Momentum-space Faddeev calculations for confining potentials

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(Received 19 December 2002; published 8 May 2003)

A momentum-space method to solve the Faddeev equations with confining potentials is presented. As an application of this method, we estimate the effect of relativistic effects in the nonstrange baryon spectrum by solving the nonrelativistic Faddeev equations for three quarks as well as a relativistic version of these equations which incorporates relativistic kinematics.

DOI: 10.1103/PhysRevC.67.055203 PACS number(s): 24.85.+p, 12.39.Jh, 14.20.Dh, 14.20.Gk

I. INTRODUCTION

The baryon spectrum is supposed to be generated by the ground and excited states of three quarks interacting through a confining potential plus additional two- and three-body short-range forces [1–4]. Therefore, for a given model of the interaction between quarks, one has to deal with a three-body problem which in principle can be solved by means of Faddeev equations. There are several reasons why one would like to perform Faddeev calculations in momentum space, as we will discuss shortly. The main difficulty, however, in performing three-quark calculations in momentum space is the confining potential that goes to infinity when $r\to\infty$ so that the Fourier transform of the potential does not exist. This has led to the result that all existing Faddeev calculations of the baryon spectrum have been performed in configuration space [3–6]. In the constituent quark model, however, the mass of the u and d quarks is about one-third of the mass of the nucleon which is of the same size or even smaller than the excitations appearing in the spectrum so that relativistic effect is not expected to be negligible. This means that for the calculation of the spectrum, one would like to use a relativistic generalization of the Faddeev equations [7–15], which in turn requires that one works in momentum space.

One usually calls relativistic Faddeev equations to those sets of equations for the three-body problem, which starting from a fully relativistic four-dimensional theory are reduced by some approximation to three-dimensional form. Thus, these equations have the same degree of complexity as the nonrelativistic Faddeev equations. The first attempts to include relativity into the Faddeev formalism [7–10] were based on the method introduced by Blankenbecler and Sugar [8]. In this approach, a four-dimensional version of the Faddeev equations is obtained by summing ladder-type diagrams in which three particles propagate. These equations are then reduced to three-dimensional form by introducing a three-particle propagator that eliminates some of the fourth components of the momenta while keeping two- and three-body unitarity. A second approach that comprises the work of Refs. [11–13] consists simply of putting all spectator particles on their mass shells, which again leads to three-dimensional integral equations. A third approach that was developed in Refs. [14,15] starts from a form of field theory in which the three particles are kept on their mass shells at every stage [15], and therefore, it is a three-dimensional theory from the very beginning. It is thus a straightforward generalization of the nonrelativistic Faddeev equations but incorporating relativistic kinematics. We will use this last approach in order to estimate the effects of relativity in the three-quark system where we will keep the treatment of the spin nonrelativistic. Recently, several groups have performed three-quark calculations within a covariant formalism based on the Bethe-Salpeter equation and where the spin has been treated dynamically [16–20]. This was done in the case of models with confinement [16,17] by first reducing the Bethe-Salpeter equation to three-dimensional form, i.e., to the Salpeter equation and then transforming the equations to configuration space. In the case of models without confinement [18–20], which are all based on the Nambu-Jona-Lasinio model [21], the interaction is of separable form so that the Bethe-Salpeter equation for three particles reduces to an effective two-body Bethe-Salpeter equation.

In Secs. II and III we will present the nonrelativistic and relativistic Faddeev formalisms, respectively, and in Sec. IV we will describe our method of solution. Finally, in Sec. V we will discuss the results and conclusions.

II. NONRELATIVISTIC FADDEEV FORMALISM

The Schrödinger equation for three particles is

$$|\psi\rangle = G_0(E)[V_1+V_2+V_3]|\psi\rangle, \tag{1}$$

where we have defined the propagator for three free particles as

$$G_0(E)=\frac{1}{E-k_1^2/2m_1-k_2^2/2m_2-k_3^2/2m_3}. \tag{2}$$

Making the Faddeev decomposition

$$|\psi\rangle = |\phi_1\rangle + |\phi_2\rangle + |\phi_3\rangle, \tag{3}$$

where

$$|\phi_i\rangle = G_0(E)V_i|\psi\rangle, \tag{4}$$

one gets the Faddeev equations

$$|\phi_i\rangle = G_0(E)t_i(E)[|\phi_j\rangle + |\phi_k\rangle], \tag{5}$$

where $t_i(E)$ is the solution of the Lippmann-Schwinger equation

$$t_i(E)=V_i+V_iG_0(E)t_i(E). \tag{6}$$

In the overall c.m. system $\vec{k}_1+\vec{k}_2+\vec{k}_3=0$, one introduces the relative Jacobi coordinates

$$\vec{p}_i=\frac{m_k\vec{k}_j-m_j\vec{k}_k}{m_j+m_k}, \tag{7}$$

$$\vec{q}_i=\frac{m_i(\vec{k}_j+\vec{k}_k)-(m_j+m_k)\vec{k}_i}{m_i+m_j+m_k}=-\vec{k}_i\,. \tag{8}$$

The propagator (2) is given in terms of these coordinates as

$$G_0(E;p_iq_i)=\frac{1}{E-p_i^2/2\eta_i-q_i^2/2\nu_i}, \tag{9}$$

with the reduced masses

$$\eta_i=\frac{m_jm_k}{m_j+m_k}, \tag{10}$$

$$\nu_i=\frac{m_i(m_j+m_k)}{m_i+m_j+m_k}. \tag{11}$$

Let us assume for the moment that the three particles have no spin or isospin (we will include these variables later). Then the basis states are $|\widetilde{\vec{p}_i\cdot\vec{q}_i}\rangle$, where $\vec{p}_i$ and $\vec{q}_i$ are the relative Jacobi momenta defined above. They are normalized as

$$\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}\rangle=\delta(\vec{p}_i-\vec{p}_i^{\,\prime})\cdot\delta(\vec{q}_i-\vec{q}_i^{\,\prime}) \tag{12}$$

and satisfy the completeness relation

$$1=\int d\vec{p}_i\cdot d\vec{q}_i|\widetilde{\vec{p}_i\cdot\vec{q}_i}\rangle\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|. \tag{13}$$

Using these basis states the Faddeev equations (5) take the form

$$\begin{aligned}\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|\phi_i\rangle=&G_0(E;p_iq_i)\sum_{j\neq i}\int d\vec{p}_i{}'\cdot d\vec{q}_i{}'d\vec{p}_j\cdot d\vec{q}_j\\&\times\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|t_i(E)|\widetilde{\vec{p}_i{}'\cdot\vec{q}_i{}'}\rangle\langle\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}|\widetilde{\vec{p}_j\cdot\vec{q}_j}\rangle\\&\times\langle\widetilde{\vec{p}_j\cdot\vec{q}_j}|\phi_j\rangle,\end{aligned} \tag{14}$$

where from Eq. (6), the two-body t-matrix obeys

$$\begin{aligned}\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|t_i(E)|\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}\rangle=&\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|V_i|\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}\rangle\\&+\int d\vec{p}_i{}''\cdot d\vec{q}_i^{\,\prime\prime}\langle\widetilde{\vec{p}_i\cdot\vec{q}_{ii}}|V_i|\widetilde{\vec{p}_i{}''\cdot\vec{q}_i^{\,\prime\prime}}\rangle\\&\times G_0(E;p_i''q_i'')\langle\widetilde{\vec{p}_i{}''\cdot\vec{q}_i^{\,\prime\prime}}|t_i(E)|\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}\rangle.\end{aligned} \tag{15}$$

The matrix elements of the potential are given by

$$\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|V_i|\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}\rangle=\delta(\vec{q}_i-\vec{q}_i{}')\cdot V_i(\vec{p}_i,\vec{p}_i{}') \tag{16}$$

with

$$V_i(\vec{p}_i,\vec{p}_i{}')=\frac{1}{(2\pi)^3}\int d\vec{r}e^{i\vec{p}_i\cdot\vec{r}}V(r)e^{-i\vec{p}_i^{\,\prime}\cdot\vec{r}}. \tag{17}$$

Substituting Eq. (16) into Eq. (15) leads to

$$\begin{aligned}&\langle\widetilde{\vec{p}_i\cdot\vec{q}_i}|t_i(E)|\widetilde{\vec{p}_i^{\,\prime}\cdot\vec{q}_i^{\,\prime}}\rangle\\&\quad=\delta(\vec{q}_i-\vec{q}_i{}')t_i(\vec{p}_i,\vec{p}_i{}';E-q_i^2/2\nu_i),\end{aligned} \tag{18}$$

where $t_i(\vec{p}_i,\vec{p}_i{}';E-q_i^2/2\nu_i)$ obeys the integral equation

$$\begin{aligned}&t_i(\vec{p}_i,\vec{p}_i{}';E-q_i^2/2\nu_i)\\&\quad=V_i(\vec{p}_i,\vec{p}_i{}')+\int d\vec{p}_i{}''V_i(\vec{p}_i,\vec{p}_i{}'')G_0(E;p_i''q_i)\\&\qquad\times t_i(\vec{p}_i{}'',\vec{p}_i{}';E-q_i^2/2\nu_i).\end{aligned} \tag{19}$$

Substituting Eq. (18) into Eq. (14) and following Ref. [22], the Faddeev equations (14) can be projected into partial waves as

$$\begin{aligned}\langle p_iq_i;\ell_i\lambda_i|\phi_i^L\rangle=&G_0(E;p_iq_i)\\&\times\sum_{j\neq i}\sum_{\ell_j\lambda_j}\int p_i'^2dp_i'p_j^2dp_jq_j^2dq_j\\&\times t_i\ell_i(p_i,p_i';E-q_i^2/2\nu_i)\\&\times\langle p_i'q_i;\ell_i\lambda_i|p_jq_j;\ell_j\lambda_j\rangle_L\langle p_jq_j;\ell_j\lambda_j|\phi_j^L\rangle,\end{aligned} \tag{20}$$

where ℓ_i is the orbital angular momentum of the pair jk, λ_i is the orbital angular momentum of particle i with respect to the pair jk, and L is the total orbital angular momentum.

The two-body amplitude $t_i\ell_i$ is given by the solution of the partial-wave integral equation

$$\begin{aligned}&t_{i}\ell_i(p_i,p_i';E-q_i^2/2\nu_i)\\&\quad=V_i(p_i,p_i')+\int_0^\infty p_i''^2dp_i''V_i(p_i,p_i'')\\&\qquad\times G_0(E;p_i''q_i)t_{i}\ell_i(p_i'',p_i';E-q_i^2/2\nu_i)\end{aligned} \tag{21}$$

with

$$V_i(p_i,p_i')=\frac{2}{\pi}\int_0^\infty r_i^2dr_ij_{\ell_i}(p_ir_i)V_i(r_i)j_{\ell_i}(p_i'r_i). \tag{22}$$

The recoupling coefficient between the partial-wave states i and j is given by [22]

$$\langle p_i' q_i;\ell_i\lambda_i|p_j q_j;\ell_j\lambda_j\rangle_L$$
$$=\frac{1}{2p_i'q_ip_jq_j}\frac{m_k}{\eta_i\eta_j}A_L^{\ell_i\lambda_i\,\ell_j\lambda_j}(p_i'q_ip_jq_j)$$
$$\times\delta\left(\frac{p_i'^2}{2\eta_i}+\frac{q_i^2}{2\nu_i}-\frac{p_j^2}{2\eta_j}-\frac{q_j^2}{2\nu_j}\right)$$
$$\times\theta[1-\cos^2(\vec{q}_i,\vec{p}_i{}')], \tag{23}$$

where θ is the Heaviside function and

$$A_L^{\ell_i\lambda_i\,\ell_j\lambda_j}(p_i'q_ip_jq_j)=\frac{1}{2L+1}\sum_{Mm_im_j}C^{\ell_i\lambda_iL}_{m_i,M-m_i,M}$$
$$\times C^{\ell_j\lambda_jL}_{m_j,M-m_j,M}\Gamma_{\ell_im_i}\Gamma_{\lambda_iM-m_i}$$
$$\times\Gamma_{\ell_jm_j}\Gamma_{\lambda_jM-m_j}\cos[-M(\vec{q}_j,\vec{q}_i)$$
$$-m_i(\vec{q}_i,\vec{p}_i{}')+m_j(\vec{q}_j,\vec{p}_j)] \tag{24}$$

with $\Gamma_{\ell m}=0$ if $\ell-m$ is odd and

$$\Gamma_{\ell m}=\frac{(-)^{(\ell+m)/2}\sqrt{(2\ell+1)(\ell+m)!(\ell-m)!}}{2^\ell[(\ell+m)/2]![(\ell-m)/2]!} \tag{25}$$

if $\ell-m$ is even. The angles $(\vec{q}_j,\vec{q}_i)$, $(\vec{q}_i,\vec{p}_i{}')$, and $(\vec{q}_j,\vec{p}_j)$ can be obtained in terms of the magnitudes of the momenta by using the relations

$$\vec{p}_i'=-\vec{q}_j-\frac{\eta_i}{m_k}\vec{q}_i, \tag{26}$$

$$\vec{p}_j=\vec{q}_i+\frac{\eta_j}{m_k}\vec{q}_j, \tag{27}$$

where ij is a cyclic pair.

The integration over dp_i' in Eq. (20) can be eliminated by using the δ of energy conservation that appears in the recoupling coefficient (23). Similarly, using Eq. (27) we have

$$p_jdp_j=\frac{\eta_j}{m_k}q_iq_jd\cos(\vec{q}_j,\vec{q}_i)\equiv\frac{\eta_j}{m_k}q_iq_jd\cos\theta, \tag{28}$$

so that Eq. (20) can be written in the final form

$$\langle p_iq_i;\ell_i\lambda_i|\phi_i^L\rangle=G_0(E;p_iq_i)\sum_{j\neq i}\sum_{\ell_j\lambda_j}\frac{1}{2}\int_{-1}^{1}d\cos\theta$$
$$\times\int_0^\infty q_j^2dq_j$$
$$\times t_i\ell_i(p_i,p_i';E-q_i^2/2\nu_i)$$
$$\times A_L^{\ell_i\lambda_i\,\ell_j\lambda_j}(p_i'q_ip_jq_j)\langle p_jq_j;\ell_j\lambda_j|\phi_j^L\rangle, \tag{29}$$

where from Eqs. (26) and (27) we obtain

$$p_i'=\sqrt{q_j^2+\left(\frac{\eta_i}{m_k}\right)^2q_i^2+\frac{2\eta_i}{m_k}q_iq_j\cos\theta}, \tag{30}$$

$$p_j=\sqrt{q_i^2+\left(\frac{\eta_j}{m_k}\right)^2q_j^2+\frac{2\eta_j}{m_k}q_iq_j\cos\theta}. \tag{31}$$

If there are no tensor or spin-orbit forces the Faddeev equations (29) can be generalized to include the spin and isospin degrees of freedom as

$$\langle p_iq_i;\ell_i\lambda_iS_iT_i|\phi_i^{LST}\rangle$$
$$=G_0(E;p_iq_i)\sum_{j\neq i}\sum_{\ell_j\lambda_jS_jT_j}\frac{1}{2}\int_{-1}^{1}d\cos\theta$$
$$\times\int_0^\infty q_j^2dq_jt_{i}\ell_iS_iT_i(p_i,p_i';E$$
$$-q_i^2/2\nu_i)A_L^{\ell_i\lambda_i\,\ell_j\lambda_j}(p_i'q_ip_jq_j)$$
$$\times\langle S_iT_i|S_jT_j\rangle_{ST}\langle p_jq_j;\ell_j\lambda_jS_jT_j|\phi_j^{LST}\rangle, \tag{32}$$

where S_i and T_i are the spin and isospin of the pair jk, S and T are the total spin and isospin, and

$$\langle S_iT_i|S_jT_j\rangle_{ST}=(-)^{S_j+\sigma_j-S}\sqrt{(2S_i+1)(2S_j+1)}$$
$$\times W(\sigma_j\sigma_kS\sigma_i;S_iS_j)$$
$$\times(-)^{T_j+\tau_j-T}\sqrt{(2T_i+1)(2T_j+1)}$$
$$\times W(\tau_j\tau_kT\tau_i;T_iT_j), \tag{33}$$

where σ_i and τ_i are the spin and isospin of particle i, and W is the Racah coefficient.

For a given set of values of LST, the integral equations (32) couple together the amplitudes of the different configurations $\{\ell_i\lambda_iS_iT_i\}$ with $(-)^{\ell_i+S_i+T_i}=1$ as required by the Pauli principle, since the wave function is color antisymmetric.

III. RELATIVISTIC FADDEEV FORMALISM

We will base our formalism on the theory proposed by Kadyshevski [15] and Vinogradov [14], where the three particles are kept on their mass shells at every stage.

If we replace the nonrelativistic kinetic energy operator by the corresponding relativistic expression, the Schrödinger equation (1) becomes

$$|\psi\rangle=G_0(W_0)[V_1+V_2+V_3]|\psi\rangle, \tag{34}$$

where W_0 is the invariant mass of the system and

$$G_0(W_0)=\frac{1}{W_0-\omega_1(k_1)-\omega_2(k_2)-\omega_3(k_3)} \tag{35}$$

with

$$\omega_i(k_i)=\sqrt{m_i^2+k_i^2}. \tag{36}$$

Following the same steps as in Eqs. (3)–(6), one obtains the Faddeev equations

$$|\phi_i\rangle = G_0(W_0)t_i(W_0)[|\phi_j\rangle + |\phi_k\rangle] \tag{37}$$

with

$$t_i(W_0) = V_i + V_i G_0(W_0) t_i(W_0). \tag{38}$$

In order to introduce the relative momenta, we will assume that the three particles are in the c.m. system, i.e.,

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0. \tag{39}$$

The relativistic analog of the Jacobi momenta (7) and (8) are the momenta $\vec{p}_i$ and $\vec{q}_i$, where $\vec{p}_i$ is the relative momentum of the pair jk measured in the c.m. frame of the pair (that is, the frame in which particle j has momentum $\vec{p}_i$ and particle k has momentum $-\vec{p}_i$), and $\vec{q}_i = -\vec{k}_i$ is the relative momentum between the pair jk and particle i measured in the three-body c.m. frame (that is, the frame in which the pair jk has total momentum $\vec{q}_i$ and particle i has momentum $-\vec{q}_i$). The invariant energy of the three particles $\omega_1(k_1) + \omega_2(k_2) + \omega_3(k_3)$ can be written in terms of the relative momenta $\vec{p}_i$ and $\vec{q}_i$ as

$$W(p_i q_i) = W_i(p_i q_i) + \omega_i(q_i), \tag{40}$$

where

$$W_i(p_i q_i) = \sqrt{\omega^2(p_i) + q_i^2} \tag{41}$$

and

$$\omega(p_i) = \sqrt{m_j^2 + p_i^2} + \sqrt{m_k^2 + p_i^2} \equiv \omega_j(p_i) + \omega_k(p_i). \tag{42}$$

The invariant volume element for three particles satisfying the condition (39) can be written in terms of the corresponding volume element for the relative momenta as

$$\frac{d\vec{k}_1}{2\omega_1(k_1)} \frac{d\vec{k}_2}{2\omega_2(k_2)} \frac{d\vec{k}_3}{2\omega_3(k_3)} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) = \frac{\omega(p_i)}{8W_i(p_i q_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)} d\vec{p}_i \cdot d\vec{q}_i . \tag{43}$$

Therefore, if the single-particle states are normalized invariantly on the mass shell, i.e.,

$$\langle \vec{k}_i | \vec{k}_i' \rangle = 2\omega_i(k_i)\delta(\vec{k}_i - \vec{k}_i'), \tag{44}$$

then the basis states $|\vec{p}_i \cdot \vec{q}_i\rangle$ are normalized as

$$\langle \vec{p}_i \cdot \vec{q}_i | \vec{p}_i' \cdot \vec{q}_i' \rangle = \frac{8W_i(p_i q_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)}{\omega(p_i)} \times \delta(\vec{p}_i - \vec{p}_i')\delta(\vec{q}_i - \vec{q}_i'), \tag{45}$$

and satisfy the completeness relation

$$1 = \int \frac{\omega(p_i)}{8W_i(p_i q_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)} \times d\vec{p}_i \cdot d\vec{q}_i |\vec{p}_i \cdot \vec{q}_i\rangle\langle \vec{p}_i \cdot \vec{q}_i|. \tag{46}$$

We will also consider states $|\widetilde{\vec{p}_i \cdot \vec{q}_i}\rangle$ that are normalized similar to the nonrelativistic states (12) and (13), i.e.,

$$\langle \widetilde{\vec{p}_i \cdot \vec{q}_i} | \widetilde{\vec{p}_i \cdot \vec{q}_i} \rangle = \delta(\vec{p}_i - \vec{p}_i')\delta(\vec{q}_i - \vec{q}_i'), \tag{47}$$

$$1 = \int d\vec{p}_i \cdot d\vec{q}_i |\widetilde{\vec{p}_i \cdot \vec{q}_i}\rangle\langle \widetilde{\vec{p}_i \cdot \vec{q}_i}|. \tag{48}$$

It is clear from Eqs. (45)–(48) that the two sets of states are related to each other as

$$|\vec{p}_i \cdot \vec{q}_i\rangle = \left[\frac{8W_i(p_i q_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)}{\omega(p_i)}\right]^{1/2} |\widetilde{\vec{p}_i \cdot \vec{q}_i}\rangle. \tag{49}$$

Using the basis states (45) and (46) the Faddeev equations (37) are written explicitly as

$$\begin{aligned}\langle \vec{p}_i \cdot \vec{q}_i | \phi_i \rangle = {} & G_0(W_0; p_i q_i) \\ & \times \sum_{j \neq i} \int \frac{\omega(p_i')}{8W_i(p_i' q_i')\omega_i(q_i')\omega_j(p_i')\omega_k(p_i')} \\ & \times d\vec{p}_i{}' \cdot d\vec{q}_i{}' \frac{\omega(p_j)}{8W_j(p_j q_j)\omega_j(q_j)\omega_k(p_j)\omega_i(p_j)} \\ & \times d\vec{p}_j \cdot d\vec{q}_j \langle \vec{p}_i \cdot \vec{q}_i | t_i(E) | \vec{p}_i{}' \cdot \vec{q}_i{}' \rangle \\ & \times \langle \vec{p}_i{}' \cdot \vec{q}_i{}' | \vec{p}_j \cdot \vec{q}_j \rangle \langle \vec{p}_j \cdot \vec{q}_j | \phi_j \rangle,\end{aligned} \tag{50}$$

where

$$G_0(W_0; p_i q_i) = \frac{1}{W_0 - W(p_i q_i)}, \tag{51}$$

and the two-body t-matrix Eq. (38) is also written explicitly as

$$\begin{aligned}\langle \vec{p}_i \cdot \vec{q}_i | t_i(E) | \vec{p}_i{}' \cdot \vec{q}_i' \rangle & = \langle \vec{p}_i \cdot \vec{q}_i | V_i | \vec{p}_i{}' \cdot \vec{q}_i' \rangle \\ & + \int \frac{\omega(p_i'')}{8W_i(p_i'' q_i'')\omega_i(q_i'')\omega_j(p_i'')\omega_k(p_i'')} \\ & \times d\vec{p}_i{}'' \cdot d\vec{q}_i'' \langle \vec{p}_i \cdot \vec{q}_i | V_i | \vec{p}_i{}'' \cdot \vec{q}_i'' \rangle G_0(W_0; p_i'' q_i'') \\ & \times \langle \vec{p}_i{}'' \cdot \vec{q}_i'' | t_i(E) | \vec{p}_i{}' \cdot \vec{q}_i' \rangle.\end{aligned} \tag{52}$$

The matrix elements of the potential can be obtained using Eqs. (49) and (16) as

$$\langle \vec{p}_i \cdot \vec{q}_i | V_i | \vec{p}_i' \cdot \vec{q}_i' \rangle = 8\omega_i(q_i) \times \left[\frac{W_i(p_i q_i)\omega_j(p_i)\omega_k(p_i) W_i(p_i' q_i)\omega_j(p_i')\omega_k(p_i')}{\omega(p_i)\omega(p_i')} \right]^{1/2} \times \delta(\vec{q}_i - \vec{q}_i{}') V_i(\vec{p}_i, \vec{p}_i{}'), \tag{53}$$

where $V_i(\vec{p}_i, \vec{p}_i{}')$ is given by Eq. (17). Substituting Eq. (53) into Eq. (52) we get

$$\langle \vec{p}_i \cdot \vec{q}_i | t_i(W_0) | \vec{p}_i' \cdot \vec{q}_i' \rangle = 8\omega_i(q_i) \times \left[\frac{W_i(p_i q_i)\omega_j(p_i)\omega_k(p_i) W_i(p_i' q_i)\omega_j(p_i')\omega_k(p_i')}{\omega(p_i)\omega(p_i')} \right]^{1/2} \times \delta(\vec{q}_i - \vec{q}_i{}') \langle \vec{p}_i | t_i(W_0; q_i) | \vec{p}_i' \rangle, \tag{54}$$

where $\langle \vec{p}_i | t_i(W_0; q_i) | \vec{p}_i{}' \rangle$ satisfies the integral equation

$$\langle \vec{p}_i | t_i(W_0; q_i) | \vec{p}_i{}' \rangle = V_i(\vec{p}_i, \vec{p}_i{}') + \int d\vec{p}_i{}'' V_i(\vec{p}_i, \vec{p}_i{}'') G_0(W_0; p_i'' q_i) \times \langle \vec{p}_i{}'' | t_i(W_0; q_i) | \vec{p}_i{}' \rangle. \tag{55}$$

If one substitutes Eq. (54) into Eq. (50) and makes the transformation

$$\langle \vec{p}_i \cdot \vec{q}_i | \phi_i \rangle = \left[\frac{W_i(p_i q_i)\omega_j(p_i)\omega_k(p_i)}{\omega(p_i)} \right]^{1/2} \langle \vec{p}_i \cdot \vec{q}_i | \psi_i \rangle, \tag{56}$$

one can project into partial waves in exactly the same form as done with Eq. (20) to get

$$\begin{aligned} \langle p_i q_i; \ell_i \lambda_i | \psi_i^L \rangle &= G_0(W_0; p_i q_i) \\ &\times \sum_{j \neq i} \sum_{\ell_j \lambda_j} \int \left[\frac{\omega(p_i') W_j(p_j q_j)\omega_k(p_j)\omega_i(p_j)}{W_i(p_i' q_i)\omega_j(p_i')\omega_k(p_i')\omega(p_j)} \right]^{1/2} \\ &\times p_i'^2 dp_i' \frac{\omega(p_j)}{8 W_j(p_j q_j)\omega_j(q_j)\omega_k(p_j)\omega_i(p_j)} p_j^2 dp_j q_j^2 \\ &\times dq_j \langle p_i | t_i^{\ell_i}(W_0; q_i) | p_i' \rangle \\ &\times \langle p_i' q_i; \ell_i \lambda_i | p_j q_j; \ell_j \lambda_j \rangle_L \langle p_j q_j; \ell_j \lambda_j | \psi_j^L \rangle, \end{aligned} \tag{57}$$

where the partial-wave two-body t matrix is the solution of the integral equation

$$\langle p_i | t_i^{\ell_i}(W_0; q_i) | p_i' \rangle = V_i(p_i, p_i') + \int_0^\infty p_i''^2 dp_i'' V_i(p_i, p_i'') G_0(W_0; p_i'' q_i) \times \langle p_i'' | t_i^{\ell_i}(W_0; q_i) | p_i' \rangle \tag{58}$$

with $V_i(p_i, p_i')$ defined by Eq. (22). The recoupling coefficient between the partial-wave states i and j is given by [23,24]

$$\langle p_i' q_i; \ell_i \lambda_i | p_j q_j; \ell_j \lambda_j \rangle_L = \frac{4\omega(p_i)\omega(p_j)}{p_i' q_i p_j q_j} A_L^{\ell_i \lambda_i \, \ell_j \lambda_j}(p_i' q_i p_j q_j) \times \delta[W(p_i q_i) - W(p_j q_j)] \theta[1 - \cos^2(\vec{q}_i, \vec{p}_i{}')]. \tag{59}$$

The angular function $A_L^{\ell_i \lambda_i \, \ell_j \lambda_j}(p_i' q_i p_j q_j)$ is defined by Eq. (24), where the relative angles between the vectors must now be calculated from [25]

$$\vec{p}_i' = -\vec{q}_j - \alpha_{ij}(q_i q_j \cos\theta)\vec{q}_i, \tag{60}$$

$$\vec{p}_j = \vec{q}_i + \alpha_{ji}(q_i q_j \cos\theta)\vec{q}_j, \tag{61}$$

with

$$\alpha_{ij}(q_i q_j \cos\theta) = \frac{W_i^2 - q_i^2 + m_j^2 - m_k^2 + 2\omega_j(q_j)\sqrt{W_i^2 - q_i^2}}{2\sqrt{W_i^2 - q_i^2}[W_i + \sqrt{W_i^2 - q_i^2}]}, \tag{62}$$

$$\alpha_{ji}(q_i q_j \cos\theta) = \frac{W_j^2 - q_j^2 + m_i^2 - m_k^2 + 2\omega_i(q_i)\sqrt{W_j^2 - q_j^2}}{2\sqrt{W_j^2 - q_j^2}[W_j + \sqrt{W_j^2 - q_j^2}]}, \tag{63}$$

and

$$W_i = \omega_j(q_j) + \omega_k(q_k), \tag{64}$$

$$W_j = \omega_i(q_i) + \omega_k(q_k), \tag{65}$$

$$\omega_k(q_k) = \sqrt{m_k^2 + q_i^2 + q_j^2 + 2q_i q_j \cos\theta}. \tag{66}$$

The integration over dp_i' in Eq. (57) can be eliminated by using the δ of energy conservation that appears in the recoupling coefficient (59). Similarly, from the relation

$$W_j(p_j q_j) = \sqrt{m_i^2 + q_i^2} + \sqrt{m_k^2 + q_i^2 + q_j^2 + 2q_i q_j \cos\theta}, \tag{67}$$

one obtains

$$p_j dp_j = \frac{W_j(p_j q_j)\omega_i(p_j)\omega_k(p_j)}{\omega^2(p_j)\omega_k(q_k)} q_i q_j d\cos\theta, \tag{68}$$

so that the relativistic Faddeev equations (57) take the final form including the spin and isospin degrees of freedom

$$\langle p_i q_i;\ell_i\lambda_i S_i T_i|\psi_i^{LST}\rangle$$
$$=G_0(W_0;p_iq_i)\sum_{j\neq i}\sum_{\ell_j\lambda_j S_j T_j}\frac{1}{2}\int_{-1}^{1}d\cos\theta$$
$$\times\int_0^\infty q_j^2dq_j\frac{1}{\omega_j(q_j)\omega_k(q_k)}$$
$$\times\left[\frac{W_i(p_i'q_i)\omega_j(p_i')\omega_k(p_i')}{\omega(p_i')}\right]^{1/2}$$
$$\times\left[\frac{W_j(p_jq_j)\omega_k(p_j)\omega_i(p_j)}{\omega(p_j)}\right]^{1/2}$$
$$\times\langle p_i|t_{i}\ell_i S_i T_i(W_0;q_i)|p_i'\rangle$$
$$\times A_L{}^{\ell_i\lambda_i\,\ell_j\lambda_j}(p_i'q_ip_jq_j)$$
$$\times\langle S_iT_i|S_jT_j\rangle_{ST}\langle p_jq_j;\ell_j\lambda_jS_jT_j|\psi_j^{LST}\rangle, \tag{69}$$

where from Eqs. (60) and (61)

$$p_i'=\sqrt{q_j^2+\alpha_{ij}^2(q_iq_j\cos\theta)q_i^2+2\alpha_{ij}(q_iq_j\cos\theta)q_iq_j\cos\theta}, \tag{70}$$

$$p_j=\sqrt{q_i^2+\alpha_{ji}^2(q_iq_j\cos\theta)q_j^2+2\alpha_{ji}(q_iq_j\cos\theta)q_iq_j\cos\theta}. \tag{71}$$

If we take the nonrelativistic limit, it is easy to see that

$$\alpha_{ij}(q_iq_j\cos\theta)\rightarrow\frac{\eta_i}{m_k}, \tag{72}$$

$$\alpha_{ji}(q_iq_j\cos\theta)\rightarrow\frac{\eta_j}{m_k}, \tag{73}$$

so that Eqs. (60) and (61) become (26) and (27) and consequently Eqs. (70) and (71) become (30) and (31). Similarly, in this limit the relativistic propagator (51) reduces to the nonrelativistic propagator (9) and

$$\frac{1}{\omega_j(q_j)\omega_k(q_k)}\left[\frac{W_i(p_i'q_i)\omega_j(p_i')\omega_k(p_i')}{\omega(p_i')}\right]^{1/2}$$
$$\times\left[\frac{W_j(p_jq_j)\omega_k(p_j)\omega_i(p_j)}{\omega(p_j)}\right]^{1/2}\rightarrow 1, \tag{74}$$

so that the relativistic Faddeev equation (69) becomes the nonrelativistic equation (32) and the relativistic Lippmann-Schinger equation (58) becomes the nonrelativistic one (21).

It is also interesting to consider the behavior of the integral equations at large momenta. Let us start with the two-body equations. The nonrelativistic Lippmann-Schwinger equation (21) differs from relativistic equation (58) only in that the propagator G_0 is given in the nonrelativistic case by Eq. (9), so that $G_0(E;p_i''q_i)\sim p_i''^{-2}$ when $p_i''\rightarrow\infty$. In the relativistic case, however, G_0 is given by Eq. (51) so that $G_0(W_0;p_i''q_i)\sim p_i''^{-1}$ when $p_i''\rightarrow\infty$. This means that the relativistic equation falls down more slowly at high momenta and, therefore, it is more sensitive to the behavior of the interaction at high momenta, which corresponds to its behavior at small distances.

In order to see the high-momentum behavior of the three-body equations (32) and (69), let us consider, for example, the case of S waves where $A_L^{\ell_i\lambda_i\,\ell_j\lambda_j}(p_i'q_ip_jq_j)=A_0^{0000}(p_i'q_ip_jq_j)=1$. If we go to the limit $q_i=q_j\rightarrow\infty$ it is easy to see that

$$\frac{1}{\omega_j(q_j)\omega_k(q_k)}\left[\frac{W_i(p_i'q_i)\omega_j(p_i')\omega_k(p_i')}{\omega(p_i')}\right]^{1/2}$$
$$\times\left[\frac{W_j(p_jq_j)\omega_k(p_j)\omega_i(p_j)}{\omega(p_j)}\right]^{1/2}\sim q_i^0, \tag{75}$$

so that again the behavior at high momenta is determined by the propagator G_0. Thus, also in the case of the three-body equations, the relativistic Faddeev equation falls down more slowly than the nonrelativistic one at high momenta and, therefore, it is more sensitive to the high-momentum behavior of the interaction which corresponds to the behavior of the interaction at small distances.

IV. THE METHOD OF SOLUTION

The problem with confining potentials of the form

$$V_C(r)=br^n,\quad n=1,2,\ \text{etc.}, \tag{76}$$

is that the Fourier transform of the potential does not exist since $V_C\rightarrow\infty$ when $r\rightarrow\infty$. Therefore, we will replace the potential (76) by the finite potential

$$V(r)=\begin{cases}b(r^n-R^n), & r\leqslant R\\ 0, & r>R\end{cases} \tag{77}$$

for which the Fourier transform is well defined. The Schrödinger equation for three particles interacting with one another through the potential (76) is

$$\left[\frac{k_1^2}{2m_1}+\frac{k_2^2}{2m_2}+\frac{k_3^2}{2m_3}+V_C(r_1)+V_C(r_2)+V_C(r_3)\right]|\Psi_C\rangle$$
$$=E_C|\Psi_C\rangle, \tag{78}$$

where $\vec{k}_i$ is the momentum operator of particle i and $\vec{r}_i$ is the relative coordinate between particles j and k. The Schrödinger equation for three particles interacting with each other through the potential (77) is, on the other hand

$$\left[\frac{k_1^2}{2m_1}+\frac{k_2^2}{2m_2}+\frac{k_3^2}{2m_3}+V(r_1)+V(r_2)+V(r_3)\right]|\Psi\rangle=E|\Psi\rangle, \tag{79}$$

so that in the limit $R\rightarrow\infty$ Eqs. (78) and (79) are equivalent, provided that the eigenvalues are related as

$$E_C=E-3bR^n. \tag{80}$$

Thus, our method comprises solving the three-body problem with the potential (77) with R sufficiently large such that E_C given by Eq. (80) does not change appreciably if R is increased further. A momentum-space method, based on ideas similar to ours for the two-body problem with singular potentials, has been developed in Refs. [27,28] and applied in Ref. [29] to the meson spectrum.

In the particular case of the nonstrange baryons where we have a system of three identical particles, the Faddeev equations (32) can be cast in the form

$$T^{\alpha}(p_i q_i;E)=\sum_{\beta}\int_{-1}^{1}d\cos\theta\int_0^{\infty}dq_j t^{\alpha}(p_i,p_i';E-q_i^2/2\nu_i)\times B^{\alpha\beta}(q_i,q_j,\cos\theta;E)T^{\beta}(p_j q_j;E), \tag{81}$$

where α stands for the quantum numbers $\ell_i\lambda_i S_i T_i$. If we make the change of variables

$$p_i=b\frac{1+x_i}{1-x_i}, \tag{82}$$

$$q_i=b\frac{1+y_i}{1-y_i}, \tag{83}$$

where b is a scale parameter, Eq. (81) becomes

$$T^{\alpha}(x_i y_i;E)=\sum_{\beta}\int_{-1}^{1}d\cos\theta\int_{-1}^{1}\frac{2b}{(1-y_j)^2}dy_j \times t^{\alpha}\left[x_i,x_i';E-\frac{1}{2\nu_i}\left(b\frac{1+y_i}{1-y_i}\right)^2\right]\times B^{\alpha\beta}(y_i,y_j,\cos\theta;E)T^{\beta}(x_j y_j;E), \tag{84}$$

where the variables x_i and y_i run from -1 to 1, therefore, one can expand the two-body amplitude in terms of Legendre polynomials as [26]

$$t^{\alpha}(x_i,x_i';e)=\sum_n P_n(x_i)t_n^{\alpha}(x_i';e), \tag{85}$$

so that the three-body amplitude is of the form

$$T^{\alpha}(x_i,y_i;E)=\sum_n P_n(x_i)T_n^{\alpha}(y_i;E). \tag{86}$$

Using Eq. (86), Eq. (84) reduces to the set of integral equations in one variable

$$T_n^{\alpha}(y_i;E)=\sum_{\beta m}\int_{-1}^{1}dy_j B_{nm}^{\alpha\beta}(y_i,y_j;E)T_m^{\beta}(y_j;E) \tag{87}$$

with

$$B_{nm}^{\alpha\beta}(y_i y_j;E)=\frac{2b}{(1-y_j)^2}\int_{-1}^{1}d\cos\theta \times t_n^{\alpha}\left[x_i';E-\frac{1}{2\nu_i}\left(b\frac{1+y_i}{1-y_i}\right)^2\right]\times B^{\alpha\beta}(y_i,y_j,\cos\theta;E)P_m(x_j). \tag{88}$$

In order to carry out the expansion in Legendre polynomials (85), we use the transformation (82) to write the Lippmann-Schwinger equation (21) as

$$t^{\alpha}(x_i,x_i';e)=V^{\alpha}(x_i,x_i')+\int_{-1}^{1}\left(b\frac{1+x_i''}{1-x_i''}\right)^2\frac{2b}{(1-x_i'')^2}\times dx_i''V^{\alpha}(x_i,x_i'')G_0(e;x_i'')t^{\alpha}(x_i'',x_i';e). \tag{89}$$

If we expand the potential as

$$V^{\alpha}(x_i,x_i')=\sum_n P_n(x_i)V_n^{\alpha}(x_i'), \tag{90}$$

where

$$V_n^{\alpha}(x_i')=\frac{2n+1}{2}\int_{-1}^{1}dx_i P_n(x_i)V^{\alpha}(x_i,x_i'), \tag{91}$$

then the two-body amplitude is of the form

$$t^{\alpha}(x_i,x_i';e)=\sum_n P_n(x_i)t_n^{\alpha}(x_i';e), \tag{92}$$

and Eq. (89) reduces to the set of linear equations

$$t_n^{\alpha}(x_i';e)=V_n^{\alpha}(x_i')+\sum_k a_{nk}^{\alpha}(e)t_k^{\alpha}(x_i';e) \tag{93}$$

with

$$a_{nk}^{\alpha}(e)=\int_{-1}^{1}\left(b\frac{1+x_i''}{1-x_i''}\right)^2\frac{2b}{(1-x_i'')^2}\times dx_i''V_n^{\alpha}(x_i'')G_0(e;x_i'')P_k(x_i''). \tag{94}$$

Since in the potential (77) one must take R large (typically $R\sim 12$ fm), the momentum-space representation of this potential $V(p_i,p_i')$ is a function that oscillates very rapidly. Therefore, in order to get an accurate representation of the potential by means of the Legendre expansion (90), it is necessary to include a large number of polynomials (we use 200 Legendre polynomials) and a similarly large number of integration points in the integrals (91) and (94). However, in the solution of the three-body equations (87) where the functions $t_n^{\alpha}(x_i';e)$ enter as input, one needs a much smaller number of Legendre components to reach convergence. We found that one needs at most 13 Legendre components in order to reach convergence.

TABLE I. The masses of the nucleon and delta (in MeV) obtained from the potential (95) for both the nonrelativistic and relativistic Faddeev equations including only the three-body configurations with $\ell_i=\lambda_i=L=0$.

Theory	M_N	M_Δ	$M_\Delta-M_N$
Nonrelativistic	1058	1332	274
Relativistic	−1071	1159	2230

This is the advantage of a Faddeev formulation, where the input of the three-body equations is not two-body potentials but two-body t matrices, which have been obtained by solving already an integral equation so that they contain all the information on the energy dependence of the two-body subsystems. This energy dependence is much smoother than the momentum dependence of the potential so that a few terms suffice to represent it. Also, one should notice that in the calculation of three-body bound states what matters is the energy dependence of the equations, since bound states are poles in the variable E.

We replaced the integral in Eq. (87) by a Gauss quadrature and found that the number of integration points required is of the order of 30–35. We took for the scale parameter $b=2\ \mathrm{fm}^{-1}$. In the case of the relativistic model, however, one needs to use $5\leqslant b\leqslant 10\ \mathrm{fm}^{-1}$, since in that case the equations effectively extend further out in momentum space.

Our method is very stable since, for example, the mass of the nucleon changes by less than 1 MeV when we vary R between 8 and 14 fm. We performed all our calculations with $R=12$ fm. Some applications of the nonrelativistic formalism have already been presented for the case of the chiral quark cluster model [30,31].

V. RESULTS

In this paper, our main interest is to explore the importance of relativistic effects. Moreover, since relativistic effects will be the largest for those systems that are composed of the lightest quarks, we will restrict our study to the nonstrange sector that involves only u and d quarks. In addition, the u and d quarks differ only in their isospin projection, so that we have a system of three identical particles and the Faddeev equations (32) and (69) which couple the three components of the wave function ϕ_i^{LST} or ψ_i^{LST} with $i=1,2,3$ will become a single equation since all three components are equal.

We will take for the quark-quark interaction the potential proposed by Bhaduri, Cohler, and Nogami [2], which is known to give a reasonably good description of the baryon spectrum within a nonrelativistic framework [2,3]. This interaction has the form

$$V_i(r_i)=\frac{1}{2}\left[\frac{-\kappa}{r_i}+\frac{r_i}{a^2}-D+\frac{\kappa}{m^2}\frac{1}{r_0^2 r_i}e^{-r_i/r_0}\vec{\sigma}_j\cdot\vec{\sigma}_k\right] \tag{95}$$

with $\kappa=102.67$ MeV/c, $a=0.0326$ $(\mathrm{MeV}^{-1}\mathrm{fm})^{1/2}$, $D=913.5$ MeV, $r_0=0.4545$, and the mass of the quark, $m=337$ MeV. Using this interaction in the Feshbach-Rubinow variational method [32], Bhaduri *et al.* obtained for the masses of the nucleon and Delta, $M_N=1052$ MeV and $M_\Delta=1354$ MeV. In order to compare with these results, we have calculated the nucleon and Delta masses considering only the three-body configurations where all orbital angular momenta are S waves. We show these results in Table I for both the nonrelativistic and relativistic theories. Our results for the nonrelativistic theory are in good agreement with those of Ref. [2], since the accuracy of the Feshbach-Rubinow method is about 20 MeV [2]. As it appears from this table the relativistic effects are very large, particularly for the nucleon, and they lower the masses. The nucleon mass is lowered by 2129 MeV, while the Delta mass is lowered by 173 MeV so that the mass splitting which is 274 MeV for the nonrelativistic theory increases to 2230 MeV for the relativistic one, i.e., to about eight times the experimental value of 292 MeV. From the results of Table I, one may conclude that a relativistic description of the baryon spectrum based on the potential (95) would not be possible. This, however, is not the case as it will be shown below.

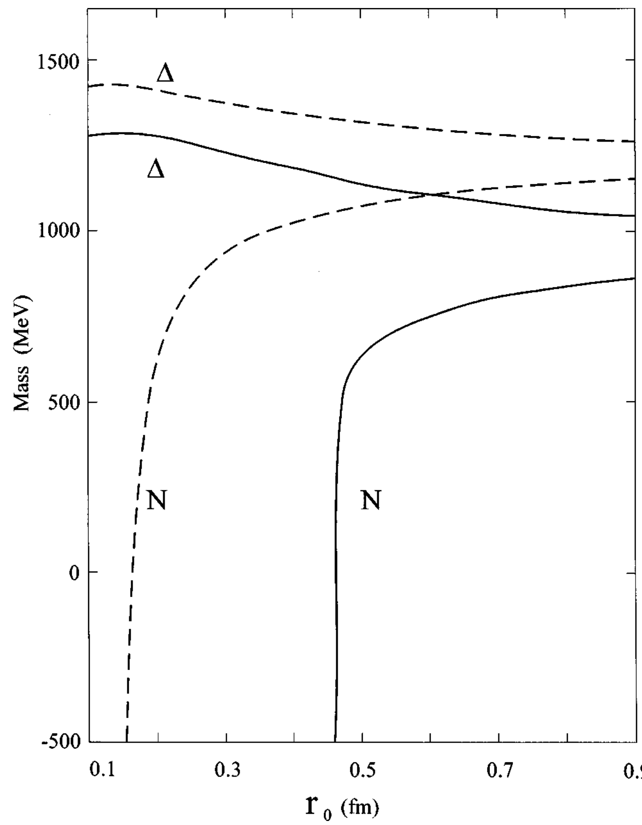

FIG. 1. The masses of the nucleon and Delta, as functions of the parameter r_0 for the nonrelativistic (dashed lines) and relativistic (solid lines) theories.

The large sensitivity to relativistic effects shown in Table I, as we will see next, is due to a particular feature of the interaction at short distances. The last term of the interaction (95) contains a smeared-out δ function with smearing parameter r_0. The δ function must be smeared-out otherwise the mass of the nucleon would collapse to $-\infty$ when $r_0\to 0$. We show this effect in Fig. 1, where we plot the masses

TABLE II. The masses of the nucleon and delta (in MeV) including all the three-body configurations with ℓ_i and λ_i up to 5. The nonrelativistic results correspond to the potential (95) and the relativistic results correspond to the potential (95) with the parameter $r_0 = 0.74$ fm.

Theory	M_N	M_Δ	$M_\Delta - M_N$
Nonrelativistic	1025	1331	306
Relativistic	780	1072	292

of the nucleon and delta as functions of the parameter r_0 for both the nonrelativistic theory (dashed lines) and the relativistic one (solid lines). As one can see from this figure the inclusion of relativity produces two main effects: first of all, as already noticed, it lowers the masses of the baryons and second, it raises the values of the parameter r_0 at which the mass of the nucleon would collapse to large negative values from ~ 0.15 fm for the nonrelativistic theory to ~ 0.45 fm for the relativistic one. Thus, while $r_0 = 0.4545$ is a reasonable value for the nonrelativistic theory, it is not so in the case of the relativistic one where instead the value of r_0 should be of the order of 0.7 fm in order to get the correct excitation energy.

The behavior of the nucleon mass in the relativistic theory is a consequence of the fact that this theory is more sensitive to the form of the interaction at short distances as we have shown in Sec. III. That is the reason why the pathological effects produced by the δ function appear in the case of the relativistic theory much sooner (i.e., at larger values of r_0) than in the case of the nonrelativistic one.

If we include all three-body configurations with ℓ_i and λ_i up to 5 and take for the relativistic theory $r_0 = 0.74$ fm, we obtain for M_N and M_Δ the results given in Table II. Our results for the nonrelativistic theory are in very good agreement with those of Ref. [3]. In both the cases the excitation energy is close to the experimental value of 292 MeV. The mass of the nucleon comes out above the experimental value for the nonrelativistic theory and below it for the relativistic one. This problem can be solved by changing the parameter D in Eq. (95) to 970 MeV in the nonrelativistic case and 807 MeV in the relativistic one. Of course, the parameter D has no effect on the excitation energies.

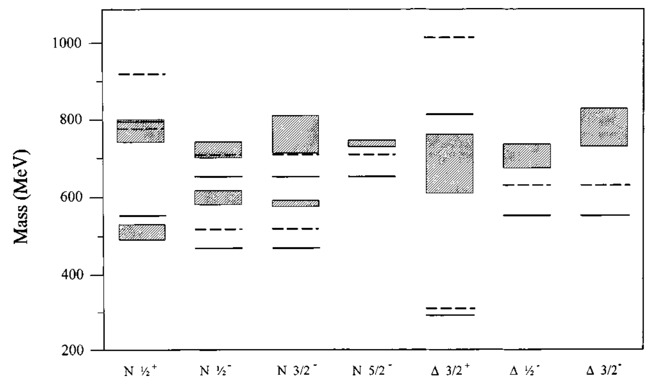

FIG. 2. Nonstrange baryon spectrum for the interaction (95) obtained from the nonrelativistic (dashed lines) and relativistic (solid lines) theories as explained in the text.

We show in Fig. 2 the excitation energy spectrum obtained using both the nonrelativistic theory (dashed lines) and the relativistic model with $r_0 = 0.74$ fm (solid lines). As it can be seen from this figure, the description provided by the relativistic model is of similar quality to that of the nonrelativistic one and in some respects even better. For example, the spacing between the positive parity states is now more reasonable and the ordering between the positive and negative parity states $N(1440)$ and $N(1535)$, which is inverted in both theories, has a smaller discrepancy in the case of the relativistic model.

Notice that the essential parameters of the interaction (95) are the strength of the one-gluon-exchange potential (the parameter κ) and the slope of the confining potential (the parameter a), which we have not touched. The parameter D fixes the overall scale of the spectrum but it has no effect on the excitation energies, and the parameter r_0 is necessary in order to avoid the collapsing of the nucleon mass. Thus, we conclude that using the potential (95) in a relativistic theory leads to a similar or somewhat better description of the baryon spectrum.

ACKNOWLEDGMENT

This work was supported in part by COFAA-IPN (México).

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