Is e^+e^- pair emission important in the determination of the ^{3}He+^{4}He S factor?

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We show that the cross section for direct $E0$ pair emission is related to the cross section for direct $E2$ photon emission, and is a negligible contribution to the total capture cross section for ^{3}He+^{4}He$\rightarrow^7$Be. $E0$ resonance emission, $E1$ pair emission, and internal conversion are also negligible. Thus there cannot be significant contributions to the ^{3}He+^{4}He$\rightarrow^7$Be capture cross section at low energies from electromagnetic emission processes other than single photon emission.

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There have been a number of measurements of the ^{3}He $+^4$He $\rightarrow^7$Be capture cross section at low energies, both by detection of the capture γ rays and by detection of the residual ^{7}Be activity. The presently recommended value of the astrophysical S factor for this reaction is $S_{34}(0)=0.53 \pm 0.05$ keV b [1]. The relatively large uncertainty in $S_{34}(0)$ stems from an apparent difference in the results of the two types of experiments, with an average of the activation experiments yielding a value for $S_{34}(0)$ which is $\sim$13% larger than the value determined from the capture γ-ray experiments [1]. While the statistics in these comparisons are suggestive, but not compelling, this apparent difference has led to the question of whether there might be some other capture reaction mechanism that could explain it, such as $E0$ pair emission [1]. At low bombarding energies, the capture reaction takes place at large radial distances, and hence processes such as $E0$ pair emission should be enhanced.

The uncertainty in $S_{34}(0)$ is the dominant error in solar model calculations of the solar neutrino production rate from ^{7}Be decay in the Sun [2]. It is also one of the largest single error contributions to the calculated ν_e production rate from ^{8}B decay [2], especially now that the uncertainty on $S_{17}(0)$, the astrophysical S factor for the ^{7}Be$(p,\gamma)^8$B reaction, is being reduced to better than $\pm 5\%$ (see Ref. [3]). A significant reduction in the uncertainty on $S_{34}(0)$ would have important consequences for solar model physics and for neutrino astrophysics. In addition, the ^{3}He+^{4}He$\rightarrow^7$Be reaction is important for big bang nucleosynthesis, since essentially all the ^{7}Li produced in the big bang comes from this reaction.

In this report we relate the cross section for direct $E0$ emission to the cross section for direct $E2$ γ-ray emission. This may be done since the operators (in nucleon coordinates) for $E0$ and $E2$ emission, $O_{E0}=\Sigma_i(e_i/e)r_i^2$ and $O_{E2}=\Sigma_i(e_i/e)r_i^2Y_{2m}(\Omega_i)$, have the same radial dependence in the long wavelength limit. As a result, $E0$ direct capture amplitudes for specific partial wave transitions may be numerically related to the corresponding $E2$ direct capture amplitudes. At low bombarding energies, this leads to a simple relation between the cross sections for $E0$ pair emission and $E2$ photon emission.

We begin by comparing $E0$ and $E2$ emissions for transitions between individual levels of finite spin. The width for $E0$ pair emission from an isolated level in a low-Z nucleus [4] (here we generalize the results of Ref. [4] to states of finite spin), and the width for $E2$ γ emission [5] are given by

$$\Gamma_{EL}=f_{EL}(E)B(EL), \tag{1}$$

where

$$f_{E0}(E)=\frac{e^4}{108\pi(\hbar c)^6}b(S)(E-2mc^2)^3(E+2mc^2)^2, \tag{2}$$

$$f_{E2}(E)=\frac{4\pi e^2}{75(\hbar c)^5}E^5, \tag{3}$$

and $B(EL)$ is the reduced transition rate [5]. Here E is the transition energy, $b(S)=(3\pi/8)(1-S/4-S^2/8+S^3/16-S^4/64+5S^5/512)$, and $S=(E-2mc^2)/(E+2mc^2)$.

For states with $J_i=J_f\geqslant 1$ and $\pi_i=\pi_f$, both $E0$ and $E2$ transitions are allowed. Although, in general, there is no simple relation between $B(E0)$ and $B(E2)$ for the same initial and final states, there is a relation for pure single-particle transitions, for which [5]

$$B(E2)_{s.p.}=\frac{5(2j_f+1)(2l_f+1)(2l_i+1)}{4\pi}\times\begin{pmatrix} l_f & 2 & l_i \\ 0 & 0 & 0\end{pmatrix}^2\begin{Bmatrix} l_f & j_f & 1/2 \\ j_i & l_i & 2\end{Bmatrix}^2|R_{2fi}|^2, \tag{4}$$

where R_{2fi} is the radial matrix element of r^2 between initial and final states. In order that $B(E0)$ be nonzero, the single-particle transition must have $j_f=j_i$ and $l_f=l_i$, in which case

$$B(E0)_{\text{s.p.}}=|R_{2fi}|^2. \tag{5}$$

As an example, consider an $E=2$ MeV $p_{3/2}\rightarrow p_{3/2}$ single-particle transition (for example, an $n\rightarrow n-1$ transition, where n is the principal quantum number). Here $B(E2)=|R_{2fi}|^2/4\pi$ and $\Gamma_{E0}/\Gamma_{E2}=4\pi f_{E0}(2\text{ MeV})/f_{E2}(2\text{ MeV})=4\times10^{-4}$.

The magnitude of the $E0/E2$ width ratio in the above example is easy to understand. The common operator dependence on r^2 indicates that both $E0$ and $E2$ photon emissions have the same degree of forbiddenness. In the $E0$ case the photon is virtual, connecting to a second electromagnetic vertex where it internally converts into an e^+e^- pair. Thus

Γ_{E0} contains an additional power of the fine structure constant, and this, together with the phase space ratio $(E-2mc^2)^3(E+2mc^2)^2/E^5$, accounts for most of the inhibition of $E0$ pair emission relative to $E2$ photon emission.

At low energies, there are no levels in ^{7}Be, and hence no resonances in ^{3}He+^{4}He$\rightarrow^7$Be capture. There are also no near subthreshold states, and as a result, direct capture dominates [6]. Hence $E0$ pair emission, if it is important, should also occur predominantly by a direct capture process.

The direct capture cross section in the channel spin coupling scheme for $E0$ pair emission and for $E2$ γ-ray emission may be written following Christy and Duck [7], and guided by the decay width relations given above:

$$\sigma_{EL}(E)=\frac{f_{EL}(E)}{\hbar v}\left(\frac{e_{eff}(EL)}{e}\right)^2\sum_{l_i,I} a_I^2 \times\frac{4\pi}{(2S_1+1)(2J_2+1)}\sum_{J_i}(2J_i+1)B(EL), \tag{6}$$

where

$$B(E2)=\frac{5(2J_f+1)(2l_f+1)(2l_i+1)}{4\pi}\times\begin{pmatrix} l_f & 2 & l_i\\ 0 & 0 & 0\end{pmatrix}^2\begin{Bmatrix} l_f & J_f & I\\ J_i & l_i & 2\end{Bmatrix}^2|R_{2fi}|^2, \tag{7}$$

and $B(E0)=|R_{2fi}|^2$ when $J_i=J_f$ and $l_i=l_f$ and zero otherwise. Here J_i is the total angular momentum in the entrance channel, J_2 is the total angular momentum of the target nucleus, S_1 is the spin of the projectile, J_f is the total angular momentum of the final state, a_I^2 is the final-state parentage coefficient for channel spin I, and here R_{2fi} is the radial transition matrix element of r^2 as defined in Ref. [7].

The usual recoil effective charge factors for $L\geqslant 1$ are

$$\left(\frac{e_{eff}(EL)}{e}\right)^2=\mu^{2L}\left(\frac{Z_1}{M_1^L}+(-1)^L\frac{Z_2}{M_2^L}\right)^2, \tag{8}$$

where $\mu=M_1M_2/(M_1+M_2)$ is the reduced mass of the entrance channel. This L dependence arises from the r^L dependence of the operator; hence $e_{eff}(E0)$ and $e_{eff}(E2)$ are equal and are given by the above formula with $L=2$.

The equations for $\sigma_{EL}(E)$ simplify when either the target or projectile has zero spin, in which case (taking $J_2=0$) $J_i=j_i, J_f=j_f$, and only one a_I^2 contributes. In the case of ^{3}He+^{4}He$\rightarrow^7$Be, $I=S_1=1/2$. A further simplification occurs at low energy, when the nuclear phase shift depends only on l_i and not on J_i or j_i [7]. Then the sum over j_i leads to

$$\sigma_{E2}(E)=\frac{f_{E2}(E)}{\hbar v}\left(\frac{e_{eff}(E2)}{e}\right)^2 a_{1/2}^2\frac{5(2j_f+1)}{2}\times\sum_{l_i}(2l_i+1)\begin{pmatrix} l_f & 2 & l_i\\ 0 & 0 & 0\end{pmatrix}^2|R_{2fi}|^2. \tag{9}$$

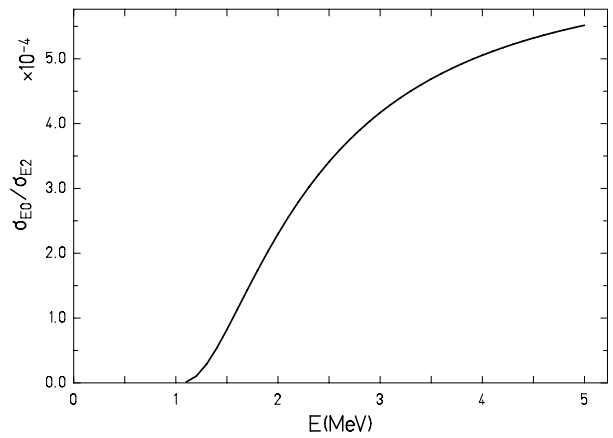

FIG. 1. Cross section ratio $\sigma_{E0}(E)/\sigma_{E2}(E)$ for p-wave direct capture ($l_i=l_f=1$) in ^{3}He+^{4}He$\rightarrow^7$Be, versus transition energy E, as given in Eq. (11).

The $E0$ cross section is nonzero only when $l_i=l_f$, in which case only $j_i=j_f$ contributes, and

$$\sigma_{E0}(E)=\frac{2\pi f_{E0}(E)}{\hbar v}\left(\frac{e_{eff}(E0)}{e}\right)^2 a_{1/2}^2(2j_f+1)|R_{2fi}|^2. \tag{10}$$

For ^{3}He+^{4}He$\rightarrow^7$Be at low energies, and $l_i=l_f=1$, the $E0/E2$ cross section ratio reduces to

$$\frac{\sigma_{E0}(E)}{\sigma_{E2}(E)}=\frac{2\pi f_{E0}(E)}{f_{E2}(E)}. \tag{11}$$

It is interesting to note that this cross section ratio is independent of j_f. It is also numerically equal to one half the width ratio in the example discussed above.

Over a wide energy range in ^{3}He+^{4}He, the $p_{1/2}$ and $p_{3/2}$ phase shifts are within 10° or so of each other [8]. For $E_{\text{c.m.}}<1$ MeV, $E2$ capture is primarily p wave ($l_i=l_f=1$) [9]. Here $E=E_{\text{c.m.}}+Q$, where $Q=1.59$ MeV for the transition to the $3/2^-$ ground state and 1.16 MeV for the transition to the $1/2^-$ first excited state. As can be seen in Fig. 1, the $E0/E2$ cross section ratio is small, less than 0.1%.

We may estimate the fractional contribution of $E0$ pair emission to the predominantly $E1$ total capture cross section if we know the fractional contribution of p-wave $E2$ capture. From Fig. 7 of Ref. [9] the p-wave $E2$ contribution can be estimated for $E_{\text{c.m.}}\sim 0.1$ to 2 MeV, for the sum of transitions to the ground state and first excited state. The fractional p-wave $E2$ contribution rises from $\sim$0.15% at $E_{\text{c.m.}}\sim 0.1$ MeV to $\sim$0.27% at $E_{\text{c.m.}}\sim 1.2$ MeV and is roughly constant at higher energies. Taking a weighted average of Eq. (11) over the ground state and first-excited-state transitions, using the experimental branching ratio of 0.4 [11] for the first-excited-state transition relative to the ground state transition, and multiplying by the p-wave $E2$ fraction yields the fractional contribution of $E0$ pair emission to the total capture cross section shown in Fig. 2 [10]. Thus direct $E0$ pair emission makes a negligible contribution to the total capture cross section for ^{3}He+^{4}He$\rightarrow^7$Be.

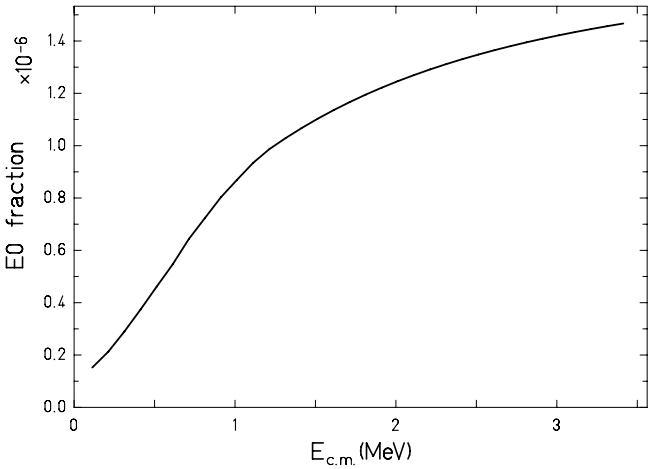

FIG. 2. Fractional contribution of direct $E0$ pair emission to the total capture cross section for $^3\mathrm{He}+^4\mathrm{He}\rightarrow{}^7\mathrm{Be}$, versus $E_{\mathrm{c.m.}}$.

What about resonant $E0$ pair emission? There are no known levels in ^{7}Be that could contribute to this process at low energy. However, even if we assume a broad ($\Gamma \sim 1$ MeV), previously undetected $E0$ resonance with a single-particle $E0$ strength [4], its cross section would be small compared to the direct $E0$ cross section estimated above. $E0$ emission may also occur by internal conversion; however, in $^3\mathrm{He}+^4\mathrm{He}$ this is also unimportant [12] as it is less probable than $E0$ pair emission except close to threshold. Collective (giant resonance) $E0$ strength is expected to lie at much higher energies. In fact, our estimated direct $E0$ cross section for $E \leqslant 4$ MeV exhausts $\sim 8\%$ of the $E0$ sum rule [13], and thus it is not possible for any broad $E0$ emission process to contribute significantly to the total cross section for $^3\mathrm{He}+^4\mathrm{He}\rightarrow{}^7\mathrm{Be}$.

In general, all electromagnetic multipole transitions may occur by pair emission. An $E0$ transition is distinguished from other multipole transitions by the absence of single photon emission rather than the presence of pair emission. Pair emission is a weak function of multipole order, and pair emission coefficients [14] (probability of pair emission divided by the probability of single photon emission) are largest for $E1$ emission. The $E1$ pair emission coefficient rises with energy and is roughly 0.13% at $E=3$ MeV for $Z=4$ [14]. Thus, e^+e^- pair emission of any multipolarity is negligible in $^3\mathrm{He}+^4\mathrm{He}$ radiative capture.

In conclusion, we have shown that the direct capture cross section for $E0$ pair emission is related to the direct capture cross section for $E2$ photon emission, and this relation is especially simple at low bombarding energies. In the case of $^3\mathrm{He}+^4\mathrm{He}\rightarrow{}^7\mathrm{Be}$, we find that the direct $E0$ pair emission cross section is of order 10^{-6} or less of the total capture cross section at low energies, and hence is unimportant in the determination of $S_{34}(0)$. Resonant $E0$ pair emission, pair emission of other multipolarity, and internal conversion must all be negligible as well. Thus, all electromagnetic processes other than direct single photon emission are unimportant in the determination of $S_{34}(0)$.

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