Nonlocality in the nucleon-nucleon interaction and three-nucleon bound states

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We address the problem of a possible nonlocality in the nucleon-nucleon interaction and its consequences for the description of the three-nucleon bound states ^{3}H and ^{3}He. A nucleon-nucleon potential model is constructed, which respects the local behavior of traditional nucleon-nucleon interactions at longer ranges but exhibits a nonlocality at shorter distances. It provides for an accurate fit of all existing nucleon-nucleon data and takes into account breaking of charge independence and charge symmetry. With this interaction model the ^{3}H and ^{3}He binding energies can be described simultaneously in perfect agreement with experimental data. No three-nucleon force is needed as in the case of purely local nucleon-nucleon potentials.

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I. INTRODUCTION

At present the problem of nonlocality of the nucleon-nucleon (*NN*) interaction must still be considered an open one. It is clear that any *NN* interaction model derived from some dynamical principle will contain some nonlocal features; as an example, one can mention meson-exchange dynamics. On the other hand, one has also learned that one can get along with purely local *NN* interaction models in describing the whole phenomenology of the *NN* system. Recent parametrization of such *NN* forces, e.g., the Argonne or Reid-like Nijmegen models, provide a perfect fit of all *NN* data. In addition, they are more easily tractable in various applications. However, when these local *NN* potentials are used in few-nucleon systems, additional three-nucleon (3*N*) forces are needed to reach a reasonable description. Here one encounters another problem, namely, the question for type and magnitude of 3*N* forces. Recently it has become a common practice to adjust the 3*N* forces by fitting the data (binding energies) of the 3*N* systems.

Unfortunately, the question of nonlocality of *NN* interaction and of the presence of 3*N* forces are intimately related. This has already been discussed by Polyzou and Glöckle [1]. As a result, one does not yet definitely know how much and which type of nonlocality can occur in the *NN* interaction and which role is left for 3*N* forces.

Here we want to investigate which nonlocality can be allowed in the *NN* interaction so that it still reproduces all *NN* data with high precision, and at the same time leads to a correct description of the three-nucleon bound sates ^{3}H and ^{3}He, without the addition of an explicit 3*N* force. In fact, we shall demonstrate that this is possible in a way compatible with the present insight into the properties of *NN* forces and the degree of accuracy of modern 3*N* calculations (including Coulomb forces).

For the purpose of our studies we first construct a *NN* interaction model that has the expected local behavior (Yukawa tail) at long ranges but comes with a nonlocality at shorter distances. The region of transition between the local and nonlocal parts is 1–3 fm, with the bulk of the nonlocality occurring at distances shorter than 1.5 fm. Similar potential models were considered before [2,3], but their nonlocalities were extended to larger ranges (of about 2–4 fm). In the present *NN* potential model we could keep the Yukawa tail down to shorter distances and still fit the *NN* data. Probably it is thus more congruent with the dynamics really prevailing in the *NN* system [4].

In practice we step out from the Yukawa tail of the local Argonne v_{18} potential and add an inner phenomenological nonlocal form. We smoothly cut off the Yukawa tail in the region of 1–3 fm and fit the nonlocal potential to the *NN* data, imposing as an additional constraint the ^{3}He binding energy. The model breaks charge independence and charge symmetry since it is required to reproduce the experimental data of all low-energy parameters (in particular the different np,pp, and nn scattering lengths) to high accuracy. The Coulomb interaction in the pp system is treated exactly both in 2*N* and 3*N* systems. Thereby, we simultaneously arrive at correct description of the triton binding energy. In all steps, both for the *NN* and 3*N* systems, we provide a detailed comparison of our partly nonlocal potential model with the local Argonne potential v_{18} and the (nonlocal) charge-dependent Bonn (CD-Bonn) potential.

In the following section we define the *NN* input into our 3*N* calculations and describe in particular the construction of the partly nonlocal *NN* potential model. In Sec. III we outline the treatment of the 3*N* bound state problem with exact treatment of the Coulomb interaction. The results for the 3*N* bound states ^{3}H and ^{3}He are presented in Sec. IV. The summary and discussion are contained in Sec. V.

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II. THE *NN* INPUT

Our 3*N* calculations are performed with two different inputs for the *NN* interaction. At first, we calculate the 3*N* bound states essentially with the Argonne (ARG) potential v_{18} [5]; for its slight modification from the original version, see the section below. Second, we replace in the $^1S_0$ and $^3SD_1$ (this is a shorter notation for $^3S_1$-$^3D_1$) partial-wave states the Argonne potential v_{18} by a potential whose inner part is a phenomenological nonlocal potential. In either case all *NN* partial waves up to total angular momentum $j=9$ are considered, however, for the nonlocal interactions the $j\leqslant 5$ case produces already the required 1-keV accuracy.

A. Modified Argonne potential v_{18}

For the purpose of the present study it is sufficient to treat the *NN* interaction in a simplified manner. Except the nonrelativistic Coulomb interaction for the distributed charge [6], all of the other electromagnetic interactions are neglected. Also, we use equal masses for the neutron and proton. In order to reproduce, under these simplifying assumptions, the singlet scattering lengths and the deuteron binding energy in complete agreement with the original ARG potential (although, for the *np* singlet scattering length we chose the slightly different experimental value), the nuclear part of the potential has to be slightly modified. For definiteness we call this interaction as modified Argonne potential v_{18} abbreviated as ARGm.

B. Nonlocal *NN* potential

In case of the nonlocal potential the full *NN* interaction consists of two parts. At longer ranges ($r\geqslant 3$ fm) it is essentially local of Yukawa type, while at shorter ranges ($r\leqslant 1$ fm) it is purely nonlocal. There is a smooth cutoff in the intermediate region of 1–3 fm. The full notation of the partial-wave decomposed *NN* potential is $\langle r(ls)j|V|r'(l's)j\rangle$. For simplicity the indices *s* and *j* are omitted and a shortened form $V_{ll'}$ is used. Of course, the parameters of the *NN* potential $V_{ll'}$ depend also on the *s* and *j* values, and it is reflected in the tables for the different partial-wave components of the *NN* interaction. The full *NN* interaction $V_{ll'}$ is defined as

$$V_{ll'}(r,r')=\delta(r-r')\cdot F_{ll'}(r)\cdot V^Y_{ll'}(r)+W_{ll'}(r,r'), \quad (1)$$

where $F_{ll'}$ is the cutoff function

$$F_{ll'}(r)=\Theta(r-R_{ll'})\{1-e^{-[\alpha_{ll'}(r-R_{ll\ell'})]^2}\}. \quad (2)$$

The first term in Eq. (1) constitutes the the local part with $V^Y_{ll'}(r)$ being the same Yukawa tail as in the Argonne potential v_{18}. The nonlocal potential is expressed explicitly as

$$W_{ll'}(r,r')=\left(\frac{\beta_l r}{\sqrt{1+\beta_l^2 r^2}}\right)^l\cdot\tilde{W}_{ll'}(x,x')\cdot\left(\frac{\beta_{l'} r'}{\sqrt{1+\beta_{l'}^2 r'^2}}\right)^{l'}, \quad (3)$$

with

$$\begin{aligned}\tilde{W}_{ll'}(x,x')=\delta_{ll'}V_l\{&e^{-[a_l(x-x_l)]^2-[a'_l(x'-x'_l)]^2}\\&+e^{-[a_l(x'-x_l)]^2-[a'_l(x-x'_l)]^2}\}+(1-\delta_{ll'})\\&\times\sum_{i=1,2}V^i_t e^{-[a^i_l(x-x^i_l)]^2-[a^i_{l'}(x'-x^i_{l'})]^2}\\&+\sum_{i=1}^{n_{ll'}}V^i_{ll'}e^{-[b^i_{ll'}(x+x'-2z^i_{ll'})]^2-[c^i_{ll'}(x-x')]^2}\end{aligned} \quad (4)$$

and $x=\gamma r^2/\sqrt{1+\gamma^2 r^2}$. For symmetry reasons, $\alpha_{ll'}=\alpha_{l'l}$, $R_{ll'}=R_{l'l}$, $V^i_{ll'}=V^i_{l'l}$, $b^i_{ll'}=b^i_{l'l}$, $c^i_{ll'}=c^i_{l'l}$, and $z^i_{ll'}=z^i_{l'l}$.

Note that the earlier form [3] is modified, since the screening term $e^{-[a_{ll'}F(y,y_{ll'})(x-x')]^2}$ is omitted. It can be interpreted as an $a_{ll'}\equiv 0$ choice. The aim of this modification is to avoid sharp changes in the $r\approx 0, r'\approx 2$ fm region caused by this term. In any case, the present new parameter values make this term obsolete.

Nonlocal potentials of similar kind, the so-called INOY (inside nonlocal, outside Yukawa) potentials were already considered in previous works [2,3]. Their nonlocalities extended to ranges of $r\approx 4$ fm. The present nonlocal *NN* interactions confine the nonlocality to shorter ranges of $r\leqslant 3$ fm. We denote them as IS (inside nonlocal, outside Yukawa short range) potentials.

Notice that in the above formulas of the nonlocal interaction, Eqs. (3) and (4), the diagonal terms contain both the central and tensor component of the *NN* interaction. The parameters $\alpha_{ll'}$, $R_{ll'}$ and β_l may depend on angular momenta, however, they were fixed to the values independent of

TABLE I. Parameters of the IS *nn*/*pp* and *np* interactions.

	$^1S_0$ (*pp*/*nn*)	$^1S_0$ (*np*)	$^3S_1$	$^3D_1$
V_l (MeV fm^{-3})	−408.0	−391.7	−255.9	0.0
a_l(fm^{-1})	2.6	2.519	2.463	
a'_l(fm^{-1})	1.650	2.0	2.0	
x_l(fm)	0.0	0.0	0.0	
x'_l(fm)	1.0	1.0	1.0	
V^i_{ll} (MeV fm^{-3})	1.839×10^4	1.217×10^4	6.672×10^3	3.811×10^3
	−237.966	−127.209	−198.714	−646.7
	−1.205	−0.7298	−0.4695	0.4105
V^2_{ll} (*nn*) (MeV fm^{-3})	−244.755			
b^i_{ll} (fm^{-1})	1.950	1.737	1.592	0.7955
	1.8	1.210	1.151	1.392
	0.55	0.5	0.5	1.225
c^i_{ll} (fm^{-1})	1.6	1.7	1.761	0.3
	1.4	1.0	1.4	1.725
	0.55	0.5	0.6	1.5
z^i_{ll} (fm)	0.0	0.0	0.0	0.0
	0.42	0.45	0.45	0.7
	1.0	1.0	1.0	1.4

TABLE II. Parameters of the varied interactions.

	ISv	ISa		ISb	
	$^1S_0$ (nn)	$^3S_1$	$^3D_1$	$^3S_1$	$^3D_1$
V_l (MeV fm^{-3})	-415.0	-182.3	0.0	-132.9	0.0
a_l (fm^{-1})	2.6	4.285		3.5	
a_l' (fm^{-1})	1.650	1.253		1.143	
x_l (fm)	0.0	0.1		0.1	
x_l' (fm)	1.0	0.9		0.9	
V_{ll}^i (MeV fm^{-3})	1.630×10^4	1.664×10^4	5.714×10^3	1.3×10^4	4.334×10^3
	-244.189	-105.797	-598.7	-140.116	-450.4
	-1.2	-0.3130	1.495	-0.2772	0.09734
b_{ll}^i (fm^{-1})	1.9	1.934	0.8227	1.843	0.8167
	2.0	2.601	1.240	2.425	1.243
	0.55	0.4548	1.429	0.4441	1.451
c_{ll}^i (fm^{-1})	1.8	2.034	0.1906	2.286	0.2037
	1.4	1.371	0.465	1.055	0.6655
	0.55	0.8	1.5	0.8	1.5
z_{ll}^i (fm)	0.0	0.0	0.0	0.0	0.0
	0.42	0.7	0.7	0.7	0.7
	1.0	1.1	1.4	1.1	1.4

the angular momenta: $\alpha_{ll'}=1$ fm^{-1}, $R_{ll'}=1$ fm, and $\beta_l=2$ fm^{-1}. The $\gamma=2$fm^{-1} value was also fixed and the $n_{ll'}=3$ value proved to be sufficiently large. The other parameters in the potential function W were determined by fitting the NN phase shifts, the effective-range parameters, the deuteron properties, and the ^{3}He binding energy. The deuteron D-state probability was assumed to be $P_D=3.6\%$, like in Ref. [3].

For the $^1S_0$ partial wave the potentials were fitted separately for np and pp states [7]. The difference of the nn and pp $^1S_0$ potentials stem only from one parameter, V_{00}^2, which is chosen to fit the nn scattering length. In addition, we provide a variant of the nn potential, denoted by ISv, in order to study the dependence of the triton binding on other properties of the nn $^1S_0$ interaction. In a similar spirit we also designed two further variants of the IS potential in $^3SD_1$ state; one, called ISa, which reproduces exactly the deuteron properties of the Argonne potential v_{18} and one, called ISb, which reproduces exactly the deuteron properties of the CD-Bonn potential [8]. Both of these versions exhibit a shorter range nonlocal part and their (deuteron) S-state wave functions are enhanced at short distances.

The parameters of all IS potentials are given in Tables I–III. In Table IV we quote the $^1S_0$ effective range parameters for all potentials considered subsequently with regard to 3N bound states. The triplet effective range parameters together with the deuteron properties are shown in Table V. The scattering lengths and the effective ranges of np and pp IS potentials are very similar to those of ARG and ARGm potentials. Only for the nn state the effective range is slightly different in order to reproduce the triton binding energy.

The IS potentials were fit to the Nijmegen phase shifts. The quality of the fit can be estimated from the comparison of all $^1S_0$ and $^3SD_1$ phase-shift parameters in Tables VI–XI, where a comparison is provided with the ARG, the ARGm, and the CD-Bonn potentials. In the $^3SD_1$ states the phase

TABLE III. Parameters of the tensor part of the nonlocal interactions.

	IS	ISa	ISb
V_t^1 (MeV fm^{-3})	-455.5	-161.7	-191.8
a_0^1 (fm^{-1})	2.5	2.829	1.858
a_2^1 (fm^{-1})	1.2	1.396	1.432
x_0^1 (fm)	0.0	0.2	0.2
x_2^1 (fm)	0.9	0.7	0.7
V_t^2 (MeV fm^{-3})	23.86	17.12	23.57
a_0^2 (fm^{-1})	1.158	1.221	1.383
a_2^2 (fm^{-1})	2.0	0.733	1.988
x_0^2 (fm)	0.9	0.7	0.7
x_2^2 (fm)	0.0	0.2	0.2
V_{02}^i (MeV fm^{-3})	-5.227×10^3	-7.062×10^3	-1.048×10^4
	-12.53	-101.9	-62.49
	-0.08051	-0.7633	-0.4598
b_{02}^i (fm^{-1})	1.843	1.285	1.590
	0.6554	0.7976	0.8268
	0.3204	0.7907	0.6218
c_{02}^i (fm^{-1})	1.0	0.7083	0.3536
	0.3944	1.380	1.218
	0.6	0.6112	0.6834
z_{02}^i (fm)	0.0	0.0	0.0
	0.6	0.7	0.7
	1.2	1.4	1.4

TABLE IV. $^1S_0$ scattering lengths and effective ranges.

	ARG	CD-Bonn	ARGm	IS	ISv
a_{pp} (fm)	−7.8064	−7.8154	−7.8064	−7.8064	
r_{pp} (fm)	2.788	2.773	2.784	2.769	
a_{nn} (fm)	−18.487	−18.968	−18.487	−18.601	−18.375
r_{nn} (fm)	2.840	2.819	2.839	2.824	2.829
a_{np} (fm)	−23.732	−23.738	−23.748	−23.748	
r_{np} (fm)	2.697	2.671	2.696	2.678	

shifts of ISa and ISb potentials are practically the same as for the Argonne v_{18} and CD-Bonn potentials, respectively.

III. TREATMENT OF THE THREE-BODY COULOMB PROBLEM

For the solution of the ^{3}He bound state we adhered to the approach in Ref. [9] which allows to treat the three-body Coulomb problem to any desired accuracy. Stepping out from the 3*N* Hamiltonian

$$H = H^0 + \sum_{\alpha=1}^{3} (v_\alpha^s + v_\alpha^C) \tag{5}$$

with separated short-range and Coulomb interactions in 2*N* subsystems α of the pairs (β,γ), the Faddeev components Ψ_α of the 3*N* wave function Ψ obey the integral equations

$$|\Psi_\alpha\rangle = G_\alpha^C(E) v_\alpha^s (|\Psi_\beta\rangle + |\Psi_\gamma\rangle) \tag{6}$$

with α,β,γ a cyclic permutation and the channel Coulomb Green's operator is defined by

$$G_\alpha^C(E) = (E - H^0 - v_\alpha^s - v_\alpha^C - v_\beta^C - v_\gamma^C)^{-1}. \tag{7}$$

By use of the resolvent equation

$$G_\alpha^C(E) = \tilde{G}_\alpha^C(E) + \tilde{G}_\alpha^C(E) U^\alpha G_\alpha^C(E) \tag{8}$$

with

$$\tilde{G}_\alpha^C(E) = (E - H^0 - v_\alpha^s - v_\alpha^C - u_\alpha^C)^{-1} \tag{9}$$

and

$$U^\alpha = v_\beta^C + v_\gamma^C - u_\alpha^C, \tag{10}$$

we can rewrite Eq. (6) as

$$|\Psi_\alpha\rangle = \tilde{G}_\alpha^C(E)[U^\alpha|\Psi_\alpha\rangle + v_\alpha^s(|\Psi_\beta\rangle + |\Psi_\gamma\rangle)]. \tag{11}$$

Herein the u_α^C is the potential

$$u_\alpha^C = Z_\alpha (Z_\beta + Z_\gamma)/y_\alpha, \tag{12}$$

where Z_α is the charge of particle α, and y_α the Jacobi coordinate between particle α and the two-body subsystem of particles β and γ.

The Faddeev integral equations are solved by applying the Coulomb-Sturmian separable expansion technique [9]. For a full-fledged calculation of ^{3}He (and similarly of ^{3}H), the resulting matrix equation has a very large dimension, and this is what makes it difficult (if not impossible) to find its solution to any desired accuracy. Therefore, we followed an alternative way to solve for the zeros of the Fredholm determinant. It makes use of the Fredholm alternative theorem, which states that if the homogeneous equation has a solution, the inhomogeneous equation with the same kernel has no solution, except if the inhomogeneity is orthogonal to all solutions of the adjoint homogeneous equation [10]. First we turn the homogeneous integral equation (11) artificially into an inhomogeneous one,

$$|\Psi_\alpha\rangle = |\Phi_\alpha\rangle + \tilde{G}_\alpha^C(E)[U^\alpha|\Psi_\alpha\rangle + v_\alpha^s(|\Psi_\beta\rangle + |\Psi_\gamma\rangle)], \tag{13}$$

by adding an arbitrary state $|\Phi_\alpha\rangle$, which has to fulfill the only condition that it is not orthogonal to $|\Psi\rangle$. Equation (13) may be solved by iteration, with the summation done by the Padé method [11]. Stable solution is usually reached after

TABLE V. Deuteron properties, triplet scattering lengths, and effective ranges given by different interactions.

	ARG	CD-Bonn	ARGm	IS	ISa	ISb
ε_D (MeV)	−2.224575	−2.224575	−2.224575	−2.224582	−2.224579	−2.224584
P_D (%)	5.76	4.85	5.764	3.600	5.767	4.849
Q_D (fm^2)	0.270	0.270	0.2699	0.2751	0.2707	0.2717
A_S (fm$^{-1/2}$)	0.8846	0.8850	0.8851	0.8850	0.8850	0.8850
A_D/A_S	0.0250	0.0256	0.02509	0.02697	0.02507	0.02563
r_{rms} (fm)	1.967	1.966	1.96735	1.96514	1.96557	1.96518
a_t (fm)	5.419	5.4196	5.4192	5.4190	5.4192	5.4191
r_t (fm)	1.753	1.751	1.7532	1.7531	1.7537	1.7534

TABLE VI. The pp $^1S_0$ phase shifts.

T_{lab} (MeV)	Nijmegen	ARG	CD-Bonn	ARGm	IS
1	32.684	32.68	32.79	32.745	32.768
5	54.832	54.74	54.85	54.746	54.826
10	55.219	55.09	55.20	55.069	55.159
25	48.672	48.51	48.63	48.465	48.509
50	38.899	38.78	38.85	38.736	38.688
100	24.97	25.01	24.91	24.929	24.860
150	14.75	15.00	14.73	14.891	14.907
200	6.55	6.99	6.58	6.854	6.920
250	−0.31	0.23	−0.29	0.078	0.108
300	−6.15	−5.64	−6.26	−5.816	−5.911
350	−11.13	−10.86	−11.56	−11.054	−11.335

TABLE VII. The nn $^1S_0$ phase shifts.

T_{lab} (MeV)	ARG	CD-Bonn	ARGm	IS	ISv
1	57.07	57.63	57.069	57.225	56.972
5	60.64	61.00	60.643	60.774	60.633
10	57.48	57.79	57.489	57.608	57.499
25	48.80	49.05	48.815	48.869	48.788
50	38.47	38.61	38.485	38.446	38.375
100	24.45	24.38	24.469	24.411	24.347
150	14.38	14.14	14.401	14.426	14.371
200	6.34	5.96	6.367	6.434	6.396
250	−0.42	−0.92	−0.398	−0.377	−0.390
300	−6.31	−6.90	−6.280	−6.391	−6.370
350	−11.53	−12.21	−11.505	−11.808	−11.746

TABLE VIII. The np $^1S_0$ phase shifts.

T_{lab} (MeV)	Nijmegen	ARG	CD-Bonn	ARGm	IS
1	62.068	62.02	62.09	62.032	62.078
5	63.63	63.50	63.67	63.509	63.620
10	59.96	59.78	60.01	59.782	59.939
25	50.90	50.61	50.93	50.611	50.870
50	40.54	40.09	40.45	40.088	40.522
100	26.78	26.02	26.38	26.021	26.811
150	16.94	15.98	16.32	15.984	16.941
200	8.94	8.00	8.31	7.996	8.859
250	1.96	1.28	1.59	1.283	1.832
300	−4.46	−4.54	−4.25	−4.546	−4.459
350	−10.59	−9.71	−9.44	−9.717	−10.176

TABLE IX. The $^3S_1$ phase shifts.

T_{lab} (MeV)	Nijmegen	ARG	CD-Bonn	ARGm	IS
1	147.747	147.75	147.75	147.740	147.741
5	118.178	118.18	118.18	118.168	118.168
10	102.611	102.62	102.62	102.607	102.604
25	80.63	80.68	80.63	80.662	80.641
50	62.77	62.89	62.73	62.878	62.839
100	43.23	43.51	43.06	43.502	43.398
150	30.72	31.19	30.47	31.188	30.871
200	21.22	21.94	20.95	21.937	21.253
250	13.39	14.45	13.21	14.448	13.293
300	6.60	8.13	6.65	8.124	6.446
350	0.502	2.65	0.92	2.637	0.424

TABLE X. The ε_1 mixing parameter.

T_{lab} (MeV)	Nijmegen	ARG	CD-Bonn	ARGm	IS
1	0.105	0.11	0.11	0.103	0.114
5	0.672	0.66	0.68	0.659	0.734
10	1.159	1.14	1.17	1.135	1.270
25	1.793	1.77	1.81	1.758	1.924
50	2.109	2.11	2.13	2.097	2.126
100	2.42	2.52	2.45	2.499	2.337
150	2.75	2.96	2.79	2.929	2.761
200	3.13	3.43	3.18	3.397	3.241
250	3.56	3.92	3.60	3.888	3.697
300	4.03	4.43	4.00	4.396	4.120
350	4.57	4.95	4.38	4.920	4.518

TABLE XI. The $^3D_1$ phase shifts.

T_{lab} (MeV)	Nijmegen	ARG	CD-Bonn	ARGm	IS
1	−0.005	−0.00	−0.01	−0.005	−0.005
5	−0.183	−0.17	−0.18	−0.181	−0.192
10	−0.677	−0.65	−0.68	−0.667	−0.708
25	−2.799	−2.72	−2.80	−2.752	−2.922
50	−6.433	−6.28	−6.44	−6.322	−6.672
100	−12.23	−12.04	−12.25	−12.096	−12.473
150	−16.48	−16.39	−16.50	−16.457	−16.537
200	−19.71	−19.82	−19.68	−19.891	−19.574
250	−22.21	−22.59	−22.12	−22.661	−22.011
300	−24.14	−24.83	−24.03	−24.908	−24.078
350	−25.57	−26.65	−25.53	−26.723	−25.887

TABLE XII. The ^{3}He binding energies as function of the angular momenta j of the *NN* interaction taken into account.

	$j\leqslant 4$	$j\leqslant 5$	$j\leqslant 6$	$j\leqslant 7$	$j\leqslant 8$	$j\leqslant 9$
ARGm	6.90620	6.91562	6.91670	6.91787	6.91796	6.91815
IS	7.71739	7.71805	7.71782	7.71825	7.71852	7.71855

20–40 iterations even if the long-range Coulomb interaction is present in a subsystem. We then vary the energy and search specifically for the energies E_i, where the solution diverges. According to the Fredholm alternative theorem the homogeneous equation (11) has its solutions exactly at these energies. Conversely, whenever the inhomogeneous equation has a finite solution, the energy E is not a bound-state energy.

In principle, the divergence of the wave function norm should be calculated; however, a properly chosen single element suits as well. It changes sign while going through the binding energy, and therefore a zero of its inverse can be found with a shorter search time. If the complete wave function is needed, the solution has to be found for all elements of the matrix equation at an energy which is as near to the binding energy as possible and where a stable solution can still be achieved. Of course, this is an approximate solution; nevertheless, in our experience its accuracy exceeds the accuracy of the other approximations applied.

Finally, we remark that the formalism outlined above assumes point charges. The *pp* potentials, however, were constructed for distributed proton charges, as required in a realistic calculation. In order to account for this difference in the three-body calculation, one has to add the difference between the distributed and point-charge Coulomb potentials to the short-range nuclear interaction in the *pp* subsystem.

IV. CALCULATIONS OF THE ^{3}H AND ^{3}He BOUND STATES

In this paper we present results for two types of calculations, namely, where the *NN* interaction in all partial waves is furnished by the modified Argonne potential v_{18} and where in the $^1S_0$ as well as $^3SD_1$ partial waves the IS potential is used while in all other partial waves the modified Argonne potential v_{18} remains.

The necessary number of the included partial waves of the *NN* interaction was studied in Ref. [3], and it was found that interactions up to the G waves ought to be included in order to get the 3*N* binding energy up to a four-digit accuracy (although, the tensor part of the $^3FH_4$ and $^3GI_5$ interactions were neglected). However, in Ref. [3] the solution was based on the separable approximation of the *NN* interactions, while in the present work the full form of the interactions are used. Therefore we follow here the usual cutoff of the *NN* interactions by the total angular momentum j (see, for example, Refs. [12,13]).

The numerical accuracy for the $j\leqslant 5$ and $j\leqslant 6$ cases was checked on the number of Laguerre polynomial basis functions [9] and their range parameter, and on the number of mesh points for the intermediate integrations. The same numerical setup was used for higher j values, which therefore may be less accurate. The convergence was checked for the modified Argonne and IS interactions. The results (shown in Table XII) indicate that the random changes above the $j\leqslant 7$ cases represent a numerical noise. We also checked that the inclusion of the Coulomb interaction for high partial waves where no nuclear interaction was taken into account (for $j>5$) has no influence on the first four digits of the ^{3}He binding energy.

TABLE XIII. The ^{3}H, ^{3}He binding energies ($j\leqslant 5$), and the ΔE energy shifts (however, for the Argonne potentials the $j\leqslant 6$ is used).

Interactions	$E(^3\mathrm{H})$ (MeV)	$E(^3\mathrm{He})$ (MeV)	ΔE (MeV)
ARG [22]	7.628	6.917	0.711
ARGm	7.6268	6.9167	0.7101
IS	8.4818	7.7181	0.7637
ISa	7.6972	6.9797	0.7175
ISb	8.0071	7.2711	0.7360
Experiment	8.48182	7.71806	0.76376

The energy shift between the triton and ^{3}He was calculated also with the ISa and ISb tensor forces (the $^1S_0$ interactions are those of the set IS). The different deuteron D-state probabilities of these tensor forces produce different 3*N* binding energies. All of the resulting 3*N* binding energies and energy shifts are shown in Table XIII.

V. SUMMARY AND CONCLUSIONS

The classical concept is that a potential must be a local one. Nonlocal potentials, however, cannot be excluded, especially if one considers that the basic source of the *NN* interaction is the exchange of mesons. Even the one-boson exchange mechanism gives rise to some nonlocality [14]. The composite structure of the nucleons also leads to nonlocality, the range of it is expected to be below 1.5 fm [4]. The theoretically sound Bonn potentials [15,14,8] are defined in momentum space, they are nonlocal and their nonlocality is of long range in coordinate space [16]. That we returned to coordinate space gives a possibility to explicitly control the range of nonlocality, but our constructed potentials are purely phenomenological ones in the inside region. It is to be noted that this phenomenology is present for all other *NN* potentials too.

Nowadays, for a potential model the minimal requirement to meet is the reproduction both the *NN* data and the 3*N* binding energies. The local interaction (or nonlocal in case of the Bonn potentials) plus 3*N* force model separates these requirements since the *NN* force reproduces only the *NN* data, while the strength of any 3*N* force can be chosen to reproduce at least one of the 3*N* binding energies [17]. This ambiguity would be resolved if the *NN* and 3*N* interactions were deduced from a unique theory and the fitted parameters would belong to both interactions. The presented nonlocal interaction model, although it is phenomenological, does not require a correctional 3*N* force in order to reproduce the 3*N* binding energies. The nonlocality gives some freedom to change the details of the interaction, which influence the 3*N*

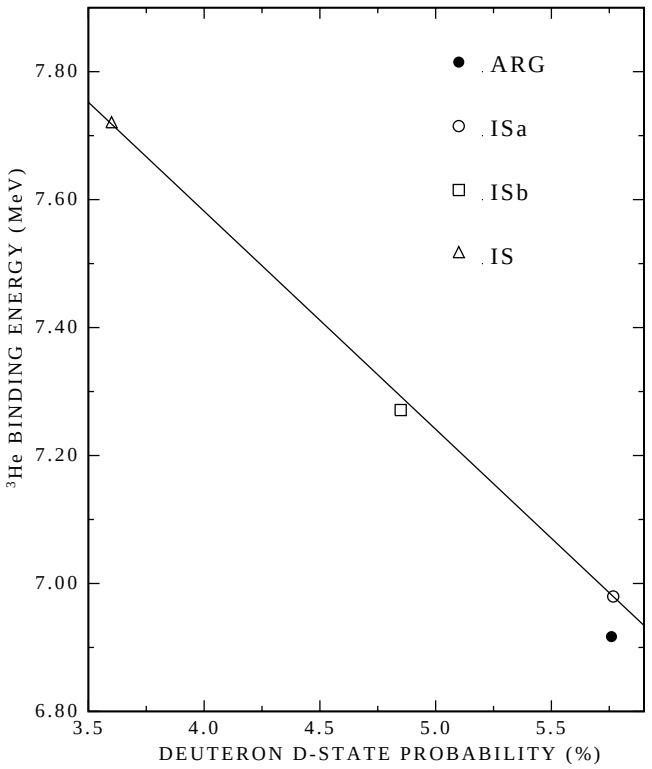

FIG. 1. The ^{3}He binding energy as a function of the deuteron *D*-state probability.

binding energy while the quality of the description of the *NN* data remains the same.

The allowed range of possible nonlocality of the *NN* interaction is questionable. In an earlier work [3] the INOY potentials had a smaller contribution of the Yukawa tail since it was cut off at a larger distance, and consequently the range of the nonlocality was larger. In the present paper we report on a successful attempt to construct an INOY interaction with a shorter range of nonlocality, which conforms to the requirements of Ref. [4].

The original and the modified Argonne potentials produce nearly the same 3*N* binding energies (Table XIII). The low-energy scattering data and the deuteron binding energy are sensitive to the electromagnetic interactions [5] and to the *n-p* mass difference, therefore in the process of modification small corrections were applied to the nuclear part of the Argonne potential v_{18} to restore its original values calculated with the inclusion of the electromagnetic interactions and *n-p* mass difference. The near identity of the 3*N* binding energies strongly indicate that the position of the $^1S_0$ virtual and the $^3SD_1$ bound state (the $^1S_0$ *nn* and *np* scattering lengths are practically determined by the virtual bound-state position) have an essential effect on the 3*N* binding energies, while other differences are not so important.

The deuteron *D*-state probability for the present IS potentials were chosen to be 3.6%. It has to be emphasized again that the choice of low deuteron *D*-state probability is essential for reproducing the 3*N* binding energies. The local potentials cannot produce both the correct phase shifts and a low (below 5%) deuteron *D*-state probability [16,18]. The earlier results [3] about the dependence of the triton binding energy and the asymptotic normalization constant A_D/A_S on the deuteron *D*-state probability is valid for the present shorter-range IS interactions too. As an example, in Fig. 1 the dependence of the ^{3}He binding energy on the deuteron *D*-state probability is shown.

The fine tuning of the interactions was performed by changing the *S*-state wave functions in the small distance region. The ^{3}He binding energy was chosen to be reproduced, because the triton depends at least on one more free parameter: on the *nn* scattering length, which is not too well known. If for the IS interaction the *nn* scattering length is chosen to be the same as that provided by the Argonne potential v_{18}, the resulting triton is underbound by about 5 keV. In order to reproduce the triton binding energy, the *nn* scattering length was chosen to be -18.601 fm (see Table IV).

However, if the CSB effect is represented by changes of other parameters of the *pp* $^1S_0$ interaction (for the *nn* $^1S_0$ interaction of the IS set, only one strength parameter is changed), the correct triton binding energy can be reproduced with a different *nn* scattering length. To illustrate it, a shorter-range nonlocal *pp* $^1S_0$ interaction with larger enhancement of the short distance wave function was fit to the *pp* data, and this parametrization was used for the ISv *nn* $^1S_0$ interaction. This ISv interaction was substituted for the *nn* $^1S_0$ interaction of the set IS. In order to reproduce the triton binding energy of 8.4818 MeV via the change of the strength parameter V_{00}^2, the scattering length of the ISv interaction has to be tuned to -18.375 fm value.

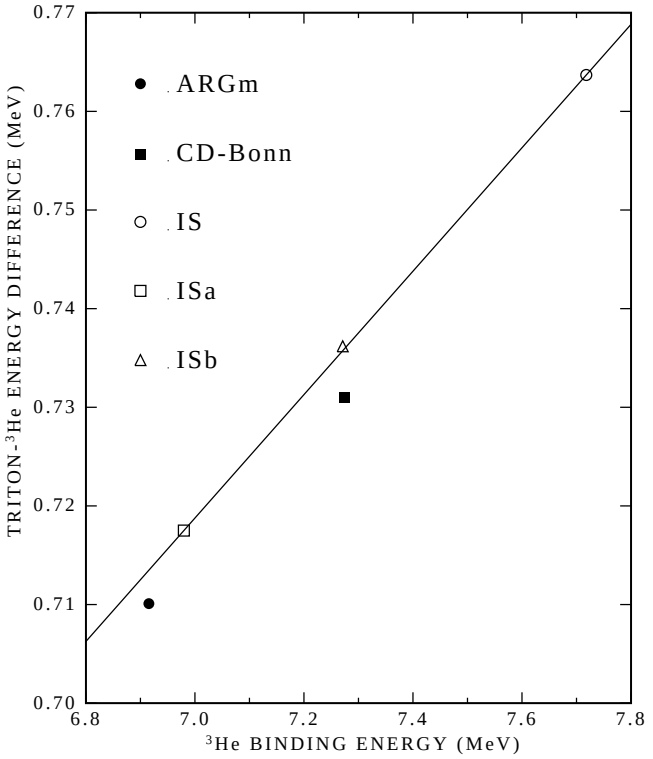

FIG. 2. The ^{3}H-^{3}He binding energy difference as a function of the ^{3}He binding energy.

We note that the effect of the nonlocality range on the 3*N* binding energies was also checked. A set of INOY interactions with the charge symmetry and charge independence breaking was constructed with a longer range used earlier [3]. The results for both the ^{3}H and ^{3}He binding energies are exactly the same as those of the IS interaction [19].

Contrary to the presented IS interaction, the Argonne-3*N* force model produces around 48 keV deficit for the triton. The CD-Bonn−3*N* force model seems to be better, because the triton is underbound only with 20 keV [17], however, the CD-Bonn potential has already a larger *nn* scattering length (-18.968 fm). In order to produce a correct triton binding energy for a 3*N* force adjusted to the ^{3}He binding energy, the *nn* scattering length has to be chosen to be -19.33 fm [21], which seems to be too large [20].

A triton calculation was performed also with the set IS, but the *nn* $^1S_0$ interaction was changed to the pure nuclear part of the *pp* interaction. The triton binding energy turned out to be 95.3 keV less, which is understandable, since the scattering length of the pure nuclear *pp* $^1S_0$ interaction of the set IS is -16.809 fm. If this CSB effect on the triton binding is removed, the Coulomb energy shift is around 0.6666 MeV (it has to be noted that the CSB effect of the higher partial-wave components of the Argonne v_{18} interaction was not removed).

The results with the IS, ISa, and ISb interactions strongly indicate that for INOY interactions the difference of the triton and ^{3}He binding energies linearly depend on the ^{3}He (or triton) binding energy (see Fig. 2). Even the result with the modified Argonne and the CD-Bonn potentials [21] is not too far from that line. Also it has to be mentioned that the 3*N* binding energies with the ISb tensor force is very near to those of CD-Bonn potential [22,21], which is an indication that these nonlocal potentials are similar.

The fact that the IS force reproduces the 3*N* binding energies proves that with nonlocal *NN* interactions this could be done without a 3*N* force. Since these nonlocal interactions are purely phenomenological ones, it could not be said that they are the real *NN* interactions, especially that different variations could be constructed (although they are rather similar if they reproduce the 3*N* binding energies). The fine effects (relativistic, electromagnetic, and *n*-*p* mass difference) were intentionally neglected, since they are small or comparable relative to the effects of the uncertainty of nuclear interaction (see, for example, the effect of the different *nn* scattering lengths).

The presented nonrelativistic nonlocal interaction IS is capable to reproduce both the 2*N* data below 350 MeV and the 3*N* binding energies with high accuracy. Besides, the shorter range IS interactions satisfy a more rigorous expectation for the range of the nonlocality. These interactions should be tested in nonrelativistic calculations of nuclear systems with higher number of nucleons (e.g., in calculating the ^{4}He binding energy).

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