and γ-soft cases. As a γ-soft model, we will use the $O(6)$ limit [10] of the interacting boson approximation (IBA) model, but the results are not particularly dependent on that model. In particular, the E2 selection rules are the same as in the Wilets-Jean model [9]. Moreover, to emulate the results of geometrical models and to calculate the $B(E2)$ values between higher spin states, we will use the limit of large boson numbers where the characteristic finite boson number effects in the IBA usually disappear and where its relevant predictions go over to those of geometric models.

We first summarize the key characteristics of the wobbling mode as obtained from rigid triaxial rotor calculations for odd-A nuclei [6–8]. We refer here to results for $-60° \leq \gamma \leq 0°$, that is, the region (sector 2) in the Lund convention corresponding to collective rotation. There are three observables that concern $E2$ transitions.

(1) Staggering in the $B(E2)$ values from the wobbling phonon band to the yrast band: specifically, the $I_{wob}\rightarrow(I+1)_{yrast}$ transitions are allowed, while the $I_{wob}\rightarrow(I-1)_{yrast}$ transitions are weak or forbidden.

(2) A large ratio of $B(E2)_{out}/B(E2)_{in}$, that is, $B[E2;I_{wob}\rightarrow(I+1)_{yrast}]/B[E2;I_{wob}\rightarrow(I-2)_{wob}]$: These ratios are typically about 0.2–0.3, which is much larger than would be expected for typical interband transitions. The enhanced $B(E2)_{out}/B(E2)_{in}$ ratios reflect the fact that the wobbling band$\rightarrow$yrast band transitions destroy a collective wobbling phonon.

(3) The $B(E2)_{out}=B[E2;I_{wob}\rightarrow(I+1)_{yrast})]$ values go as $1/I$ rather than $1/I^2$ as in cranking model calculations. These characteristic features are similar in odd [6,8] and even-even [1,7] nuclei.

Finally, there is another prediction for the wobbling mode that has caused some concern since it seems contrary to experimental observation in cases of wobbling in the odd-A Lu isotopes. Theoretically, the energy of the wobbling band states above the yrast band should not be a constant but rather linear in spin. Specifically, $\hbar\omega_{wobb}\propto\hbar\omega_{rot}\propto I$ (this specific spin dependence assumes the moments of inertia J_x, J_y, J_z are constant).

II. RESULTS

To contribute towards the elucidation of the wobbling phenomenon, we now present calculations for these signatures for a γ-soft even-even nucleus. We take the yrast band as the base configuration. The levels $(2_2^+, 3_1^+, 4_2^+, \ldots)$ are normally considered to be a kind of quasi-γ band, although this terminology is not precise (due to extreme K mixing at large γ). As spin increases ($I\gtrsim 9\hbar$), the odd spin levels of the quasi-γ band take on the character of the one phonon wobbling mode. (We will discuss the transition from a traditional γ-soft vibrator to this "wobbling" regime as a function of spin below.) The levels are labeled by the τ quantum number [the characteristic γ-soft quantum number of $O(5)$, which is equivalent to the λ quantum number of Wilets-Jean] τ is somewhat like a phonon quantum number, although the spin content of the τ multiplets is restricted relative to the traditional spherical oscillator multiplets. In the familiar quadrupole vibrator the one-, two-, and three-phonon levels comprise a 2^+ level, a $0^+-2^+-4^+$ triplet, and the 0^+, 2^+, 3^+, 4^+, 6^+ quintuplet, respectively. In the γ-soft case, the two-phonon ($\tau=2$) multiplet is a 2^+, 4^+ doublet and the three-phonon ($\tau=3$) multiplet contains 0^+, 3^+, 4^+, and 6^+ levels.

In the simplest version of each model the multiplet states are degenerate. In the spherical vibrator, the energies go as the phonon number n, so that, for example, $R_{4/2}\equiv E(4_1^+)/E(2_1^+)=2.0$. In the γ-soft limit the energies go as $\tau(\tau+3)$, giving $R_{4/2}=2.5$. The key levels in each τ multiplet for the present discussion are the yrast levels with $I_{yrast}=2\tau$ (e.g., 4^+ for $\tau=2$), and the odd I "γ-band" levels with $I_\gamma^{odd}=2\tau-3$ (e.g., 3^+ for $\tau=3$). These labels for a γ-soft model are included in Fig. 1. A key characteristic of γ-soft nuclei is the general $E2$ selection rule, $\Delta\tau=\pm1$.

From Fig. 1, we can immediately see that one of the characteristic wobbling signatures emerges directly from the τ-selection rules for γ-soft nuclei, namely, the staggering in $B(E2)_{out}$ values for I_γ^{odd} to the yrast $I+1$ and $I-1$ states. The former (such as the $3_1^+\rightarrow 4_1^+$ transition just discussed) are allowed ($\Delta\tau=3-2=1$) while the latter (such as $3_1^+\rightarrow 2_1^+$) are strictly forbidden ($\Delta\tau=3-1=2$).

To calculate the magnitudes of the $B(E2)$ values in the Wilets-Jean model [9], we use the analytic expressions for the $O(6)$ limit [10] of the IBA for an asymptotic boson number of $N\rightarrow\infty$. Since the Wilets-Jean model is characterized by $V(\gamma)=$const for $\gamma=0°$ to $\gamma=60°$, the appropriate comparison to a rigid triaxial rotor is to the Davydov-Filippov model [11] with $\gamma=30°$. Analytic formulas for the relevant transitions can be deduced from Ref. [13]. We stress that all the present results apply specifically to nuclei with large γ ($\gamma\sim30°$). Results for smaller asymmetries are not analytic in γ-soft models. Numerical calculations for such cases are underway and will be reported later.

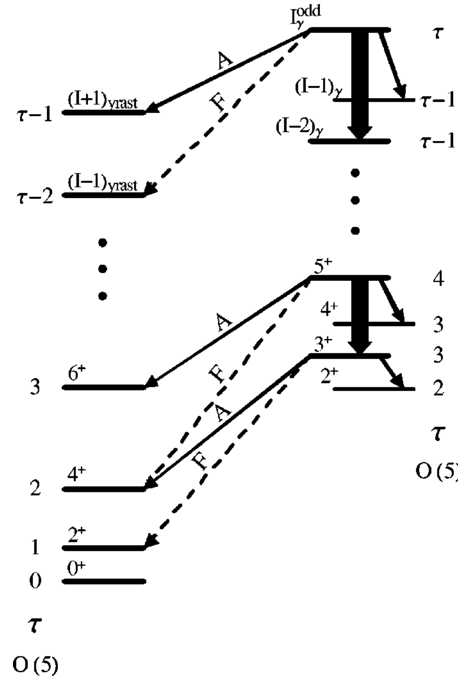

FIG. 1. Schematic level scheme of the ground and quasi-γ bands showing the allowed (A) and forbidden (F) $E2$ transitions in γ-rigid and γ-soft models. In the latter, these transitions obey a $\Delta\tau=1$ selection rule, and the τ quantum numbers associated with the $O(5)$ group of a γ-flat potential are shown.

Although our focus is the odd spin members of the quasi-γ band, which simulate the wobbling mode at intermediate spins, it is informative to look first at the even spin states. We show the results for $B(E2)$ values for transitions from these states in Fig. 2(a). The allowed, very collective, $I\rightarrow(I-2)$ in-band transitions, for both yrast and quasi-γ bands, are similar in the γ-soft and γ-rigid models, differing only by 10–20 % above $I_\gamma\gtrsim10$. However, for the (much less collective) $I_\gamma^{even}\rightarrow I_{yrast}$ transitions, the γ-soft model gives $B(E2)$ values about one order of magnitude larger than in the γ-rigid model. $I_\gamma^{even}\rightarrow(I-2)_{yrast}$ transitions are forbidden in both models (since $\Delta\tau=2$ in the γ-soft case, and since these transitions vanish in the γ-rigid model for $\gamma=30°$). Figure 2(a) shows another interesting feature we will return to below, namely, the gap that develops between in-band [e.g., $I_\gamma\rightarrow(I-2)_\gamma$] and interband (e.g., $I_\gamma\rightarrow I_{yrast}$) as a function of spin. These classes of transitions, in both γ-soft and γ-rigid models, have comparable $B(E2)$ values at low spin but diverge sharply above $I_\gamma\sim10$, favoring the stretched $E2$ in-band transitions.

We now turn to the $E2$ transitions from the odd-spin

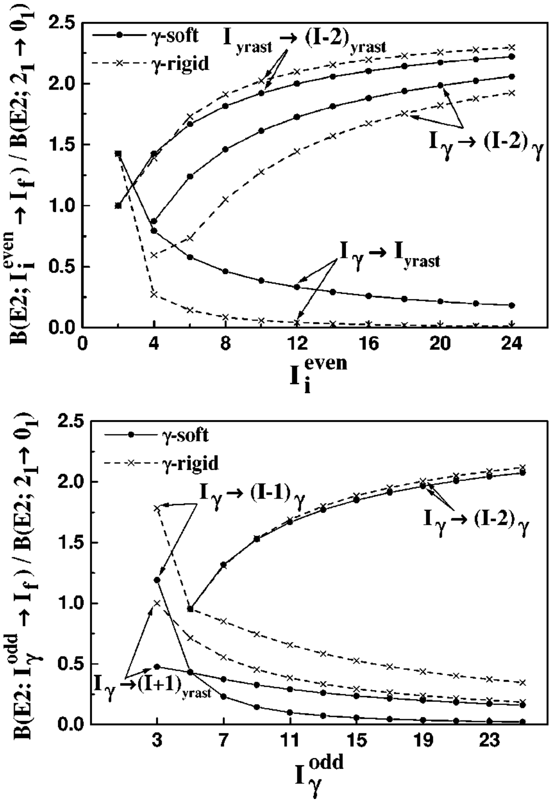

FIG. 2. (a) $B(E2)$ values from even spin members of the quasi-γ band for γ-rigid and γ-soft models. (b) Similarly for odd-I initial states of the quasi-γ band. The $B(E2)$ values here are normalized to the $B(E2;2_1^+\rightarrow 0_1^+)$ value.

members of the quasi-γ band, which are the γ-soft analogues of the wobbling mode in the spin regime above $I\sim 9$. As shown in Fig. 2(b), the strongly collective in-γ-band stretched $E2$ transitions, $I_\gamma^{odd}\rightarrow(I-2)_\gamma$, are virtually identical in the two models. So too are the $I_\gamma^{odd}\rightarrow(I+1)_{yrast}$ transitions which are allowed in both models and whose $B(E2)$ values are within about $\lesssim 25\%$ of each other for $I_\gamma \gtrsim 9$. Notice, however, that the in-band $\Delta I=1$ transitions $I_\gamma^{odd}\rightarrow(I-1)_\gamma$ are very different for γ-soft and γ-rigid models.

Let us consider the implications of these results. There are three that are of prime interest.

(1) The similarity of the $B[E2;I_\gamma^{odd}\rightarrow(I+1)_{yrast}]$ values in γ-soft and γ-rigid models means that this key wobbling observable does not determine the γ dependence of the potential for highly axially asymmetric nuclei. Moreover, the magnitudes of the ratio $B(E2)_{out}/B(E2)_{in}=B[E2;I_\gamma^{odd}\rightarrow(I+1)_{yrast}]/B[E2;I_\gamma^{odd}\rightarrow(I-2)_\gamma]$ are very similar, and near the values ~ 0.2 found in odd-A wobblers. Further, since the $I_\gamma^{odd}\rightarrow(I-1)_{yrast}$ transitions are strictly forbidden in both models, both models predict similar $B(E2)$ staggering. These ideas are shown in Fig. 3. Finally, as also shown in the inset to Fig. 3, the $B(E2)_{out}$ values closely follow a $1/I$ dependence and not the $1/I^2$ cranking model dependence.

(2) Figure 2(b) reveals a new way of distinguishing γ-soft and γ-rigid nuclei, namely, the $B[E2;I_\gamma^{odd}\rightarrow(I-1)_\gamma]$ value,

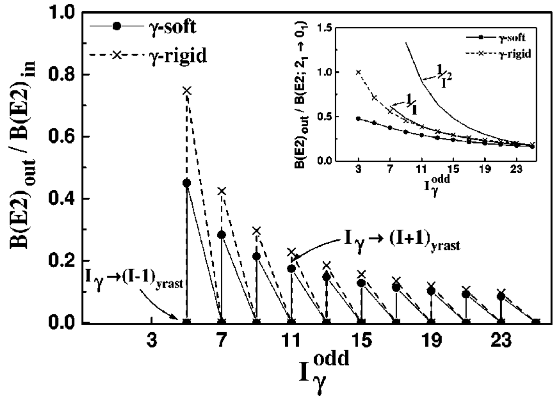

FIG. 3. Staggering in $B(E2)_{out}/B(E2)_{in}=B[E2;I_\gamma^{odd}\rightarrow(I\pm 1)_g]/B[E2;I_\gamma^{odd}\rightarrow(I-2)_\gamma]$ in γ-soft and γ-rigid models. The inset shows the allowed $B[E2;I_\gamma^{odd}\rightarrow(I+1)_g]$ values in comparison with $1/I$ and $1/I^2$ spin dependencies.

which is much stronger in the γ-rigid case. The convenient branching ratio $B[E2;I_\gamma^{odd}\rightarrow(I-1)_\gamma]/B[E2;I_\gamma^{odd}\rightarrow(I-2)_\gamma]$, shown in Fig. 4, gives a clear distinction, amounting to about an order of magnitude, for $I_\gamma^{odd}\gtrsim 9$.

(3) The results in Figs. 2(a,b) show an interesting, and characteristic, difference between the "in-band" and "out-of-band" transitions as a function of spin. At low spins, typically two or more decay routes are allowed for a given initial state. Thus, for example, the 5_γ^+ level decays to the 6_{yrast}^+ level as well as to the 4_γ^+ and 3_γ^+ states. These allowed transitions have roughly comparable $B(E2)$ values since all three transitions destroy a single τ phonon. However, at higher spins (above $I_\gamma\sim 9$), the stretched $E2$ transition (e.g., $9_\gamma^+\rightarrow 7_\gamma^+$) dominates and all others ($9_\gamma^+\rightarrow 10_{yrast}^+, 9_\gamma^+\rightarrow 8_\gamma^+$) are small. This behavior illustrates one of the most characteristic differences between low and high spin spectroscopy. At low spin, decay occurs by multiple, competitive routes, while at high spin, in-band decays clearly dominate.

It is interesting to apply the idea in point 3 to the development of the wobbling mode with spin. At low spin, all the

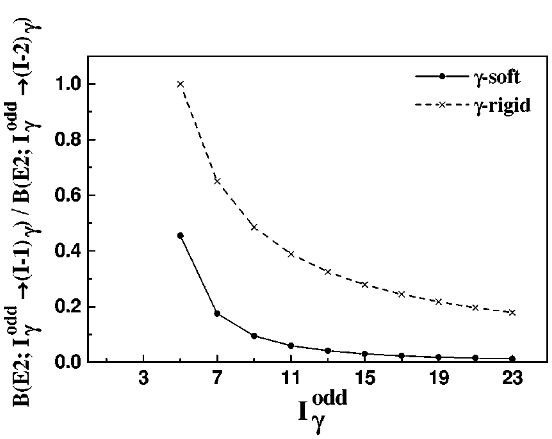

FIG. 4. Illustration of a branching ratio from odd spin members of the quasi-γ band that provides an observable that distinguishes γ-soft and γ-rigid shapes for nuclei with large axial asymmetry ($\gamma\sim 30°$).

spin allowed transitions are of *τ-phonon changing*, $\Delta\tau=\pm 1$ character. At higher spins ($I\gtrsim 9$), the allowed transitions separate into two families, highly collective transitions typical of intraband $B(E2)$ values and increasingly weaker transitions corresponding to *wobbling-phonon changing* transitions.

Thus, while the predictions of the γ-rigid and γ-soft models are almost identical, the results in Figs. 2(b) and 4 may provide a simple signature for shape evolution with spin and, therefore, a test of microscopic predictions. For example, in Refs. [6–8], the odd-A nuclei are calculated to be axially symmetric at low spin, with a triaxial shape developing only at higher rotational spins ($I\sim 9$). In even-even nuclei, such a shape evolution can be distinguished from one in which the nucleus is axially asymmetric at all spins. In the former case, at low spin, the $B(E2)_{out}/B(E2)_{in}$ values would be ratios of interband ($\gamma-g$ band) to intra-γ-band transitions and would therefore be very weak ($\sim$0.05). In the wobbling spin regime the out-of-band transition is a collective wobbling phonon-changing transition and $B(E2)_{out}/B(E2)_{in}$ grows, to roughly 0.2.

In contrast, if a nucleus is axially asymmetric for all spins, then, as seen in Fig. 2(b), at low spin *both* the $B(E2)_{out}$ and $B(E2)_{in}$ transitions are *τ-phonon-changing* transitions and are therefore comparable [$B(E2)_{out}/B(E2)_{in}$ values $\sim$0.5]. At higher spin the $B(E2)_{out}$ transitions become *wobbling-phonon changing* transitions and the $B(E2)_{in}$ values remain of collective in-band character, and thus $B(E2)_{out}/B(E2)_{in}$ decreases.

Thus, for a shape evolution with spin from symmetric to asymmetric, both γ-soft and γ-rigid models give small $B(E2)_{out}/B(E2)_{in}$ values ($\sim$0.05) that *grow* with spin into the wobbling regime and then *decrease*. For a shape that remains axially asymmetric at all spins, $B(E2)_{out}/B(E2)_{in}$ is large at low spins ($\sim$0.5) and decreases monotonically with spin. Hence, the low spin behavior of the $B(E2)_{out}$ values, or the $B(E2)_{out}/B(E2)_{in}$ ratio, might help to distinguish which interpretation of shape evolution with spin is applicable.

Finally, we consider the comparison of phonon energies in the rigid triaxial wobbler with the energy differences between the quasi-γ band and yrast band levels in the γ-soft nucleus. In Ref. [1] the wobbling frequency is given by $\hbar\omega_{wobbling}=\hbar\omega_{rot}f(J_x,J_y,J_z)$, which increases with spin I. Therefore, we would expect a systematic divergence between the energies in the wobbling band and in the yrast band. For the γ-soft case the $\tau(\tau+3)$ dependence of energies in the Wilets-Jean model, gives

$$E(I_\gamma^{odd})-E[(I+1)_{yrast}]=\frac{(I_\gamma^{odd}+5)E(2_1^+)}{4}, \qquad (1)$$

which is also linear in I. Thus the similarity of the two models (γ-rigid and γ-soft) is again manifest and extends to both energies and $B(E2)$ values.

III. CONCLUSIONS

To summarize, axial asymmetry arising from γ softness gives similar predictions as a γ rigid rotor with $\gamma=30°$ for transitions that are allowed and collective. Strong and significant differences, however, arise for weaker interband transitions such as those between even spin states, $I_\gamma\rightarrow I_{yrast}$, for $I_\gamma\gtrsim 6$, where the $B(E2)$ values for γ-soft nuclei are almost an order of magnitude stronger, and for $I_\gamma\rightarrow(I-1)_\gamma$, intra-$\gamma$-band transitions where the $B(E2)$ values for I_γ^{odd}, γ-rigid nuclei are much larger. These transitions therefore provide distinguishing characteristics of γ softness and γ rigidity that have not been extensively discussed before. The relative spin dependence of in-band and out-of-band transitions is also interesting. At low spin, these $B(E2)$ values are comparable [see Figs. 2(a), (b)] while at higher spin (for $I_\gamma\gtrsim 10$), the $\gamma\rightarrow$yrast $B(E2)$ values drop well below the $\Delta I=2$ in-band $B(E2)$ values. Moreover, for the characteristic transitions that identify the wobbling mode states (namely, $I_\gamma^{odd}\rightarrow(I+1)_{yrast}$ transitions), the γ-soft and γ-rigid models are very similar. Finally, the $I_\gamma^{odd}\rightarrow(I+1)_{yrast}$ transitions follow an $1/I$ spin dependence and the $B(E2)_{out}/B(E2)_{in}$ staggering is identical for γ-soft and γ-rigid models.

We stress that these results are definitely *not* intended to assert that wobbler nuclei are γ soft. Rather, we only mean to suggest that the *existing* $E2$ signatures may not be sufficient to *empirically* establish whether these nuclei are rigidly triaxial or γ soft. We also caution that our comparison is for even-even nuclei, while existing evidence [2–5] for wobbling motion is in odd-A nuclei.

It is hoped that the present contribution may spur the search for definitive experimental characteristics of wobbling, and of the γ dependence of the potential involved, in even-even nuclei. Microscopic calculations of wobbling nuclei seem, in fact, to suggest γ rigidity, and may well be correct, but they are not a substitute for appropriate experimental evidence. Ultimately, a conclusion in favor of rigid triaxiality at medium spin in wobbling nuclei would perhaps be more interesting since it would imply a fascinating shape (phase?) transition from γ soft to γ rigid with spin.

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