Screening effects on $^1S_0$ pairing in neutron matter

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The $^1S_0$ superfluidity of neutron matter is studied in the framework of the generalized Gorkov equation. Vertex corrections to the pairing interaction and the self-energy corrections are introduced and approximated on the same footing in the gap equation. A suppression of the pairing gap by more than 50% with respect to the BCS prediction is found, which deeply changes the scenario for the dynamical and thermal evolution of neutron stars.

Neutron superfluidity in neutron-star matter, though an old subject, is still of great actuality and vividly debated (for a review, see Ref. [1]). The reason for this stems from the fact that superfluidity is an extraordinarily subtle process when it comes to quantitative predictions starting from the bare nucleon-nucleon (NN) interaction. On the other hand, for neutron stars, such quantitative predictions are necessary, since the manifestations of the superfluidity are only rather indirect through glitches and relaxation phenomena and cooling rates. One, therefore, lacks direct experimental information on the magnitude of neutron pairing. However, there is no doubt that the dynamics and the thermodynamics of neutron stars are strongly influenced by the superfluid character of neutron matter and it is therefore important to get pairing properties of neutron-star matter under better control from the microscopic point of view. Of course, neutron matter superfluidity is not completely decoupled from the one prevailing in finite nuclei. Though we have experimental information for these objects, also in this case the fully microscopic explanation of the observed phenomena is far from being completely settled. It may be argued that in finite microscopic systems, where the surface plays a very important role, the situation can be quite different from the homogenous case. However, like for other quantities of nuclear physics, it should be possible to disentangle volume and surface effects also for pairing properties in finite nuclei and it is therefore our belief that the topic of superfluidity in neutron matter, nuclear matter, and finite nuclei should be studied in an interrelated way.

In this work we again concentrate on neutron matter in pursuing previous studies. However, it is planned to parallel this work for nuclear matter in the near future. In the past we have mainly been concentrating on the influence of either dynamic self-energy corrections [2] (see also Ref. [3]) or vertex corrections to the neutron matter pairing problem [4]. All investigations in this direction invariably led to the conclusion that dynamic self-energy corrections yield a quite strong reduction of $^1S_0$ pairing in neutron matter. However, to be consistent, self-energy corrections have to be followed by vertex corrections on the same footing. This is the objec-

tive of the present investigation. It will be seen that the vertex corrections have a tendency to further reduce the gap, but to a lesser extent than in the case of self-energy corrections. Our approach is based on the Gorkov Green's function formalism where we develop systematically self-energy and effective pairing interaction to lowest order in the particle-hole bubble insertion. Though intuitively quite reasonable and physically motivated, we should mention that this is neither based on an expansion in a small parameter nor does it rely on some variational principle. Nonetheless, our results will be quite comparable to those of other works, notably to those of Clark *et al.* [5] who based their investigation on the correlated basis function approach which, to a large extent, is variational. In spite of the similarity between our results and the results of Ref. [5], we think that disentangling the separate contributions from self-energy and vertex corrections yields rich additional physical insight into the complex phenomenon we are going to study.

The superfluid phase of a homogenous system of fermions is described by the pairing field $\Delta_k(\omega)$, which is the solution of the generalized gap equation

$$\Delta_{\vec{k}}(\omega) = \sum_{\vec{k}'} \int \frac{d\omega'}{2\pi i} \mathcal{V}_{\vec{k},\vec{k}'}(\omega,\omega') F_{\vec{k}'}(\omega'). \tag{1}$$

Here $\mathcal{V}$ is the sum of all irreducible NN interaction terms and $F_k(\omega)$ the anomalous propagator [6–8]. In most pairing calculations [1] the effects of the medium polarization have been included in the self-energy [2,3], and not in the pairing potential. A more general study requires the medium corrections to be treated on an equal footing also in the vertex corrections as well as in the self-energy. This is the main concern of the present paper. Accordingly, the expansion of the interaction block $\mathcal{V}$ and the self-energy Σ have both been truncated to second order of the interaction. The corresponding diagrams are depicted in Fig. 1.

The limitation to lowest order bubble insertion may seem as a strong restriction because resummation of the bubble series into random-phase approximation (RPA) can have an important influence. However, in the exploratory work where
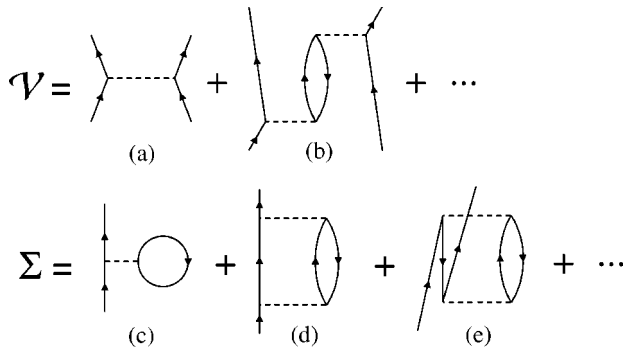
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FIG. 1. The diagrams of *NN* interaction and self-energy discussed in the text. The exchange terms are understood.

FIG. 2. (a) Rearrangement contributions to the self-energy, where k is fixed to k_F. (b) The HF mean field is plotted vs momentum k at $k_F = 0.5$, 0.8, and 1.1 fm^{-1}.

self-energy and vertex corrections are treated consistently for the first time, we think that higher order effects unnecessarily complicate the approach and that the lowest order effects in the density should at least give the correct tendency even up to densities around saturation. This opinion is corroborated by our experience in this matter. For example, in Ref. [9] the nucleon optical potential was calculated for finite nuclei via the local density approximation (LDA) using the graphs of Fig. 1 for Σ and obtained, with the Gogny force for the vertices, quite reasonable agreement with empirically determined expressions. Furthermore, in Ref. [10] the incoherent two particle–two hole contribution to the spreading width of giant resonances in nuclei was evaluated in LDA. This gives an important fraction of the giant resonance widths and additional RPA correlations did not reverse this tendency. We therefore believe that our lowest order approach will also qualitatively grasp the situation of the present problem with quantitative corrections concerning higher order terms.

The matrix elements of the bare interaction V exhibit hard-core divergences that have to be removed by dressing them with the short-range particle-particle correlations (ladder diagrams). This can be done either in a microscopic approach by replacing V with the G-matrix or in a semiphenomenological approach by replacing V with an effective interaction.

In the calculations of all diagrams, the superfluid propagators have been replaced by the normal phase propagators. This amounts to neglecting second-order corrections in the gap, which are actually negligible as shown in a fully self-consistent calculation discussed below.

In general, the interaction as well as the self-energy are complex quantities, which implies the energy gap to be a complex function. The introduction of the imaginary part of the self-energy amounts to taking into account the effects of the quasiparticle spectral function, which have been studied elsewhere [11,12]. The complex nature of the potential is due to finite time propagation and decay of processes shown in Fig. 1. However, in this investigation $\mathcal{V}$ will be assumed to be a real function, which implies the gap function to be real also.

For the present calculation we adopted the semiphenomenological approach and chose the Gogny force D1 as effective interaction at each coupling vertex shown in Fig. 1. The replacement of the vertices in Fig. 1 by the Gogny force,
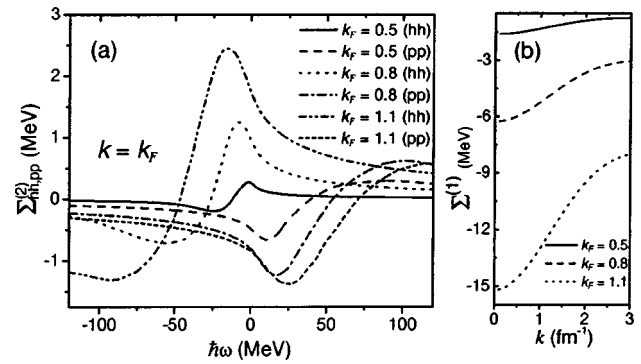
which can be considered as a phenomenological representation of a G matrix, may seem unjustified for the lowest order term in Fig. 1(a), since perturbation theory tells us that it should be the bare *NN* interaction. However, it is well known [13] that limiting the k space, the bare interaction has to replaced by an effective one and choosing the cutoff to be at k_F the effective force in Fig. 1(a) can be shown to be again equivalent to the G matrix. In neutron matter only the density independent part of the Gogny force survives with a range in k space of about k_F at the saturation density. Therefore, from this point of view, it may not be unreasonable to take the Gogny force even for the lowest order term in Fig. 1(a), which is further backed by the fact that the gap with the Gogny force (without polarization terms) is quite close to the one calculated from a bare *NN* force [14]. Also there exists now an exact low momentum mapping of the bare *NN* force by Kuo *et al.* [15], called $V_{\text{low-}k}$. From such a calculation it can indeed be seen that $V_{\text{low-}k}$ is very close to the Gogny force, except for k-vertices close to saturation where $V_{\text{low-}k}$ is more repulsive [16]. In any case, the qualitative influence of the polarization on the lowest order solution of the gap equation, given by retaining only graphs Figs. 1(a) and 1(c), should not depend very much on whether we use in Fig. 1(a) the bare interaction or the Gogny force.

The self-energy $\Sigma_k(\omega)$ in neutron matter has been calculated within the above described approximation. The first-order contribution is reported in Fig. 2(b) for three values of the nuclear matter density in the range where the pairing is expected to be largest. The second-order contributions $\Sigma_{pp}^{(2)}$ and $\Sigma_{hh}^{(2)}$ [graphs (d) and (e) in Fig. 1, respectively] are depicted in Fig. 2(a) as a function of the energy. Only the energy dependence of these terms will be discussed, since the Gogny force implicitly contains the static part, which will be removed from the gap equation, as described later.

$\Sigma_{hh}^{(2)}$ exhibits a pronounced maximum in the vicinity of the Fermi energy due to the high probability amplitude for particle-hole excitations near ε_F. It is in very good agreement with the results obtained from Brueckner-Hartree-Fock calculation with G matrix [2], justifying, *a posteriori*, the use of the Gogny force.

The second-order potential is given by the one-bubble exchange term, plotted in Fig. 1(b), which is the first one of the ring diagram series. Physically it represents the screening to
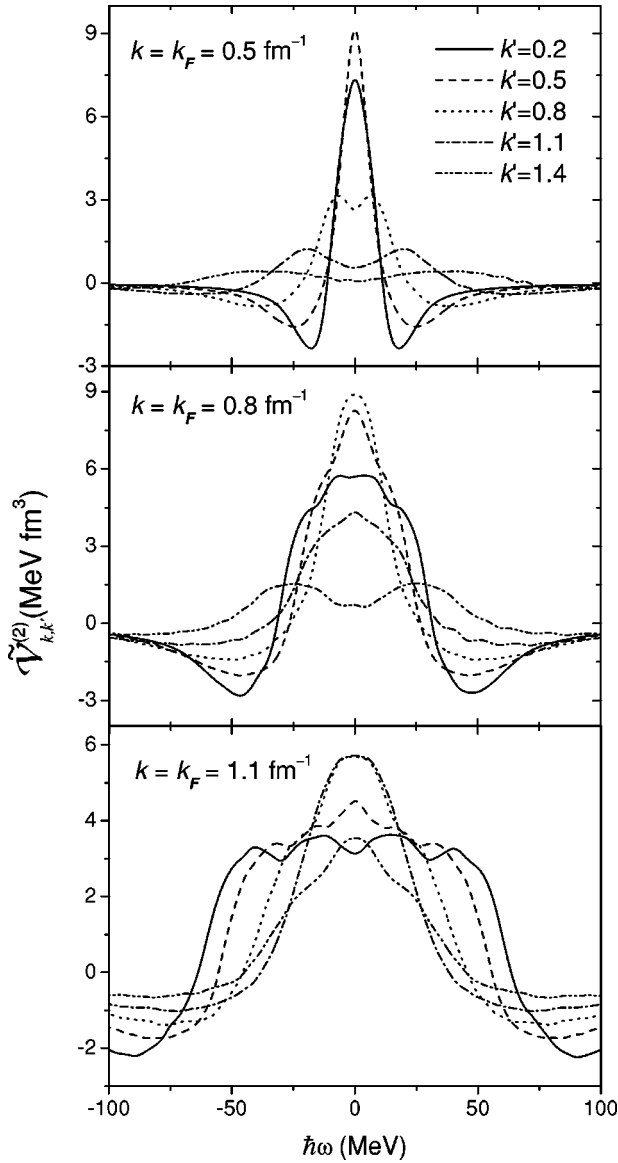
FIG. 3. Screening potential vs energy at $k_F = 0.5$, 0.8, and 1.1 fm^{-1}, separately.

the pairing due to the medium polarization. Again we take the Gogny force for all vertices in Figs. 1(a),(b). Our prediction for $\mathcal{V}^{(2)}_{kk'}(\omega)$ at three typical densities is reported in Fig. 3. We plot the symmetric part

$$\tilde{\mathcal{V}}^{(2)}_{k,k'}(\omega,\omega'_k) \equiv \frac{1}{2(2\pi)^3}[\mathcal{V}(\omega,\omega'_k) + \mathcal{V}(\omega,-\omega'_k)], \quad (2)$$

which is the only one relevant for the pairing gap. The strength of $\tilde{\mathcal{V}}$ is concentrated around the Fermi energy ($\omega = 0$) with a peak value at $k = k' = k_F$ and a width increasing with the density. Its ω dependence is shaped by the polarization part, i.e., Lindhard functions [17], which at $\omega = 0$ (static limit) is repulsive at any momentum and density, but it becomes attractive for $|\omega| \gg \varepsilon_F$. One therefore expects a reduction of the gap due to screening.

The gap equation, Eq. (1), is to be coupled to the closure equation for the Green's function

$$\rho = \sum_k \int \frac{d\omega}{2\pi i} e^{i\omega 0^+} G_k(\omega), \quad (3)$$

fixing the chemical potential in the superfluid phase.

The anomalous propagator

$$F_k(\omega) = \frac{\Delta(\omega)}{[\hbar\omega - \varepsilon_k(\omega)][\hbar\omega + \varepsilon_k(-\omega)]} \quad (4)$$

has two poles, symmetric with respect to the imaginary axis of the complex ω plane [3], which are the roots of the equation

$$\pm\omega_k = \varepsilon_k(\pm\omega_k). \quad (5)$$

The quasiparticle energy ε_k is given by

$$\varepsilon_k(\omega) = \Sigma_{\vec{k}}^-(\omega) + \sqrt{[\varepsilon_k^0 + \Sigma_{\vec{k}}^+(\omega)]^2 + \Delta_{\vec{k}}^2(\omega)}, \quad (6)$$

where

$$\Sigma_{\vec{k}}^{\pm}(\omega) = \frac{1}{2}[\Sigma_{\vec{k}}(\omega) \pm \Sigma_{\vec{k}}(-\omega)]. \quad (7)$$

The two poles are located close to the real axis on opposite sides of the imaginary axis. Leaving aside a general integration of the gap equation, we adopt the pole approximation, relying on replacing the full propagator by its pole part:

$$F_k(\omega) = \frac{Z_k\Delta_k(\omega)}{\varepsilon_k(\omega) + \varepsilon_k(-\omega)}\left[\frac{1}{\omega - \omega_k + i\eta} - \frac{1}{\omega + \omega_k - i\eta}\right]. \quad (8)$$

In general, the residue Z_k at the poles is defined as

$$Z_k = \left(1 - \frac{\partial\varepsilon_k(\omega)}{\partial\omega}\bigg|_{\omega = \omega_k}\right)^{-1}, \quad (9)$$

calculated at the Fermi surface. In the calculations we took the limit of Z_k for $\Delta \to 0$, which corresponds to the quasiparticle strength. The factor Z is keeping the full dynamical dependence of the self-energy reported in Fig. 2. Afterwards we may subtract the static part $\Sigma^{(2)}(\omega = 0)$ from the self-energy. In the calculation of the gap the inclusion of the latter only brings a variation of less than 1% since the gap is not sensitive to the static self-energy far from ϵ_F. Inserting Eq. (8) for the anomalous propagator into the gap equation, after ω integration we obtain

$$\Delta_k(\omega) = -\frac{1}{2}\int k'^2 dk' \tilde{\mathcal{V}}_{k,k'}(\omega - \omega_{k'})\frac{Z_{k'}\Delta_{k'}(\omega_{k'})}{\varepsilon_k(\omega_{k'}) + \varepsilon_k(-\omega_{k'})}. \quad (10)$$

Notice that the reason why only the ω-even part of the interaction contributes to the integral can be traced to time reversal invariance of the superfluid ground state for which the anomalous propagator as well as the gap function are even functions of ω. The remarkable advantage of this ap-
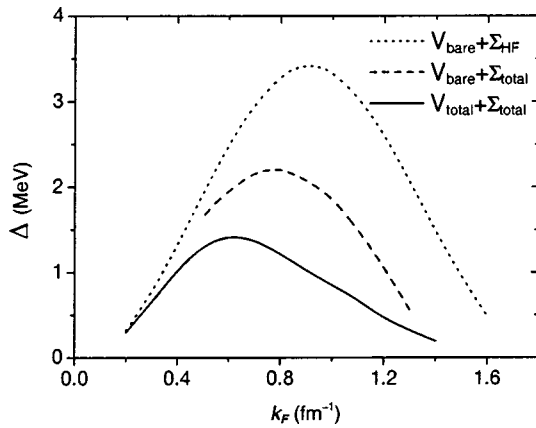
FIG. 4. Energy gap in the present approximation. For comparison the predictions from the pure BCS model (dotted line) and from BCS plus self-energy effects (dashed line) are plotted.

FIG. 5. Energy gap as a function of ω at $k_F = 0.6$ fm^{-1}.

proximation is that the gap depends only parametrically on ω and its energy dependence is only related to the energy dependence of the interaction. The on-shell gap $\Delta_k(\omega_k)$ fulfills the equation

$$\Delta_k(\omega_k) = -\frac{1}{2}\int k'^2 dk' \, \tilde{\mathcal{V}}^S_{k,k'}(\omega_k - \omega_{k'}) \frac{Z_{k'}\Delta_{k'}(\omega_{k'})}{\varepsilon_k(\omega_{k'}) + \varepsilon_k(-\omega_{k'})},$$

(11)

equivalent to the gap equation in the static limit.

The approximate version of the gap equation, Eq. (10), has been solved using as input the self-energy and pairing potential discussed in the preceding section. The focus was on the $^1S_0$ neutron-neutron pairing for neutron matter, which is by far the most important component of pairing as to possible implications in nuclear systems. The energy gap at the Fermi surface ($k = k_F$ and $\omega = 0$) as a function of k_F is reported in Fig. 4. The domain of existence of the superfluid state is mainly at low densities with a peak value of about 1.4 MeV at $k_F = 0.6$ fm^{-1}. In the same figure we also report the result in the BCS limit (for a review see Ref. [1]) (neither self-energy effects nor screening) and the result with self-energy effects but without screening [2]. From the comparison of the three predictions one sees that the main suppression of Δ is due to the strong ground-state correlations that lead to a Z factor much less than unity. However, screening of the pairing interaction produces an additional suppression. It has to be noticed that the screening potential also shifts the peak value of the gap to lower density, where the suppression is less sizable. As final result, obtained solving Eq. (10), we report in Fig. 5 the gap as a function of the energy for $k_F = 0.6$ fm^{-1}. We point out here the relevance of such a feature for the study of pair correlations in dynamical processes such as the expanding and disassembling phases of the fragmentation events in heavy-ion collisions [18].

Screening effects on the pairing interaction have also been studied in different contexts. One of the earliest calculations has been performed in the framework of the second-order correlated basis perturbation theory [5], where a pairing suppression by a factor of 4 is predicted. In that work the mo-
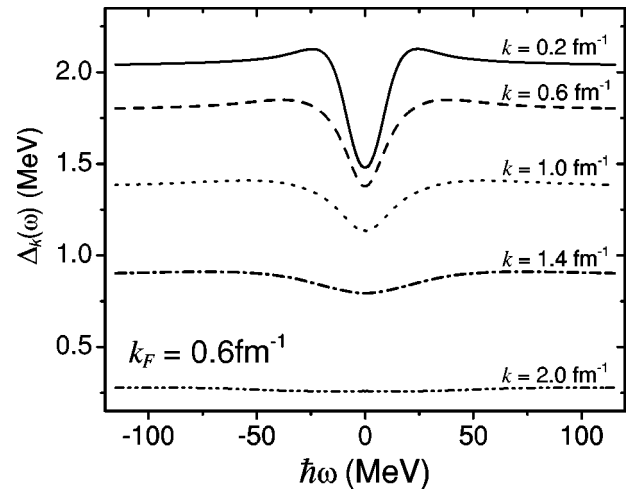
mentum dependence is mostly neglected, which amounts to overestimating the suppression. Closer to the present approach is the polarization potential [19,4] calculated from the induced interaction theory [20], which gives substantially the same gap when treated within the Landau parameter approximation [13]. Finally, we should mention a parallel study on finite nuclei [21], where the polarization potential is given by the coupling to surface vibrations. While the self-energy plays the same role as in neutron matter, the phonon exchange produces an enhancement of the pairing at variance with the dominant repulsive effect of the spin fluctuations in neutron matter [22].

In conclusion, this work constitutes a continuation of a previous one (Ref. [2]) where we investigated self-energy effects on the pairing gap in infinite neutron matter. Here we treated consistently the additional inclusion of vertex corrections. On the same footing we considered the lowest order particle-hole polarization bubble both in the self-energy and in the screening of the pairing force in great detail. In a different language one may say that we investigated the coupling of the pairing mode to incoherent four quasiparticle states. We expect that this approximation yields at least the qualitative trend, whereas collectivity in the four quasiparticle channel may change things quantitatively. Instead of the G matrix we used the phenomenological Gogny force at all coupling vertices. We verified that this replacement has only very little influence on the numerical results. The screened pairing interaction is, in principle, energy dependent but in the quasiparticle approximation used here, this dependence on energy becomes only a parametrical one which greatly facilitates the numerical task of solving the gap equation. The outcome of the inclusion of vertex corrections is that the gap as a function of k_F maintains approximately its bell-shaped form, but with respect to the self-energy corrections only, a further substantial reduction of the gap is induced by screening the bare NN interaction in the gap equation. Therefore, with respect to the lowest order approach, i.e., without any polarization effects, that is, with bare interaction and k mass only, the gap at its maximum is now reduced by about 50%. This strong reduction is a common feature of all pre-

vious calculations and in this sense our investigation is a confirmation of what has been found by other authors earlier, even though the approaches differ in detail. Nonetheless, this strong reduction of the pairing due to the polarization remains intriguing. If the same situation should prevail in nuclear matter, an estimate via the local density approximation [8] would lead to a by far too low value of the gap in finite nuclei. However, the influence of polarization terms in nuclear matter, due to the different quantum numbers involved, may be quite different from the one in neutron matter. It may be a very interesting problem for studies in the near future to see whether the polarization corrections in the different channels, i.e., n-n pairing in neutron matter, n-n pairing in nuclear matter, and n-p pairing in nuclear matter, can, at least qualitatively, explain the experimental finding from finite nuclei.

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